A Short Course on Political Economics, taught by Roger Myerson at Peking University, July 2005

OVERVIEW OF TOPICS

Day 1. Multiple equilibria and the foundations of institutions:
An impossibility theorem of social choice. (Muller-Satterthwaite thm. Condorcet cycle.)
Culture, justice, and Schelling's focal-point effect. (War-of-attrition model)
A model of a prince's moral hazard problems and constitutional constraints.

Day 2. Binary voting:
Inhibiting potential challengers: the selectorate model.
The probabilistic voting model and utilitarianism. The bipartisan set.
Turnout with costly voting.
[Ledyard's model of costly voting and utilitarianism: omitted]
The Condorcet jury theorem and the swing voter's curse.

Day 3. Multicandidate elections:
Diversity of candidates in symmetric equilibria of election games.
Barriers to entry and nonsymmetric equilibria of election games.
The M+1 law of single nontransferable vote.
[Citizen-candidate model]

Day 4. Voting in legislatures:
Sophisticated solutions of binary agendas.
[Proposal power: Baron-Ferejohn, Diermeier-Feddersen: omitted]
Groseclose-Snyder lobbying model and Diermeier-Myerson legislative organization model.
Austen-Smith and Banks model of elections and post-election coalitional bargaining.

Day 5. Accountability and separation of powers:
Ferejohn's model of retrospective voting and accountability.
Tiebout competition, asset mobility, and the curse of natural resources.
Federalism and incentives for success of democracy.

These notes are available online at
`http://home.uchicago.edu/~rmyerson/research/pekingu.pdf`
Computational models for use with these notes can be found at
`http://home.uchicago.edu/~rmyerson/research/pekingu.xls`
For a longer reading list see
`http://home.uchicago.edu/~rmyerson/econ361.htm`
Surveys papers:
1. "Fundamentals of social choice theory" *Northwestern U. working paper* (1996),
`http://home.uchicago.edu/~rmyerson/research/schch1.pdf`
2. "Analysis of democratic institutions" *J of Economic Perspectives* 9(1):77-89 (1995),
`http://home.uchicago.edu/~rmyerson/research/perspec.pdf`
3. "Economic analysis of political institutions", *Advances in Economic Theory and Econometrics* 1:46-65 (1997), `http://home.uchicago.edu/~rmyerson/research/japan95.pdf`
4. "Theoretical comparison of electoral systems,"*European Economic Review* 43:671-697 (1999)
`http://home.uchicago.edu/~rmyerson/research/schump.pdf`

Notes from first lecture at Peking University, 18 July 2005.

An impossibility theorem of social choice
Can a political institution abolish multiple equilibria?
A variant of Arrow's impossibility theorem says No.

Let N denote a given set of individual voters.
Let Y denote a given set of social-choice options, of which the voters must select one.
We assume that N and Y are both nonempty finite sets.
Let L(Y) denote the set of strict transitive orderings of the alternatives in Y.
Let $L(Y)^N$ denote the set of profiles of preference orderings, one for each voter.
We may denote such a preference profile by a profile of utility functions $u = (u_i)_{i\in N}$, where each u_i is in L(Y). So if the voters' preference profile is u, then the inequality $u_i(x) > u_i(y)$ means that voter i prefers alternative x over alternative y.
($u_i(x) = \#\{y\in T|$ x is preferred to y under i's preference in u$\}$.)

A <u>social choice function</u> is any function $F:L(Y)^N \rightarrow Y$, where F(u) denotes the alternative in Y to be chosen if the voters' preferences were as in u.
Let $F(L(Y)^N) = \{F(u)|\ u\in L(Y)^N\}$.
Given any game form $H:\times_{i\in N} S_i\rightarrow Y$ (where each S_i is a nonempty strategy set for i),
let E(H,u) be the pure Nash equilibrium outcomes of H with preferences u. That is,

$$E(H,u) = \{H(s)|\ s \in \times_{i\in N} S_i,\ \text{and, } \forall i\in N, \forall r_i\in S_i, u_i(H(s)) \geq u_i(H(s_{-i},r_i))\}.$$

<u>Theorem (Muller-Satterthwaite)</u> Suppose that a social choice function $F:L(Y)^N\rightarrow Y$ and a game form $H:\times_{i\in N} S_i\rightarrow Y$ satisfy

$$\#F(L(Y)^N) > 2 \ \text{ and }\ E(H,u) = \{F(u)\}\ \forall u\in L(Y)^N.$$

Then there is some h in N such that $u_h(F(u)) = \max_{x\in F(L(Y)^N)} u_h(x),\ \forall u\in L(Y)^N$.

That is, if an institution H admits more than two possible outcomes and always yields a unique pure-strategy Nash equilibrium, then H must be a dictatorship.
Different democratic institutions may have very different sets of equilibria, but we cannot expect any to abolish multiplicity or randomization of Nash equilibria,
and so democratic outcomes may depend on more than just the voters' preferences.
<u>Lemma (monotonicity)</u> Suppose $E(H,u) = \{F(u)\}\ \forall u\in L(Y)^N$. Then for any u and v,
if $\{(i,y)\in N\times Y|\ v_i(y) > v_i(F(u))\} \subseteq \{(i,y)\in N\times Y|\ u_i(y) > u_i(F(u))\}$, then $F(v) = F(u)$.

<u>Example: the Condorcet cycle.</u> Social options are Y = {a,b,c}, voters are N = {1,2,3}.
$u_1(a)=2 >u_1(b)=1 >u_1(c)=0$; $u_2(b)=2 >u_2(c)=1 >u_2(a)=0$; $u_3(c)=2 >u_3(a)=1 >u_3(b)=0$.
If H is symmetric with respect to social options (<u>neutrality</u>) and voters (<u>anonymity</u>) then its pure-strategy equilibrium outcomes are either E(H,u)=Y (multiple equilibria) or E(H,u)=Ø (only randomized equilibria).

Kaushik Basu's Analytical Development Economics begins with a question, illustrating the economic importance of culture:

Why should a passenger pay a taxi driver? (out of car, alone, short visit)
Why should a pedestrian not pay a taxi driver who happens to be standing nearby?

Basu suggests that cultural training makes people want to pay what they owe.
In economic analysis, it is better to think of culture, not as a shared influence on people's preferences, but as shared traditions for identifying focal equilibria.
(Any institution would perform well if all could be taught to max total social welfare.
Endogenous preferences could trivialize problems of poverty, teaching Poor to like it.)
Part of culture is justice: people's shared understanding about what is due to each.

Payoffs to players 1 (driver) and 2 (passenger) respectively in a Rival-claimants game:

	Player 2 claims	Player 2 leaves
Player 1 claims	$-c, -c$	V, 0
Player 1 leaves	0, V	0, 0

Here V is value of some prize, c is cost of conflict. (V=20, c=1)

Equilibria:

1. (1 claims, 2 leaves), which yields payoffs (V,0).
2. (1 leaves, 2 claims), which yields payoffs (0,V).
3. Each independently randomizes, claiming with probability $V/(V+c)$, which yields payoffs (0,0), because $0 = [-c]V/(V+c) + [V]c/(V+c)$.

Justice may determine the focal equilibrium that they all expect and so rationally play:
the (V,0) equilibrium if V is justly owed to player 1 (he drove the passenger)
the (0,V) equilibrium if player 2 has property rights (a pedestrian here).

Consider now a dynamic open-ended version of this game, in which play continues if both claim, and c is cost of conflict for one round (short, say 1 minute).
For each equilibrium of the one-stage game, there is an equilibrium of the dynamic game where this behavior is expected at every round.
Repeated randomized equilibrium is War of Attrition, with expected payoffs (0,0). The length of conflict is random but expected cost exhausts the value of the prize.
With V=20 and c=1, each player's probability of leaving at next minute is 0.048,
expected duration of conflict is 9.76 minutes, prob'y of both leaving is only 0.024.

Continuous-time version: c is cost per unit time when both claim.
Symmetric eqm: each claims for a random time that is exponential with mean V/c.
P(leave after time t) = $e^{-tc/V}$. E[leave now] = 0 = $V - c(V/c)$ = E[claim forever].

So multiple equilibria in a rival-claimants game is not a technical difficulty,
but is exactly what we need to get the right answer for driver-&-passenger/pedestrian.
Justice determines the focal equilibrium; "give each his due" to avoid conflict.

