Online Appendix to *Modeling Theories of Women's Underrepresentation in Elections*

Contents

A Proofs

A.1 Results Applying to All Models

We begin with some formal development that applies to both variants of the model.

Recall that each candidates has quality θ, independently drawn from distribution F with strictly positive density f. As we observed in the main text, an equilibrium will involve two cutoffs, $\hat{\theta}_M$ and $\hat{\theta}_W$. The interpretation is that a male candidate runs if and only if his quality θ is greater than or equal to $\hat{\theta}_M$, and a female candidate runs if and only if her quality is greater than or equal to $\hat{\theta}_W$.

Given these entry decisions, we can calculate the share of women and men in the pool of candidates. If women potential candidates use the cutoff $\hat{\theta}_W$, then the number of female candidates is:

$$\lambda_W = \frac{1}{2}\left(1 - F(\hat{\theta}_W)\right). \tag{1}$$

Similarly, the number of male candidates is:

$$\lambda_M = \frac{1}{2}\left(1 - F(\hat{\theta}_M)\right). \tag{2}$$

The total number of candidates is:

$$\lambda = \lambda_W + \lambda_M. \tag{3}$$

So the share of female candidates is $\dfrac{\lambda_W}{\lambda}$ and the share of male candidates is $\dfrac{\lambda_M}{\lambda}$.

If potential candidates with gender $\gamma \in \{W, M\}$ use cutoff $\hat{\theta}_\gamma$, the probability a candidate has quality less than or equal to θ conditional on being of gender γ is:

$$F^\gamma(\theta) = \begin{cases} 0 & \text{if } \theta < \hat{\theta}_\gamma \\ \frac{F(\theta)-F(\hat{\theta}_\gamma)}{1-F(\hat{\theta}_\gamma)} & \text{if } \theta \geq \hat{\theta}_\gamma. \end{cases}$$

The associated density is:

$$f^\gamma(\theta) = \begin{cases} 0 & \text{if } \theta < \hat{\theta}_\gamma \\ \frac{f(\theta)}{1-F(\hat{\theta}_\gamma)} & \text{if } \theta \geq \hat{\theta}_\gamma. \end{cases} \tag{4}$$

It will also be useful to be able to discuss the distribution of quality plus noise, $\theta + \epsilon$, conditional on running. The usual convolution formula for sums of independent random variables gives the cdf for the sum:

$$H^{\gamma}(\theta) \equiv \int F^{\gamma}(\theta - \epsilon) g(\epsilon)\, d\epsilon. = \int G(\theta - \epsilon) f^{\gamma}(\epsilon)\, d\epsilon.$$

This cdf is strictly increasing in θ since G is. H^{γ} has a density given by:

$$h^{\gamma}(\theta) = \int f^{\gamma}(\theta - \epsilon) g(\epsilon)\, d\epsilon.$$

We will use several facts about how these distributions are stochastically ordered. Let $\tilde{x}$ and $\tilde{y}$ be random variables with distributions F_x and f_y, respectively, and densities f_x and f_y, respectively. Recall that $\tilde{x}$ *(strictly) first-order stocahstically dominates* $\tilde{y}$ if $F_x(z) \leq F_y(z)$ for all z (with strict inequality for some z). In this case, for any nondecreasing function u, we have $\int u(z)\, dF_x(z) \geq \int u(z)\, dF_y(z)$, with strict inequality if u is strictly increasing on an interval containing a z where $F_x(z) < F_y(z)$.

We start with a pair of general results. The first is relatively standard, but we include a proof because the usual references assume the two random variables have a common support.

Lemma 2 *Suppose $\tilde{x}$ and is a random variable with distribution F and density f that is strictly positive on $[\underline{x}, \infty)$, and $\tilde{y}$ is a random variables with distribution G and density g that is strictly positive on $[\underline{y}, \infty)$. If $\underline{x} > \underline{y}$ and, for all $z > z'$,*

$$f(z)g(z') \geq f(z')g(z), \tag{5}$$

then $z > \underline{y}$ implies $F(z) < G(z)$.

Proof. Notice first that, for $z \in (\underline{y}, \underline{x}]$, we have $G(z) > 0 = F(z)$. Thus is suffices to show the inequality for $z > \underline{x}$.

Integrate to get:

$$\begin{aligned}
f(z)G(z) &= \int_{-\infty}^{z} f(z)g(z')\,dz' \\
&= \int_{\underline{y}}^{z} f(z)g(z')\,dz' \\
&> \int_{\underline{x}}^{z} f(z)g(z')\,dz' \\
&\geq \int_{\underline{x}}^{z} f(z')g(z)\,dz' \\
&= \int_{-\infty}^{z} f(z')g(z)\,dz' \\
&= F(z)g(z),
\end{aligned}$$

where the strict inequality is from $\underline{x} > \underline{y}$ and the weak inequality is from 5.

A similar argument gives:

$$(1 - F(z))g(z) = \int_{z}^{\infty} f(z'')g(z)\,dz'' \geq \int_{z}^{\infty} f(z)g(z'')\,dz'' = f(z)(1 - G(z)).$$

Since $z > \underline{x}$, neither $G(z)$ nor $1 - G(z)$ are zero. Thus we can combine these two displayed inequalities to get

$$\frac{1 - F(z)}{1 - G(z)} \geq \frac{f(z)}{g(z)} > \frac{F(z)}{G(z)}.$$

Cross-multiply to get:

$$G(z) - F(z)G(z) > F(z) - F(z)G(z),$$

or $F(z) < G(z)$ ■

Lemma 3 *For $\theta > \theta'$,*

$$f^W(\theta)f^M(\theta') \geq f^W(\theta')f^M(\theta).$$

Proof. There are four cases.

