Abstract. We analyze a simple model of moral hazard in local public investments which could be efficiently managed by local officials who are accountable in autonomous local politics. In a centralized autocracy, however, the local residents who can observe the quality of a local public investment may be unable to communicate their views to the national elite who must approve any decision to dismiss the responsible official. Thus, an autocratic national government cannot make a credible commitment to efficient local investments without granting some fundamental guarantee of local political rights. We also consider a model of a unitary democratic state to see why elected national leaders would choose to maintain inefficient centralized control of local government in some regions, even though informed voters would prefer a candidate who promised decentralized accountability.

1. Introduction

The prosperity of any community depends on local public investments in infrastructure, legal protection, and other public services that are provided by government. The provision of these public goods and services requires that substantial authority and resources must be delegated to local officials of government, but these officials then could profit by abuse of their power. The effective functioning of government therefore depends on a system of controls and incentives to deter abuse of power by government agents. Responsibility for the distribution of agents' rewards and sanctions in a government ultimately belongs to its political leaders, who must act to hold power within the (implicit or explicit) constitutional rules of the nation's political system. Thus, the constitutional structure of government can affect the way that its leaders will solve fundamental moral-hazard agency problems in public investment, and so differences in the wealth of nations may be derived from differences in their political systems.

In this paper, we consider a simple problem of moral hazard in local public services that has an efficient solution which is politically feasible only with a constitutional division of powers between local and national governments. This model is designed to show how a centralized political system may prevent a nation from realizing the benefits of some local public investments which could enrich an otherwise similar nation with a different political system.

A skeptic might question the possibility of such a model by asking why an autocratic national leader with absolute power would not want to undertake any public investment that would enrich his nation and expand its tax base by more than the cost of the investment. In reality, however, even an autocrat cannot exercise power throughout a nation without a network
of appropriately motivated agents. Thus (as I have argued in Myerson, 2008, 2015), even a seemingly absolute leader must face fundamental political constraints that are derived from the need to maintain his agents' and supporters' confidence that he will reliably reward them for their service. The main point of this paper is to show how such basic political constraints can prevent a centralized autocratic government from undertaking beneficial local public investments that would be readily feasible for a constitutionally decentralized government.

Many have argued that political decentralization and community empowerment may be essential for successful economic development. Banfield's (1958) classic study of the *Moral Basis of a Backward Society* ultimately concluded (in chapter 9) that local development was inhibited by an extreme centralization of power to national ministries and an appointed local prefect. Fortmann (1983) studied how development in poor rural communities can be frustrated by a lack of local political accountability for those with power to spend public funds. Ostrom (1990) examined the vital importance of organizations with locally accountable leadership for the efficient management of common-pool resources. Mansuri and Rao (2013) provide a detailed overview of theory and evidence for development strategies that are based on local community empowerment (see also a review by Myerson, 2014).

Some comparative political studies have found beneficial effects of autonomous local government, particularly in countries which have strong competitive political parties at the national level (see Crook and Manor, 1998, Enikolopov and Zhuravskaya, 2007, Faguet, 2012, and Ponce-Rodriguez et al., 2012). But there have not been enough cross-national comparative studies of political decentralization, partly because it has been difficult to obtain comprehensive global data on subnational political institutions. (One good resource is United Cities and Local Governments, 2007.) Broad perspectives on political decentralization and economic development can be found in Bardhan and Mookherjee (2006), Faguet (2014), and Faguet and Pöschl (2015).

In a cogent critique of the large literature on potential advantages of federalism, Triesman (2007) has suggested that the national leadership of a centralized unitary state could just as well apply different government policies in different regions, to accommodate regional differences or to find better policies by local experimentation. Against this critique, we argue here that, in a centralized state with no autonomous local politics, problems of maintaining trust between the national leader and key supporters may prevent the leader from appropriately disciplining officials whose performance can be properly evaluated only by local residents.
Section 2 here develops a simple model of moral-hazard problems in local public services, and it shows incentive-efficient solutions that are feasible when local government officials are accountable to the residents in autonomous local politics. Section 3 shows how, in a centralized autocratic regime that admits no constitutional protection for local political expression, fundamental problems of trust in the ruler's coalition can prevent the government from achieving such efficient local public investments. Section 4 considers how these results may be extended to centralized democratic governments, introducing a second model to show how democratic competition may induce leaders to inefficiently centralize local public services in much of the country. Section 5 concludes.

2. A simple model of moral hazard in local public services

Imagine a remote town or district in a large nation. Let \( n \) denote the number of residents who live in this town, far from the nation's capital. Each resident in the town can own an enterprise that requires the resident to make a long-term private investment of \( C \). Each year, each resident's enterprise may yield a return worth either \( S > 0 \), if the resident's enterprise is successful this year, or \( 0 \) otherwise. These returns are independent across residents and years, but the probability of success for each resident's enterprise each year depends on some local public services that must be administered by a local public official or magistrate. We may think of these public services as the administration of a local justice system that adjudicates disputes and protects property rights for residents, or as the maintenance of a local transportation network that is used only by the residents. The more that the local official spends on these local public services, the greater is the probability of success for any resident's enterprise.

Specifically, suppose that, in any year when the total spent on the town's local public services is \( nk \) (so that \( k \) is the local public spending per resident), each resident's enterprise will have an independent probability \( \pi(k) \) of success. Everyone is risk neutral and discounts future income with an annual discount factor \( \beta \). Here \( n > 0 \), \( 0 < \beta < 1 \), \( S > (1-\beta)C > 0 \), and \( \pi(k) \) is an increasing, concave, and differentiable function of \( k \geq 0 \), with \( \pi(0)=0 \) and \( 0 \leq \pi(k) \leq 1 \) for all \( k \).

