

A MODEL OF MORAL-HAZARD CREDIT CYCLES,

by Roger Myerson, June 2010

Paper: <http://home.uchicago.edu/~rmyerson/research/bankers.pdf>

These notes: <http://home.uchicago.edu/~rmyerson/research/banknts.pdf>

Model: <http://home.uchicago.edu/~rmyerson/research/bankers.xls>

Can microeconomics of banking explain macroeconomic fluctuations?

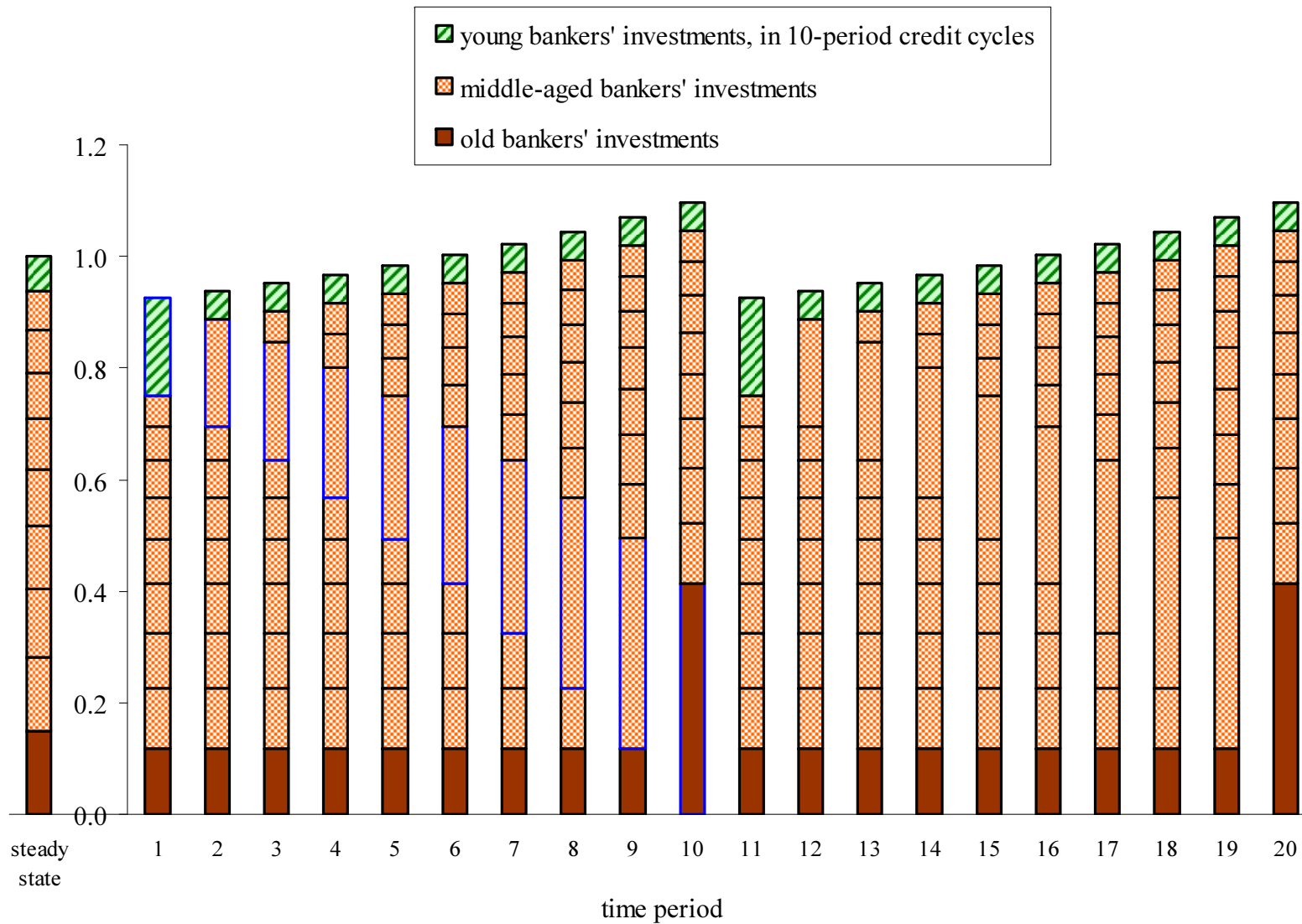
This paper offers a simple model to show how macroeconomic fluctuations can be driven purely by problems of moral hazard in financial intermediation.

Other factors are omitted here: no money, no long-term illiquid assets, stationary nonstochastic environment.

Bankers' contractual positions or wealth form the state of this dynamic system.

