MORAL-HAZARD CREDIT CYCLES WITH RISK-AVERSE AGENTS

Roger Myerson

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This paper: http://home.uchicago.edu/~rmyerson/research/rabankers.pdf
Journal of Economic Theory 153:74-102 (2014).

Previous paper: "A model of moral-hazard credit cycles," Journal of Political Economy 120(5):847-878 (2012).

These notes: http://home.uchicago.edu/~rmyerson/research/rabanknts.pdf

Computational model: http://home.uchicago.edu/~rmyerson/research/rabankers.xls

Macroeconomic fluctuations driven by moral hazard

Moral hazard in financial intermediation can cause macroeconomic fluctuations in a stationary nonstochastic environment, even when agents are risk averse.

Because of moral hazard, financial agents need long-term relationships with investors, and these relationships can create complex macroeconomic dynamics.

My previous paper showed how such credit cycles can be sustained over an infinite time horizon when agents are risk neutral (a natural simplifying assumption).

Optimal contracts for risk-neutral agents look rather extreme, with agents receiving incentive payments only at the end of successful careers.

When agents are risk averse, the analysis becomes more complicated, but the resulting optimal contracts seem more realistic.

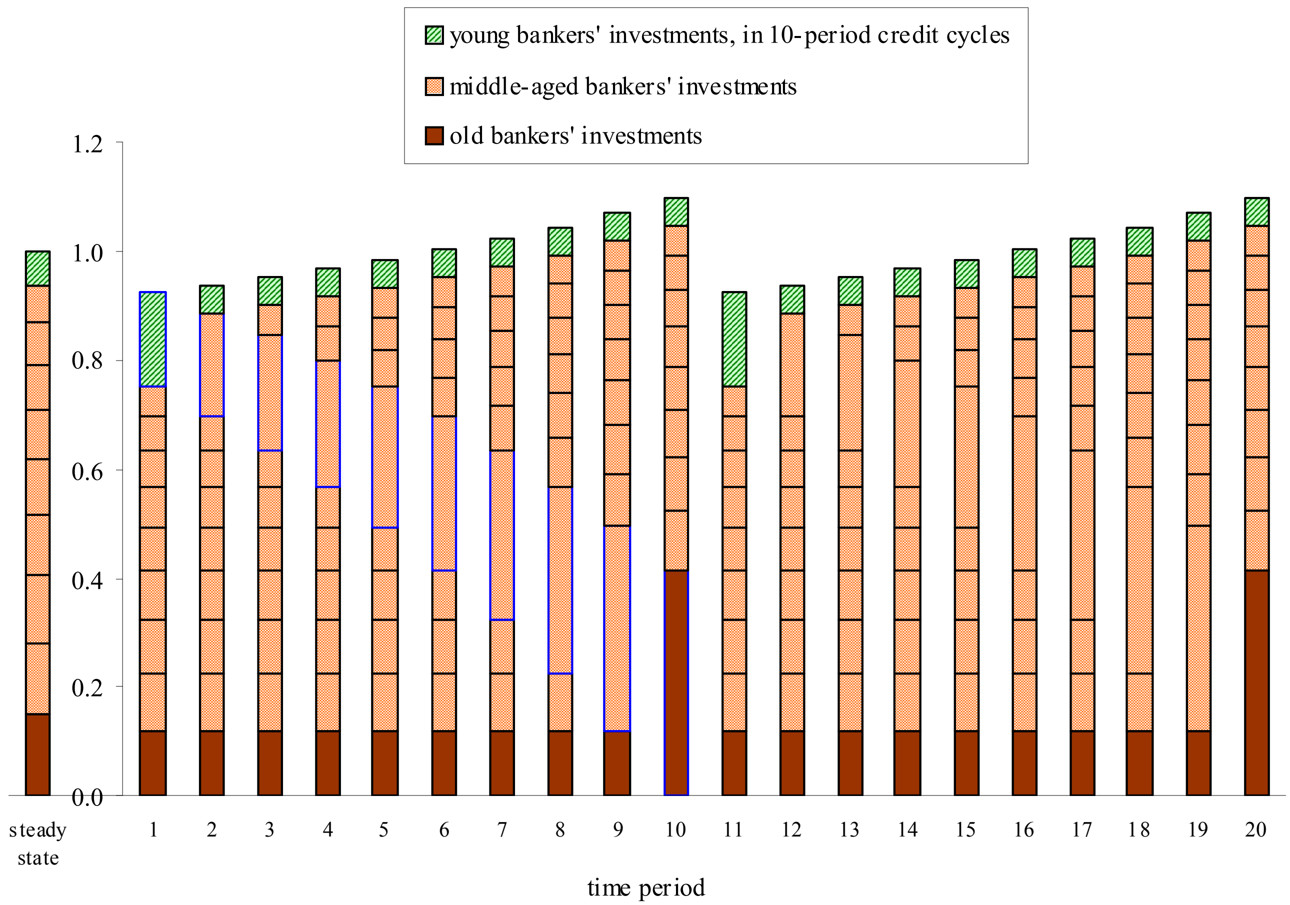
Risk-averse financial agents get substantial rewards in each period when they supervise investments.

Remarkably, we also find that the economy can become even more unstable when financial agents are risk averse.

Snapshot of risk-neutral model from the previous paper "A model of moral-hazard credit cycles" at http://home.uchicago.edu/~rmyerson/research/bankers.pdf

Investments handled by different cohorts of agents with 10-period careers, starting with agents investing 80% of steady state.

Total investments managed by each cohort of bankers grows in proportion to the expected present discounted value of their total end-of-career bonuses (which is not uncertain for the whole cohort).



Summary of the model with risk-averse agents

Investments are managed each period by agents subject to moral hazard.

Agents live 3 periods (t=young, t+1=old, t+2=retired); in each of the first 2 periods, the agent can manage one investment with subsequent accountability.

Consuming $c_s \geq 0$ in period t+s yields current utility $(c_s)^\theta/\theta$ for some $\theta < 1$.

Lifetime utilities are summed with discount factor δ: $\mathbf{[(c_0)^\theta + \delta(c_1)^\theta + \delta^2(c_2)^\theta]/\theta}$

The agent is hired by risk-neutral investors with the same discount factor δ.

Given investment h_s to manage in period t+s, the agent can manage well or can mismanage it and divert $\boldsymbol{\gamma h_s}$ into her current income available for consumption.

The investment will succeed and yield $h_s(1+p_{t+s})/\delta$ next period if the agent manages it well, but the investment will fail and yield 0 if the agent mismanages it.

So the expected (t+s)-discounted return from an investment h_s is $\mathbf{h_s(1+p_{t+s})}$ if it is well managed, but is 0 if it is mismanaged.

Agents cannot borrow against future income, but a mismanaging agent can save to consume each period v_1=$\mathbf{(c_1+\gamma h_1)/(1+\delta)}$ from s=1, $\mathbf{(c_0+\gamma h_0)/(1+\delta+\delta^2)}$ from s=0.