The moral-hazard problem here is that the local official cannot be prevented from stealing any funds for local public services and then fleeing abroad (where the former official would be immune from prosecution). Thus, when the per-capita investment level is \( k \), the official's expected discounted value of all rewards from office cannot be less than \( nk \). That is, the official must expect that, by good behavior, she can earn moral hazard rents that have an
expected present discounted value of $k$ per resident. With the annual discount factor $\beta$, this promise can be achieved by paying the official an annual salary of $nr(k)$ (or $r(k)$ per resident) where

$$r(k) = (1-\beta)k.$$ 

If the residents of the town hire the local official to administer their local public services with a public-services budget of $nk$ plus an annual salary of $nr(k)$ for the official, then the average resident's expected annual net benefit from local government will be

$$U(k) = \pi(k)S - (1-\beta)C - k - r(k).$$

Here $(1-\beta)C$ is the annual return that is needed to amortize the cost of the resident's private investment $C$. The benefit per resident is maximized by per-capita public spending $k_1$ such that

$$\pi'(k_1) = (2-\beta)/S.$$ 

To avoid a trivial no-investment solution, let us assume that local public services at this efficient level would be worth more than the total cost to the residents, that is

$$\pi(k_1)S > (1-\beta)C + k_1 + (1-\beta)k_1.$$ 

Let us assume, however, that the official's actual expenditure on the public investment cannot be directly observed or monitored by anyone else. That is, even if the official is given a budget of $nk_1$ to spend on local public services, the official could actually spend any lesser amount $nk$ on public services and could secretly divert the remaining $n(k_1 - k)$ to personal consumption. Under this assumption, the successes and failures of residents' private enterprises in the town provide the only evidence for the actual public investment $nk$.

When local public spending is $k$ per resident, the number of successful enterprises in the town will be a Binomial random variable with parameters $n$ and $\pi(k)$. So the fraction of successful enterprises in the town will be an approximately Normal random variable with mean $\pi(k)$ and standard deviation $\sigma(k) = [\pi(k)(1-\pi(k))/n]^{0.5}$. For any $k$, this standard deviation $\sigma(k)$ is less than $0.5/n^{0.5}$, which goes to 0 as $n$ becomes large. When the town's population $n$ is large, this evidence is good enough to sustain a close approximation to the efficient solution $k_1$.

To achieve asymptotic efficiency as $n$ gets large, it is enough to consider local political incentive plans of the following general form. Each year, the local official is given a budget $b$ per resident to invest in local public services. At the end of the year, the residents will vote to evaluate the official's performance based on whether their enterprises succeed or not. If the fraction of residents whose enterprises succeed is at least the threshold $\theta$, then the official will be
paid the salary \( \rho \) and will be renewed to serve again next year. Otherwise, if the fraction of successes is less than \( \theta \), the official will be dismissed without further pay, and then the town will elect a new local official on the same terms for next year.

With such a plan in a town of \( n \) residents, let \( Q(k, \theta, n) \) be the probability that the official would be paid and re-elected in any given year if the official actually spent \( k \) per resident in the town of \( n \) residents. That is, \( Q(k, \theta, n) \) is the probability that the Binomial-\((n, \pi(k))\) random number of successes will be at least \( n\theta \)

\[
Q(k, \theta, n) = \sum_{h \geq n\theta} \pi(k)^h (1-\pi(k))^{n-h} n! / ((n-h)!h!).
\]

With a per-capita budget \( b \), salary \( \rho \), and renewal threshold \( \theta \), the official's optimal expected discounted payoff \( \bar{W} \) and induced public investment level \( \bar{k} \) would satisfy

\[
\bar{W} = (b - \bar{k}) + Q(\bar{k}, \theta, n)(\rho + \beta \bar{W}) = \max_{k \in [0,b]} (b - k) + Q(k, \theta, n)(\rho + \beta \bar{W}).
\]

The following proposition says that, with a budget that equals the efficient spending level of \( k_1 \) per resident, the threshold \( \theta \) and salary \( \rho \) can be chosen as functions of the town's size \( n \) so that, for large \( n \), the official will be induced to spend almost all of the budgeted funds appropriately. Thus, the residents of a large town can expect net returns from local public spending that are close to the maximal value of \( U(k_1) \), with an incentive system in which the official's renewal depends on getting votes of approval from at least a \( \theta \) fraction of the residents in an annual poll.

**Proposition 1.** For any \( \varepsilon > 0 \), renewal thresholds \( \theta(n) \) and official salaries \( \rho(n) \) can be set as functions of local population \( n \) so that \( \lim_{n \to \infty} \rho(n) \leq r(k_1) + \varepsilon \) and, with the efficient investment budget \( b = k_1 \), the induced public investment levels \( \bar{k}(n) \) satisfy \( \lim_{n \to \infty} \bar{k}(n) = k_1 \) and \( \lim_{n \to \infty} Q(\bar{k}(n), \theta(n), n) = 1 \).

The proof is in the appendix, but we should remark here that parameter values which can satisfy this proposition include \( \theta(n) = \pi(k_1) - \log(n) / n^{0.5} \) and \( \rho(n) = (1 / Q(k_1, \theta(n), n) - \beta)k_1 + \varepsilon \). The threshold \( \theta(n) \) is chosen so that \( \pi(k_1) - \theta(n) \) is a small fraction but is a large multiple of \( \sigma(k_1) \), and the salary \( \rho(n) \) is chosen so that the official strictly prefers spending \( k_1 \) appropriately over stealing all of \( k_1 \).

We may call this a simple model of moral hazard in local public services, not only because it is characterized by just a few parameters, but also because the efficient solution for maximizing the residents' expected net benefit seems so straightforwardly achievable with a
form of democratic local government. (If the threshold \( \theta \) is different from \( 1/2 \), the results can still be achieved by majority voting if the voters on the expected long side randomly abstain with an appropriate probability, as in the equilibria of Feddersen and Pesendorfer, 1996.)

This local political game, in which the incumbent official each year can decide to spend or divert local tax revenues and the residents can vote to retain or dismiss the official, actually has multiple equilibria. We have been focusing on good equilibria in which the official appropriately invests public funds and the residents privately invest in their enterprises and vote to retain the official when they succeed. Such a good equilibrium represents a relationship of trust between the responsible official and the local voters to whom she is accountable. The behavior that this trust entails is rational as long as everyone understands that everyone else will act according to this relationship. But there are other possible equilibria in which this trust does not exist.

In a distrustful equilibrium of the local political game, the residents would not vote to retain the incumbent official even when their private enterprises have succeeded, and so the incumbent official would have no incentive to spend anything on local public services. When the residents understand that the official would corruptly divert any local public budget to her own private consumption, then they would prefer to replace the incumbent official by anyone else whom they could trust to act according to the good equilibrium.

In other dynamic equilibria, the relationship between an official and the voting residents could switch from trust to distrust as a result of some random event that does not directly affect anyone's payoff but is publicly observable. Such a loss of trust could occur, for example, as result of an embarrassing accident that is publicly interpreted as a scandal for the official. The possibility of such events would depend on the local political culture. But any such event that causes residents to doubt whether they should trust the official can become a self-fulfilling prophecy, causing each side to react in a way that confirms the other's loss of trust.

