1. Suppose the conjugate-gradient algorithm is applied to a symmetric matrix but one that is not positive definite. What can happen?

2. Let $A$ be a square matrix whose first row and column are dense. The only other nonzero elements are on the diagonal.
   
   (a) What is the permutation that minimizes bandwidth?
   
   (b) Assuming the matrix is positive definite and symmetric, what is the fill in for the resulting factors?
   
   (c) If it is not positive definite, what is the fill in?

3. What is the solution of
   \[
   \min_{\alpha} \|a - \alpha e\|_i
   \]
   for $i = (1, 2, \infty)$, where $a$ is a vector?

4. What is the SVD of $a$, where $a$ is a vector?

5. What is the SVD of $[a, b]$, where $a$ and $b$ are vectors?

6. Define $F(x) = x^T + \frac{1}{2}x^THx$, where $H$ is a positive-definite symmetric matrix with a multiple eigenvalue $\lambda$. Show that the conjugate-gradient algorithm converges in one step when $\nabla f(x_0)$, where $x_0$ is the initial estimate of the solution, lies in the space spanned by the eigenvectors corresponding to $\lambda$. 