The expected profit rate p_t is related to aggregate investment I_t in period t by a given decreasing investment-demand function $I_t = \mathbf{I(p_t)} \geq 0$.

We assume $\gamma < 1$, so that mismanaged investments are never worthwhile.

Example: $\theta = 0.5$, $\delta = 0.5 = 0.966^{20}$, $\gamma = 0.25$, $I(p) = 1.5\max\{1-p/0.7, 0\}$.

Optimal incentive plans for young agents at time t

In hiring a young agent at time t, the investors' optimal incentive problem is:

choose (h_0,c_0,h_1,c_1,c_2) to

maximize $h_0 p_t + \delta h_1 p_{t+1} - c_0 - \delta c_1 - \delta^2 c_2$

subject to $(h_0,c_0,h_1,c_1,c_2) \geq 0,\ h_0 = 1$ (*nonnegativity, normalization*)

$(c_1^{\theta} + \delta c_2^{\theta})/\theta \geq (1+\delta)[(c_1+\gamma h_1)/(1+\delta)]^{\theta}/\theta$ (*incentive constraint at t+1*)

$(c_0^{\theta} + \delta c_1^{\theta} + \delta^2 c_2^{\theta})/\theta \geq (1+\delta+\delta^2)[(c_0+\gamma h_0)/(1+\delta+\delta^2)]^{\theta}/\theta$ (*incentive constraint at t*)

If investors could expect positive profits, they would want to invest more, driving rates of return down until investments in young agents earn zero expected profits.

Given p_{t+1}, let $\mathbf{Y(p_{t+1})}$ be the profit rate p_t such that the optimal value here is 0, so that such that investors can just expect to break even in hiring a young agent at t.

With $h_0=1$, $Y(p_{t+1}) = c_0 + \delta c_1 + \delta^2 c_2 - \delta h_1 p_{t+1}$ at an optimal solution (indep of p_t).

Let $\mathbf{G(p_{t+1})} = h_1/h_0$ for an optimal solution (which is independent of p_t with $h_0=1$).

$G(p_{t+1})$ is the expected growth in agents' responsibilities from young at time t to old at time t+1. By the envelope theorem, $Y'(p_{t+1}) = -\delta G(p_{t+1})$.

Let $\mathbf{H(p_{t+1})} = h_1/v_1$, where v_1 is the constant-equivalent consumption satisfying $(1+\delta)(v_1)^{\theta}/\theta = (c_1^{\theta} + \delta c_2^{\theta})/\theta$ for an optimal solution.

$H(p_{t+1})$ is the old agents' responsibilities at time t+1 per unit of constant-equivalent consumption promised to them after a successful investment when young.

Equilibria of the dynamic model with rational expectations

We assume that young agents can be hired in any period t.

Let $\mathbf{J_t}$ denote the total investments managed by young agents in any period t.

If profits p_t in a period t were greater than $Y(p_{t+1})$, then investors could expect positive surplus by hiring young agents to invest more, thus decreasing p_t.

So in equilibrium, in any period t, we must have $p_t \leq Y(p_{t+1})$, and if $J_t > 0$ then we must have $p_t = Y(p_{t+1})$.

The aggregate investment responsibilities of young agents from period t are expected to grow to $J_t G(p_{t+1})$ in period t+1 when they are old agents.

Initial contracts may promise some constant-equivalent payoffs V_0 to old agents in period 1.

Given this initial condition, an ***equilibrium*** is a sequence of profit rates $(p_1,p_2,p_3,\ldots)$ and young-agent investment responsibilities $(J_1,J_2,J_3,\ldots)$ such that

$I(p_1) = V_0H(p_1)+J_1,\ \ I(p_{t+1}) = J_tG(p_{t+1}) + J_{t+1},\ \ \forall t \in \{1,2,3,\ldots\}.$

$J_t \geq 0,\ \ 0 \leq p_t \leq Y(p_{t+1}),\ \text{and}\ \ [p_t - Y(p_{t+1})]J_t = 0,\ \ \forall t \in \{1,2,3\ldots\}.$

Dynamic instability of equilibria with risk averse agents

There exists a unique $p^* \in [0,Y(0)]$ such that **$p^* = Y(p^*)$**. Suppose $I(p^*) > 0$.
A steady state equilibrium with $p_t = p^*$ and $J_t = J^* = I(p^*)/[1+G(p^*)]$ $\forall t$
exists from the initial condition $V_0 = V^* = I(p^*)/[H(p^*)(1+1/G(p^*))]$.

If $G(p^*) \leq 1$, the steady state is stable for all initial conditions. *(with θ=0.5, δ>0.85)*

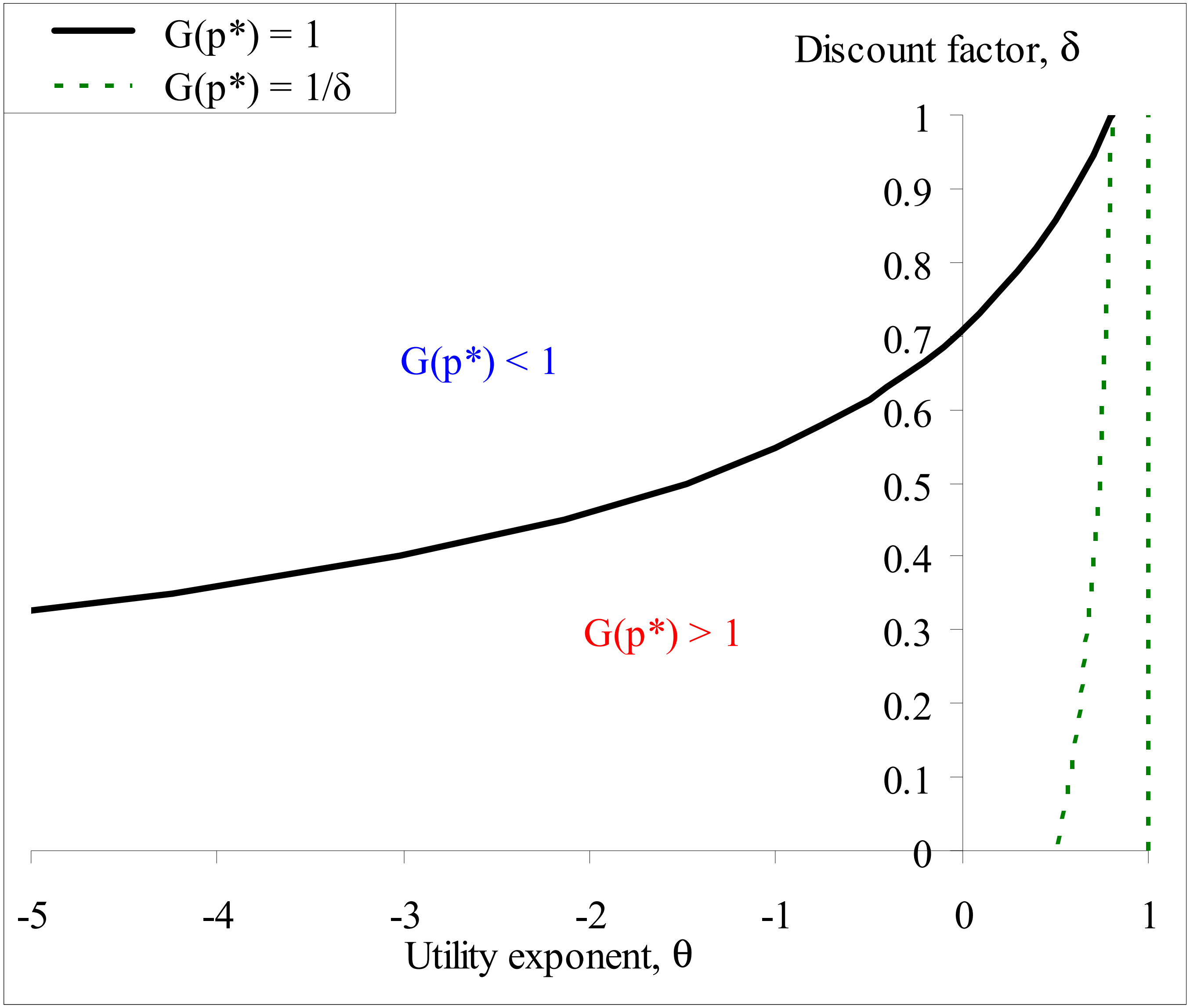
Recall $h_0 p_t + \delta h_1 p_{t+1} = h_0[p_t + \delta G(p_{t+1})p_{t+1}]$ and so $Y'(p_{t+1}) = -\delta G(p_{t+1})$.
So the dynamics of p_t=$Y(p_{t+1})$ in equilibrium will depend critically on $\delta G(p^*)$.
With $\theta \leq 0.5$, we always find $G(p^*) < 1/\delta$, so that $\delta G(p^*) < 1$.

