CAPITALIST INVESTMENT AND POLITICAL LIBERALIZATION

by Roger Myerson, 12/2008, http://home.uchicago.edu/research/caplib.pdf

F = (fixed-resource endowment). Additional invested capital is durable and mobile.

$Y(F+k)$ is net output production (flow) with any investment $k \geq 0$.

Suppose $Y(\bullet)$ is differentiable, strictly concave, $Y'(0) = +\infty$, $Y'(+\infty) = 0$.

Capitalists' rate of time discounting is r. They must get rent rk on investment k.

ρ = (rate of time discounting for an authoritarian ruler).

With investment k, authoritarian ruler gets value $(Y(F+k)-rk)/\rho$.

θ = (fraction of investment that the ruler could expropriate).

With worst reputational threat, expropriation would yield $\theta k + Y(F)/\rho$.

Capitalist investment k is <u>feasible without liberalization</u> iff

$(Y(F+k)-rk)/\rho \geq \theta k + Y(F)/\rho$.

Equivalently, $Y(F+k)-(r+\rho\theta)(F+k) \geq Y(F)-(r+\rho\theta)F$.

Let $K_r = \text{argmax}_{\kappa\geq 0}\, Y(\kappa) - r\kappa$,

$K_{r+\rho\theta} = \text{argmax}_{\kappa\geq 0}\, Y(\kappa) - (r+\rho\theta)\kappa$.

$(Y(F+k)-rk)/\rho \geq \theta k + Y(F)/\rho$ iff $Y(F+k)-(r+\rho\theta)(F+k) \geq Y(F)-(r+\rho\theta)F$.
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<u>Thm 1</u>. $\exists h_0$ such that k is feasible without liberalization iff $0 \leq k \leq h_0$.
In this interval, the optimum for ruler is $k = \min\{h_0, K_r-F\}$.
If $F \geq K_{r+\rho\theta}$, then $h_0 = 0$.
If $F < K_{r+\rho\theta}$, then $F+h_0 > K_{r+\rho\theta}$ and $Y(F+h_0)-(r+\rho\theta)(F+h_0) = Y(F)-(r+\rho\theta)F$.
When $F < K_{r+\rho\theta}$, a small increase in F would cause both h_0 and $F+h_0$ to decrease.

<u>Cor 1.</u> Given $(Y(),r,\rho,\theta)$, if $Y(K_r)-(r+\rho\theta)K_r < Y(0)$ then the ideal investment K_r-F is not feasible without liberalization for any fixed endowment $F < K_r$.
But if $Y(K_r)-(r+\rho\theta)K_r \geq Y(0)$, then there exists some f_0 such that the ideal investment K_r-F is feasible without liberalization iff $F \leq f_0$.
This bound f_0 satisfies $Y(f_0)-(r+\rho\theta)f_0 = Y(K_r)-(r+\rho\theta)K_r$ and $f_0 < K_{r+\rho\theta} < K_r$.
(So F near $K_{r+\rho\theta}$ always makes ideal K_r infeasible without liberalization.)

<u>Cor 2.</u> If the production function is $Y(\kappa) = A\kappa^{\alpha}$, for some $A>0$ and $0<\alpha<1$, then the ideal investment K_r-F is feasible without liberalization for F=0 iff $\alpha \leq r/(r+\rho\theta)$.
(So even at F=0, ideal K_r is infeasible without liberalization when $\alpha>r/(r+\rho\theta)$.)

Liberalization λ is probability of ruler losing power if he tried to expropriate capital.
False-alarm scandals occur at rate ψ.
Ruler's present discounted value is $V(k,\lambda) = (Y(F+k)-rk)/(\rho+\psi\lambda)$.
Trying to expropriate would yield $W(k,\lambda) = (1-\lambda)(\theta k + Y(F)/\rho)$.
Optimal regime (k,λ) maximizes $V(k,\lambda)$ s.t. $V(k,\lambda) \geq W(k,\lambda)$, $k\geq 0$, $0\leq\lambda\leq 1$.