If such random scandals in local politics generated some probability \( q \) of the incumbent local official being voted out of office in any year even with good performance, then the official could be trusted to make a public investment \( nk \) only if the office's annual salary \( nr \) satisfied

\[
nr/(1-\beta (1-q)) \geq nk.
\]

So the official salary expense per resident would have to be at least \( \hat{r}(k,q) = (1-\beta (1-q))k \). Then the residents' net benefit \( \hat{U}(k,q) = \pi(k)S - (1-\beta)C - k - \hat{r}(k,q) \) would be maximized by a per-capita public investment \( \hat{k}(q) \) such that
\[ \pi'(\hat{k}(q)) = \frac{(2-\beta(1-q))}{S}. \]

As \( \pi \) is concave and \( \beta > 0 \), the residents' optimal per-capita public investment \( \hat{k}(q) \) is a decreasing function of the exogenous political instability factor \( q \), and \( \hat{k}(q) < k_1 = \hat{k}(0) \) for any \( q > 0 \). The residents could still expect to benefit from local government as long as \( \hat{U}(\hat{k}(q), q) > 0 \).

With a local government that is less democratic but still politically autonomous, the town could actually achieve returns that are greater in aggregate, although the typical resident would benefit less. Let us consider an extreme case where a local aristocrat can offer to serve as the local official on terms that the other residents of the town can only accept or reject, with rejection implying that there will be no local public services at all. With all the bargaining power, the local aristocrat could ask all residents to pay their share of the public service budget \( k \) plus (almost) all of their expected net benefits from these public services \( \pi(k)S - (1-\beta)C - k \).

(Equivalently, if the aristocrat's personal wealth is greater than \( nk \), she could offer to pay the budgeted public-service cost from her own funds but then ask the residents to pay their expected benefits \( \pi(k)S - (1-\beta)C \).) Thus, by offering to manage public services that cost \( nk \), the aristocrat could hope to earn from each resident each year (assuming no instability \( q=0 \)) the net payoff

\[ \pi(k)S - (1-\beta)C - k = U(k) + r(k). \]

The aristocrat's optimal \( k \) would maximize this payoff subject to the incentive constraint

\[ \frac{\pi(k)S - (1-\beta)C - k}{(1-\beta)} \geq k. \]

This optimum is achieved by per-capita public spending \( k_2 \) such that

\[ \pi'(k_2) \geq \frac{1}{S}, \quad \pi(k_2)S - (1-\beta)C \geq (2-\beta)k_2, \]

with at least one equality

so that \[ \pi'(k_2) - \frac{1}{S}[\pi(k_2)S - (1-\beta)C - (2-\beta)k_2] = 0. \]

As \( \pi \) is concave and \( \beta < 1 \), this aristocratic public-spending level \( k_2 \) is strictly greater than the level \( k_1 \) that would be democratically chosen by the residents at large.

With large \( n \), the local aristocrat can credibly promise to spend on local public services a budgeted amount \( b \) that is close to \( k_2 \) per resident, by an incentive plan that is similar to the one in Proposition 1 above. Here the renewal of the aristocrat's service each year would depend on the approval of at least some fraction \( \theta \) that is close to \( \pi(k_2) \), with a total of budget and salary \( b + \rho \) that does not exceed the residents' expected benefits, which would equal \( \theta S - (1-\beta)C \) when a fraction \( \theta \) succeed. (Formally, for any \( \epsilon > 0 \), we can pick \( (b, \theta, \rho) \) with \( k_2 \geq b \geq k_2 - \epsilon \),

\[ \pi(b) > \theta \geq \pi(b) - \epsilon, \quad \theta S - (1-\beta)C - b \geq \rho > (1-\beta)b, \]

such that, when the official gets annual per-capita budget \( b \), renewal threshold \( \theta \), and salary \( \rho \), the induced levels of public spending \( \hat{k}(n) \)
will satisfy \( \pi(\bar{k}(n)) \geq \theta \) for all sufficiently large \( n \) and \( \lim_{n \to \infty} Q(\bar{k}(n), \theta, n) = 1 \). By such an incentive-compatible mechanism, the expected net benefits in the town could be increased to as much as \( U(k_2) + r(k_2) \) per resident, but the largest share of these benefits would accrue to the aristocratic official.

In all such mechanisms, however, it is essential that the local official must be politically accountable to the residents of the town. We are assuming here that only a resident can directly observe whether his or her private enterprise has succeeded or not in any given year. Under this basic assumption, the evidence for the official's public-service spending is dispersed among the residents of the town. If the official's future rewards were not dependent on residents' testimony as to whether they have benefitted from local public services, then the official would have no incentive to spend anything on public services, given that she could instead secretly divert all budgeted funds to her own personal consumption. But if such misbehavior were anticipated, the residents would have no incentive to invest in the town or approve any taxes for the official.

3. Political difficulties for an autocrat to centralize local moral-hazard rents

In our model, the official who controls the budget for local public services must get substantial moral-hazard rents. These rents make the official's position a valuable prize for which candidates should be willing to pay. A political leader who has the power to appoint such officials could potentially benefit by selling the offices, if not for cash then for political support.

So when the leader of the national government can appoint the officials who manage local public spending, the allocation of these local offices may depend on national political considerations. Under any political system, a national leader must occasionally face rivals for power, and success in these contests will depend on the leader's ability to mobilize active support from many individuals. Expectations of future rewards can be essential for motivating supporters' costly political efforts, and so any successful national leader needs a reputation for distributing patronage benefits to those who have provided vital support in a contest for power. Thus, national leaders may have a strong incentive to allocate valuable offices as patronage rewards for key supporters.

Our main question is, if a national leader sells such offices of local government, whether for cash or political service, can the appointed officials still be induced to provide efficient public services to the local residents? In this section, we consider this question under the assumption that the national leader is an autocratic ruler of a centralized state where contests for power are as
in the model of Myerson (2008). (See Svolik, 2012, for more on authoritarian politics.)

In the model of Myerson (2008), the autocratic ruler of a state has the full power to allocate some given stream of tax revenue as long as he holds power. At random times, however, the ruler must face a challenger in battle, and the winner then will be ruler until the next challenger appears. In this battle, the ruler and challenger must each recruit supporters or captains, and the probability of winning for each leader is an increasing function of the ratio of his number of supporting captains in the battle divided by his opponent's. Each captain has a positive personal cost of providing support in battle, and so a captain will not actively support a leader without some promise of future rewards, which the leader can pay only after a victory. But after victory, the ruler will be able to get all the tax revenue without any support from any captains until the next challenger appears. Thus, once his power has been secured, a ruler could be tempted to take more of the revenue for himself, rather than share it with his captains. However, the captains would not support the leader's bid for power in the first place if they anticipated that the leader would subsequently deny their promised rewards.