1. If $\theta < \hat{\theta}_W$, then both sides of the inequality are zero.

2. If $\theta > \hat{\theta}_W$ and $\theta' < \hat{\theta}_M$, then both sides of the inequality are zero.

3. If $\theta > \hat{\theta}_W$ and $\theta' \in [\hat{\theta}_M, \hat{\theta}_W)$, then the left-hand side of the inequality is positive and the right-hand side is zero.

4. If $\theta > \hat{\theta}_W$ and $\theta' \geq \hat{\theta}_W$, then the two sides of the inequality are both positive and they are equal.

■

Lemmas 2 and 3 immeditely yield:

Corollary 1 *If $\hat{\theta}_W > \hat{\theta}_M$, then, for all $\theta > \hat{\theta}_M$, we have $F^W(\theta) < F^M(\theta)$.*

Lemma 4 *Suppose $\tilde{x}$ and $\tilde{y}$ are random variables and $\tilde{x}$ (strictly) first-order stochastically dominates $\tilde{y}$. If ε is a random variable that is independent of $\tilde{x}$ and $\tilde{y}$, then $\tilde{x} + \varepsilon$ (strictly) first-order stochastically dominates $\tilde{y} + \varepsilon$.*

Proof. Let F_x, F_y, and F_ε be the CDFs of the random variables. Using the convolution formula to get the CDFs of $\tilde{x} + \varepsilon$ and $\tilde{y} + \varepsilon$, we to show that, for all z:

$$\int F_x(z - \varepsilon)\, dF_\varepsilon(\varepsilon) \leq \int F_y(z - \varepsilon)\, dF_\varepsilon(\varepsilon).$$

Which is true because $\tilde{x}$ (strictly) first-order stochastically dominates $\tilde{y}$ implies $F_x(z - \varepsilon) \leq F_y(z - \varepsilon)$, for all z and ε. ■

A.2 Election Aversion and/or Perception Gap

This section proceeds as follows. First, we characterize the equilibrium cutoffs and show that $\hat{\theta}_W > \hat{\theta}_M$ with either differential costs or the perception gap. Second, we derive the distributions of quality conditional on election and on a tie, and show that the distribution of qualities for women is better in each case. Third, we use these results to prove Proposition 1.

Lemma 5 *The pair $(\hat{\theta}_W, \hat{\theta}_m)$ are equilibrium cutoffs if and only if:*

$$\frac{\lambda_W}{\lambda} \int F^W(\phi^W(\hat{\theta}_W) - \epsilon) g(\epsilon)\, d\epsilon + \frac{\lambda_M}{\lambda} \int F^M(\phi^W(\hat{\theta}_W) - \epsilon) g(\epsilon)\, d\epsilon = c^W \tag{6}$$

$$\frac{\lambda_W}{\lambda} \int F^W(\phi^M(\hat{\theta}_M) - \epsilon) g(\epsilon)\, d\epsilon + \frac{\lambda_M}{\lambda} \int F^M(\phi^M(\hat{\theta}_M) - \epsilon) g(\epsilon)\, d\epsilon = c^M \tag{7}$$

Proof. Fix cutoffs $(\hat{\theta}_W, \hat{\theta}_M)$.

A candidate with perceived quality θ and preference shock μ believes they defeat an opponent with perceived quality θ' and preference shock ν' if and only if:

$$\theta + \nu \geq \theta' + \nu'.$$

Recalling that $\epsilon = \nu' - \nu$, we can rewrite this condition as:

$$\theta \geq \theta' + \epsilon.$$

Recall that ϵ has density g. Thus the probability that a candidate of perceived type θ believes they win, conditional on being selected, is:

$$\Pr(\text{Elected} \mid \theta) = \frac{\lambda_W}{\lambda} \int F^W(\theta - \epsilon) g(\epsilon)\, d\epsilon + \frac{\lambda_M}{\lambda} \int F^M(\theta - \epsilon) g(\epsilon)\, d\epsilon, \tag{8}$$

where λ_W, λ_M, and λ are as in Equations 1–3.

A potential candididate of gender γ and type θ runs if and only if $\Pr(\text{Elected} \mid \phi^\gamma(\theta)) - c^\gamma \geq 0$. Since each F^γ and ϕ^γ are continuous and strictly increasing in θ, so is $\Pr(\text{Elected} \mid \phi^\gamma(\theta))$. Thus $(\hat{\theta}_W, \hat{\theta}_M)$ are equilibrium cutoffs if and only if $\Pr(\text{Elected} \mid \phi^\gamma(\hat{\theta}_\gamma)) = c^\gamma$ for both γ. ■

Lemma 6 *For any x and y,*

$$\operatorname{sgn}\left(\int \left(F(x-\epsilon) - F(y-\epsilon)\right) g(\epsilon)\, d\epsilon\right) = \operatorname{sgn}(x-y).$$

Proof. When $x = y$, the integrand is the zero function, so the integral is zero. Differentiate with respect to x to get:

$$\int \left(f(x-\epsilon)\right) g(\epsilon)\, d\epsilon > 0,$$

where the inequality follows from f being strictly positive on its support. ■

Lemma 7 *If*

1. *$c^W > c^M$ and $\phi^\gamma(\theta) = \theta$ for all θ and all γ;*
2. *$c^W = c^M$ and $\phi^W(\theta) < \phi^M(\theta)$ for all θ;*
3. *$c^W > c^M$ and $\phi^W(\theta) < \phi^M(\theta)$ for all θ,*

then $\hat{\theta}_W > \hat{\theta}_M$.