Let $Q(k) = [Y(F+k)-rk]/[\theta k + Y(F)/\rho]$, and $q(\lambda) = (\rho+\psi\lambda)(1-\lambda)$.
So $V(k,\lambda)\geq W(k,\lambda)$ iff $Q(k) \geq q(\lambda)$.
Let $\Lambda(k)$ denote the smallest $\lambda\geq 0$ such that $Q(k) \geq q(\lambda)$.
Optimal k maximizes $V(k,\Lambda(k))$ over $k\geq 0$.
Notice $q(\lambda) = \rho+(\psi-\rho)\lambda-\psi\lambda^2$, which is maximized at $(1-\rho/\psi)/2$,
and $q(0) = \rho = q(1-\rho/\psi)$, $q(1)=0$.
So if $Q(k) \geq \rho$ then $\Lambda(k) = 0$.
If $Q(k) < \rho$ then $\Lambda(k) = \{\psi-\rho + [(\psi-\rho)^2 + 4\psi(\rho-Q(k))]^{0.5}\}/(2\psi)$.

<u>Thm 2.</u> If $(k,0)$ is optimal, then $k=\min\{h_0, K_r-F\}$, as in Thm 1.
If (k,λ) is optimal with $\lambda>0$, then $Q(k) = q(\lambda)$, $Y'(F+k) = r + \theta\psi(1-\lambda)^2$,
$\lambda = \Lambda(k) > 1-\rho/\psi$, and $k > h_0$.

Recall that the worst endowment without liberalization was $F = K_{r+\rho\theta}$.
Incentives for liberalization may be greatest with such intermediate endowments.

Cor 3. If $F = K_{r+\rho\theta}$ and $\psi < \rho$ then the optimal regime has $\lambda > 0$.

Strong liberalization becomes optimal in cases where investments yield approximately constant returns for national output.
Approximate linearity implies that, when $Y' > r$, large investments can yield surplus returns, but then strong liberalization is needed to protect these investments.

Theorem 3. Consider production functions of the form $Y(F+k) = A(F+k)^{\alpha}$, where $A>0$ and $0<\alpha<1$, so that the parameters of our models are $(r,\rho,\theta,\psi,F,A,\alpha)$.
Consider a sequence of models where $(r,\rho,\theta,\psi,A,\alpha)$ converge to finite positive limits, and the fixed endowments F satisfy $\lim Y'(F)/r > 1$.
If $\lim \alpha = 1$ then the optimal (k,λ) satisfy $\lim Y'(F+k)/r = 1$ and $\lim \lambda = 1$.

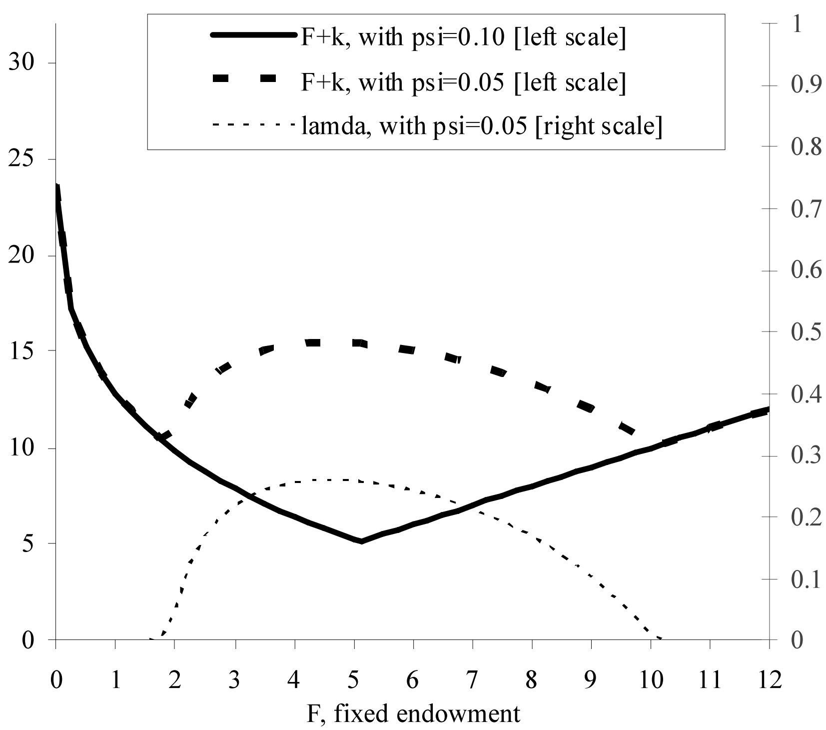
Consider adding fixed revenue for the government, independent of invested capital. Adding a fixed revenue means adding a positive constant $z>0$ to the production function, changing it from $Y(\kappa)$ to $\hat{Y}(\kappa) = Y(\kappa)+z$ for all $\kappa \geq 0$, keeping all other parameters unchanged.