In such contests for power, absolute leaders whose actions are truly above anyone's judgment could not credibly recruit more than a few supporters. To recruit a larger force of supporters, a leader must somehow commit himself to pay his debts for past service. But who can constrain a leader who has won supreme power in an authoritarian state? The answer is that, to hold power and wield it, even a sovereign ruler will need supporters and agents who have confidence that their loyal service will be well rewarded. The ruler may lose the trust of one supporter, but he cannot hold power against future challengers if he loses all supporters' trust. Thus, a leader can credibly recruit a larger force of supporters when they are organized into a group who monitor each others' treatment by the leader, so that a failure by the leader to appropriately reward any one supporter could cause them all to distrust the leader in the future.

That is, a leader can build stronger support when his key supporters effectively form a court that judges his treatment of all supporters, so that the leader would be unable to hold power against their judgment. Myerson (2008) showed that, in a simple model of political competition, if any leaders are able to organize their supporters into such a court, then a challenger for power could not recruit any supporters without instituting the same constraints on himself.

From this perspective, we should expect that even an autocratic ruler must be subject to a personal constitutional constraint to maintain his reputation, at least within an elite circle of supporters, for reliably rewarding their service. To this court, the ruler must regularly justify his
allocation of major patronage rewards and his decisions to punish or promote key supporters. Although nobody wants to speak openly about the possibility of losing trust in the ruler, the courtiers around him must feel a social imperative to watch for (and gossip about) any evidence that the ruler may have failed to appropriately reward a key supporter.

In the simple model of Myerson (2008), the autocrat's tax revenue was assumed to be exogenously fixed, but here we should consider the possibility that investments in public services could increase the tax base. Let us assume that residents of a town can be more highly taxed than peasants scattered across the countryside, but people will not remain in a highly taxed town unless it provides some compensating benefits from local public services. Within our model, then, the quantity $\pi(k)S - (1-\beta)C$ can be interpreted as the additional annual tax burden that each of the $n$ residents in the town would be willing to pay (instead of hiding in the countryside) if the government could be expected to provide local public services costing $k$ per resident each year. With such a tax increase, the ruler could pay the cost of these local public services, could pay the local official's salary of $r(k)$ per resident, and could still get a net per-resident fiscal benefit worth

$$U(k) = \pi(k)S - (1-\beta)C - k - r(k).$$

The above formula treats the local official's salary as a cost to the ruler, which it would be if the official were just some resident of the town who happens to be trusted by her neighbors. But the ruler could potentially gain still more if these valuable moral-hazard rents were allocated instead to someone to whom the ruler already owes a comparable political debt. The patronage appointment of a key supporter to this local office, with its expected annual salary of $nr(k)$, could be used to discharge a political debt worth $nr(k)/(1-\beta) = nk$. When the annual salary expense of $r(k)$ per resident can thus be credited against the ruler's political debts, then ruler would get a broader (fiscal plus political) annual benefit per resident worth

$$U(k)+r(k) = \pi(k)S - (1-\beta)C - k.$$

From this perspective, it seems obvious that the national ruler should choose to exercise appointment power over such local offices, if there is no constitutional constraint against such central control of local government. Furthermore, it seems that the ruler would then choose to support a level of local public services $k_2$ (maximizing $U(k)+r(k)$), which is higher than the level $k_1$ (maximizing $U(k)$) that would be chosen by autonomous local democracy in the town.

But let us now ask whether can a ruler actually commit himself to hold these local officials properly accountable in a centralized autocratic regime? In our model, an official must regularly face a risk of dismissal if too few residents benefit from the official's local public
services. Under an optimal incentive-compatible mechanism, there must be some small probability of an official being dismissed even when she invests her budgeted public funds correctly. Indeed, there must be a strictly positive probability \((1−\pi(k))^n > 0\) of the event that all residents' enterprises fail even when the official has appropriately spent the public funds \(nk\). Such an event may be unlikely but, when it occurs, the dismissed official must not expect any further rewards from the state, because the moral-hazard constraint requires that the local official must be punished for such evidence of corruption. That is, a supporter's appointment to this office must be taken as fully discharging any political debt of the ruler worth up to \(nk\) to the supporter, even if some unlikely unfortunate outcome subsequently causes the official to be dismissed with little or no accumulated salary benefits.

When the local office becomes vacant, however, it again becomes a valuable prize that the ruler could again sell or allocate in exchange for the discharge of another political debt. Thus, the autocratic ruler can actually derive positive benefits from replacing a local official. Such benefits from re-selling the office could tempt an unconstrained autocrat to manipulate reports of local public services to justify an incumbent's dismissal. A powerful autocrat has many ways to induce people to say what he wants to hear.

The autocrat's potential benefits from re-selling an office must be countered by the implicit threat that too many dismissals could create dangerous and costly distrust in the elite circle of courtiers around the ruler. Indeed, we may take the definition of autocracy to be that the ruler can be held politically accountable only by this elite inner circle. The autocrat's ability to suppress popular political discontent implies that local residents' cannot impose political costs on the ruler or his regime, even when they are severely disappointed by inferior public services. Ex ante, the residents would not make private investments in towns where good public services cannot be credibly promised, but ex post they have no power to punish the autocratic ruler when his officials disappoint them.

For local officials to be held properly accountable, the regime must apply a strategy of dismissing an official when reports indicate that the official has provided inferior public services. Even without permitting freedom of popular political expression, an autocratic ruler can maintain networks of informants to him private reports on local residents' views about the quality of their local public services. But what is to prevent the ruler from deviating to some other strategy, such as dismissing officials instead when his reports indicate that they have provided superior public services? The ruler can be committed to a strategy of holding local officials properly
accountable only if such strategic deviations could be effectively punished by the ruler's court.

Any positive incentive for the autocratic ruler to hold local officials accountable must come from the circle of elite courtiers around him, as only they can impose political costs on their ruler. The courtiers can impose political costs on their ruler, but these costs cannot depend on information that the ruler controls, which he could manipulate to reduce his costs. Thus, the central question of accountability is whether evidence on the quality of local public goods can be credibly communicated to the elite courtiers on whom the autocrat must rely.