A fable about the foundations of institutions.
Consider an island where people are matched to play rival-claimants games each day.

	Player 2 claims	Player 2 leaves
Player 1 claims	$-c, -c$	V, 0
Player 1 leaves	0, V	0, 0

Many kinds of equilibria:
1. Always play symmetric randomized eqm, E$ = 0 (anarchic state of nature).
2. In some matches, symmetry may be broken by exogenous shared understanding of who should claim here (traditional ownership rights).
3. Where claiming rights are unclear, players may ask a mutual friend to arbitrate.
For fairness, perhaps he may randomize, toss a coin. But loser might ask to toss again.
4. For unquestionably final random arbitration, with no higher appeal, players may consult a divine oracle. (General use of the Divine for focal coordination in societies.)
5. In an assembly, players may establish rules that define claiming rights in more situations (legislation). Shared understanding of these rules gives them force.
6. A generally recognized leader may allocate claiming rights in cases not covered by tradition and legislation. Shared understanding of his authority gives his rulings force.
Leaders may be identified by any election procedure, and the scope and duration of their authority may have any limits recognized by the players (constitutionalism).
Could lose leadership if generally observed to have claimed "too much" for himself.
Such leaders and assemblies may be seen as divinely legitimated.

Lessons: An institution is a game that is sustained as an eqm in a bigger game.
As Hardin (1989) has observed, the danger of anarchy makes constitution-selection a coordination game with multiple equilibria. So political institutions may be arbitrary, depending on people's shared understanding, not on individual characteristics, and so economic analysis may treat institutional structure as an exogenous policy variable.
Constitution: an equilibrium selection to solve all equilibrium-selection problems.
If payoff is reproductive fitness then, by Malthusian dynamics, cultures with systems of rights and authority for effective coordination should have spread over the world.
In common games, expect culture and authority to select eqm (not math'l refinement).
Players' shared understanding of claiming rights may be called their system of justice, although this "justice" could be unfair (bullies, hierarchy).
In international games, if each side sees its focal eqm as universal, get inconsistency?

Model 2: Agency problems between the prince and his governors.
The prince needs some number of governors to control his principality.
Any governor always has three options: to be a good governor, or to be corrupt, or to openly rebel against the prince.
Let G denote the expected payoff to a governor when he rebels.
A governor's substantial local authority may make G quite large.
The prince cannot directly observe whether a governor is good or corrupt, but he can observe any costly governmental crises that may occur under a governor's rule.
When the governor is good, crises will occur in his province at a Poisson rate α.
When the governor is corrupt, crises will occur at a Poisson rate β, where $\beta > \alpha$.
An corrupt governor also gains an additional secret income worth γ per unit time.
The governor observes any crisis in his province shortly before the prince does.
We assume that the prince's costs from governmental crises are very high, and so the prince wants to induce his governors to be good (as well as loyal).

Even a good governor could incur an unusually large number of crises, in which case the prince might replace him and offer the job to someone else.
The position of governor may be quite valuable, but candidates have only some limited wealth K, and so they cannot pay more than K for the job. Suppose $K < G$.
Again, we assume that each individual is risk neutral and has discount rate δ.

The prince's policy towards his governors can be characterized by the expected income y that a governor is paid (not counting any income from corruption) and the probability q that a governor is dismissed when a crisis occurs in his province.
Then the expected discounted value of payoffs for a good governor is $U = y/(\delta+\alpha q)$.
The expected discounted value of payoffs for a corrupt governor is $(y+\gamma)/(\delta+\beta q)$.
Deterring corruption requires that $y/(\delta+\alpha q) \geq (y+\gamma)/(\delta+\beta q)$.
When a crisis occurs, the governor's expected value will decrease to $(1-q)U$, until he learns whether he is about to be dismissed. Danger of rebellion is worst here.
Deterring rebellion requires $(1-q)U \geq G$.
As $0 \leq q \leq 1$, this implies $U \geq G$. So candidates are willing to pay K [<G] for a governorship when a vacancy occurs, which happens at Poisson rate αq.
Then the prince's expected net cost rate from the governorship is $D = y - \alpha q K$.
The prince wants to choose y and q to minimize D subject to these constraints.

With $y = (\delta+q\alpha)U$, this problem is equivalent to choosing (U,q) to minimize $D = \delta U + \alpha q(U-K)$ subject to $U-G \geq qU \geq \gamma/(\beta-\alpha)$.
The unique optimal solution is $U = G + \gamma/(\beta-\alpha)$ and $q = \gamma/[(\beta-\alpha)U]$.
K only reduces D, without affecting q, U, or y.

Full derivation of optimal policy in Model 2:
We have the definition $U = y/(\delta+q\alpha)$, and so $y = (\delta+\alpha q)U$.
The incentive constraint against corruption is $U \geq ((\delta+q\alpha)U + \gamma)/(\delta+\beta q)$,
which can be simplified to the equivalent inequality $qU \geq \gamma/(\beta-\alpha)$.
The incentive constraint against rebellion can be written $U-G \geq qU$.
So the two incentive constraints are satisfied iff $U-G \geq qU \geq \gamma/(\beta-\alpha)$.
This implies $U \geq G + \gamma/(\beta-\alpha)$.

The prince's expected cost rate is $D = y-\alpha qK = (\delta+\alpha q)U-\alpha qK = \delta U + \alpha q(U-K)$.
So with any feasible value of U (satisfying $U \geq G > K$), the prince would prefer the smallest feasible q, which is achieved by letting $q = \gamma/[(\beta-\alpha)U]$.
Then the prince's expected cost rate is
$D = \delta U + \alpha(U-K)\gamma/[(\beta-\alpha)U] = \delta U + \alpha\gamma/(\beta-\alpha) - \alpha K\gamma/[(\beta-\alpha)U]$.
But this formula for D is an increasing function of U, because $\beta > \alpha$.
So the prince's optimal policy should minimize U.
With our incentive constraints, the lowest possible U is $U = G+\gamma/(\beta-\alpha)$.
Thus, the optimal policy has $U = G+\gamma/(\beta-\alpha)$,
$q = \gamma/[(\beta-\alpha)U] = (U-G)/U = \gamma/[(\beta-\alpha)G + \gamma]$,
$y = (\delta+\alpha q)U = \delta G + (\delta+\alpha)\gamma/(\beta-\alpha)$,
$D = y-\alpha qK = \delta G + (\delta+\alpha)\gamma/(\beta-\alpha) - \alpha K\gamma/[(\beta-\alpha)G + \gamma]$.

The prospect of charging K for offices does not affect the optimal policy (y,q), but it reduces the prince's expected net cost rate.
The risk of dismissal q here makes the governor's expected loss from a crisis $\gamma/(\beta-\alpha)$, which exactly deters the governor from increasing the crisis rate $\beta-\alpha$ for the γ profits.

Numerical example: With $\delta = 0.05$, $\alpha = 0.2$, $\beta = 0.4$, $\gamma = 1$, and $G = 5$, we get
$U = 5 + 1/(0.4-0.2) = 10$, $q =(10-5)/10 = 0.5$, $y = 10\times(0.05+0.2\times0.5) = 1.5$.

Extension:
Suppose the governor can pay a penalty $t \leq \tau$ when a crisis occurs, given a bound τ.
The prince's problem is: choose (U,y,q,t) to minimize $D = y-\alpha t-\alpha qK$ subject to
$U = (y-\alpha t)/(\delta+\alpha q) \geq (y+\gamma-\beta t)/(\delta+\beta q)$, $(1-q)U-t \geq G$, $0 \leq q \leq 1$, $t \leq \tau$.
With $y = \delta U+\alpha(qU+t)$, this problem is equivalent to: choose (U,q,t) to
minimize $D = \delta U + \alpha q(U-K)$ subject to $U-G \geq qU+t \geq \gamma/(\beta-\alpha)$, $0 \leq q \leq 1$, $t \leq \tau$.
The optimal solution has $U = G + \gamma/(\beta-\alpha)$, $qU+t = \gamma/(\beta-\alpha)$, $y = \delta U+\alpha\gamma/(\beta-\alpha)$,
$q = [\gamma/(\beta-\alpha) - t]/U$, $t = \min\{\tau, \gamma/(\beta-\alpha)\}$, $D = \delta U + \alpha[\gamma/(\beta-\alpha) - t](1 - K/U)$.
The penalty t reduces q and D, but not U or y.