Thm 4. Adding a fixed revenue $z>0$ would allow greater investments to be feasible with any given liberalization λ such that $\max\{0,1-\rho/\psi\} < \lambda < 1$.
But when the optimal regime has positive liberalization λ, adding a fixed revenue would decrease both the optimal liberalization and the optimal investment.
When the optimal regime involves no liberalization ($\lambda=0$), adding a fixed revenue would not change the optimal regime.

Proof: Adding $z>0$ to Y increases V by $z/(\rho+\psi\lambda)$ and increases W by $z(1-\lambda)/\rho$.
With $\lambda > \max\{0,1-\rho/\psi\}$, we get $(1-\lambda)(\rho+\psi\lambda) < \rho$ and $z/(\rho+\psi\lambda) > z(1-\lambda)/\rho$.
Let $K(\lambda) = \Lambda^{-1}(\lambda)$. Adding $z>0$ increases $K(\lambda)$.
$K'(\lambda) = q'(\lambda)/Q'(K(\lambda)) = (\psi-\rho-2\psi\lambda)\,(\theta k+Y(F)/\rho)/[Y'(F+K(\lambda))-r-\theta q(\lambda)]$.
$d/d\lambda\ LN(V(K(\lambda),\lambda)) = (1/W)\ d/d\lambda\ [(1-\lambda)(\theta K(\lambda) + Y(F)/\rho)]$
$\quad = \theta\,(2\psi\lambda+\rho-\psi)/[r+\theta q(\lambda)-Y'(F+K(\lambda))] - 1/(1-\lambda)$, decreasing in z (thru K).
So increasing z decreases optimal λ and k, with $Y'(F+k) = r + \theta\psi(1-\lambda)^2$.

$\max_{k \geq 0,\, \lambda \in [0,1]} \; V(k,\lambda) = (Y(F+k) - rk)/(\rho + \psi\lambda)$ s.t $V(k,\lambda) \geq (1-\lambda)(\theta k + Y(F)/\rho)$.

<u>Example.</u> Consider an island with production $Y(F+k) = (F+k)^{0.4}$,
investors' discount rate $r=0.05$, authoritarian-ruler's discount rate $\rho=0.1$,
expropriatable fraction $\theta=1$, and scandal rate $\psi=0.1$.
The ideal investment $K_r=32$ (with $Y(K_r)=4$) is not feasible without liberalization.
With $F=0$, optimum is $k = 23.61$, $\lambda = 0$; then $Y(F+k) = 3.54$, $V(k,\lambda) = 23.61$.
With $F=K_{r+\rho\theta} = 5.128$, optimum is $k=0$, $\lambda=0$; then $Y(F+k) = 1.92$, $V(k,\lambda) = 19.23$.



The curse of natural resources even harms the ruler here.

$\lambda=0$ is optimal $\forall F$ when $\psi=0.10$ here.

But reducing the scandal rate to $\psi=0.05$ makes positive liberalization λ optimal for intermediate endowments $1.9 < F < 10.1$.

Example. Consider an archipelago where production is $(F+k)^{0.4}L^{0.5}$.
Each island has labor $L=1$, fixed capital $F=5$, $r=0.05$, $\rho=0.1$, $\theta=1$, $\psi=0.1$.
When L are serfs, we get $Y=(F+k)^{0.4}$, optimum $k=0.258$, $\lambda=0$; $Y=1.94$, $V=19.29$.
A free labor fringe has marginal product $0.5Y/L = 0.971$.

Suppose one island frees its serfs, extensively recruits mobile free labor at wage w:
$Y(F+k) = \max_{L\geq 0} (F+k)^{0.4}L^{0.5} - wL = (F+k)^{0.8}/(4w)$, with $L = (F+k)^{0.8}/(2w)^2$.
With $w=0.971$, we get $Y(F+k) = 0.2574(F+k)^{0.8}$,
optimum is $k=1126$, $\lambda=0.931$, with $L=73.5$, yielding $Y = 71.36$, $V(k,\lambda) = 77.95$.
The possibility of matching capitalist investments with additional labor creates a strong incentive for liberalization ($77.95 > 19.29$).
But $73.5>1$. If all free serfs and compete for free labor, the wage must increase.

With $w=1.777$, we have $Y(F+k) = 0.1407(F+k)^{0.8}$ and get two optimal regimes:
liberal optimum $k=47.73$, $\lambda=0.903$, with $L=1.89$, $V=5.10$;
nonliberal optimum $k=0$, $\lambda=0$, with $L=0.29$, $V=5.10$. ($K_r-F=57.8-5=52.8$.)
Average matches supply $L=1$ when 44% of islands are liberal, 56% nonliberal.