Fact. Suppose **$1 < G(p^*) < 1/\delta$**. Either (*regular case*) **$\exists(q_0,q_1)$ such that**
$q_0 = Y(q_1)$, $I(q_0)G(q_1) = I(q_1)$, $q_1 < p^* < q_0$, and $p < Y(Y(p))$ $\forall p \in [q_1,p^*)$;
or (*alternative case*) $\exists(r_0,r_1)$ with $J_0 \geq 0$ and $J_1 \geq 0$ such that
$r_0 = Y(r_1)$, $r_1 = Y(r_0)$, $J_0 + G(r_0)J_1 = I(r_0)$, $J_1 + G(r_1)J_0 = I(r_1)$.
The alternative case has not been found with $1 < G(p^*) < 1/\delta$.

In the regular case, there is an extreme cyclical equilibrium, from initial $V_0 = 0$, that alternates between $p_t = q_0$ with $J_t = I(q_0)$ and $p_{t+1} = q_1$ with $J_{t+1} = 0$.

Fact. With $1 < G(p^*) < 1/\delta$ and regularity, from any initial condition V_0 other than V^*, there exists an equilibrium that enters the extreme (q_0,q_1) cycle within finitely many periods.

Conditions for existence of cycles around the steady state (G(p*)>1) depend on the discount factor δ and the utility exponent θ.



Dependence of steady-state contracts on the utility exponent θ

with δ=0.5, γ=0.25, and h_0 = 1.

(Deviations from steady state amplify for $-1.47 \leq \theta \leq 0.73$)

θ	p*	h_0	c_0	h_1	c_1	c_2
1	0.125	1	0	2	0	1
0.9	0.147	1	0.000	2.151	0.012	1.200
0.8	0.172	1	0.002	2.083	0.089	1.218
0.7	0.194	1	0.009	1.955	0.170	1.157
0.6	0.214	1	0.022	1.834	0.232	1.087
0.5	0.231	1	0.038	1.727	0.275	1.021
0.3	0.260	1	0.070	1.554	0.329	0.909
0.1	0.282	1	0.098	1.424	0.356	0.824
−0.1	0.299	1	0.123	1.324	0.371	0.758
−0.5	0.325	1	0.160	1.183	0.381	0.664
−1	0.346	1	0.193	1.071	0.383	0.591
−2	0.372	1	0.232	0.946	0.376	0.510
−4	0.395	1	0.269	0.838	0.364	0.441

A numerical example

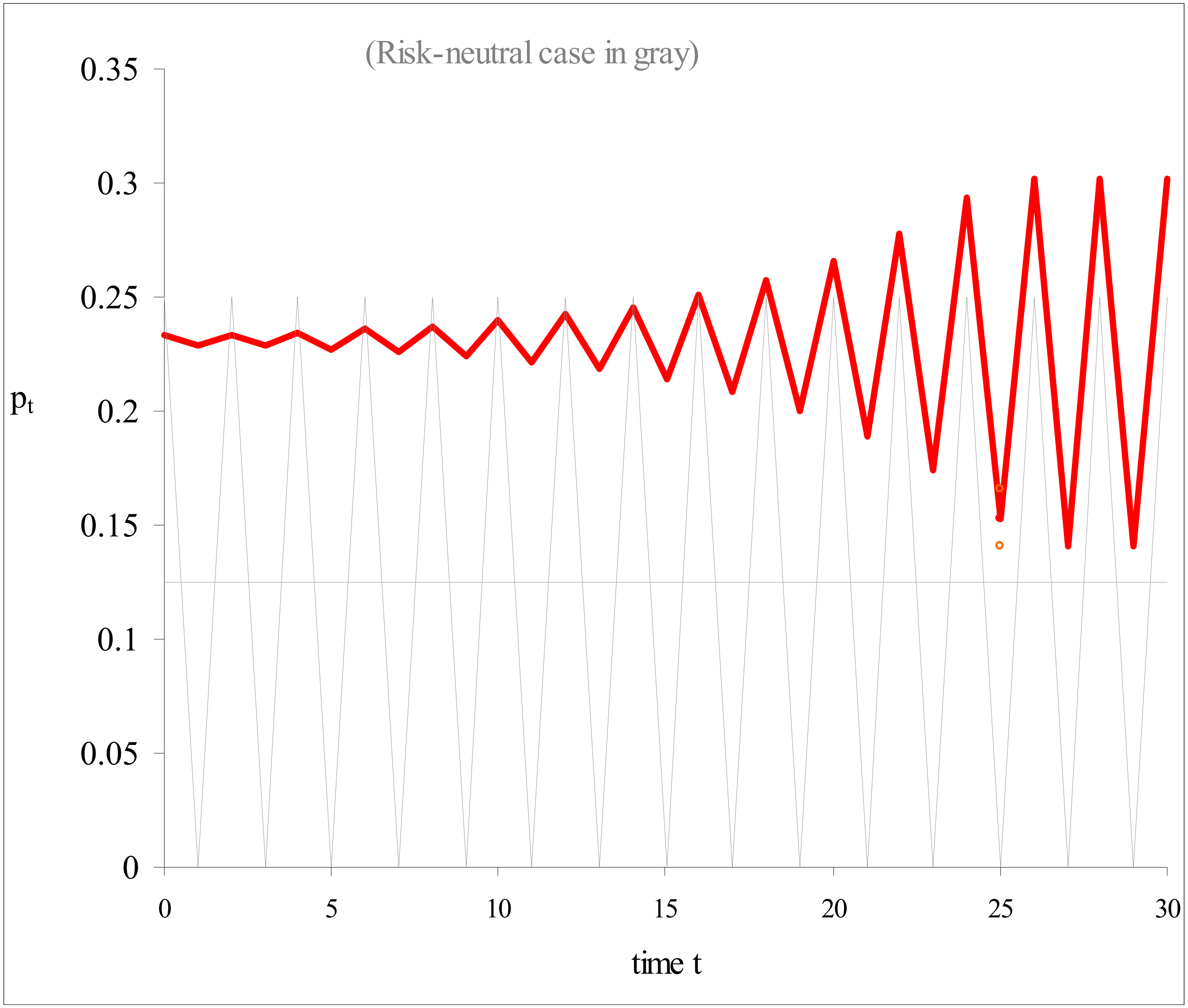
For example, let $\theta = 0.5$, $\delta = 0.5$, $\gamma = 0.25$, $I(p) = 1.5 \max\{1 - p/0.7, 0\}$.
The steady-state profit rate is $p^* = 0.231$. Then $G(p^*) = 1.727 > 1$,
$I(p^*) = 1.004$, $J^* = I(p^*)/[1+G(p^*)] = 0.368$, $V^* = 0.174$.