Discussion of Model 2:
$U = G + \gamma/(\beta-\alpha)$, $D = y-\alpha t-\alpha qK = \delta U + \alpha[\gamma/(\beta-\alpha) - t](1 - K/U)$.

Governors' moral hazard compels the prince to allow them high expected payoffs, as a Shapiro-Stiglitz (1984) efficiency wage, so that dismissal is costly for governors.
But the threat of dismissal must be moderated by randomization, or else it would incite a governor to rebel when he sees a crisis coming in his province.
This randomization can be implemented by a "fair" trial at the prince's court.

The prospect of high payoffs makes the governor's office a valuable asset that a prince should not waste, especially when he needs resources to motivate supporters.
So unless there are big inequalities among people's ability, the prince should not give the office to the most talented person if he cannot pay anything for the office.
The prince's ideal would be to sell the office for $K=U$, so that capital from new governors would cover the entire expected discounted cost of paying them.
But when $G+\gamma/(\beta-\alpha)$ is large, there may not exist candidates who can pay so much.

Candidates for governor must pay in assets that the prince could not simply seize.
Candidates may pay in private wealth (timocracy), or by years of service in lower offices (bureaucracy), or by earnings from past support of his dynasty (aristocracy).
In some societies, people without political connections may be unable to protect large private wealth, but any prince needs a reputation for protecting his supporters' assets.
So the prince's supporters may tend to become a self-perpetuating aristocracy.

There is a fundamental tension between selling office and randomizing dismissal.
When $K>0$, the prince would always prefer to dismiss a governor after a crisis ($q=1$).
As in Model 1, prince needs a council to guarantee his promises to major supporters.
For credibility, governors' fair trials must be monitored by the prince's council.

Example: Septimus Severus began recruiting generals from talented common soldiers, but previous emperors recruited generals exclusively from the senatorial class.
The Senate could help guarantee fair trials only for members of their class.
Rebellions by generals plagued the Roman Empire after Septimus Severus.

Prince could gain by making punishment more productive (fine instead of dismissal), but not more severe. Torturing ex-governors would not reduce prince's cost, because governor's expected payoff after a crisis is bounded below by G (option to rebel).
If a dynastic principle (grant of office binding on heirs of governor and prince) could reduce the effective δ for governors, then it would decrease the prince's costs D.

Perspectives on constitutional government:
Many of our insights about a monarch and his active supporters can be extended to the politicians and high officials in more complex constitutional governments.
A game theorist in political economics often looks at a constitution as the rules of the game the politicians must play to achieve power and govern the nation.
But when we ask how a constitution is sustained, we must begin by recognizing that general acceptance of any constitution must be one of many equilibria in a more fundamental coordination game where disagreement is the worst outcome, civil war.

The rules of a constitution can be enforced only by actions of individual people, and these individuals must have a positive motivation to enforce the constitutional rules.
So a constitution can be effective only when there are specific individuals who expect to be highly rewarded as long as they act to enforce constitutional rules, but who would lose these privileges if they did not fulfill their constitutional responsibilities.
These individuals are the officials of the constitutional government.
So a constitution specifies (1) a set of political offices, (2) powers and responsibilities of these offices, and (3) the procedures for selecting future holders of these offices.
But to understand a constitution as a self-enforcing system, we should extend our definition of a constitution to include also a specification of (4) the privileged individuals who are to hold these offices initially, when the constitution is established.

Under any system, a political leader can win power and hold power only with the help of active supporters who trust that he will reward them after his victory.
The rules which define what a leader must do to maintain his supporters' trust
may be considered as a kind of "personal constitution" for the leader.

The first officials of a new constitution need supporters to win this privilege.
So the fate of a new constitution may depend critically on the pre-existing personal constitutions that bind its first political leaders with their primary supporters.
Provisions of the new constitution may be unenforceable when they ask these leaders to violate their pre-existing personal constitutions with their supporters.
We should not assume that the rules of a new regime are written on a blank slate.

In negotiating a new political system, established leaders cannot begin by abandoning the source of their power: those who supported them in expectation of future rewards.
So local warlords may oppose national unification unless they and their captains are promised privileged positions in the new political system (Louis XIV, Afganistan).

A democratic constitution would be imperiled if its highest national office were won by a politician who could be confident that his active supporters would still trust him after he openly violated the constraints of the constitution.
A new democratic constitution can find candidates for national leadership who already have a reputation for governing democratically only if local democracy started earlier.

Conclusions:
We have applied basic efficiency-wage models of agency theory to probe the essential role of rewards and privileges for the agents who maintain a political leader in power.

Under any system, a political leader can win and hold power against challengers only with the costly efforts of many supporters who must expect rewards from his success.
Powerful officials who cannot be perfectly monitored must also expect great rewards, so that threat of dismissal can deter them from abuse of office or open rebellion.
Costs of rewarding high officials can be recouped by selling offices (for money or service), but then the leader gets a positive incentive to dismiss and resell an office.
So a successful leader always needs to credibly reassure his supporters and agents that rewards will be allocated according to some mutually understood rules.

These rules, which define what a leader must do to maintain his supporters' trust, may be considered as a kind of personal constitution for the leader, even if he is an absolute monarch who is not formally constrained by any other constitution.
In this personal constitution, the leader is constrained to share the benefits of his power with a privileged group of loyal agents and supporters, and new recruiting into this group may be restricted to protect the privileges of its current members.
There must be some council or forum where the leader's compliance with these rules can be monitored by his supporters. To them at least he must give a kind of justice.
We have not asked why he should give justice to others who are not active supporters.
But his personal constitution could also commit the leader to comply with other norms and promises, if his violation of them would be treated like his cheating a supporter.

In democracy, leaders should extend their base of support to include voting masses.
But a core of active supporters, small enough to monitor, is essential for any leader.
So the performance of democracy may depend critically on the personal constitutions that bind its political leaders with their active supporters.

Political leaders can also be constrained by norms of legitimacy.
A challenger with a reputation for rewarding support still needs to attract enough active supporters so that his promises can have a high expected value
(probability of winning an office multiplied by the value of resources it controls).
That is, potential supporters of challengers have a coordination game:
they do not want to support someone whom nobody else is supporting.
Factors that focus attention on one challenger are called legitimacy and charisma
("charisma" if intrinsic to the individual challenger, "legitimacy" if extrinsic).
Any action by an incumbent leader that is considered illegitimate can increase the probability of successful challenges to his power.

So at the foundations of modern constitutional democracy, we may find the same solutions to the problems of leadership that Xenophon described for successful kings.

Notes from second lecture at Peking University, 19 July 2005.

The **selectorate model** has been developed by Bueno de Mesquita, Siverson, Smith, and Morrow in their book Logic of Political Survival, see pages 104-126. (See also their article in the American Political Science Review, 1999.)
The version covered in class can be found on the Selectorate page of the course spreadsheet <pekingu.xls>.

The **probabilistic voting model** is widely used in many theory papers and books. For example, see Persson and Tabellini Political Economics (2000) chapters 3 and 5 for more sophisticated probabilistic voting models that also include campaign contributions. The utilitarian optimum result depends on voters having personal biases for or against party 1 that are drawn from a sufficiently wide interval $[-\delta,\delta]$.
In class, we emphasized the Condorcet cycle (ABC example), as shown in the ProbabilisticVoting page of the course spreadsheet.
In the limiting case of $\delta=0$, a unique equilibrium where both parties choose policies randomly in the "bipartisan set" has been derived by Laffond, Laslier, and Le Breton, Games and Economic Behavior (1993); see also my Social Choice survey paper.

Costly voting

Let's analyze the question of whether large turnout can be rational when voting is costly.
Consider a population that consists of 2,000,000 leftist voters and 1,000,000 rightist voters.
There are two candidates. Candidate 1 is a leftist, and candidate 2 is a rightist.
Each voter gains \$1 when the winner is like him, but voting costs \$0.05.
Let $\tilde{S}_i$ = [number of votes for candidate i], for i = 1,2.
The candidate with the most votes wins. In case of a tie, the winner is selected by a coin toss.
We say that a voter <u>pivots</u> in the election if adding his vote in (instead of abstaining) would change the outcome of the election.
Let piv_i denote the event that one more vote for candidate i would change the outcome.
So piv_1 occurs in two ways:

if $\tilde{S}_1=\tilde{S}_2$ with probability 0.5 (when i would lose the toss),
if $\tilde{S}_1+1=\tilde{S}_2$ with probability 0.5 (when i would win the toss).

