Econ 20700. Assignment 7. (*not to be handed in*)

1. Consider a two-person game where player 1 chooses T or B, and player 2 chooses L or R. When they play this game, player 2 also knows whether her type is A or B. The players' utility payoffs (u_1,u_2) depend on their actions and on player 2's type as follows:

	2'sType = A			2'sType = B	
	L	R		L	R
T	4,0	0,2	T	4,0	0,4
B	0,4	2,0	B	0,2	2,0

(a) Suppose first that, in this game, player 1 thinks that 2's type is equally likely to be A or B. Find a Bayesian equilibrium.

(b) How would the Bayesian equilibrium change in a game where player 1 thinks that player 2 has probability 1/6 of being type A, and has probability 5/6 of being type B?

2. Players 1 and 2 each must decide whether to fight for a valuable prize. If both players decide to fight, then they both lose \$1, and nobody gets the prize (it is destroyed). If one player decides to fight but the other does not, then the player who is willing to fight gets the prize. A player who does not fight is guaranteed a payoff of 0. Everybody knows that the prize is worth $V_2 = \$2$ to player 2. But the prize may be worth more to player 1. Let V_1 denote the value of the prize to player 1. In terms of V_1, the players' payoffs (u_1,u_2) will depend on their actions as follows:

	Player 2	
Player 1	NotFight	Fight
NotFight	0, 0	0, 2
Fight	V_1, 0	−1, −1

Let us explore some different assumptions about this value V_1.

(a) Suppose first that the value of the prize to player 1 is $V_1 = \$3$, and everybody knows this. Find all equilibria of this game, including a mixed-strategy equilibrium in which both players have a positive probability of fighting.

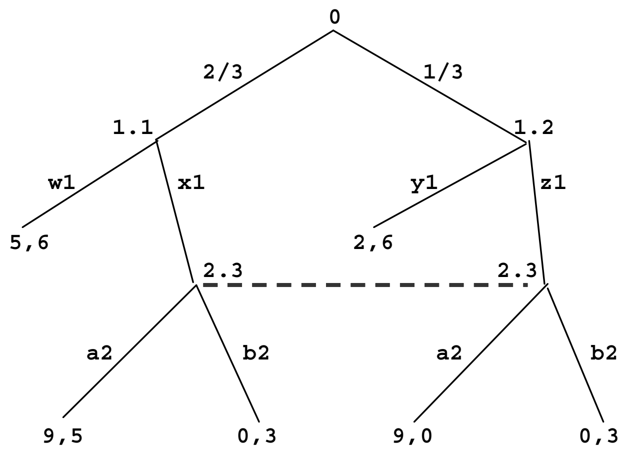
(b) Suppose next that the value of the prize to player 1 is either $V_1 = \$2$ or $V_1 = \$3$. Player 1 knows his actual value, but player 2 thinks each of these possibilities has probability 1/2. Find a Bayesian equilibrium of this game where player 2 randomizes between fighting and not fighting.

(c) Finally, suppose that the value of the prize to player 1 is $V_1 = \$2+\tilde{t}_1$ where $\tilde{t}_1$ can be any number between 0 and 1. Player 1 knows its actual value, but player 2 thinks of $\tilde{t}_1$ as a uniform random variable on the interval from 0 to 1. Find a Bayesian equilibrium of this game where player 2 randomizes between fighting and not fighting.