In the case of an official who is responsible for maintaining the nation's main highway network or essential public services in the nation's capital, the autocrat's courtiers may be able to see for themselves the evidence of administrative malfeasance by an appointed official. But it is much harder for elite courtiers to monitor the performance of a local official who is responsible for residents' basic legal protection in a remote town. Testimony of a few selected residents is not enough, as it is generally possible in any large town to find some residents who would be willing to testify either way about a local official's performance. It is not enough for the ruler to maintain a network of secret informants to report evidence of malfeasance by his officials, as the ruler could solicit negative reports or communicate them selectively against any official whom he wants to replace. A vote of the residents proves nothing when the autocrat can have the counting process manipulated any way he wants. For the ruler's court to evaluate whether a local official is being held properly accountable, either some courtiers must have their own separate reporting networks that reach into the town, or political communication channels from town residents to the capital must have some protected independence against manipulation by the ruler. But we may assume that an autocratic ruler does not permit such independent channels for communicating information about people's political grievances to the national elite. Thus, in a centralized autocracy, the costs that are imposed on the ruler by his courtiers cannot depend on residents' opinions about the quality of local public goods in remote towns.

To formalize this argument, let $J$ denote the set of local offices, indexed by $j$. In any given year, let $g(j)$ denote the value that the ruler could earn from replacing the incumbent official $j$ by another supporter who would offer new political services in exchange for appointment to office $j$. This value $g(j)$ could be somewhat less than the moral-hazard rents associated with this office, perhaps by some random amount that depends on how much the ruler would value new political service or support this year. It may also depend on any information that the ruler has about official $j$. But the courtiers have less information than their ruler, and so
they may view these g(j) values as random variables drawn from some probability distribution. The set of dismissed officials D is observed by the courtiers, who can compel the ruler to pay some political penalty κ(D) that depends on this set. Then, knowing g(•) and κ(•), the ruler should choose the dismissal set D⊆J to maximize

\[ \sum_{j \in D} g(j) - \kappa(D) \]

For the courtiers to ensure the ruler will not dismiss many officials, the marginal κ penalty for each office should greater than the moral-hazard rents of that office unless the dismissal set D is a small fraction of all offices. For example, the ruler might have no political cost of dismissing one local official each year but might incur a high political cost to maintain his supporters' trust for every additional dismissal. But the political costs κ(D) cannot depend on the actual quality of local public services in the towns, because the residents who observe this quality cannot credibly communicate their views to the elite courtiers around the autocratic ruler.

If the benefit and cost functions g(•) and κ(•) treated all offices symmetrically, then the ruler could be neutral on the choice of which official to replace, and so he would be willing to dismiss the official who (according to his informants) has provided the worst local public services this year. But under these neutrality assumptions, the ruler would be equally willing to dismiss the official who has provided the best public services instead (and indeed he might prefer to do so if he feared that popular local officials could become challengers for national power).

The ruler's expected net benefit g(j) from replacing any official j may depend on the ruler's private political information. Let us assume that, from the courtiers' perspective, each g(j) includes an independent random term that is drawn from a continuous distribution over some interval and is observed only by the ruler. That is, for any j and any given profile of g values for all other offices, suppose that the conditional probability distribution of g(j) as assessed by the courtiers would be continuous over some interval of positive length. Then for any cost function κ that the courtiers can impose, the probability of the ruler being indifferent among two different dismissal sets would be zero. Thus, the set of officials who are dismissed by the ruler would be determined completely by political considerations: the ruler's net benefits g(j) of replacing any official j by someone offering new political service, and the political costs κ(D) imposed by the elite courtiers around the ruler. Then the ruler's optimal dismissal set D cannot depend on local resident's views of their local public services, even though such views may be reported to the ruler by his network of secret informants.
Proposition 2. In a nation where people have no protected independent channels for expression of political grievances, an autocrat who has unrestricted power to appoint and dismiss local officials cannot be credibly committed to hold them accountable for the quality of public services that are observed only by local residents.

In the determination of the \( \kappa \) cost function itself, neutrality among incumbent officials could also be subverted by influential courtiers raising even slightly the ruler's political cost of dismissing an official who has better connections with them. Such biases should be expected once we recognize that a ruler cannot take responsibility for monitoring and rewarding every agent of the state. The autocrat's key supporters must themselves be leaders of their own teams or factions, which they must motivate by their own essential reputations for reliably distributing promotions and rewards to those who have served them. So if the ruler chose one local official to replace, it would more likely be the one whose advocates at court are weakest, rather than the one whose reported public services are worst.

Thus, the autocratic regime would not provide any incentive for officials to spend any money on local public services in remote towns, and so rational residents should not make private investments that depend on good local public services there. However, our model from section 2 can offer a measure of the potential fiscal benefits that a ruler might earn if he could credibly grant some constitutional rights for autonomous local governments to take responsibility for their local public services.

When local governments have political autonomy (as we assumed in section 2), local officials can be held accountable by local residents for the observed quality of their local public services without threatening the political reputation of the national ruler. Autonomy means that the national ruler must give up the right to allocate these local offices and their valuable moral-hazard rents. But improved local public services may enrich communities throughout the nation in ways that ultimately increase the national government's tax base as well. Within our model, if the ruler could commit to allowing a town to elect its own autonomous local government in exchange for their paying a special tax to the national treasury, the town's \( n \) residents should be willing to pay an annual tax up to \( \max_{k \geq 0} U(k) = U(k_1) \). If the ruler could appoint the first head of the local government which will spend \( nk \) annually, the position could be sold to a supporter for a debt worth \( nk \), and then town could be taxed \( nU(k) \) per year, and so the ruler could earn a value worth up to \( \max_{k \geq 0} U(k)+nk/(1-\beta) = U(k_2)+r(k_2) \) in annual revenue per resident.
Thus, in principle, a national leader could derive sustained fiscal benefits from devolving responsibility for local public services to constitutionally autonomous local governments. But such plans can only work if the local officials are then responsible to autonomous local politics, and so the ruler must be constitutionally prohibited from interfering in this local politics. Ex post, such a prohibition would be against the ruler's interest, but it could be enforced if any violation by the ruler would cause his supporters to distrust him in the future. In practice, however, two fundamental political considerations can deter national leaders from introducing such political decentralization where it has not previously existed in a nation.

First, political decentralization weakens the national leader's essential ability to reward supporters with appointments to valuable local offices. Without local accountability, local officials would be effectively responsible for a narrower range of public services that are observable by others in the national elite, but these responsibilities could still entail valuable moral-hazard rents. When local governments become politically autonomous, then national leader gives up the right to use these offices as patronage rewards for his loyal supporters. It is generally risky for a leader to disappoint supporters of rewards that they had been expecting.