Proof. Substitute in the definitions of λ_W and λ_M from Equations 1 and 2, and subtract Equation 7 from Equation 6 to get:

$$\frac{1}{2\lambda} \int \Big(F(\phi^W(\hat{\theta}_W) - \epsilon) - F(\phi^M(\hat{\theta}_M) - \epsilon) \Big) \, g(\epsilon) \, d\epsilon = c^W - c^M.$$

Now consider the three cases:

1. Since the right-hand side is positive and $\lambda > 0$, the integral must be positive. Thus Lemma 6 implies $\hat{\theta}_W > \hat{\theta}_M$.

2. Since the left-hand side is zero, the integral must be zero. Thus Lemma 6 implies $\phi^W(\hat{\theta}_W) = \phi^M(\hat{\theta}_M)$. Since $\phi^W(\theta) < \phi^M(\theta)$ for all θ, this requires $\hat{\theta}_W > \hat{\theta}_M$.

3. Follows from combining the two arguments above.

■

Now we derive the relevant conditional densities. Conditional on winning an election, the quality of a candidate of gender γ has a distribution with density:

$$f^\gamma(\theta \mid \text{Elected}) = \frac{\Pr(\text{Elected} \mid \theta) f^\gamma(\theta)}{\int \Pr(\text{Elected} \mid \tilde{\theta}) f^\gamma(\tilde{\theta}) \, d\tilde{\theta}}. \tag{9}$$

Lemma 8 *Fix $\theta > \theta'$.*

$$f^W(\theta \mid \textit{Elected}) f^M(\theta' \mid \textit{Elected}) \geq f^W(\theta' \mid \textit{Elected}) f^M(\theta \textit{Elected}).$$

Proof. Substituting from Equation 9, the inequality is equivalent to:

$$f^W(\theta) f^M(\theta') \geq f^W(\theta') f^M(\theta).$$

The result now follows from Lemma 3. ■

Conditioning on a tie between a woman and a man is more delicate, since ties have probability zero. Thus Bayes' rule does not apply directly. Instead, we define $f^\gamma(\theta \mid \text{Tie})$ as follows:

$$f^\gamma(\theta \mid \text{Tie}) = \lim_{\delta \to 0} f^\gamma(\theta \mid -\delta < \theta - \theta' - \epsilon < \delta).$$

This yields:

$$f^W(\theta \mid \text{Tie}) = \frac{h^M(\theta)f^W(\theta)}{\int h^M(\tilde{\theta})f^W(\tilde{\theta})\,d\tilde{\theta}}$$

and

$$f^M(\theta \mid \text{Tie}) = \frac{h^W(\theta)f^M(\theta)}{\int h^W(\tilde{\theta})f^M(\tilde{\theta})\,d\tilde{\theta}}.$$

Lemma 9 *For $\theta > \theta'$,*

$$f^W(\theta \mid \mathit{Tie})f^M(\theta' \mid \mathit{Tie}) \geq f^W(\theta' \mid \mathit{Tie})f^M(\theta \mid \mathit{Tie}).$$

Proof. There are four cases.

1. If $\theta < \hat{\theta}_W$, then both sides of the inequality are zero.

2. If $\theta > \hat{\theta}_W$ and $\theta' < \hat{\theta}_M$, then both sides of the inequality are zero.

3. If $\theta > \hat{\theta}_W$ and $\theta' \in [\hat{\theta}_M, \hat{\theta}_W)$, then the left-hand side of the inequality is positive and the right-hand side is zero.

4. If $\theta > \hat{\theta}_W$ and $\theta' \geq \hat{\theta}_W$, then the two sides of the inequality are both positive and they are equal.

■

Now we can prove Proposition 1.

Proof of Proposition 1.

1. Lemma 7 immediately implies $\lambda_W < \lambda_M$.

2. From Equation 8, the probability of being elected conditional on θ, $\Pr(\text{Elected} \mid \theta)$, does not depend on gender. The measure of winners of gender γ, then, is:

$$\lambda^\gamma \int_{\hat{\theta}_\gamma}^{\infty} \Pr(\text{Elected} \mid \theta) f^\gamma(\theta)\,d\theta.$$

   Canceling λ^γ and the denominator of f^γ, this can be rewritten:

$$\frac{1}{2}\int_{\hat{\theta}_\gamma}^{\infty} \Pr(\text{Elected} \mid \theta) f(\theta)\,d\theta.$$

Now the result follows from $\hat{\theta}_M < \hat{\theta}_W$.

3. Lemmas 2 and 8 imply that $F^W(\theta \mid \text{Elected}) < F^M(\theta \mid \text{Elected})$ for all $\theta > \hat{\theta}_M$. Thus $\int \tilde{\theta}\, dF^W(\tilde{\theta} \mid \text{Elected}) > \int \tilde{\theta}\, dF^M(\tilde{\theta} \mid \text{Elected})$.

4. A candidate with quality θ wins with probability $\Pr(\text{Elected} \mid \theta)$, defined in Equation 8. This probability is strictly increasing in θ. The result then follows from Lemmas 2 and 3.

■

A.3 Voter Discrimination at the Ballot Box

This section proceeds as follows. First, we characterize the equilibrium cutoffs and show that $\hat{\theta}_W = \hat{\theta}_M + b$ (Lemma 1 from the main text). Second, we derive a stochastic order result under the additional assumption that F has an increasing hazard rate. Third, we prove the remaining results from the main text.