Econ 20700. Assignment 6. *Due May 21, 2018.*

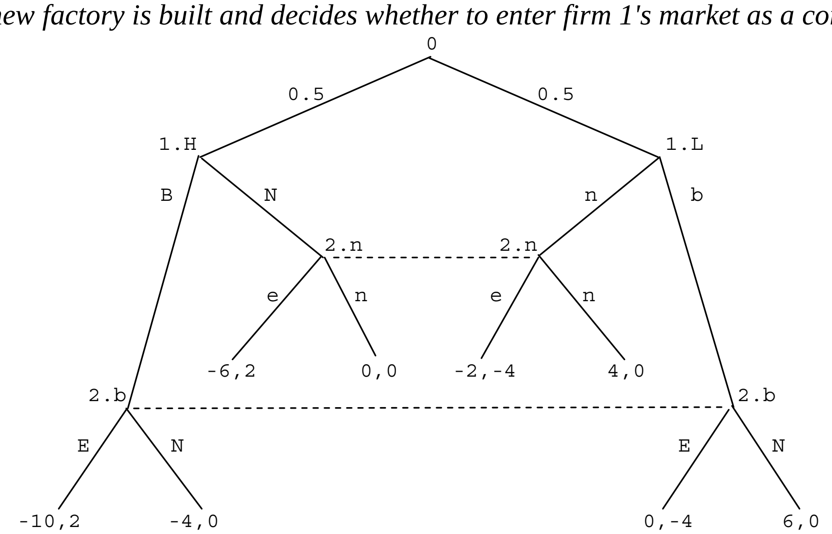
1. Consider the three-player game in Osborne's Figure 331.2 (exercise 331.1).
(a) Show its normal representation in strategic form.
(b) Show that this strategic game has a pure-strategy Nash equilibrium that corresponds to a sequential equilibrium of the given extensive-form game. Be sure to indicate what beliefs would make this a sequential equilibrium.
(c) Show that this game also has a pure-strategy Nash equilibrium that does not correspond to a sequential equilibrium of the given extensive-form game.

2. Consider the following extensive-form game, where player 1 observes the chance move, but player 2 does not observe it. If 2 gets to move, she only knows that 1 chose either x1 or z1.



(a) Find a sequential equilibrium in which the (prior) probability of player 2 getting to move is 1.
(b) Find a sequential equilibrium in which the probability of player 2 getting to move is 0. (You must describe what player 2 would believe and do if she got to move.)
(c) Find a sequential equilibrium in which the probability of player 2 getting to move is strictly between 0 and 1.
(d) Show the normal representation of this game in strategic form.

3. Consider the following extensive-form game, which begins with a chance move. (*Interpretation: firm 1 has high or low costs and must decide whether to build a new factory; then firm 2 observes whether the new factory is built and decides whether to enter firm 1's market as a competitor.*)



(a) Show the normal representation of this game in strategic form, and find all pure-strategy Nash equilibria of this game.
(b) For each Nash equilibrium that you found in part (a), explain whether this equilibrium corresponds to a sequential equilibrium of the extensive-form game. If so, at every information set (including sets of probability zero) you should indicate what beliefs would make this a sequential equilibrium.
(c) Apply iterative elimination of weakly dominated strategies to the normal representation. Does this analysis eliminate any of the Nash equilibria that you found in part (a)?

Econ 20700. Assignment 5. *Due May 14, 2018.*

1. Consider a repeated game where 1 and 2 repeatedly play the game below infinitely often.

	a_2	b_2
a_1	8, 8	1, 2
b_1	2, 1	0, 0

The players want to maximize their δ-discounted average value of payoffs, for some $0<\delta<1$.
Consider the following state-dependent strategies: The possible states are state 1 and state 2.
In state 1, we anticipate that player 1 will play b_1 and player 2 will play a_2.
In state 2, we anticipate that player 1 will play a_1 and player 2 will play b_2.
The game begins at period 1 in state 1. The state of the game would change after any period where the outcome of play was (a_1,a_2), but otherwise the state always stays the same.
What is the lowest value of δ such that these strategies form an equilibrium?

2. Consider a repeated game where 1 and 2 repeatedly play the game below infinitely often.

	a_2	b_2
a_1	3, 3	0, 5
b_1	5, 0	−4,−4

The players want to maximize their δ-discounted average value of payoffs, for some $0<\delta<1$.
(a) Find the lowest value of δ such that you can construct an equilibrium in which the players will actually choose (a_1,a_2) forever, but if any player i ever chose b_i at any period then they would play the symmetric randomized equilibrium of the one-stage game forever afterwards.
(b) What is the lowest value of δ such that you can construct an equilibrium in which the players will actually choose (a_1,a_2) forever, but if some player i unilaterally deviated to b_i at any period then that player i would get payoff 0 at every round thereafter? Be sure to precisely describe state-dependent strategies that form this equilibrium.

3. Consider a repeated game where 1 and 2 repeatedly play the game below infinitely often.

	a_2	b_2
a_1	0, 8	2, 0
b_1	8, 0	0, 2

Each player i wants to maximize his or her δ_i-discounted average value of payoffs, for some δ_1 and δ_2, where each $0<\delta_i<1$.
Find the lowest values of δ_1 and δ_2 such that you can construct an equilibrium in which the players will actually alternate between (a_1,a_2) and (b_1,a_2) forever, but if any player ever deviated then they would play the randomized equilibrium of the one-stage game forever afterwards.