When the profit rate each period is p*, a young agent managing $h_0 = 1$ gets first-period consumption $c_0 = 0.038$.
If successful in her first period then, in her second period, this agent would manage $h_1 = 1.727$ while consuming $c_1 = 0.275$, and then, if successful again, the agent would consume $c_2 = 1.021$ in retirement.

The extreme cyclical equilibrium has $q_0 = 0.302$, $I(q_0) = 0.853$ (15% below $I(p^*)$), $q_1 = 0.141$, $I(q_1) = 1.198$ (19% above $I(p^*)$).
Optimal contracts for agents hired when $p_t = q_0$ in the extreme cycle have, with $h_0 = 1$:
$c_0 = 0.052$, $h_1 = 1.404$, $c_1 = 0.314$, $c_2 = 0.768$.

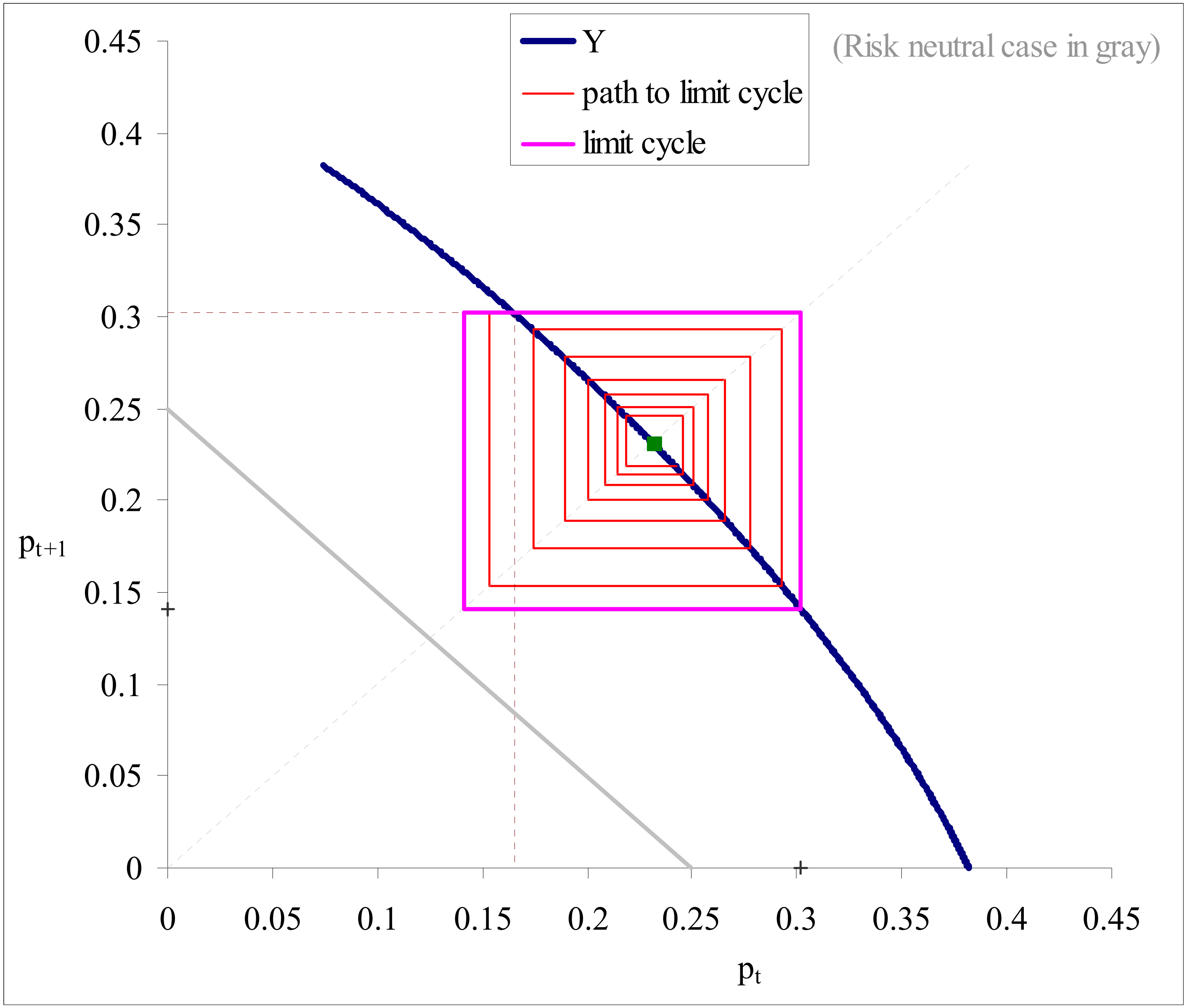
Profit rates in a dynamic equilibrium that reaches the maximal cycle at time 20. (With: θ=0.5, δ=0.5, γ = 0.25, I(p) = 1.5max{1−p/0.7, 0}. V_0 3% from V*.)

The widest cycle and steady state with risk neutral agents are shown in gray.



Divergence of profit rates $p_t = Y(p_{t+1})$ in a cob-web diagram.