We may consider an intermediate form of decentralization in which the national leader retains the power of appointment to local offices but gives residents a constitutionally protected right to dismiss their appointed officials by majority vote in regular independently supervised local elections. With such accountability, local officials can be trusted with larger budgets, making their positions more valuable, and the ruler can still take credit for their appointment. But serious political risks are created by empowering independent voters to decide whether appointed officials have been performing adequately. Under this system, the ability of the ruler's patronage appointees to enjoy their promised moral-hazard rents depend not just on the ruler's appointment but also on the voters' approval. So independent voters would have effective power to undermine the ruler's reputation for reliably rewarding loyal service, which is the vital political foundation for the autocratic regime. The autocratic ruler would be politically vulnerable if a potential rival could turn these voters against the regime, perhaps by inducing them to focus on alternative distrustful equilibria of the local voting games.

Second, politically autonomous local governments can become a proving ground for new political leadership and so may eventually generate new competition for national leadership. To win a valuable public office in local politics, a candidate must develop a reputation for reliably rewarding a cadre of active supporters and for responsibly providing public services that can
benefit residents throughout the community. Strong candidates for power at the national level also have such qualities. If the responsibilities of leadership in local government are qualitatively comparable to those in national government, then a leader of local government who has provided better public services could readily become a popularly trusted candidate for national leadership. Thus (as was argued by Myerson 2006), autonomous local politics can become a source of new competitive challengers for power at the national level. Established national leaders would naturally prefer not to face such competitive challengers for their power.

Thus, a fiscally beneficial decentralization of power may be politically too costly for the incumbent national leadership. We should not be surprised that national leaders have often chosen to retain centralized control of local government, even when decentralization could strengthen their country's economic development.

4. Extensions to the case of centralized democracy

In the previous section we considered the difficulty of overseeing local public services for a centralized national government that is headed by an autocratic ruler. Now let us consider the case where the government is similarly centralized but the national leader is democratically elected. Many of our previous arguments about centralized autocratic regimes can be straightforwardly extended to centralized democratic regimes.

The democratic requirement for a national leader to face regular electoral challenges only increases the vital importance of a national leader's ability to mobilize many active supporters in contests for power. Thus, a democratic national leader, no less than an autocrat, needs a reputation for reliably rewarding past supporters with the greatest possible fund of patronage benefits. The ability to promise moral-hazard rents from the management of public spending can be a valuable asset for the leader to recruit key supporters in an election campaign, and so the democratic national leader has a strong incentive to assert power of appointment over valuable local offices and to allocate them according to national political concerns. But the moral-hazard rents can accomplish their necessary agency function only if the national leader can then commit to make an appointed official's continuation in office conditional on the appropriate management of the office's resources. The central question now is whether democratic national leaders can credibly commit to such local accountability more than autocrats.

We should note first that, even if the current national leader made such a commitment for his time in office, democratic competition would subject the official's future retention to national
political risks that are independent of her local performance and thus must increase the cost of required moral-hazard rents. Democracy generates a positive probability of a transfer of national power at the next scheduled election, in which event the next national leader would want to replace all patronage appointees. In our model, if there were a probability \( q \) of such national political change in any given year, then a local official's annual moral-hazard rents for spending \( k \) per resident would have to be \( \hat{r}(k, q) = (1 - \beta(1 - q))k \), which is an increasing function of \( q \).

But once installed in power, whether by democratic election or not, any national leader may have an ex-post incentive to re-allocate a valuable office as a reward for new national political service, using some manipulated expression of local discontent as a cover. To credibly commit against such ex-post incentives, key supporters of a democratic leader must also be watchful against the possibility of other supporters being so cheated out of their promised rewards, and any suggestion of such cheating must have the potential to undermine the leader's essential political reputation for reliably rewarding past service. Thus any national leader, whether democratically elected or not, may feel a political imperative to retain a patronage appointee in office even against some expression of local discontent with the quality of public services, unless key supporters in the capital can verify for themselves that the local discontent is genuine.

In this regard, one advantage of democracy is that freedom of speech and freedom of the press enable residents of remote towns to publicly express complaints about the quality of their local public services. Thus, democratic freedoms can help key supporters of the national leader to verifiably monitor widespread expressions of discontent in remote towns, and so can improve the central government's political ability to oversee local public services.

Under democracy we also find that successful leaders of autonomous local governments can become strong candidates for national leadership, which is against the interests of the incumbent national leader. This basic point can create a powerful bias against reforms for political decentralization in any centralized government, autocratic or democratic. In this regard, however, there are some noteworthy differences between these two kinds of regimes. On the one hand, compared to an autocrat who has never won a competitive election, a democratic national leader who has previously won with endorsement from a majority of voters might feel somewhat less vulnerable to competition from popularly trusted local leaders. But on the other hand, a national autocrat might have less fear of future competition from locally elected officials in very small districts, whose responsibilities cannot be compared to the vastly larger responsibilities of
national leadership. In a constitutional democracy, the ruling national coalition includes members of the legislature who may view elected mayors of even small towns as potential competitors for power. So in some cases, a democratic national government may resist reforms to decentralize power more than an autocratic government would. In Pakistan, for example, democratic local governments have been established three times under military rule, and each time these local governments were dissolved when democracy was restored at the national level (Cheema, Khan, and Myerson, 2015).

The most fundamental implication of democracy is that an elected national leader actually needs to earn the trust and approval of a large subset of the voters in the nation. Such democratic accountability could provide some incentive for a national leader to make local officials' positions dependent on their providing a quality of public services that can earn local residents' approval. So we might hope that, even in a unitary state that has no constitutional requirement for local officials to be held accountable to local voters, a national leader who must win a democratic election might promise such local accountability as part of a competitive strategy to win more voters' approval.

In a democratic election, however, the success of a campaign may depend both on promises of government policies that can benefit large groups of voters and on active support from a smaller number of wealthy and influential individuals. Candidates' competitive decisions about whether to promise local accountability of local officials or to use local offices as patronage prizes may depend on the relative importance of these factors in election campaigns. Campaign strategies in equilibrium could also depend on other details of the constitutional democratic system. But it may be instructive here to consider a general analytical model, to get a basic framework for thinking about these factors.

Let us consider a competition between two candidates for national leadership, in a nation that is administratively divided into many local districts that have equal size. To focus on the problem of supervising local administration in a centralized state, suppose that the national leader's principal power is the power to appoint and dismiss the local officials or magistrates who control public spending in each district, and the candidates have no significant differences on other questions of national policy.