<u>A randomized equilibrium.</u> First, we can find an equilibrium in which
each leftist votes with a small probability p, and each rightist votes with small probability q.
Let λ = 2000000p. Then the probability of k votes for the leftist candidate is

$$\left(\frac{2000000!}{k!\,(2000000-k)!}\right) p^{k}(1-p)^{2000000-k} \approx \frac{\lambda^k}{k!}\left(1-\frac{\lambda}{2000000}\right)^{2000000} \approx e^{-\lambda}\frac{\lambda^k}{k!}.$$

So $\tilde{S}_1$ has a probability distribution that is approximately Poisson with mean λ.
Let μ = 1000000q. Then $\tilde{S}_2$ has a prob'y distrib'n that is approximately Poisson with mean μ.
<u>Fact</u> When $(\tilde{S}_1,\tilde{S}_2)$ are independent Poisson random variables with with means (λ,μ),

$$P(piv_1) = 0.5\ P(\tilde{S}_1=\tilde{S}_2) + 0.5\ P(\tilde{S}_1+1=\tilde{S}_2)$$
$$= \sum_{k=0}^{\infty} e^{-\lambda}\frac{\lambda^k}{k!}\ e^{-\mu}\frac{\mu^k}{k!}\left(0.5+0.5\frac{\mu}{k+1}\right) \approx \frac{e^{2\sqrt{\lambda\mu}-\lambda-\mu}}{4\sqrt{\pi\sqrt{\lambda\mu}}}\left(\frac{\sqrt{\lambda}+\sqrt{\mu}}{\sqrt{\lambda}}\right).$$

Similarly, $$P(piv_2) \approx \frac{e^{2\sqrt{\lambda\mu}-\lambda-\mu}}{4\sqrt{\pi\sqrt{\lambda\mu}}}\left(\frac{\sqrt{\lambda}+\sqrt{\mu}}{\sqrt{\mu}}\right).$$

It can be verified that the most likely way that either pivot event can occur is when both parties' vote totals are near the geometric mean of their expected values, so that $k \approx \sqrt{\lambda\mu}$ in the above sum.
These approximations also rely on Stirling's formula: $k! \approx (k/e)^k\sqrt{2\pi k}$.

Notice $P(piv_1)/P(piv_2) = \sqrt{\mu/\lambda} = \sqrt{E(\tilde{S}_2)/E(\tilde{S}_1)}$.
Solving $P(piv_1) = P(piv_2) = 0.05$, we get $\lambda = \mu \approx 32$. So there is an equilibrium in which
each of 2000000 leftists votes with probability $p \approx 32/2000000$,
and each of 1000000 rightists votes with probability $q \approx 32/1000000$.

Are there other equilibria? Yes, Palfrey and Rosenthal Public Choice (1983) found many, some with large turnout. Here is an example of such a large-turnout equilibrium of this game.
Among 2000000 leftists, 1000000 leftists are expected to vote (the in-leftists), and another 1000000 are expected to abstain (the out-leftists).
Each of 1000000 rightists randomizes, abstaining with probability $r \approx 2.3/1000000$.
So $\tilde{X}$ = [number of abstaining rightist voters] is approximately Poisson with mean $\theta=2.3$, and
$P(\tilde{X}=0) = e^{-\theta} = 0.100$,
$P(\tilde{X}=1) = e^{-\theta}\,\theta/1 = 0.230$,
$P(\tilde{X}=2) = e^{-\theta}\,\theta^2/2 = 0.265$,
$P(\tilde{X}=3) = e^{-\theta}\,\theta^3/6 = 0.203,\ldots$
P(in-leftist pivots) = $0.5\,P(\tilde{X}=0)+0.5\,P(\tilde{X}=1) = 0.5(0.100+0.230) > 0.05$.
P(out-leftist pivots) = $0.5\,P(\tilde{X}=0) = 0.5(0.100) \le 0.05$.
P(rightist pivots) = $0.5\,P(\tilde{X}=0) = 0.5(0.100) = 0.05$.
[Actually, P(out-leftist pivots) is slightly smaller than P(rightist pivots)=0.05 because
P(out-leftist pivots) = P(no rightists abstain)
< P(no rightists other than last abstain) = P(rightist pivots).]
In this equilibrium, the different behavior of in-leftists and out-leftists depends on the fact that, when a leftist voter looks at the other voters in his environment, an out-leftist sees in his environment one more leftist who is expected to vote than an in-leftist sees in his environment.
So this perverse equilibrium depends critically on every voter knowing his personal role and knowing exactly how many other voters are expected to vote in each way (or abstain).
Palfrey and Rosenthal (APSR 1985) showed that adding population uncertainty eliminates such perverse equilibria.

The mathematically simplest way of adding population uncertainty is to assume that the numbers of each group in the population are independent Poisson random variables (stdev = mean^0.5).
So suppose #Leftists is Poisson mean=2000000, #Rightists is Poisson mean=1000000, independent, each voter applies same type-dependent randomized strategy independently.
Then number of votes for each candidate are independent Poisson random variables.
We can get all pivot probabilities equal to 0.05 only if these means are 32 (approximately).

Fact Suppose the number of voters is a Poisson random variable with mean n and each voter has an independent probability τ_i of voting for candidate i, for each i in {1,2}. Then the magnitude of the pivot probability for each i is $\lim_{n\to\infty} \mathrm{LN}(P(\mathrm{piv}_i))/n = 2\sqrt{\tau_1\tau_2}-\tau_1-\tau_2 = -(\sqrt{\tau_1}-\sqrt{\tau_2})^2$.

The **Condorcet jury theorem** asserts that, with large numbers of independent voters, majority rule achieves correct decisions with asymptotic efficiency.
Austen-Smith and Banks (APSR, 1996) and Feddersen and Pesendorfer (APSR, 1996) have transformed this 200-year-old literature by assuming rational strategic voting.
(Myerson GEB 1998 shows a general Poisson version of the Condorcet jury theorem.)
We do an example here. (See also the Jury pages of the course spreadsheet, pekingu.xls).

There are two possible states: Bad or Good (= quality of Citizen Capet).
Voters must choose Yes (acquit Capet) or No (condemn Capet).
Voters all get payoff 0 from No, +1 from Yes if Good, -1 from Yes if Bad.
Common-knowledge prior: $P(\text{Good}) = 0.50$.
#Voters is Poisson random variable with mean n.
Each voter independently gets a signal which will be Positive or Negative:
$P(\text{Positive}|\text{Good}) = 0.20$, $P(\text{Positive}|\text{Bad}) = 0.10$.
(Drop old assumption of "$P(\text{Positive}|\text{Good}) > 0.5$, $P(\text{Positive}|\text{Bad}) < 0.5$".)
Notice $P(\text{Good}|\text{Positive}) = 2/3 = 0.667$, $P(\text{Good}|\text{Negative}) = 8/17 = 0.471$.

<u>Sincere</u> scenario: "Positives vote Yes, Negatives vote No, and so No wins with high probability, even in Good state, by expected margin of 80-20 when n = 100.
Not an equilibrium, because pivotal votes would be much more likely in Good state than in Bad state. In fact, this scenario gives
$P(\text{N-pivotal}|\text{Bad}) \approx 1.46 * 10^{-19}$, $P(\text{N-pivotal}|\text{Good}) \approx 6.89 * 10^{-11}$,
and so $P(\text{Good}|\text{N-pivotal}) > .99999999$.
Knowing this, even Negatives would prefer to vote Yes.
...But everybody voting Yes is not an equilibrium either!

An equilibrium exists between these two scenarios: all Positives vote Yes;
Negatives randomize between No with prob'y ≈ 0.588, Yes with prob'y ≈ 0.412,
when n is large. Then expected vote shares are approximately
53% Yes and 47% No in Good state, 47% Yes and 53% No in Bad state.
So the voting outcome is probably correct in each state.
For large n, the Negative voters voting No with probability p = 0.588 equates the two states' pivot magnitudes: $-[(0.9p)^{1/2} - (0.1+0.9(1-p))^{1/2}]^2 = -[(0.2+0.8(1-p))^{1/2} - (0.8p)^{1/2}]^2 = -0.00173$.