Lemma 10 *The pair $(\hat{\theta}_W, \hat{\theta}_M)$ are equilibrium cutoffs in the model with voter discrimination at the ballot box if and only if:*

$$\frac{\lambda_W}{\lambda} \int F^W(\hat{\theta}_W - \epsilon) g(\epsilon)\, d\epsilon + \frac{\lambda_M}{\lambda} \int F^M(\hat{\theta}_W - b - \epsilon) g(\epsilon)\, d\epsilon = c \tag{10}$$

$$\frac{\lambda_W}{\lambda} \int F^W(\hat{\theta}_M + b - \epsilon) g(\epsilon)\, d\epsilon + \frac{\lambda_M}{\lambda} \int F^M(\hat{\theta}_M - \epsilon) g(\epsilon)\, d\epsilon = c \tag{11}$$

The proof closely follows that of Lemma 5, modified to account for the effect of bias on the probability of winning.

Proof. Fix cutoffs $(\hat{\theta}_W, \hat{\theta}_M)$.

Consider a female candidate with quality θ and preference shock ν. She defeats a female opponent with quality θ' and preference shock ν' if and only if:

$$\theta + \nu \geq \theta' + \nu'.$$

She defeats a male opponent with quality θ' and preference shock ν' if and only if:

$$\theta + \nu \geq \theta' + b + \nu'.$$

Thus, the probability a female candidate with quality θ wins is:

$$\Pr(\text{Elected} \mid \theta, W) = \frac{\lambda_W}{\lambda} \int F^W(\theta - \epsilon) g(\epsilon)\, d\epsilon + \frac{\lambda_M}{\lambda} \int F^M(\theta - b - \epsilon) g(\epsilon)\, d\epsilon. \tag{12}$$

A similar argument shows that the probability a male candidate of type θ wins is:

$$\Pr(\text{Elected} \mid \theta, M) = \frac{\lambda_W}{\lambda} \int F^W(\theta + b - \epsilon) g(\epsilon)\, d\epsilon + \frac{\lambda_M}{\lambda} \int F^M(\theta - \epsilon) g(\epsilon)\, d\epsilon. \tag{13}$$

A potential candidate of gender γ runs if and only if $\Pr(\text{Elected} \mid \theta, \gamma) - c \geq 0$. Since each F^γ is continuous and strictly increasing in θ, so are each $\Pr(\text{Elected} \mid \theta, \gamma)$. Thus $(\hat{\theta}_W, \hat{\theta}_M)$ are equilibrium cutoffs if and only if $\Pr(\text{Elected} \mid \hat{\theta}_\gamma, \gamma) = c$ for both γ. ■

Proof of Lemma 1. The left-hand sides of Equations 10 and 11 are equal if

$$\hat{\theta}_W = \hat{\theta}_M + b.$$

Moreover, for any $\hat{\theta}_M$, there is at most one $\hat{\theta}_W$ such that Equation 11 holds. Therefore, any solution to this system of equations must have $\hat{\theta}_W = \hat{\theta}_M + b$. ■

For the next results, we will need an expression for the expected quality of a candidate of gender γ, conditional on that candidate having quality greater than or equal to some number α. Denote that quality by $\mathbb{E}^\gamma[\theta \mid \theta \geq \alpha]$.

Denote the maximum of $\hat{\theta}_\gamma$ and α by $\hat{\theta}_\gamma \vee \alpha$. Then:

$$\mathbb{E}^\gamma[\theta \mid \theta \geq \alpha] = \int_{\hat{\theta}_\gamma \vee \alpha}^{\infty} \tilde{\theta} \frac{f(\tilde{\theta})}{1 - F(\hat{\theta}_\gamma \vee \alpha)}\, d\tilde{\theta}.$$

Immediately from this equation, we get:

Lemma 11 *1. $\mathbb{E}^\gamma[\theta \mid \theta \geq \alpha]$ is increasing in α, strictly so if $\alpha > \hat{\theta}_\gamma$.*

2. Suppose $\alpha < \hat{\theta}_W$. Then:

$$\mathbb{E}^W[\theta \mid \theta \geq \alpha] > \mathbb{E}^M[\theta \mid \theta \geq \alpha].$$

3. *Suppose* $\alpha \geq \hat{\theta}_W$. *Then:*

$$\mathbb{E}^W[\theta \mid \theta \geq \alpha] = \mathbb{E}^M[\theta \mid \theta \geq \alpha].$$

To win election, a woman's quality must be greater than some *hurdle*. This hurdle depends on whether she faces a woman or man opponent. If she faces a woman of quality θ, the hurdle is $\theta + \epsilon$. If she faces a man, the hurdle is $\theta + b + \epsilon$. Thus, the hurdle a woman faces has CDF:

$$\tilde{H}^W(\theta) = \lambda_W \int F^W(\theta - \epsilon) g(\epsilon)\, d\epsilon + \lambda_M \int F^M(\theta - b - \epsilon) g(\epsilon)\, d\epsilon.$$

Similarly, the hurdle a man faces has CDF:

$$\tilde{H}^M(\theta) = \lambda_W \int F^W(\theta + b - \epsilon) g(\epsilon)\, d\epsilon + \lambda_M \int F^M(\theta - \epsilon) g(\epsilon)\, d\epsilon.$$

Denote the densities associated with each of these as $\tilde{h}^W$ and $\tilde{h}^M$.

Lemma 12 $\tilde{H}^W$ *strictly FOS-dominates* $\tilde{H}^M$.