We might assume that the two candidates compete by promising to each district j the per-capita level of public spending k_j that they would direct for their appointed local officials, subject to some overall public budget constraint. Rational informed voters should then vote for the
candidate who has offered more public spending in their district. For any given vector of offers from one candidate, the other candidate could win within the same overall budget by offering no public spending in at least one district where his opponent has offered the most, and then by promising to spend slightly more than his opponent in other districts that have a majority of voters. Thus, in a centralized national government without any constitutional protection for local public budgets, democratic competition can naturally create incentives for candidates to utterly neglect some regions of the country. Furthermore, a best-response dynamic of offering only slightly more that one's opponent in a majority of districts, while offering nothing to the district where the opponent has offered the most, could ultimately lead (in the dynamic of Kramer, 1977) to candidates offering very little even to their most favored districts. (Ferejohn, 1986, reaches a similar conclusion in his Proposition 6.)

To avoid such a result of bidding in a continuous scale, let us instead consider a model where each district has a legally fixed public budget $K>0$, funded out of national revenues, and each candidate for national leadership can only decide, for each district, whether or not to guarantee good local government by promising that their local magistrate will be held accountable to local public opinion. In effect, a candidate can promise to delegate his power over a local official to the local voters. But in any districts where the candidate does not make such a promise, the local voters will expect the candidate to sell the office to a rich donor, who then will steal their entire local public budget if the candidate is elected. Each candidate can make this decision independently for each district. Then in each district, informed voters will expect their level of local public services to be either high ($K$) or low ($0$), depending on whether the elected national leader has promised to make their local magistrate accountable to local politics or not. But suppose also (following Baron, 1994, and Grossman and Helpman, 1996) that the funds which are raised by corruptly selling local offices will be spent in the campaign on advertisements that will influence the votes of other voters who are uninformed or impressionable. When donors view the two candidates as equally likely to win the election, the campaign funds that each candidate can raise by selling offices will be proportional to the number of districts where the candidate has not promised good accountable local government.

Suppose that some fraction $\alpha$ of the national electorate are uninformed voters. For simplicity let us assume that, among these uninformed voters, the fraction who vote for candidate $i$ will be equal to the amount spent by candidate $i$ in the campaign divided by the total campaign spending of both candidates. The remaining $1-\alpha$ fraction of the electorate are rational informed
voters who will vote for the candidate who promises better government in their district. The informed voters will split equally in any district where the two candidates have promised the same local public spending (both K or both 0). In this game, let \( x_i \) denote the fraction of districts which candidate \( i \) decides to sell to a donor, so that \( 1-x_i \) is the fraction of districts in which candidate \( i \) is promising local political accountability for good local government. Then the net difference of total votes for candidate 1 minus total votes for candidate 2, will be

\[
V_1 - V_2 = \alpha(x_1 - x_2)/(x_1 + x_2) + (1-\alpha)[(1-x_1) - (1-x_2)].
\]

Candidate 1 chooses \( x_1 \) to maximize this difference, and candidate 2 chooses \( x_2 \) to minimize it.

By analyzing first-order conditions for each candidate \( i \)'s best response \( x_i \) to the other's \( x_{-i} \)

\[
0 = \partial(V_1-V_2)/\partial x_i = 2\alpha x_{-i}/(x_1+x_2)^2 - (1-\alpha)
\]

we can derive the unique symmetric equilibrium of this two-person zero-sum game.

**Proposition 3.** In the absence of any constitutional requirement to devolve local public services to autonomous local governments, the two-candidate electoral contest for national leadership has an equilibrium in which each candidate would maintain inefficient centralized control of local public services in a fraction \( x_1 = x_2 = \min\{0.5\alpha/(1-\alpha), 1\} \) of all districts.

For example, when a 60% majority of the electorate are rational informed voters but the other \( \alpha=40\% \) are uninformed voters who respond to campaign spending, then there is an equilibrium in which each candidate sells 1/3 of the districts to rich donors instead of providing good local government there. Thus, under the assumptions of this model, competitive democratic national politics could still yield regions where no informed residents expect any benefits from their government.

The possibility of such politically neglected regions should be considered extremely dangerous for a nation's territorial integrity, if there is any outside option for disaffected regions to secede. Of course this result depended on our assumption that candidates for national leadership are competing only on their promises for local public services which can differ across regions. But under the constitution of Ukraine, for example, the popularly elected President has primary responsibility for appointing the local administrative heads in the regions and districts of Ukraine, while national government policy is primarily directed by a prime minister who is responsible to the national assembly. The extreme regional polarization of presidential politics in Ukraine may be at least partly explained by the strategic incentives that are exhibited by this simple model of political competition in a centralized democratic government.
In the above model, the funds that a candidate could raise in a campaign by selling the promise of an office should actually be equal to the value of the office to the donor (here K) multiplied by the probability that the candidate will win the election. Thus, our above assumption, that the candidates can raise campaign funds that are equally proportional to their numbers of corruptly sold offices, may depend on a basic assumption that donors would perceive each candidate as equally likely to win independently of how many offices each sells. (This could be true, for example, if the donors pay the candidates for their promised offices before learning how many other offices are being sold.) This game could have other equilibria, however, if a candidate's unilateral deviation from some predicted scenario would make donors perceive him as a likely loser. Once a candidate perceived as likely to lose, nobody would pay anything for the candidate's promises of appointments, and so (against any $x_i \leq \alpha/(1-\alpha)$) the candidate would lose the election with no campaign funds and no uninformed votes, thus justifying the donors' perception. (Morton and Myerson, 2012, showed that such discontinuous equilibria can be pervasive in games with impressionable voters.)

For example, this game could have an equilibrium where any candidate's decision to sell any office to a donor would be considered so scandalous that the candidate would be considered a likely loser, who then could not raise any funds from such promises. But the forces in this alternative equilibrium may be considered as effectively equivalent to having a constitutional prohibition against selling offices. Indeed, any constitutional rule can be enforced by such self-justifying adverse expectations against a leader who was seen to violate it (Myerson, 2008).

5. Conclusions

When the quality of local public investments can be evaluated only by local residents, the management of these investments poses a fundamental political problem for a centralized unitary state. Officials can be given incentives to manage local public investments appropriately only if their career rewards are dependent on the broad approval from the local residents who can observe the quality of these investments. In a centralized unitary state, however, such local accountability may be incompatible with the natural imperative for a national leader to use local government offices as valued rewards for loyal supporters. Any dismissal from a valuable office that has been used as a patronage prize can create dangerous distrust among key supporters of the national leader, as they must recognize that the leader could potentially benefit of re-selling offices that become vacant. To maintain his supporters' trust, the national leader cannot dismiss
an appointed official without reasons that are broadly evident to his key supporters.