For case of $n = 100$, the equilibrium is slightly different from $n \to \infty$ limit:
Negatives vote No with probability 0.594, Yes with probability 0.406.
Then the expected vote totals are
47.5 No and 52.5 Yes if state = Good,
53.5 No and 46.5 Yes if state = Bad.
Pivot probabilities are then
Pr(Y-pivot|Good) = 0.0345, Pr(N-pivot|Good) = 0.0362,
Pr(Y-pivot|Bad) = 0.0325, Pr(N-pivot|Bad) = 0.0303,
So a voter with a Negative signal should assess the expected gains
of adding a Yes vote to be 0.471 * 0.0345 - 0.529 * 0.0325 = -0.001,
of adding a No vote to be -0.471 * 0.0362 + 0.529 * 0.0303 = -0.001 .
Thus, voters with Negative signals are willing to randomize, as the claimed equilibrium requires.
But they would prefer to abstain!

This is the swing voter's curse, discovered by Feddersen and Pesendorfer.
(Given that some vote is pivotal, learning that your vote is pivotal makes it more likely that fewer others voted on your side, which suggests that the quality of your side may have been worse than you thought.)

If abstention is allowed then the equilibrium with $n = 100$ is as follows:
Negative voters vote No with prob'y 0.173, and abstain otherwise, while Positive voters vote Yes.
Then the expected vote totals are
13.9 No and 20 Yes if state = Good,
15.6 No and 10 Yes if state = Bad.
Pivot probabilities are then
Pr(Y-pivot|Good) = 0.036, Pr(N-pivot|Good) = 0.043,
Pr(Y-pivot|Bad) = 0.048, Pr(N-pivot|Bad) = 0.039 .
So a voter with a Negative signal should assess the expected gains
of adding a No vote to be -0.471 * 0.043 + 0.529 * 0.039 = 0,
but of adding a Yes vote to be 0.471 * 0.036 - 0.529 * 0.048 < 0.
A voter with a Positive signal should assess the expected gains
of adding a Yes vote to be 0.667 * 0.036 - 0.333 * 0.048 > 0,
but of adding a No vote to be -0.667 * 0.043 + 0.333 * 0.039 < 0.
So Negative voters randomize between No and Abstain, Positives vote Yes.

For large n, Negative voters voting No with probability $p = 0.172$ equates the two state's pivot magnitudes $-[(0.9p)^{1/2} - 0.1^{1/2}]^2 = -[0.2^{1/2} - (0.8p)^{1/2}]^2 = -0.00589$.

Notes from third lecture at Peking University, 20 July 2005.

Diversity of candidates in symmetric equilibria of election games

Much of the first half of the lecture followed the treatment of "model 1" in the 1996 paper "Economic analysis of political institutions: an introduction."

The discussion of rank-scoring rules, and the public-goods variant of the model was taken from sections 2 and 3 of "Theoretical Comparisons of Electoral Systems" (Myerson, European Economic Review, 1999).

The game where candidates can choose distributions of cash benefits that are promised to individual voters is developed in a paper I published in American Political Science Review 1993. Gary Cox's original work on the threshold of diversity is in the American Journal of Political Science 1987 and 1989.

The equilibrium distributions for different six-candidate rank-scoring rules are computed on the FavoredMinorities page of the course spreadsheet. (But to make it work you must first select the range E16:F116 and use the Data:Table command, selecting the column-input cell E2.)

The Brazilian election data is from a paper published by Ames in Journal of Politics, 1995.

Barriers to entry and nonsymmetric equilibria of election games

The discussion of the "above the fray" and "one bad apple" examples is taken from sections 4 and 5 of the paper "Comparison of Scoring Rules in Poisson Voting Games" (Journal of Economic Theory 2002).

Then the discussion followed the treatment of "model 2" in the 1996 paper "Economic analysis of political institutions: an introduction." This material was also covered in sections 4 and 5 of of "Theoretical Comparisons of Electoral Systems" (European Economic Review, 1999).

This model of an election with candidates who differ on a simple binary policy question and on a corruption dimension was originally developed in a paper by Myerson in Games and Economic Behavior 1993.

A discussion of the **M+1 law** for multi-seat legislative elections under the single nontransferable vote (SNTV) system, as was used in Japanese legislative elections, can be found in section 4 of the paper "Theoretical Comparison of Scoring Rules" . See the papers by Reed in British J. of Political Science, 1991 and by Cox in APSR 1994.)

Short proof of the weak M+1 law for plurality voting with a large Poisson electorate:

Suppose that there are K candidates, numbered 1,2,...,K, in an election with single nontransferable vote where the top M candidates win. In case of a tie for M'th and M+1'th place, a random ordering of the candidates is generated, and the set of M winners is completed by selecting from the borderline-winning candidate in this order.
Here $M < K$.
Each voter has a type which is drawn independently from some finite set according to some fixed probability distribution.
Each type of voter has a strict utility ranking of the candidates, with $u_i(t)$ denoting the utility of candidate i winning for a voter of type t.
A voter's payoff from the election is the sum of the winners for him.
In the n'th voting game, the number of voters is a Poisson random variable with mean n.
A large equilibrium is a convergent sequence of equilibria of these voting games as $n \to \infty$.
By convergent, we mean that the expected fraction of the electorate who vote for each candidate i is converging to some limit τ_i. Choosing a subsequence if necessary, we may assume that other probabilities are also convergent to well-defined limits as $n \to \infty$.

Now consider a large equilibrium. Without loss of generality, we may assume that the candidates are numbered so that $\tau_1 \geq \tau_2 \geq \ldots \geq \tau_K$.
The {i,j} race is close when adding one vote for i or j could make one of them replace the other in the set of winners.
Let $\tilde{x}_i$ denote the number of votes for candidate i (a random variable).

If there is no close race, then adding one more vote in cannot matter to anybody.
In equilibrium, each voter must vote cast the ballot that would maximize his conditional expected utility gain, relative to not voting, given that there is at least one close race.

A race between two candidates is <u>serious</u> iff its conditional probability of being close, given that there is some close race, is strictly positive in the limit as $n \to \infty$.
A candidate is <u>serious</u> iff he is involved in at least one close race.
A voter's conditional expected gain, given that there is a close race, from voting for his favorite serious candidate would be strictly positive in the limit.
So each voter's ballot in equilibrium must give him a strictly positive conditional expected gain, given that there is a close race.
Thus we get:
<u>Lemma 1.</u> In the large equilibrium, nobody votes for candidates who are not serious. That is, if h is not a serious candidate then $\tau_h = 0$.

From any standard paper on Poisson voting games, we get

Fact For any two candidates i and j such that $\tau_i > \tau_j$, the magnitude of the event that $1+\tilde{x}_j \geq \tilde{x}_i$ is $-(\sqrt{\tau_i}-\sqrt{\tau_j})^2$.

A close race between candidates M and M+1 can occur when their vote-totals are within one of each other and all other candidates' vote-totals are near their expected values. Thus:

Lemma 2. The magnitude of a close race involving candidates M and M+1 is $-(\sqrt{\tau_M}-\sqrt{\tau_{M+1}})^2$.

Consider now some candidate $j > M+1$.
If candidates 1,..,M all got strictly more votes than $1+\tilde{x}_j$, then candidate j would not be in a close race. Thus, when candidate j is in a close race, there at least one candidate i in {1,...,M} such that $1+\tilde{x}_j \geq \tilde{x}_i$. The magnitude of this event is $-(\sqrt{\tau_i}-\sqrt{\tau_j})^2 \leq -(\sqrt{\tau_M}-\sqrt{\tau_j})^2$.
So the magnitude of the event that j is in a close race is not more than $-(\sqrt{\tau_M}-\sqrt{\tau_j})^2$.
But if $\tau j < \tau_{M+1}$ then this magnitude is strictly less than the magnitude of a close race between candidates M and M+1. Thus we get

Lemma 3 For any j in {M+2,...,K}, if $\tau_j < \tau_{M+1}$ then candidate j is not serious.

Consider now some candidate i < M.
If candidates M+1,...,K all got strictly less votes than $\tilde{x}_j-1$, then candidate i would not be in a close race, because he would be a guarenteed winner even with one more vote. Thus, when candidate i is in a close race, there at least one candidate j in {M+1,...,K} such that $1+\tilde{x}_j \geq \tilde{x}_i$. The magnitude of this event is $-(\sqrt{\tau_i}-\sqrt{\tau_j})^2 \leq -(\sqrt{\tau_i}-\sqrt{\tau_{M+1}})^2$.
So the magnitude of the event that j is in a close race is not more than $-(\sqrt{\tau_i}-\sqrt{\tau_{M+1}})^2$.
But if $\tau_i > \tau_M$ then this magnitude is strictly less than the magnitude of a close race between candidates M and M+1. Thus we get

Lemma 4 For any i in {1,...,M−1}, if $\tau_i > \tau_M$ then candidate i is not serious.