Proof. Define a function $\mathcal{H} : \mathbb{R}^3 \to \mathbb{R}$ by

$$\mathcal{H}(\theta, \beta_1, \beta_2) = \lambda_W \int F^W(\theta + \beta_1 - \epsilon) g(\epsilon)\, d\epsilon + \lambda_M \int F^M(\theta - \beta_2 - \epsilon) g(\epsilon)\, d\epsilon.$$

Since $\int F^\gamma(z - \epsilon) g(\epsilon)\, d\epsilon$ is strictly increasing in z, $\mathcal{H}$ is strictly increasing in β_1 and strictly decreasing in β_2.

But $\tilde{H}^W = \mathcal{H}(\cdot, 0, b)$ and $\tilde{H}^M = \mathcal{H}(\cdot, b, 0)$. Thus, for all z, $\tilde{H}^W(z) < \tilde{H}^M(z)$. ■

We can now prove Proposition 2.

Proof of Proposition 2.

1. Follows immediately from Lemma 1.

2. The measure of winners of gender γ is:

$$\lambda^\gamma \int_{\hat{\theta}_\gamma}^{\infty} \Pr(\text{Elected} \mid \theta, \gamma) f^\gamma(\theta)\, d\theta.$$

Canceling λ^γ and the denominator of f^γ, we can rewrite this as

$$\frac{1}{2}\int_{\hat{\theta}_\gamma}^{\infty} \Pr(\text{Elected} \mid \theta, \gamma) f(\theta)\, d\theta.$$

The result now follows from the following chain of inequalities:

$$\begin{aligned}\int_{\hat{\theta}_M}^{\infty} \Pr(\text{Elected} \mid \theta, M) f(\theta)\, d\theta &> \int_{\hat{\theta}_W}^{\infty} \Pr(\text{Elected} \mid \theta, M) f(\theta)\, d\theta \\ &> \int_{\hat{\theta}_W}^{\infty} \Pr(\text{Elected} \mid \theta, W) f(\theta)\, d\theta,\end{aligned}$$

The first inequality follows from $\hat{\theta}_M > \hat{\theta}_W$. The second inequality follows from the fact that, for a fixed θ, $\Pr(\text{Elected} \mid \theta, W) < \Pr(\text{Elected} \mid \theta, M)$, which is immediate from a comparison of Equations 12 and 13.

3. The expected quality of a woman winner is $\int \mathbb{E}^W[\theta \mid \theta > \alpha]\tilde{h}^W(\alpha)\, d\alpha$. The expected quality of a man winner is $\int \mathbb{E}^M[\theta \mid \theta > \alpha]\tilde{h}^M(\alpha)\, d\alpha$. We have:

$$\begin{aligned}\int \mathbb{E}^W[\theta \mid \theta > \alpha]\tilde{h}^W(\alpha)\, d\alpha &> \int \mathbb{E}^W[\theta \mid \theta > \alpha]\tilde{h}^M(\alpha)\, d\alpha \\ &> \int \mathbb{E}^M[\theta \mid \theta > \alpha]\tilde{h}^M(\alpha)\, d\alpha,\end{aligned}$$

where the first inequality is Lemma 12 and the second inequality if Lemma 11.

4. The probability a candidate of type θ and gender γ wins conditional on running, is $\Pr(\text{Elected} \mid \theta, \gamma)$, defined in Equations 12 and 13.

We can write the probability of election, conditional on gender, as:

$$\begin{aligned}\Pr(\text{Elect} \mid W) &= \frac{\lambda_W}{\lambda}\int_{\hat{\theta}_W}^{\infty} H^W(\theta) f^W(\theta)\, d\theta + \frac{\lambda_M}{\lambda}\int_{\hat{\theta}_W}^{\infty} H^M(\theta - b) f^W(\theta)\, d\theta \\ \Pr(\text{Elect} \mid M) &= \frac{\lambda_W}{\lambda}\int_{\hat{\theta}_M}^{\infty} H^W(\theta + b) f^M(\theta)\, d\theta + \frac{\lambda_M}{\lambda}\int_{\hat{\theta}_M}^{\infty} H^M(\theta) f^M(\theta)\, d\theta.\end{aligned}$$

Write the difference in probability of reelection as:

$$\begin{aligned}\Delta = &\left[\frac{\lambda_W}{\lambda}\int_{\hat{\theta}_W}^{\infty} H^W(\theta) f^W(\theta)\, d\theta + \frac{\lambda_M}{\lambda}\int_{\hat{\theta}_W}^{\infty} H^M(\theta - b) f^W(\theta)\, d\theta\right] \\ &- \left[\frac{\lambda_W}{\lambda}\int_{\hat{\theta}_M}^{\infty} H^W(\theta + b) f^M(\theta)\, d\theta + \frac{\lambda_M}{\lambda}\int_{\hat{\theta}_M}^{\infty} H^M(\theta) f^M(\theta)\, d\theta\right].\end{aligned}$$

And write the difference in probability of reelection if behavior by politicians is as in equilibrium when voters discriminate at the ballot box, but the voters do not discriminate, as:

$$\tilde{\Delta} = \left[\frac{\lambda_W}{\lambda}\int_{\hat{\theta}_W}^{\infty} H^W(\theta) f^W(\theta)\, d\theta + \frac{\lambda_M}{\lambda}\int_{\hat{\theta}_W}^{\infty} H^M(\theta - 0) f^W(\theta)\, d\theta\right] \\ - \left[\frac{\lambda_W}{\lambda}\int_{\hat{\theta}_M}^{\infty} H^W(\theta + 0) f^M(\theta)\, d\theta + \frac{\lambda_M}{\lambda}\int_{\hat{\theta}_M}^{\infty} H^M(\theta) f^M(\theta)\, d\theta\right].$$