Reducing ψ can decrease optimal λ. An island with $\psi=0.05$ has $k=48.44$, $\lambda=0.874$;
$\lim_{\psi\to 0} \lambda = 1 - [Y(K_r)-r(K_r-F)]/[Y(F)+\rho\theta(K_r-F)] = 0.832$.

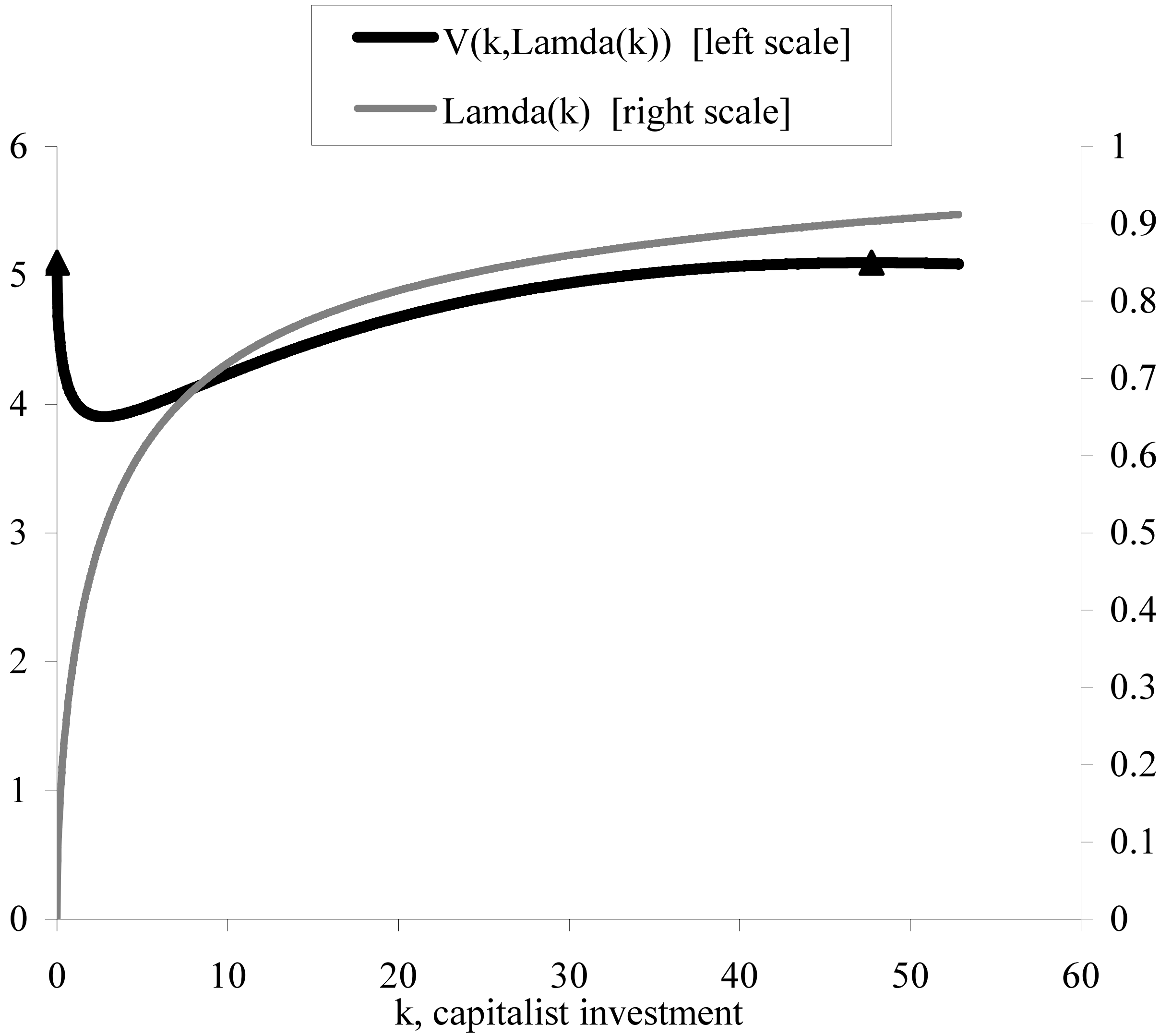


Figure 2. Required liberalization Λ(k) and ruler's value V(k,Λ(k)), for $Y(F+k)=0.1407(F+k)^{0.8}$ with F=5, r=.05, ρ=.1, ψ=.1, θ=1; from w=1.777.