We obviously cannot have $\tau_i > \tau_M$, because then i would not be serious and so τ_i would be 0, contradicting $\tau_i > \tau_M \geq 0$. Thus:

Theorem For each i in {1,2,...,M}, τ_i must be equal to τ_M. For each j in {M+1,...,K}, τ_j must be either τ_{M+1} or 0.

References: W. Riker, "The two-party system and Duverger's law," APSR 76:753-766 (1982). S. Reed, "Structure and behavior: extending Duverger's law to the Japanese case," British J of Political Science 20:335-356 (1990). G. Cox, "Strategic voting equilibria under single non-transferable vote," APSR 88:608-621 (1994).

Citizen-Candidate Model T. Besley and S. Coate, "An economic model of representative democracy," Quarterly J of Economics 112:85-114 (1997).

N = {citizens}
Y = {policy space}
For each $i \in N$, $u_i: Y \to \mathbb{R}$ is i's utility for policies.
δ = cost of becoming a candidate
Let θ_i be i's ideal point $\theta_i = \text{argmax}_x\, u_i(x)$.
First, each citizen decides independently whether to become a candidate.
Then all citizens learn $K = \{\text{candidates}\} \subseteq N$, and each votes for one candidate.
The candidate with the most votes is the winner (ties resolved by randomization) and the government policy is the ideal point of the winner.
So if j is winner then each citizen i gets payoff $u_i(\theta_j)$ if $i \notin K$, or $u_i(\theta_j) - \delta$ if $i \in K$.
If $K = \emptyset$ then the outcome is some given x_0 in Y, and i gets $u_i(x_0)$.
The game is analyzed by looking at subgame-perfect equilibria in pure (nonrandom) strategies, after eliminating dominated strategies (voting for the least-preferred candidate) in each subgame after K is determined. The existence of such equilibria can be proven.
We consider cost δ to be small, taking limit as $\delta \to 0$.

An equilibrium in which exactly one candidate enters can only be near or at (as $\delta \to 0$) a Condorcet-winning policy position.

Equilibria where exactly two candidates i and j enter (Duvergerian) can exist for any {i,j} such that the number of citizens who prefer i's ideal policy over j's is equal to the number who prefer j's ideal over i's.

Equilibria with three or more tied winners are hard to sustain, for the same reason as in Feddersen AJPS 1992: If a pure-strategy equilibrium generates a tie among k candidates, then no voter can strictly prefer any two of the tied candidates over the k-way randomization, because he could break the tie in favor of whichever candidate he was not expected to vote for (in the eqm).

But we can construct equilibria where three or more candidates enter even though most are expected to lose, because the presence of these spoilers can change the focal equilibrium in the subgame after candidates' entry. Remember, for any pair of candidates, there exists an equilibrium in the plurality-voting election where this pair is considered to be the only serious race, and so everybody votes for the one in this pair whom he prefers.
Consider a simple Hotelling example where Y = [0,100], citizens have ideal points that are distributed uniformly over the interval 0 to 100, and each citizen's policy-payoff is minus the distance of policy from his ideal point.
Pick any x such that $2 < x < 98$. We can construct an equilibrium in which seven candidates enter with ideal points {0,1,2,x,98,99,100}.
On the equilibrium path, the only serious race is 0 versus x, and x wins.
But if any candidate other than x dropped out, then the post-entry subgame equilibrium would switch to one where the only serious race is between two extreme candidates, the least moderate remaining on the side of the unexpected dropout, and the most moderate on the other side. (E.g.: if 0 dropped out, then the serious race would be between 1 and 98, and 98 would win).
An unexpected extra entrant could be ignored (or could lead to an eqm where the 2 or 98 wins, whichever is worse for the unexpected entrant).

Notes from fourth lecture at Peking University, 21 July 2005.

[We actually began the fourth lecture with the M+1 law, deferred from last time.]

Our discussion of **binary agendas and sophisticated solutions** was adapted from sections 1.4 and 1.5 of "Foundations of Social Choice Theory."

The model of lobbying and incentives for legislative organization and party discipline (from Diermeier and Myerson, American Economic Review, 1999) was discussed following the paper "Economic Analysis of Political Institutions: an Introduction" where this is "model 3."

Finally, we discussed **an integrated model of elections and legislative bargaining** (due to Austen-Smith and Banks, American Political Science Review, 1988) following the paper "Economic Analysis of Political Institutions: an Introduction" where this is "model 4."

Lobbying and incentives for legislative organization

(Diermeier & Myerson, AER 1999; Groseclose & Snyder, APSR 1996)
Legislatures develop different internal structures in different countries.
USA: Many independent committees, each with blocking power only.
UK: One committee (cabinet) normally determines passage of bills into law.
Why? "Party discipline in UK prevents independent committees."
But might this remark reverse cause and effect?
Strength of party discipline is not exogenous.

There is good reason to expect that party discipline can be affected by the electoral system (Carey Shugart, Electoral Studies,1995).
Closed-list PR should make stronger parties than single-member districts.
But USA and UK both use plurality voting in single-member districts.
For structural explanations, we must look to legislative structures.

In his history of the development of British party discipline, Cox (Efficient Secret) finds that legislative party discipline developed ahead of electoral party discipline.
Shugart & Carey (Presidents & Assemblies) find correlation between president's legislative powers and weak parties. Independent gatekeeping committees in legislatures tend to correlate with weak party structure.

We haves many theories to explain why committees are powerful in USA:
Committee power as commitment device, to enforce and maintain distributive agreements of a majority coalition (Shepsle 1979, Weingast & Marshall 1988).
Committee power as incentive for information-gathering (Gilligan&Krehbiel).
Committee proposal power as a seniority system that encourages reelection of incumbents (McKelvey & Riezman 1992, based on Baron & Ferejohn 1989).
But none of these models apply more to USA than UK.

Groseclose & Snyder's (APSR 1996) model of vote-buying by lobbyists:
Two lobbyists: agent 1 for new bill, agent 0 for status quo.
V = (agent 0's value for blocking a bill)
W = (agent 1's value for passing a bill)
Perfect information assumed. Agents learn V and W,
then agent 1 can offer bribes to legislators to pass bill,
and then agent 0 can offer bribes to kill bill.
Each legislator acts for the agent who has offered more
(for 1 if positive tie, for 0 if tie at \$0).

If agent 1 makes bribes, they should be $(x(i))_{i\in L}$ that solve:

minimize $\sum_{i\in L} x(i)$ subject to $x(i) \geq 0$, $\forall i \in L$ = {legislators}

and $\sum_{i\in C} x(i) \geq V$ for each coalition C that can block a bill.

The optimal value is $\sum_i x(i) = rV$ for some <u>hurdle factor</u> r.
Legislators get total payoffs rV if $W \geq rV$, get 0 otherwise.

Diermeier & Myerson (AER 1999) note that the hurdle factor depends on legislative organization:
r = 1 when a disciplined majority obeys single leader.
r = 2 with simple independent majority voting (no discipline)
r = 3 when a gatekeeper is added to an undisciplined majority-rule legislature
$r = 1/(1-Q)$ when a supermajority Q is required (without gatekeeper).

In a serial multicameral legislature, the overall hurdle factor is the sum of the hurdle factors in the separate chambers.
Let s = (hurdle factor in the "House"),
let t = (total hurdle factors in other chambers). So $r = s + t$.

Suppose chambers fix internal structures independently, before learning V & W.
Let $D(r) = P(\text{bill can pass})\, E(V|\text{bill can pass}) = P(W > rV)\, E(V|\ W > rV)$
Then E(payments to House) = $s\, D(s + t)$.
If $s + t = r_0$ where r_0 maximizes E(total payments to legislators) = $r\, D(r)$,
then $0 = (s+t)D'(s+t) + D(s+t)$ and so $\partial/\partial s\ \ s\, D(s + t) = -t\, D'(s + t) > 0$.
So bicameral separation creates incentive to raise hurdle factors.