Econ 20700. Assignment 4. *Due May 7, 2018.*

1. Do exercises 59.1 and 189.1 in Osborne.

2. Player 1 chooses a number a_1 between 0 and 1 ($0 \le a_1 \le 1$), and player 2 also chooses a number a_2 between 0 and 1 ($0 \le a_2 \le 1$). Their payoffs (u_1,u_2) depend on the chosen numbers (a_1,a_2) and a known parameter γ as follows:

$$u_1(a_1,a_2) = \gamma\, a_1a_2 - (a_1)^2 \,,$$
$$u_2(a_1,a_2) = 2a_1a_2 - a_2.$$

(a) Given γ=1.5, find all (pure) Nash equilibria of this game if the players choose their numbers independently.

(b) Given γ=1.5, find a subgame-perfect equilibrium of this game if player 1 chooses a_1 first, and then player 2 chooses a_2 after observing a_1.

(c) Given γ=0.8, find all (pure) Nash equilibria of this game if the players choose their numbers independently.

(d) Given γ=0.8, find a subgame-perfect equilibrium of this game if player 1 chooses a_1 first, and then player 2 chooses a_2 after observing a_1.

3. Find all Nash equilibria (pure and mixed) of the following 2×3 game:

		Player 2	
Player 1	L	M	R
T	0, 4	5, 6	8, 7
B	2, 9	6, 5	5, 1

4. Do exercise 145.1 in Osborne. (*Hint:* In the symmetric equilibrium, each of the n players is willing to bid any amount between 0 and K. The probability distribution for a player's bid can be characterized by its cumulative distribution function.)

Econ 20700. Assignment 3. *Due April 23, 2018.*
In Chapter 5, do exercises:
156.2ac,
173.2 and show the strategic form that represents Figure 173.1,
176.1 simplify the problem by assuming that a player who does not pass must bid exactly 1 more than the other player's most recent bid (say the other's "most recent bid" is 0 at the start of the game), analyze the cases (v,w)=(2,3) and (v,w)=(3,4) and (v,w)=(3,5),
177.1.

In problem 176.1, when you are asked to find all subgame-perfect equilibria, you can restrict your attention to subgame-perfect equilibria without randomization.

Econ 20700. Assignment 2. *Due April 11, 2018.*
In Osborne chapter 4, do exercises:
114.1, 114.3, 120.2 (only consider mixed strategies that randomize among M and B),
128.1, 130.2, 141.2b, 142.1 (look for a symmetric randomized equilibrium where A and B both have positive probability).
*34.3(a) with randomization: For the first game in 34.3 (without the middle road), find a symmetric randomized equilibrium in which each driver independently chooses between the X route, with some probability p, and the Y route, with probability 1-p. What is p in equilibrium?

Econ 20700. Assignment 1. *Due April 2, 2018.*
In Osborne chapter 2, do exercises:
42.1,
42.2 (notice that each player gets <u>half</u> of $f(x_1,x_2)$, and each x_i satisfies $0 \le x_i \le 1$),
47.1,
52.2,
34.3 (just find <u>one</u> equilibrium for each of the two games),
and the following small attrition game:

<u>Small Attrition game.</u> There are two players numbered 1 and 2. Each player i must choose a number c_i in the set {0, 1, 2}, which represents the number of days that player i is prepared to fight for a prize that has value V=$9. A player wins the prize only if he is prepared to fight strictly longer than the other player. They will fight for as many days as both are prepared to fight, and each day of fighting costs each player $1. Thus, the payoffs for players 1 and 2 are as follows:
Player 1's payoff is $u_1(c_1,c_2) = 9-c_2$ if $c_1 > c_2$, but $u_1(c_1,c_2) = -c_1$ if $c_1 \le c_2$.
Player 2's payoff is $u_2(c_1,c_2) = 9-c_1$ if $c_2 > c_1$, but $u_2(c_1,c_2) = -c_2$ if $c_2 \le c_1$.
Show a 3×3 matrix that represents this game.
What dominated strategies can you find for each player in this game?
What pure-strategy equilibria can you find for this game?