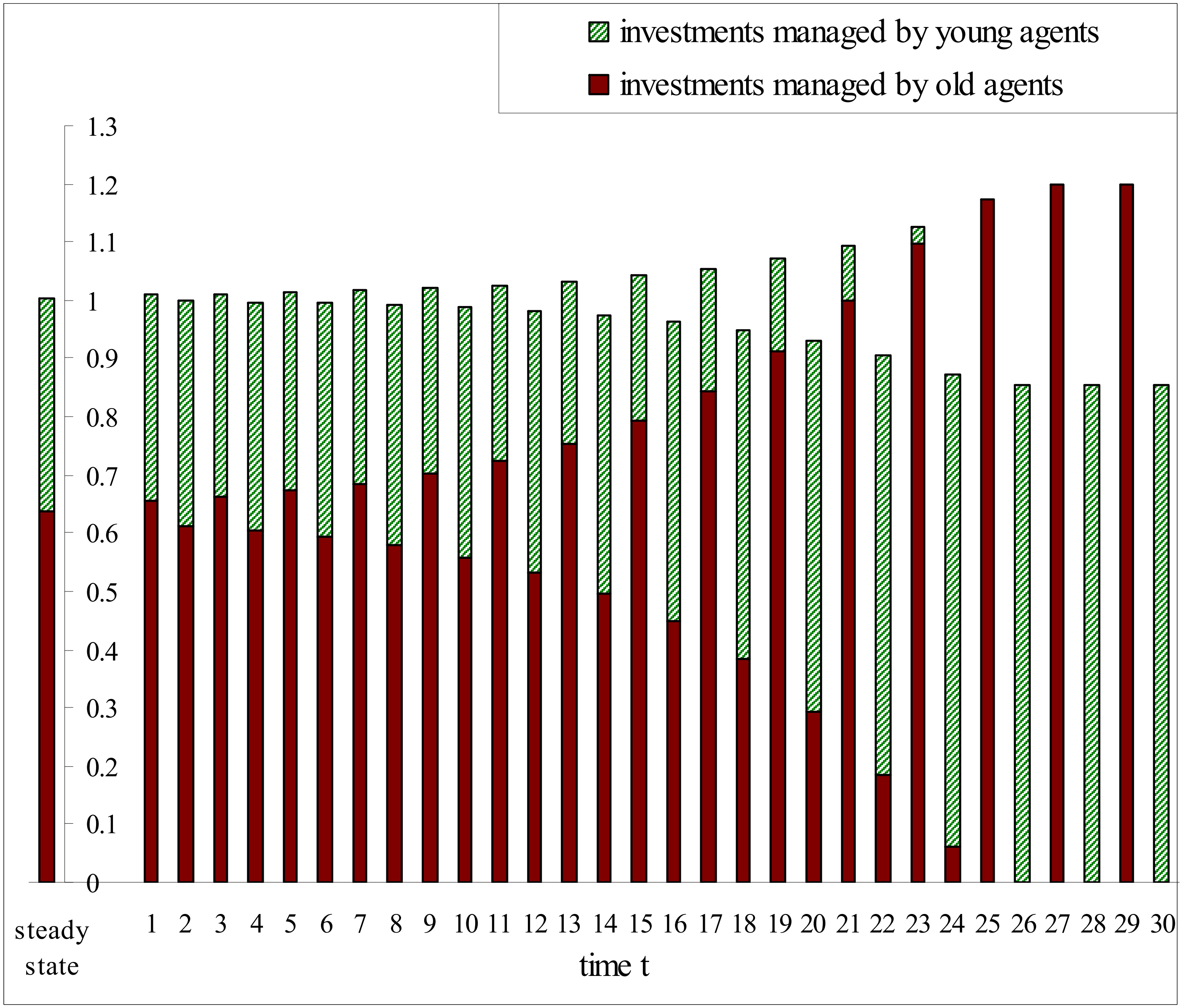
(Parameters: $\theta=0.5$, $\delta=0.5$, $\gamma = 0.25$, $I(p) = 1.5\max\{1-p/0.7, 0\}$.)



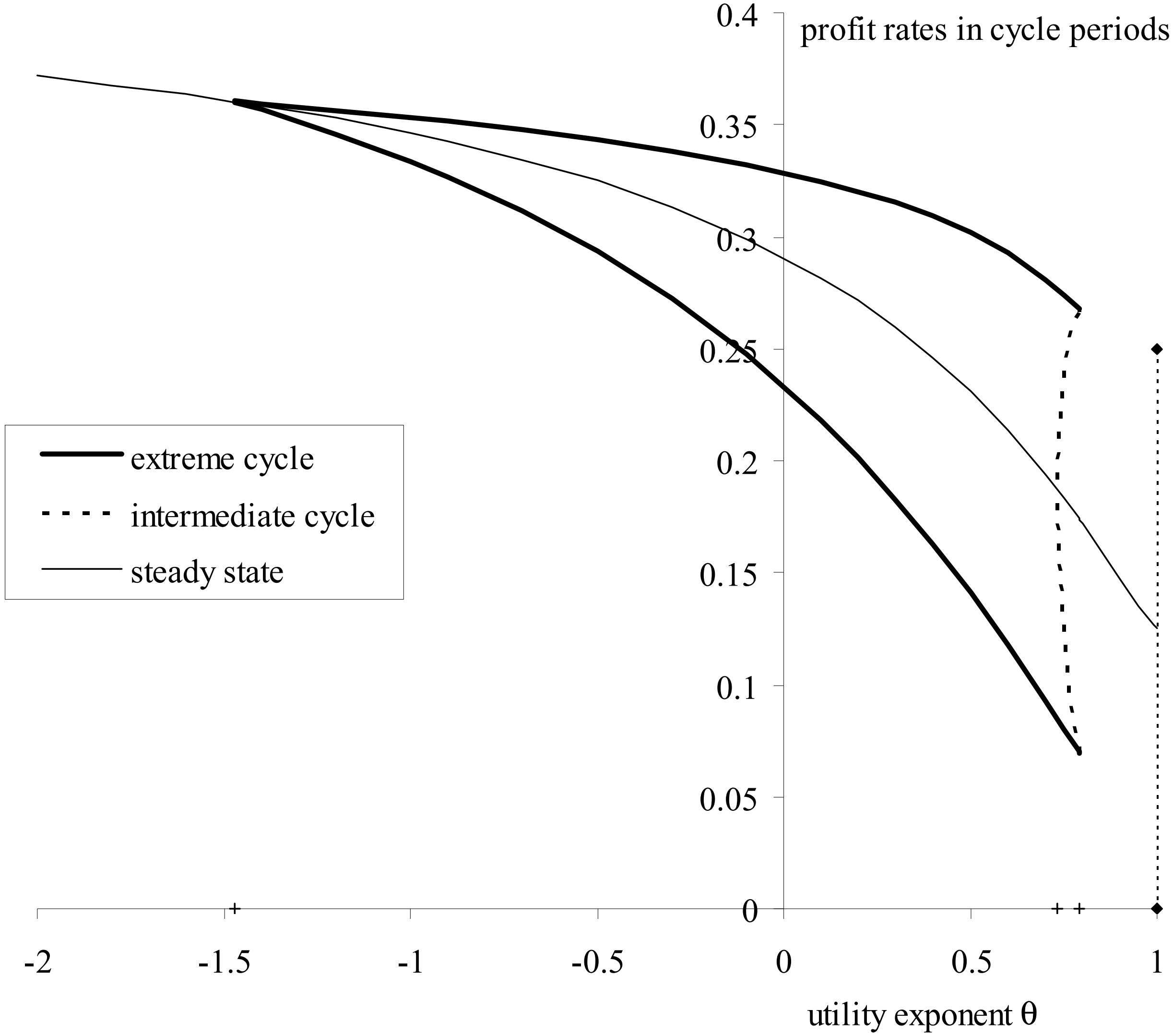
Dynamics of $p_t = Y(p_{t+1})$ around p* depend critically on $Y'(p_{t+1}) = -\delta G(p_{t+1})$.