Suppose V and W are independent exponential random variables, mean 1.
Then $s\, D(s + t) = \int_0^\infty \int_{(s+t)v}^\infty v\, e^{-w}\, dw\, e^{-v}\, dv = s/(s + t + 1)^2$.
So $\text{argmax}_{s\geq 1}\ s\, D(s+t) = t + 1$.
Simple unicameral case: $t = 0$ so $s = 1$ is optimal (strong majority leader).
With presidential veto: $t = 1$ so $s = 2$ optimal (no discipline).
Bicameral legislature: competition to have slightly higher hurdles...

An integrated model of elections and legislative bargaining. Austen-Smith&Banks APSR 1988 develop an integrated model of both electoral and legislative politics, to analyze the question: "Does PR truly create a legislature that is a proportional mirror of the people's interests groups."

The space of possible government policies is [0,1]. Benefits of power: $$ worth $G \geq 1$.
There are many voters, who have ideal points θ uniformly distributed in [0,1].
Three parties in {1,2,3} announce policies (x_1,x_2,x_3) in [0,1] simultaneously and independently.
Then voters each vote for a party, and seats are allocated by proportional representation (PR) with $\alpha = 0.05$ minimum.
After election, parties bargain to form a governing majority coalition.
An offer specifies a policy y in [0,1] and a division of spoils (g_1,g_2,g_3) with all $g_i \geq 0$, $g_1+g_2+g_3=G$.
The largest party offers first, if it fails to get a majority then second-largest offers,
and if it fails to get a majority then smallest offers.
If all three fail to get a majority, then the outcome is caretaker government and new elections, which gives very negative payoffs to all parties.
Otherwise, each party i gets payoff $g_i - (y - x_i)^2$ (share of spoils minus policy embarassment).
(Getting no seats is also very negative outcome.)
Voter with ideal θ gets payoff $-(y - \theta)^2$ if government policy is y.

In equilibrium, if the largest party does not have a majority,
then it offers to form coalition with smallest; if rejected then second largest would offer to largest;
if both rejected then smallest would offer to closest; but first offer is accepted in equilibrium.
Government policy is the average of coalition partners' positions.
There is an equilibrium in which parties 1,2,3 respectively choose $x_1 = 0.20$, $x_2 = 0.80$, $x_3 = 0.50$;
and each voter with ideal point θ votes for party 1 if $\theta \in [0,0.475)$, for party 3 if $\theta \in [0.475, 0.525]$, for party 2 if $\theta \in (0.525, 1]$.
So government policy outcome is equally likely to be 0.35 or 0.65 .
(Voting for most-preferred among this (x_1,x_2,x_3) would not be an equilibrium for the voters.)

If voters in $[0.475, 0.475+\varepsilon)$ voted for 1, the policy outcome would be 0.20, because party 2 would fall below the threshold α and would get no seats.
If voters in $[0.475-\varepsilon, 0.475)$ sincerely voted for 3, the policy outcome would be 0.80 for sure, because 2 would surely be the largest party.
If party 1 moved closer to center then, in this off-path subgame, voters could focus on an equilibrium in which everyone votes for party 2 or party 3 (so 3 gets majority, but 1 gets nothing).

The model has other equilibria, but this one suffices to show that extremists could be systematically overrepresented in a legislature that has been elected by proportional representation.
A reform that removes this advantage for being the largest party would decrease voters' incentive to vote for a big party.

Notes from fifth lecture at Peking University, 22 July 2005.

Ferejohn's "Incumbent Performance and Electoral Control" (Public Choice 1986).
This model analyses politics as a kind of principal-agent problem where the principal is the voters, or more precisely one representative voter, and the agent is the leader of government.
In each election, the voter looks back at the incumbent leader's performance over the previous period and re-elects him if this performance was good enough. On the other hand, if his performance has been below some threshold, then the voter dismisses the leader and replaces him by a challenger drawn from some pool of politicians.
This backward-looking approach to voting is called retrospective voting (as opposed to prospective voting that is about whose campaign promises look best for the next term.

The parameters of the model are as follows:
δ denotes the discount factor per period.
W denote the benefits to the leader each period (unlike an economic principal-agent problem, it is natural to assume that such benefits of high political office might be exogenously fixed).
Each period a random variable $\tilde{\theta}$ is drawn independently from a uniform distribution on the interval between 0 and 1. $\tilde{\theta}$ is a measure of the favorability of the political environment this period.

At the beginning of each period, the leader observes $\tilde{\theta}$ and then chooses an effort level A.
Then the leader's payoff for this period is $W-A$.
The voter's payoff this period is $A\tilde{\theta}$.
The voter observes this multiplicative product, but not A or $\tilde{\theta}$ separately.
Then the voter decides whether to re-elect the incumbent leader or dismiss him (in which case his payoff becomes 0 at all future periods). (Here let us assume that dismissed incumbents never return to office. Ferejohn considered more general cases.)

Let us consider voter's policies of the form re-elect iff $A\tilde{\theta} \geq K$, for some cutoff $K>0$.
Given such a policy, the leader's optimal policy must be of the following form:
choose effort $A=K/\tilde{\theta}$ if $\tilde{\theta} \geq \theta_0$, and $A=0$ if $\tilde{\theta}<\theta_0$,
where θ_0 is some number in [0,1].
At the beginning of any period, before learning $\tilde{\theta}$, the leader's expected discounted value is
$$V = W + \int_{\theta_0}^{1} (-K/\theta + \delta V)\, d\theta.$$
So $V = W + K\,\mathrm{LN}(\theta_0) + (1-\theta_0)\delta V$.
The highest level of $\tilde{\theta}$ such that the leader is willing to choose zero effort
$-K/\theta_0 + \delta\theta_0 = 0$, if the solution is less than 1, or else $\theta_0 = 1$.
So $\theta_0 = K/(\delta V)$.
These two equations can be solved for V and θ_0, and then we can compute the voter's expected utility $EU = (1-\theta_0)K$.

For example, when δ=0.9 and W=1, the optimal policy for the voter is to use cutoff K=0.4288, which yields θ_0=0.360, V=1.33, EU=0.275.
With a cutoff K that is too low (always re-electing), the incumbent does little effort.
With a cutoff K that is too high (never re-electing), the incumbent never exerts any effort.
Indeed, expert observers have sometimes worried that citizens in new democracies have excessively high expectations of the benefits of democracy, so that their cutoff for re-election may be too high and their new leaders may be disinclined to work hard for re-election.

With full observability, the voters could do better.
In the δ=0.9 and W=1 example, a better policy for the voter with full information would be to require effort A=2.07 when $\tilde{\theta} \geq \theta_0$=0.63, which yields EU=0.63 and satisfies the incentive constraint that $A \leq \delta V = \delta(W - A(1-\theta_0))/(1-\delta)$.
The incumbent's expected value in this example is then V = 2.06.

Notice this policy asks more of leaders in rarer occasions, but motivates them by allowing them more free benefits of government in other cases.
Without full observability, if there were a separation of powers among different elected officials who have the same $\tilde{\theta}$ and all observe it, then this outcome could be achieved as one of (very) many equilibria of the noncooperative game among the officials when the voter uses the policy: re-elect any official as long as no other official i yields a higher performance $A_i\tilde{\theta}$.

Ferejohn also discusses the case where one elected leader needs the approval of many voters who live in different districts, and the leader can choose different effort levels for different districts. Then the noncooperative game among the different voters has a Bertrand-pricing equilibrium where their thresholds of re-election K_j are all zero, because the leader would exert effort only in the 51% of the districts where the threshold is lowest.

"Federalism and incentives for success of democracy,"
R. Myerson, Quarterly Journal of Political Science, 1:3-23 (2006).
Excerpts from notes at: http://home.uchicago.edu/~rmyerson/research/fednotes.pdf

3. Federal democracy.
N = number of provinces.
In each period, elect national president, then governor in each province,
each serves corruptly or responsibly.
b_1 = president's benefit (each period) when he serves responsibly,
b_1+c_1 = president's benefit from serving corruptly,
b_0 = governor's benefit when he serves responsibly,
b_0+c_0 = governor's benefit from serving corruptly,
0 = politician's payoff out of office.
w_1 = welfare for national voters with president serving responsibly,
0 = welfare for national voters with president serving corruptly,
x_1 = expected transition cost for voters when changing to a new president,
w_0 = welfare for provincial voters with governor serving responsibly,
0 = welfare for provincial voters with governor serving corruptly,
x_0 = expected transition cost for voters when changing to a new governor,
(but no cost when replacing a governor who's been promoted to president).
ρ = discount factor per period.
ε = probability that any new politician is always-responsible virtuous type.
Elections at each level are determined by voters' expected payoffs from this level of government, ignoring any effects from the other level of government.
(Spse national elections are not influenced by local effects in any one province of its governors becoming president; and provincial elections are not influenced by the national benefits of searching for better presidential candidates.)
Transition costs x_i may be due to new leader learning on job, or to thefts by outgoing leader, or to activists' costs of opposing an incumbent.