We can now decomponse the probability of reelection, writing it as the sum of a direct effect and a selection effect:

$$\Delta = \underbrace{\Delta - \tilde{\Delta}}_{\text{direct effect}} + \underbrace{\tilde{\Delta}}_{\text{selection effect}}. \tag{14}$$

To see that the direct effect is negative, subtract and cancel like terms to get that $\Delta - \tilde{\Delta}$ is equal to:

$$\frac{\lambda_M}{\lambda}\int_{\hat{\theta}_W}^{\infty} \left(H^M(\theta - b) - H^M(\theta)\right) f^W(\theta)\, d\theta - \frac{\lambda_W}{\lambda}\int_{\hat{\theta}_M}^{\infty} \left(H^W(\theta + b) - H^W(\theta)\right) f^M(\theta)\, d\theta.$$

This is negative because H^M and H^W are strictly increasing.

To see that the selection effect is positive, rewrite $\tilde{\Delta}$ as:

$$\tilde{\Delta} = \int_{\hat{\theta}_W}^{\infty} \left(\frac{\lambda_W}{\lambda}\cdot H^W(\theta) + \frac{\lambda_M}{\lambda}\cdot H^M(\theta)\right) f^W(\theta)\, d\theta \\ - \int_{\hat{\theta}_M}^{\infty} \left(\frac{\lambda_W}{\lambda}\cdot H^W(\theta) + \frac{\lambda_M}{\lambda}\cdot H^M(\theta)\right) f^M(\theta)\, d\theta.$$

Since H^M and H^W are strictly increasing, $\theta \mapsto \frac{\lambda_W}{\lambda}\cdot H^W(\theta) + \frac{\lambda_M}{\lambda}\cdot H^M(\theta)$ is increasing. And from Corollary 1, f^W FOS dominates f^M, which establishes that the selection effect is positive.

All that remains, then, is to show that there exist parameters such that either effect dominates. Notice, this is equivalent to showing that there exist parameters such that Δ is positive and such that Δ is negative. We show this with two examples.

Example 1 Let $b > 0$ and F have an increasing hazard rate—that is, the function

Λ_F mapping z to $\Lambda_F(z) = \dfrac{f(z)}{1-F(z)}$ is increasing. We show that, in this case, Δ is negative.

We will use the following result.

Lemma 13 *Suppose F has an increasing hazard rate. Then a random variable with distribution $F^M(x-b)$ first-order stochastically dominates a random variable with distribution $F^W(y)$.*

Proof. A random variable with distribution F_1 *hazard rate dominates* a random variable with distribution F_2 if $\Lambda_{F_1}(z) \leq \Lambda_{F_2}(z)$ for all z or, equivalently, if the ratio

$$\frac{1-F_1(z)}{1-F_2(z)}$$

is increasing. Hazard rate dominance implies, but is not implied by, first-order stochastic dominance (Wolfstetter, 1999).

Let $\tilde{x}$ be a random variable with distribution $F^M(x-b)$, and let $\tilde{y}$ be a random variable with distribution $F^W(y)$. If F has an increasing hazard rate, then $\tilde{x}$ hazard rate dominates $\tilde{y}$. To see this, observe that

$$\frac{1-F^M(z-b)}{1-F^W(z)} = \frac{1-F(\hat{\theta}_W)}{1-F(\hat{\theta}_M)} \cdot \frac{1-F(z-b)}{1-F(z)}.$$

Differentiate the log of the right-hand side to see that the function is increasing if and only if

$$\frac{f(z)}{1-F(z)} \geq \frac{f(z-b)}{1-F(z-b)}.$$

■

Combining terms, using the fact that $\hat{\theta}_W = \hat{\theta}_M + b$, and doing a change of variables to put them on the same support, we can rewrite the probabilities of election conditional on running as:

$$\Pr(\text{Elect} \mid W) = \frac{1}{\lambda} \int_{\hat{\theta}_M+b}^{\infty} \left(\lambda_W H^W(\theta) + \lambda_M H^M(\theta - b)\right) f^W(\theta)\, d\theta$$
$$\Pr(\text{Elect} \mid M) = \frac{1}{\lambda} \int_{\hat{\theta}_M+b}^{\infty} \left(\lambda_W H^W(\theta) + \lambda_M H^M(\theta - b)\right) f^M(\theta - b)\, d\theta.$$

The result now follows from the fact that Δ is the difference of these two probabilities, Lemma 13, and the fact that $\lambda_W H^W(\theta) + \lambda_M H^M(\theta - b)$ is increasing in θ.

Example 2 Let $b = 0.25$, $c = 0.2$, F be Pareto with minimum 0.1 and shape parameter $q = 3.7$, and G be standard normal. Here we will show that Δ is positive.

Calculation shows that, in this case, a man who runs wins with marginal probability in the interval $(0.4503, 0.4504)$, while a woman who runs wins with a larger marginal probability, in the interval $(0.4508, 0.4509)$.[8]

■

A.4 Regression Discontinuity

Proof of Proposition 3.

1. In the model with differential costs and/or the perception gap, Lemmas 3 and 9 imply that $F^W(\theta \mid \text{Tie}) < F^M(\theta \mid \text{Tie})$ for all $\theta > \hat{\theta}_M$. Thus $\int \tilde{\theta}\, dF^W(\tilde{\theta} \mid \text{Tie}) > \int \tilde{\theta}\, dF^M(\tilde{\theta} \mid \text{Tie})$.