Thus, we have argued, if the quality of a local public investment cannot be monitored by key members of the national ruling coalition, then the public investment may be effectively infeasible for a unitary state without some constitutionally protected political decentralization. We analyzed a simple model of moral hazard in local public investments which could be efficiently managed by local officials who are accountable to autonomous local politics, but then we found that an autocratic national government could not make a credible commitment to such efficient local investments without granting some fundamental guarantee of local political rights. In a centralized autocracy, the local residents who can observe the quality of a local public investment may be unable to communicate their information to the national elite who must approve any decision to dismiss the responsible official.

National leaders may have compelling political reasons for wanting to maintain centralized control of local public services, even when it harms national economic development. One reason is that the leader's power is increased by the ability to allocate valuable offices of local government as rewards for loyal service and support. Another reason is that autonomous local government can become a source of popularly trusted competitors for national power, which is against the interests of incumbent national leaders. In a model of democratic elections in a unitary state, we found that competitive candidates for national leadership would choose to maintain inefficient centralized control of local government in some regions, even though informed voters would prefer a candidate who promised decentralized accountability.

However, a constitutional system with autonomous local government can become politically stable once it is established. When governors and mayors have been locally elected, they become local power-brokers from whom competitive candidates for national leadership must regularly seek support. Then it would be very costly for any national leader to threaten the constitutional powers of these elected local officials. In such a federal democracy with constitutionally protected powers for local government, the efficient solution to the moral hazard problem that we considered here can be straightforwardly implemented.

The founding leaders of the United States of America had no choice but to accept substantial decentralization of power in their new nation, because autonomous local governments in America had been established over a century before the first national elections. Since then, democratic local governments in America have had their share of problems and corruption, but they have overseen local public investments that provided the basic framework for building the
richest nation on earth.

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Appendix

Proposition 1. For any $\varepsilon>0$, renewal thresholds $\theta(n)$ and official salaries $\rho(n)$ can be set as functions of local population $n$ so that $\lim_{n \to \infty} \rho(n) \leq r(k_1) + \varepsilon$ and, with the efficient investment budget $b=k_1$, the induced public investment levels $\bar{k}(n)$ satisfy $\lim_{n \to \infty} \bar{k}(n) = k_1$ and $\lim_{n \to \infty} Q(\bar{k}(n),\theta(n),n) = 1$.

Proof. Choose the thresholds $\theta(n)$ to satisfy $\lim_{n \to \infty} \theta(n) = \pi(k_1)$ and $\lim_{n \to \infty} (\pi(k_1) - \theta(n))n^{0.5} = +\infty$. For example, we can let $\theta(n) = \pi(k_1) - \log(n)/n^{0.5}$. Then let
\[ \rho(n) = (1/Q(k_1,\theta(n),n) - \beta)k_1 + \varepsilon. \] This implies
\[ Q(k_1,\theta(n),n)(\rho(n) + \beta k_1) = k_1 + \varepsilon Q(k_1,\theta(n),n) > k_1, \]
and so appropriately investing \( k_1 \) is strictly better for the official than stealing it all, and
\( \bar{W}(n) > k_1. \) From \( \lim_{n \to \infty} (\pi(k_1) - \theta(n))n^{0.5} = +\infty, \) we get \( \lim_{n \to \infty} (\pi(k_1) - \theta(n))/\sigma(k_1,n) = +\infty, \) and so \( \lim_{n \to \infty} Q(k_1,\theta(n),n) = 1 \) and \( \lim_{n \to \infty} \rho(n) = (1-\beta)k_1 + \varepsilon = r(k_1) + \varepsilon. \) The official's optimal investment level \( \bar{k}(n) \) and expected payoff \( \bar{W}(n) \) then satisfy
\[ \bar{W}(n) = (k_1 - \bar{k}(n)) + Q(\bar{k}(n),\theta(n),n)(\rho(n) + \beta \bar{W}(n)) \]
\[ = \max \{(k_1 - k) + Q(k,\theta(n),n)(\rho(n) + \beta \bar{W}(n)) \mid 0 \leq k \leq k_1\} \]
\[ \geq Q(k_1,\theta(n),n)(\rho(n) + \beta \bar{W}(n)) \geq Q(k_1,\theta(n),n)(\rho(n) + \beta k_1) = k_1 + \varepsilon Q(k_1,\theta(n),n). \]
Notice that this last term converges to \( k_1 + \varepsilon \) as \( n \to \infty, \) and so \( \liminf_{n \to \infty} \bar{W}(n) \geq k_1 + \varepsilon. \)

We now show that there cannot be any subsequence on which \( \bar{k}(n) \) converges to a limit strictly less than \( k_1. \) If such a subsequence existed then we would get
\[ \liminf_{n \to \infty} \pi(\bar{k}(n)) < \pi(k_1) = \lim_{n \to \infty} \theta(n), \] and so
\[ \liminf_{n \to \infty} (\pi(\bar{k}(n)) - \theta(n))/\sigma(\bar{k}(n)) = -\infty, \] and \( \liminf_{n \to \infty} Q(\bar{k}(n),\theta(n),n) = 0. \)
But with \( \bar{k}(n) \geq 0, \) this would imply
\[ \liminf_{n \to \infty} \bar{W}(n) = \liminf_{n \to \infty} (k_1 - \bar{k}(n)) + Q(\bar{k}(n),\theta(n),n)(\rho(n) + \beta \bar{W}(n)) \leq k_1 + 0. \]
This contradiction of our conclusion from the preceding paragraph implies \( \lim_{n \to \infty} \bar{k}(n) = k_1. \)

Finally we show that that there cannot be any subsequence on which \( Q(\bar{k}(n),\theta(n),n) \) converges to a limit strictly less than \( 1. \) If such a subsequence existed then we would get
\[ \liminf_{n \to \infty} \bar{W}(n) = \liminf_{n \to \infty} (k_1 - \bar{k}(n)) + Q(\bar{k}(n),\theta(n),n)(\rho(n) + \beta \bar{W}(n)) \]
\[ = 0 + \liminf_{n \to \infty} Q(\bar{k}(n),\theta(n),n)(\rho(n) + \beta \bar{W}(n)) \]
\[ < \lim_{n \to \infty} (\rho(n) + \beta \bar{W}(n)) = \lim_{n \to \infty} (k_1 - k_1) + Q(k_1,\theta(n),n)(R(n) + \beta \bar{W}(n)), \]
which would contradict the optimality of \( \bar{k}(n) \) over all \( k \) such that \( 0 \leq k \leq k_1. \)