With N large, P(no province has a virtuous governor) = $(1-\varepsilon)^N \leq e^{-\varepsilon N}$ is small,
so there are likely to be some provinces where politicians have good reputations
(assuming candidates are recruited independently from pop'n in each province).

At either level (national or provincial), we may say that democracy:
succeeds if voters expect leader to serve responsibly always with prob'y 1;
is frustrated if the leader would always get re-elected even after acting corruptly.
Voters' expected benefit from democracy at either level is
their expected value of payoffs at this level in the given eqm *minus*
what their expected value would be if their current leader were guaranteed his office forever:
E[NPV of voters' payoffs at this level in eqm]$-$P(leader is virtuous)*$w_i/(1-\rho)$.

Basic assumptions: $\varepsilon w_0/(1-\rho) < x_0 < w_0/(1-\rho)$, $b_0+c_0 < b_0/(1-\rho)$,
$\varepsilon w_1/(1-\rho) < x_1 < w_1/(1-\rho)$, $b_1+c_1 < b_1/(1-\rho)$, and $b_1 > b_0 + c_0$.

Theorem 1. If either level of government existed in isolation as a unitary democracy,
then there would exist a good equilibrium where unitary democracy succeeds and voters' expected benefit from democracy is strictly positive,
but there would also exist a bad equilibrium where unitary democracy is frustrated and voters' expected benefit from democracy is 0.

Now consider sequential equilibria of the federal democracy.

$\exists$ eqm where provincial democracy succeeds but national democracy is frustrated
(corrupt governors would not be re-elected, so all governors act responsibly; national voters understand that any governor would become corrupt with prob'y $1-\varepsilon$ after election to the presidency, so corrupt presidents are re-elected).

$\exists$ eqm where provincial democracy is frustrated but national democracy succeeds
(a rare governor who serves responsibly can be identified as virtuous,
but his provincial voters still get no expected benefit from democracy,
and his virtue doesn't make him more attractive to national voters, who expect any president to act responsibly for re-election; so governors have no motive to be responsible).

But such mixed equilibria require voters to have inconsistent expectations about functioning of democracy at different levels, and so seem less likely to be focal.
$\exists$ eqm where provincial and national democracy both succeed
(presidents and governors always act responsibly, else they would not be re-elected).
But no eqm has sure frustration at both levels.

Theorem 2. In any sequential equilibrium of the federal game, as long as some province has a governor who has not yet acted corruptly, democracy cannot be frustrated both at the national level and at all provincial levels.
When national democracy is frustrated, in any province where the voters have just (re)elected a governor with no record of past corruption, their expected benefit from democracy must be strictly positive.

Tiebout (JPE 1956) suggested that local governments could be motivated to provide efficient public goods by the desire to increase their tax base by attracting residents who are free to move from one locality to another.
Epple and Zelenitz (JPE 1981) asked the question: "does Tiebout need politics." Conversely: Can local political leaders be deterred from corrupt profit-taking by citizens who can vote with their feet as effectively as by citizens who can vote democratically to replace their leaders?
They find that the answer to this question is No, because local leaders have the ability to tax away rents of fixed local assets like land, and demand for local land is not infinitely elastic to tax cuts or improvements in local public goods because of congestion effects.
Let's consider a related model, showing how natural resources can worsen local government.

Consider a local government with a given unelected leader, who can offer leadership services.
The leader inelastically provides a given quantity of government services G (keeping the peace and adjudicating disputes).
The district ruled by this leader contains some fixed supply of land L.
By saving or borrowing, workers may accumulate capital which they can also bring to district.
A worker with 1 unit of labor and k units of capital can command a total income of $\omega+\rho k$ in other districts, where ω is the wage rate and ρ is the rate of return on capital.
If the district attracts n units of labor with k units of capital, total output in the district will be
$A\,(L+k)^{\alpha}\, n^{\beta}\, G^{\gamma}$,
where (A,α,β,γ) are given parameters such that $A>0$, $\alpha>0$, $\beta>0$, $\gamma>0$ and $\alpha+\beta+\gamma=1$.
The district leader can get $V(n,k,L) = A\,(L+k)^{\alpha}\, n^{\beta}\, G^{\gamma} - \omega n - \rho k$
by taxing away all output in excess of workers' best alternative elsewhere.
But there is a problem: the leader could expropriate capital from the workers in the district and invest it or rent it abroad to earn ρk.
If the district leader lost his reputation for protecting capital, then he would get only workers without capital, in which case his income would be
$v_0(L) = \max_{m\geq 0} V(m,0,L) = A\, L^{\alpha}\, m^{\beta}\, G^{\gamma} - \omega m$.
The solution to this optimization problem is $m = (L^{\alpha}G^{\gamma}A\beta/\omega)^{1/(1-\beta)}$
and $v_0(L) = (1-\beta)[AL^{\alpha}G^{\gamma}(\beta/\omega)^{\beta}]^{1/(1-\beta)}$.
To deter the leader from expropriating workers' capital, the districts economic plan (n,k) must satisfy the following incentive constraint $V(n,k,L) \geq v_0(L) + \rho k$.
In an optimal plan for the district, V(n,k,L) is maximized over (n,k) subject to this constraint.

Once the total of land plus capital L+k is specified, the optimal labor force is
$n(L+k) = ((L+k)^{\alpha}G^{\gamma}A\beta/\omega)^{1/(1-\beta)}$.
Let $U(L+k) = A\,(L+k)^{\alpha}\,(n(L+k))^{\beta}\, G^{\gamma} - \omega n(L+k)$.
So $U(L+k) = V(n(L+k),k,L)+\rho k$, the leaders' income with optimal labor before paying capital.
Notice also that $v_0 = U(L)$.
Substituting for n(L+k), we have $U(L+k) = (1-\beta)[A(L+k)^{\alpha}G^{\gamma}(\beta/\omega)^{\beta}]^{1/(1-\beta)}$.
So the leader's problem is to
choose $k\geq 0$ to maximize $U(L+k)-\rho k$ subject to $U(L+k)-\rho k \geq U(L)+\rho k$.

Let H_1 denote the total quantity of land plus capital that would maximize the leader's profits if there were no incentive constraints and he could borrow freely at the interest rate ρ.
So $H_1 = \text{argmax}_{h\geq 0}\, U(h)-\rho h$. By calculus, we get $H_1 = A^{1/\gamma}\, (\alpha/\rho)^{(1-\beta)/\gamma}\, (\beta/\omega)^{\beta/\gamma}\, G$.
Let H_2 denote the total quantity of land plus capital that would maximize the leader's profits if there were perfect capital markets, with no incentive constraints, but the interest rate was 2ρ.
So $H_2 = \text{argmax}_{h\geq 0}\, U(h)-2\rho h$. Then $H_2 = A^{1/\gamma}\, (\alpha/(2\rho))^{(1-\beta)/\gamma}\, (\beta/\omega)^{\beta/\gamma}\, G < H_1$.

Results:
If $L > H_2$ then $k=0$ is the optimal solution to the leader's problem, because he cannot borrow any positive amount of capital.
There may be some interval $[0,H_3)$ such that L in this low interval allows the leader to borrow the unconstrainted optimal amount $k = H_1-L$.
In any case, $H_3<H_2$. But this low interval may be empty, that is, we may get $H_3=0$.
When $L \in [H_3,H_2]$, $U(L+k)-\rho k$ is an increasing function of k over the feasible set, and so leader's optimum satisfies the incentive constraint with equality $U(L+k)-\rho k = U(L)+\rho k$.
When $L \in [H_3,H_2]$, the optimal solution (n,k) has $k > 2(H_2-L)$.
So decreasing the endowment of natural resources L from H_2 increases k by more than two units for each unit of L lost, and so $L+k$ actually increases by more than the amount that L decreases.