2. Write the difference in expected quality of a woman and main who win a tied election as:
$$\Delta' = \frac{\int \theta h^M(\theta - b) f^W(\theta)\, d\theta}{\int h^M(\theta - b) f^W(\theta)\, d\theta} - \frac{\int \theta h^W(\theta + b) f^M(\theta)\, d\theta}{\int h^W(\theta + b) f^M(\theta)\, d\theta}.$$
And write the difference in expected quality condition on a tie if entry decisions are as they are in equilibrium with voters discriminating, but voters don't in fact discriminate at the ballot box as:
$$\tilde{\Delta}' = \frac{\int \theta h^M(\theta) f^W(\theta)\, d\theta}{\int h^M(\theta) f^W(\theta)\, d\theta} - \frac{\int \theta h^W(\theta) f^M(\theta)\, d\theta}{\int h^W(\theta) f^M(\theta)\, d\theta}.$$
We can decompose the difference in expected ability conditional on a tie as:
$$\Delta' = \underbrace{\Delta' - \tilde{\Delta}'}_{\text{direct effect}} + \underbrace{\tilde{\Delta}'}_{\text{selection effect}}. \tag{15}$$
The same proof as point 1 of this proposition shows that $\tilde{\Delta}' > 0$, so the selection effect is positive.

[8] Mathematica code for all calculations available in the replication archive.

All that remains, then, is to show that there exist parameters such that Δ' is positive and such that Δ' is negative. We show this with two examples.

Example 3 Let $b > 0$ and f and g be log-concave. We show that, in this case, Δ' is positive.

We will use the following results. Let $f^\gamma(\theta \mid \text{Tie})$ be the density of ability of candidates of gender γ, conditional on a tie election. We start by establishing some stochastic order results for $f^M(\theta \mid \text{Tie})$ and $f^W(\theta \mid \text{Tie})$.

Lemma 14 *Suppose f is log-concave. Then $z > z'$ implies:*

$$f^M(z-b)f^W(z') \geq f^M(z'-b)f^W(z),$$

with strict inequality if $z' \geq \hat{\theta}_W$.

Proof. If $z' < \hat{\theta}_W$, then Lemma 1 implies both sides of the inequality are zero. So suppose $z > z' \geq 0$, and define the function $\ell(z)$ by:

$$\ell(z) = \frac{f^M(z-b)}{f^W(z)}.$$

The result follows from the claim that ℓ is strictly increasing.

To prove that claim, note that:

$$\ell(z) = \frac{(1-F(\hat{\theta}_W))}{(1-F(\hat{\theta}_M))} \cdot \frac{f(z-b)}{f(z)}.$$

Take logs and differentiate the right-hand side to get:

$$\frac{f'(z-b)}{f(z-b)} - \frac{f'(z)}{f(z)} > 0,$$

where the inequality follows from logconcavity and $b > 0$. ■

Lemma 15 *Suppose f and g are logconcave. Then h^W and h^M are logconcave.*

Proof. Logconcavity of f implies logcavity of f^W and of f^M, by Bagnoli and Bergstrom (2005, Theorem 7). Logconcavity of f^W (f^M) and of g implies logconcvity of h^W (h^M), by Miravete (2011, Lemma 2).[9] ■

Corollary 2 *Suppose f and g are logconcave. Then the function $z \mapsto \dfrac{h(z)}{h(z+b)}$ is increasing.*

Lemma 16 *Suppose f and g are logconcave. Then the function $z \mapsto \dfrac{h^M(z-b)}{h^W(z)}$ is increasing.*

Proof. Let $\tilde{x}$ be a random variable with density $f^M(x-b)$, and let $\tilde{y}$ be a random variable with density $f^W(y)$. Lemma 14 says $\tilde{x}$ dominates $\tilde{y}$ in the likelihood ratio order. Thus $\tilde{x}+\epsilon$ likelihood ratio dominates $\tilde{y}+\epsilon$ (Keilson and Sumita, 1982, Theorem 2.1(d)). ■

Lemma 17 *In the model with voter discrimination at the ballot box, $\theta > \theta'$ implies $f^W(\theta \mid Tie) f^M(\theta' \mid Tie) \geq f^W(\theta' \mid Tie) f^M(\theta \mid Tie)$.*

Proof. There are four cases.

(a) If $\theta < \hat{\theta}_W$, then both sides of the inequality are zero.

(b) If $\theta > \hat{\theta}_W$ and $\theta' < \hat{\theta}_M$, then both sides of the inequality are zero.

(c) If $\theta > \hat{\theta}_W$ and $\theta' \in [\hat{\theta}_M, \hat{\theta}_W)$, then the left-hand side of the inequality is positive and the right-hand side is zero.

(d) If $\theta > \hat{\theta}_W$ and $\theta' \geq \hat{\theta}_W$, then the two sides of the inequality are both positive, and we can rewrite the target inequality as:

$$\frac{f^W(\theta \mid \text{Tie})}{f^M(\theta \mid \text{Tie})} \geq \frac{f^W(\theta' \mid \text{Tie})}{f^M(\theta' \mid \text{Tie})}. \tag{16}$$

[9] Miravete (2011) has compact support as a maintained assumption, but the arguments do not rely on that fact. See Miravete (2002).