(In the risk neutral case, $G(p_{t+1}) = 1/\delta$.)

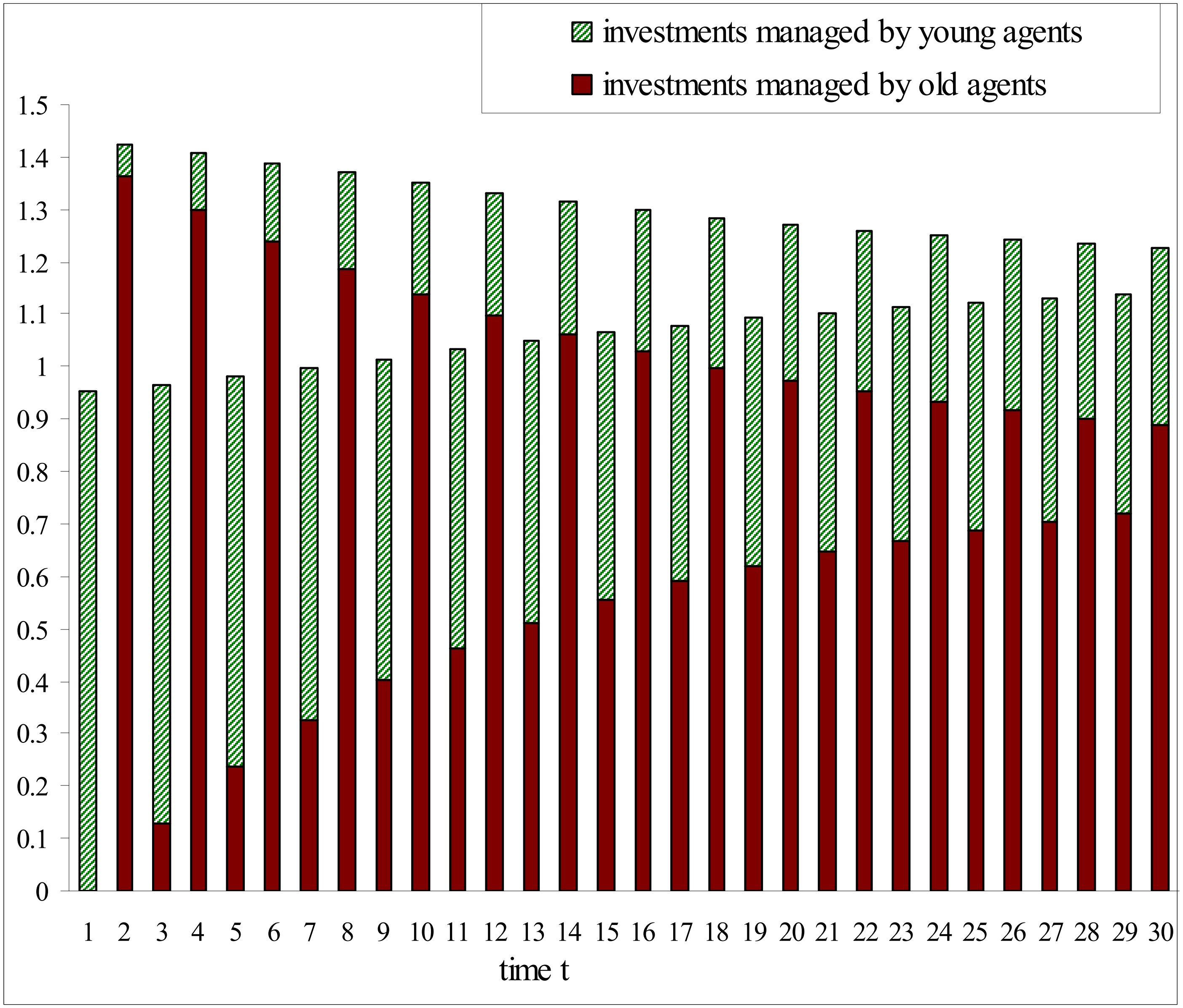
Development of generational inequalities between investments managed by young and old agents (J_t, $I(p_t)-J_t$), with θ=0.5, δ=0.5, γ=0.25, $I(p)=1.5\max\{1-p/0.7,0\}$.



Bifurcation diagram: profit rates in cycle periods, in the extreme cycle, in the steady state, and in an unstable intermediate cycle (for θ near 0.75), for varying utility exponents θ, with δ=0.5, γ=0.25, I(p)=1.5max{1−p/0.7,0}.



Investments managed by young and old agents $(J_t, I(p_t)-J_t)$ in a dynamic equilibrium with $V_0 = 0$, $\boldsymbol{\theta=0.9}$, $\delta=0.5$, $\gamma=0.25$, $I(p) = 1.5\max\{1-p/0.7, 0\}$.



Efficiency

Let $\mathbf{W(K)} = \int_0^K I^{-1}(k)dk$, which can be interpreted as a measure of total welfare for investors and other consumer or suppliers when total investment is K.

For $c = (c_0,c_1,c_2)$, let $\mathbf{f_1(c)}$ and $\mathbf{f_0(c)}$ be largest feasible h_1 and h_0, satisfying

$c_1^{\theta} + \delta c_2^{\theta} = (1+\delta)[(c_1+\gamma f_1(c))/(1+\delta)]^{\theta}$, [*binding incentives at t+1*]

$c_0^{\theta} + \delta c_1^{\theta} + \delta^2 c_2^{\theta} = (1+\delta+\delta^2)[(c_0+\gamma f_0(c))/(1+\delta+\delta^2)]^{\theta}$ [*binding incentives at t*].

(These functions $f_1(c)$ and $f_0(c)$ are concave and homogeneous of degree 1 in c.)

[5] Choose $c(t) = (c_0(t), c_1(t), c_2(t)) \geq 0$ for all t to
maximize $\sum_{t\geq 1} \delta^{t-1}[W(f_0(c(t))+f_1(c(t-1))) - c_0(t) - c_1(t-1) - c_2(t-2)]$
subject to $c_1(0)^{\theta} + \delta c_2(0)^{\theta} = (1+\delta)V_0^{\theta}$.