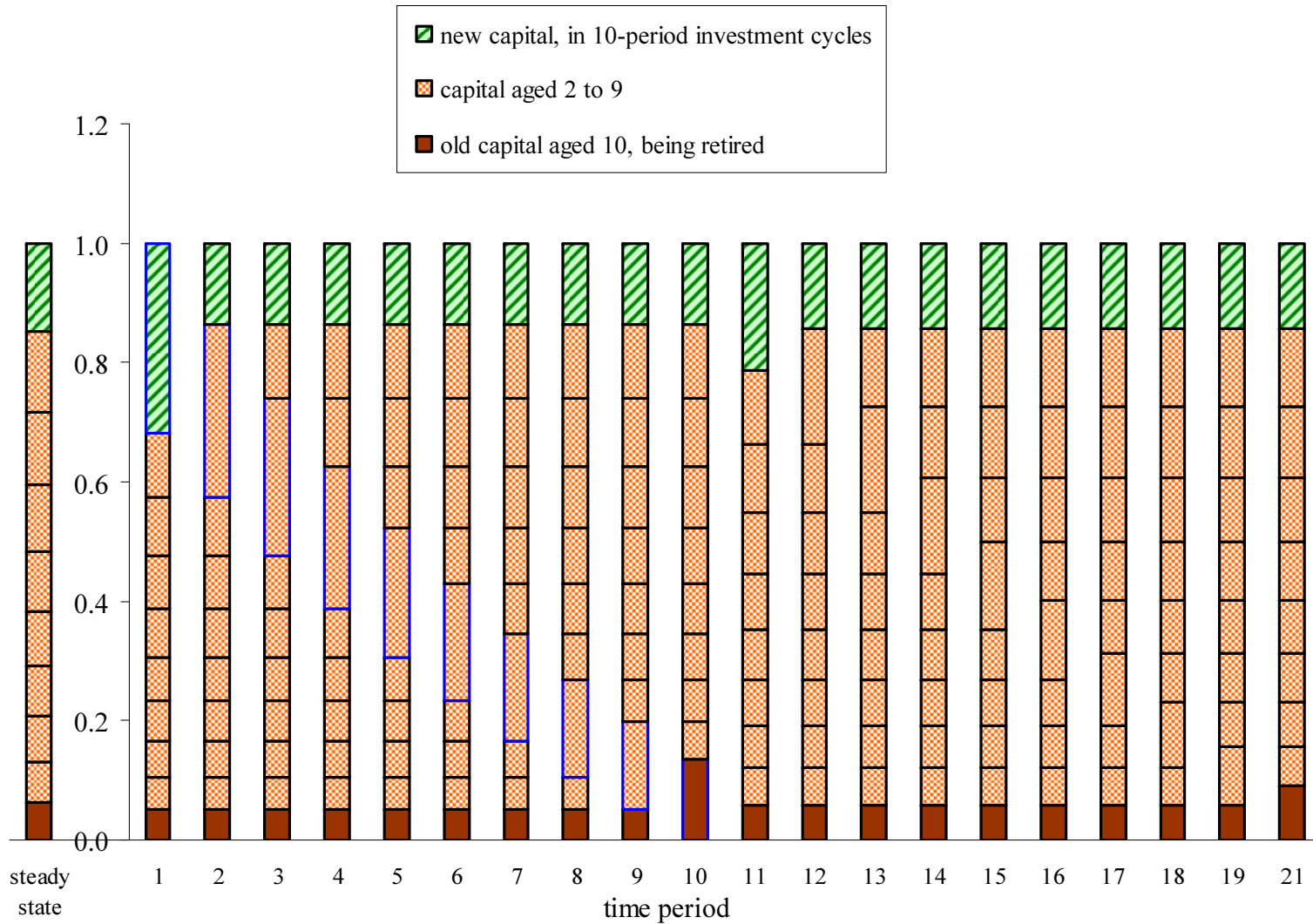
We find multiple dynamic equilibria, including boom-and-bust credit cycles.

In such cycles, when investment is weak, a bailout or stimulus that uses poor workers' taxes to subsidize rich bankers can actually make the workers better off.



Investment amounts handled by different cohorts of bankers with 10-period careers, starting at time 1 with bankers investing only 80% of steady-state amounts.

($\rho=0.1$, $n=10$, $\alpha=0.95$, $\beta=0.6$, $\gamma=0.05$, $\eta=0$, $\psi=1.74$, $\pi = 0.422$). Older cohorts are lower.



The usual assumptions about depreciation of capital do not yield such instability. (Here capital depreciates 10% per year, scrapped after 10; initial 20% shortage.)

A model of investment with moral hazard by entrepreneurs and bankers

There is one commodity (grain). It can be consumed or invested, lasts one period.

Individuals' wealths are much smaller than the minimal size of a good investment, so they must pool their wealth and rely on a banker to identify good projects.

Each agent has risk-neutral utility for consumed grain, with **time discount rate** ρ .

If an investment of h at time t succeeds then, at time $t+1$, it returns $r_{t+1}h$, else 0.