To establish this, define a function ℓ by:

$$\ell(z) = \frac{f^W(\theta \mid \text{Tie})}{f^M(\theta \mid \text{Tie})}.$$

Substitute from the definitions of the two densities to get:

$$\begin{aligned}\ell(z) &= K\frac{h^M(z-b)f^W(z)}{h^W(z+b)f^M(z)} \\ &= K\frac{h^M(z-b)}{h^W(z)}\frac{h^W(z)}{h^W(z+b)}\frac{f^W(z)}{f^M(z)},\end{aligned}$$

where K is a constant. The function $z \mapsto \dfrac{h^M(z-b)}{h^W(z)}$ is increasing by Lemma 16. The function $z \mapsto \dfrac{h^W(z)}{h^W(z+b)}$ is increasing by Corollary 2. The function $z \mapsto \dfrac{f^W(z)}{f^M(z)}$ is increasing by Lemma 3. All three of these functions are positive on $[\hat{\theta}_W, \infty)$, so ℓ is increasing on that interval.

■

In the model with voter discrimination at the ballot box, conditioning on a tie, the densities of candidate quality by gender are:

$$f^W(\theta \mid \text{Tie}) = \frac{h^M(\theta - b)f^W(\theta)}{\int h^M(\tilde{\theta} - b)f^W(\tilde{\theta})\,d\tilde{\theta}}$$

and

$$f^M(\theta \mid \text{Tie}) = \frac{h^W(\theta + b)f^M(\theta)}{\int h^W(\tilde{\theta} + b)f^M(\tilde{\theta})\,d\tilde{\theta}}.$$

Lemmas 3 and 17 imply that if the densities are log-concave, then $F^W(\theta \mid \text{Tie}) < F^M(\theta \mid \text{Tie})$ for all $\theta > \hat{\theta}_M$. Thus $\int \tilde{\theta}\,dF^W(\tilde{\theta} \mid \text{Tie}) > \int \tilde{\theta}\,dF^M(\tilde{\theta} \mid \text{Tie})$, which implies that Δ' is positive.

Example 4 Let $b = 0.5$, $c = 0.2$, F be Pareto with minimum 0.1 and shape parameter $q = 3$, and G be standard normal. Here we will show that Δ' is negative.

Calculation shows that, in this case, a man who wins in a tie has expected quality in the interval $(4.55, 4.56)$, while a woman who wins in a tie has expected quality in the interval $(4.33, 4.34)$, so Δ' is negative.

■

B Computational Examples

Example 5 ***Voter Discrimination at the Ballot Box Alone***
Consider the model with voter discrimination at the ballot box but no differential costs or perception gap. Suppose the distribution of θ is Pareto with minimum 0.1 and shape parameter q. This distribution has a decreasing hazard rate for all values of q. The noise ϵ is distributed standard normal, and so has a log-concave density. Let $b = 0.25$ and $c = 0.2$.

If $q = 3.7$, then a man who runs wins with marginal probability in the interval $(0.4503, 0.4504)$, while a woman who runs wins with a larger marginal probability, in the interval $(0.4508, 0.4509)$.[10] If $q = 3.8$, then a man who runs wins with marginal probability in the interval $(0.4491, 0.4492)$, while a woman who runs wins with a smaller marginal probability, in the interval $(0.4490, 0.4491)$. Thus a continuity argument ensures us that for some $q \in (3.7, 3.8)$, men and women win with identical probability, conditional on running.

Example 6 ***Combined Model***
Consider a version of the model with both voter discrimination and differential costs (we could do the same with a perception gap). Suppose the distribution of both θ and ϵ are distributed standard normal, and so have increasing hazard rates. Let $b = 0.25$.

If women and men use cutoffs $\hat{\theta}_W = 1.12$ and $\hat{\theta}_M = 0.5$, respectively, then the probability that a woman wins (conditional on running) if her quality is exactly $\theta = \hat{\theta}_W = 1.12$ is in the interval $(0.38, 0.39)$ and the probability a man wins (conditional on running) if his quality is exactly $\theta = \hat{\theta}_M = 0.5$ is in the interval $(0.265, 0.27)$. Thus a continuity argument shows that there is a pair (c^W, c^M) with $0.38 < c^W < 0.39$ and $0.265 < c^M < 0.27$ such that there is an equilibrium with $\hat{\theta}_W = 1.12$ and $\hat{\theta}_M = 0.5$. Given such an equilibrium, we can calculate the probability a man wins conditional on running minus the probability a woman wins conditional on running. Doing so shows that it is contained in $(0.004, 0.005)$. So, at this equilibrium, men win at a slightly higher rate than women.

If women and men use cutoffs $\hat{\theta}_W = 1.14$ and $\hat{\theta}_M = 0.5$, respectively, then the probability that a woman wins (conditional on running) if her quality is exactly $\theta = \hat{\theta}_W = 1.14$ is in the interval $(0.39, 0.4)$ and the probability a man wins (conditional on running) if his quality is exactly $\theta = \hat{\theta}_M = 0.5$ is in the interval $(0.26, 0.265)$. Thus a continuity argument shows that there is a pair (c^W, c^M) with $0.39 < c^W < 0.4$ and $0.26 < c^M < 0.265$ such that there is

[10] Mathematica code for all calculations available in the replication archive.

an equilibrium with $\hat{\theta}_W = 1.12$ and $\hat{\theta}_M = 0.5$. Given such an equilibrium, we can calculate the probability a man wins conditional on running minus the probability a woman wins conditional on running. Doing so shows that it is contained in $(-0.0005, -0.0004)$. So, at this equilibrium, men win at a slightly lower rate than women.

Continuity now immediately implies that there is some (c^W, c^M) such that there is an equilibrium with $\hat{\theta}_W \in (1.12, 1.14)$ and $\hat{\theta}_M = 0.5$ where women and men win at the exact same rate. In such an equilibrium, women are clearly under-represented in the pool of available candidates and have higher average quality conditional on winning. Hence, this example shows that a model with both differential costs and voter discrimination at the ballot box can account for all three empirical facts.