***Fact*. Given V_0, an equilibrium $\{p_t,J_t\}$ corresponds to an optimal solution of [5]** with $I(p_t) = f_0(c(t)) + f_1(c(t-1))$, $J_t = f_0(c(t))$, and $G(p_{t+1})J_t = f_1(c(t))$ for all $t\geq 1$.

Proof. The objective in [5] function is concave in the c(t) vectors.

With $W'(f_0(c(t))+f_1(c(t-1))) = I^{-1}(f_0(c(t))+f_1(c(t-1))) = p_t$, a plan is optimal if it maximizes $\sum_{t\geq 1} \delta^{t-1}[p_t\, f_0(c(t)) + p_t\, f_1(c(t-1)) - c_0(t) - c_1(t-1) - c_2(t-2)]$.

But this objective decomposes into the problems solved in equilibrium by investors choosing the incentive plans c(t) for new agents in each period t.

Stabilization policies

Consider our example (θ=0.5, δ=0.5, γ = 0.25, $I(p) = 1.5\max\{1-p/0.7, 0\}$) with the worst initial condition V_0=0, which would lead straight to the extreme cycle ($q_0 = 0.301$, $I(q_0) = 0.853$, $q_1 = 0.141$, $I(q_1) = 1.197$).

To get to steady state ($p^*=0.231$, $I(p^*)=1.004$, $J^*= 0.368$) by period 3, we could have in period 1 "Keynesian" investment K_1 that is managed by young agents on one-period contracts, so that they do not add to a boom in period 2.

Such inefficiently managed investment requires a subsidy that could be financed by a tax τ on investment in periods 1 and 2.

With this stabilization, investors' net expected profit rate after taxes would be p^* in every period, and so K_1 and τ must satisfy the equations:

$(Y(0)-p^*)K_1 = \tau[I(p^*+\tau) + \delta I(p^*+\tau)]$ (*budget balance for Keynesian subsidy*)

$I(p^*+\tau) = J^* + G(p^*)[I(p^*+\tau) - K_1]$ (*young agents manage J* in period 2*).

Solution for our example: $K_1 = 0.576$, $\tau = 0.067$, and so total investment in periods 1 and 2 is $I_t = I(p^*+\tau) = 0.861 > I(q_0)$, with $J_1 = I(p^*+\tau) - K_1 = 0.285$, $G(p^*)J_1 = 0.492$, $J_2 = I(p^*+\tau) - G(p^*)J_1 = J^*$.

NonKeynesian stabilization by balanced taxes and subsidies in 2 periods requires $\tau_1 I(p^*+\tau_1)+\delta\tau_2 I(p^*+\tau_2) = 0$, $I(p^*+\tau_2) = J^*+G(p^*)I(p+\tau_1)$, so τ_1=0.20, τ_2= –0.17.

Instability near steady state when agents have n-period careers

At steady state, optimal contracts for agents with n-period careers solve:

choose $(h_0,\ldots,h_{n-1},c_0,\ldots,c_n) \geq 0$ to maximize $\sum_{s\in\{0,\ldots,n-1\}} \delta^s h_s$ subject to

$$\left(\sum_{r\in\{s,\ldots,n\}} \delta^{r-s}\right)^{1-\theta} (c_s+\gamma h_s)^{\theta}/\theta \leq \sum_{r\in\{s,\ldots,n\}} \delta^{r-s} (c_r)^{\theta}/\theta, \ \forall s\in\{0,\ldots,n-1\},$$

and $\sum_{s\in\{0,\ldots,n\}} \delta^s c_s = 1$.

Then the steady state profit rate is $p^* = 1/(\sum_{s\in\{0,\ldots,n-1\}} \delta^s h_s)$.

Small deviations of new-agent investments ΔJ_t that are compatible with continuing steady state would be a bounded solution to: $\sum_{s\in\{0,\ldots,n-1\}} h_s \Delta J_{t-s} = 0, \ \forall t>n$.

Small shocks can force the economy to leave steady state if any (complex) root x has magnitude > 1 for the equation: $\sum_{s\in\{0,\ldots,n-1\}} h_s x^{n-1-s} = 0$ [9].

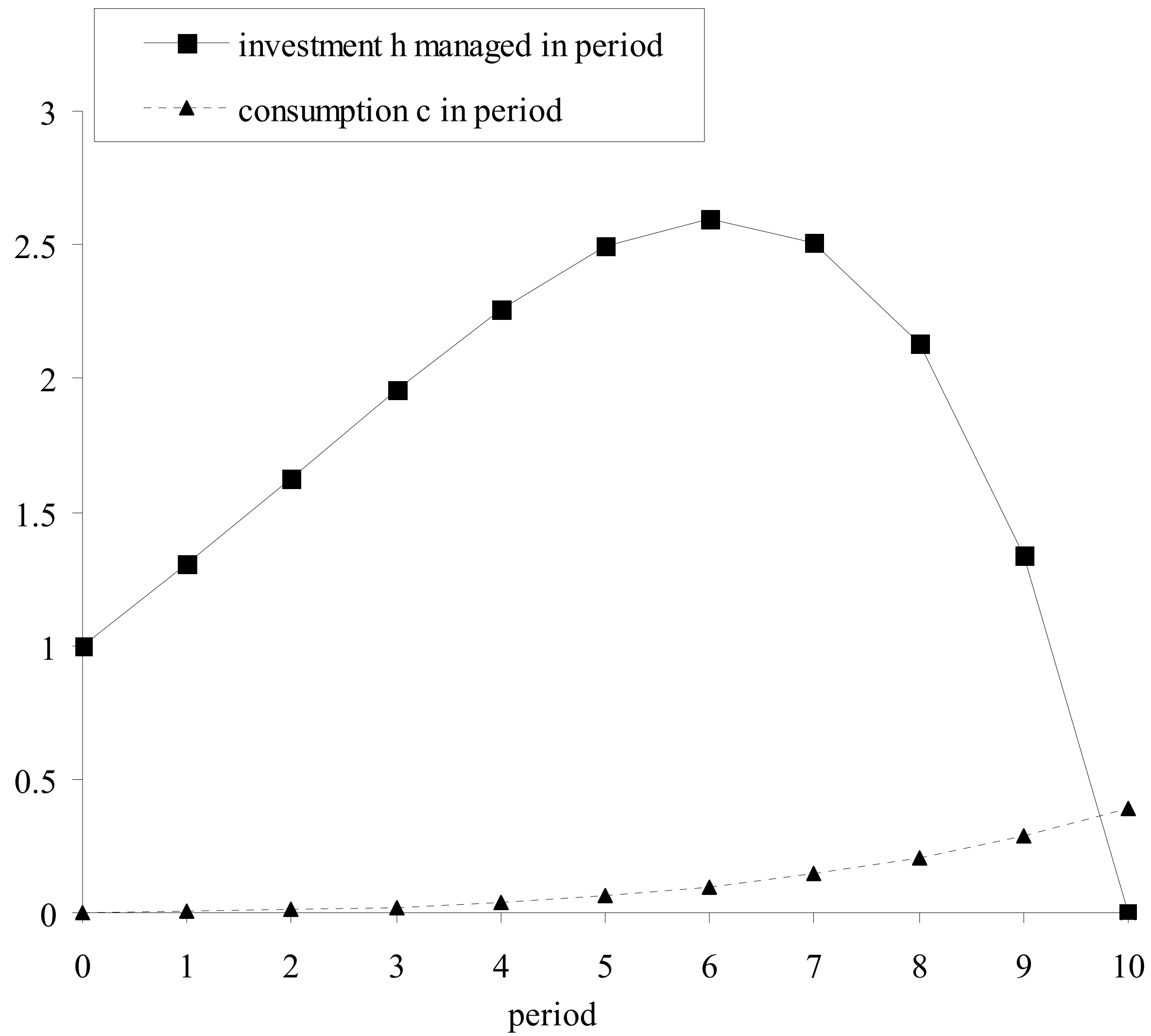
Small deviations of profit rates Δp_t that stay near steady state would yield a bounded solution to: $\sum_{s\in\{0,\ldots,n-1\}} h_s \delta^s \Delta p_{t+s} = 0, \ \forall t>0$.

Small shocks must drive the economy substantially away from steady state if all roots y have magnitude > 1 for the equation: $\sum_{s\in\{0,\ldots,n-1\}} h_s \delta^s y^s = 0$ [11].

With $\theta = 0.5$, $\delta = 0.25^{1/10} = 0.871$, $\gamma = 0.25$ and $n = 10$, we get: $p^* = 0.0409$, $(h_0,h_1,\ldots h_9)/h_0 = (1, 1.302, 1.628, 1.957, 2.259, 2.492, 2.6, 2.509, 2.127, 1.338)$.

Then the largest roots x of [9] are $0.264\pm1.015i$, which have magnitude 1.042, and the smallest roots y of [11] are $0.770\pm0.712i$, which have magnitude 1.049.

Steady-state investments and consumption for agents with n=10 period careers, with $\theta = 0.5$, $\delta = 0.25^{1/10} = 0.871$, $\gamma = 0.25$, and $p^* = 0.0409$.



Benchmark model: Agents are **risk neutral**, have 2-period careers, but the agents' discount factor $\boldsymbol{\delta_1}$ is different from outside investors' discount factor $\boldsymbol{\delta_0}$.

With limited liability, risk-neutral agents' rewards will be fully back-loaded after two successes, and so the expected growth of agents' responsibilities is $G = 1/\delta_1$.

This corner solution to the agency problem does not depend on the investors' expected profit rates p_t in a neighborhood of the steady state p^*.

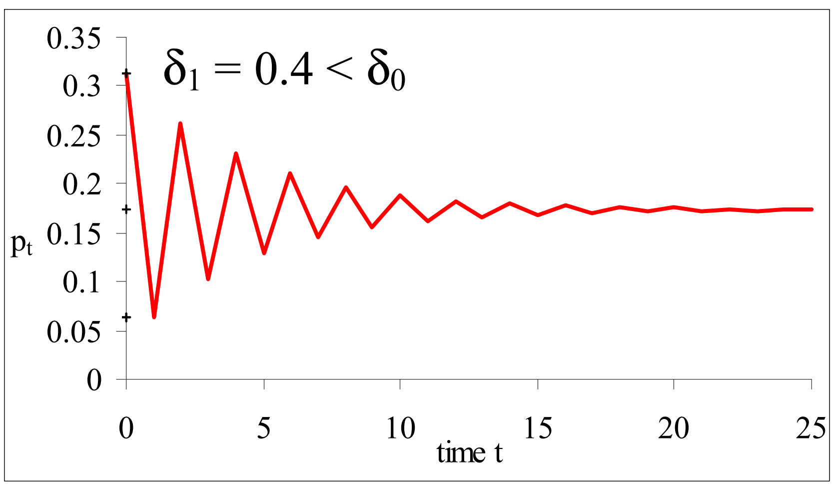
In equilibrium, the 0-profit condition for investors to hire new agents at period t yields $p_t + \delta_0 G p_{t+1} = p^* + \delta_0 G p^*$, and so $p_{t+1} - p^* = -(\delta_1/\delta_0)(p_t - p^*)$.

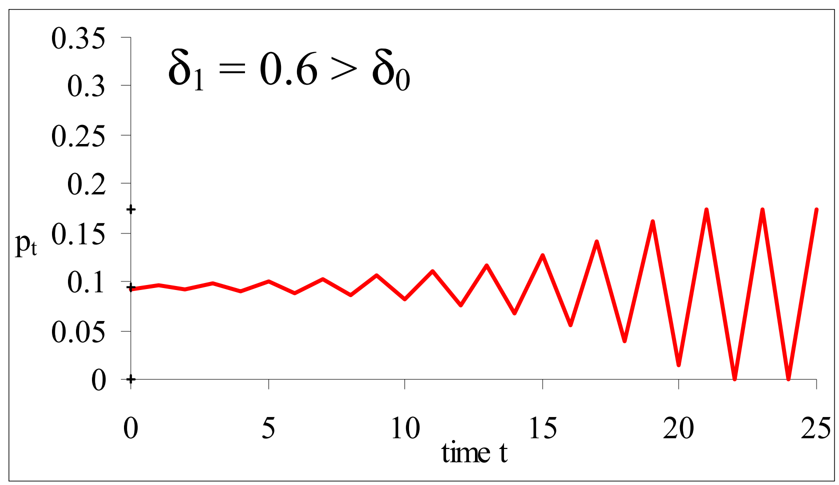
Thus, we get constant cycling of returns p_t around the steady state p^* if $\delta_1 = \delta_0$.

Deviations of p_t from p^* tend to shrink over time if $\delta_1 < \delta_0$.

If $\delta_1 > \delta_0$, deviations from p^* grow as long as new agents are hired each period.

With δ_0=0.5, $\gamma = 0.25$, $I(p) = 1.5\max\{1-p/0.7, 0\}$:





Conclusions

Employers commit to long-term incentives for agents in responsible positions.
New hiring must take account of expected future returns.
The dynamic state of the economy includes commitments to mid-career agents.

In the recessions of our model, productive investment is reduced by a scarcity of trusted mid-career agents for financial intermediation (*bankers*).
Competitive recruitment of new agents cannot fully remedy such undersupply, because agents can be efficiently hired only with long-term contracts in which their responsibilities are expected to grow during their careers.
So a large adjustment to reach steady-state financial capacity in one period would create oversupply in future periods.

For new agents to be hired, low profit rates must be followed be higher expected rates in a later period, which must be followed in turn by lower expected rates.
Early payments to risk-averse agents can reduce their growth of responsibilities, making their employers' expected profits less sensitive to future profit rates, so that future deviations of profit rates from steady state must exceed current deviations as long as new young agents are being hired.
Thus, in the absence of any fiscal policies for macroeconomic stabilization, the economy can develop large cyclical inequalities across generations.