A decreasing investment-demand function R specifies **rate of return** $r_{t+1} = R(I_t)$ that successful investments yield when aggregate investment has been I_t . [slide 10]

Good investments of sizes $h \geq 1$ (billion) are found by entrepreneurs, who must manage them, but anybody can find bad investments in all sizes ≥ 0 .

Probability of success is α if investment is good, β if bad; $\alpha > \beta$.

From investment h at t , **entrepreneur could divert γh** for $t+1$ consumption, but this diversion would make a good project bad. (No effect on bad project.)

Banker could divert ηh by not verifying quality of investment h .

Bad projects are unprofitable when $\gamma + \eta + \beta r_{t+1} < 1 + \rho$. For this in eqm, assume:

$$[2] \quad \gamma / (1 - \beta / \alpha)^3 + \eta / (1 - \beta / \alpha)^2 < 1 + \rho \quad (\gamma \text{ and } \eta \text{ are not too large}).$$

Each individual lives $n+1$ periods, so bankers can invest up to n periods.

$$[\rho=0.1, \alpha=0.95, \beta=0.6, \gamma=0.05, \eta=0, n=10, r=R(I)=1.74-0.233\alpha I]$$

Moral-hazard incentive constraints and rents for entrepreneurs and bankers

Consider an investment project of size h , and consider a contract in which the entrepreneur and banker respectively get **payoffs e and b** in case of success.

Limited liability: entrepreneurs and bankers cannot get worse than 0 if fail.

The entrepreneur could divert $h\gamma$ and make the project bad, reducing the probability of success from α to β . The entrepreneur's moral-hazard constraint is $\alpha e \geq h\gamma + \beta e$.

Let $E = \gamma / (\alpha - \beta)$.

So the minimal compensation for a successful entrepreneur is $e = h\gamma / (\alpha - \beta) = hE$.

This amount hE may be called the entrepreneur's moral-hazard rent.

Banker could choose a bad project, divert $(\eta + \gamma)h$, get e kickback if it succeeds.

The banker's moral-hazard constraint is $\alpha b \geq (\eta + \gamma)h + \beta(b + e)$.

Thus, the minimal pay for the banker after success is

$$b = h(\eta + \gamma + \beta E) / (\alpha - \beta) = h[\eta / (\alpha - \beta) + \alpha\gamma / (\alpha - \beta)^2] = hB$$

where $B = (\eta + \gamma + \beta E) / (\alpha - \beta) = \eta / (\alpha - \beta) + \alpha\gamma / (\alpha - \beta)^2$.

When a project of size h succeeds, the banker's expected payoff must be worth at least hB . This amount hB may be called the banker's moral-hazard rent.

(Entrepreneurs find their projects at age $n-1$, consume all their rents next period.)

Long-term investment with a banker who invests n periods

Each individual lives $n+1$ periods. Consider a consortium that hires an individual born at time 0 to be their banker, to pick good investments at times $t=0, \dots, n-1$.

Consortium will pay the banker some amount b_n in her last period n if all her projects succeed, dismiss to 0 after failure (**maximal punishment & backloading**).

The investment at time t will be some h_t as long as the banker has not failed.

Suppose that the banker starts at time 0 with a minimal investment $h_0=1$.

At time 0, the consortium's expected discounted net profit is

$$\sum_{t \in \{0, \dots, n-1\}} \alpha^{t+1} h_t [r_{t+1} - E - (1+\rho)/\alpha] / (1+\rho)^{t+1} - \alpha^n b_n / (1+\rho)^n.$$

The banker's expected payoff from success at time $t+1$ is $b_n [\alpha / (1+\rho)]^{n-(t+1)}$.

To satisfy the banker's moral-hazard incentive constraint at every period t , b_n and h_t must satisfy $b_n [\alpha / (1+\rho)]^{n-(t+1)} \geq h_t B = h_t [\eta / (\alpha - \beta) + \alpha \gamma / (\alpha - \beta)^2]$.

For good investments to be worthwhile, rates of return r_{t+1} must satisfy

$$[3] \quad r_{t+1} \geq E + (1+\rho)/\alpha = \gamma / (\alpha - \beta) + (1+\rho)/\alpha, \quad \forall t \in \{0, \dots, n-1\}.$$

Then given b_n , the optimal investment at each t is $h_t = b_n [\alpha / (1+\rho)]^{n-(t+1)} / B$.

With $h_0=1$, we get $b_n = B [(1+\rho)/\alpha]^{n-1}$, so $h_t = [(1+\rho)/\alpha]^t, \quad \forall t$.

In equilibrium, any such consortium must expect zero net profit, that is

$$\sum_{t \in \{0, \dots, n-1\}} [r_{t+1} - E - (1+\rho)/\alpha] \alpha / (1+\rho) - B \alpha / (1+\rho) = 0, \quad \text{and so}$$

$$[4] \quad \sum_{t \in \{0, \dots, n-1\}} [r_{t+1} - E - (1+\rho)/\alpha] = B.$$

Terms of employment for bankers hired by an investing consortium

The banking surplus is $\sigma_{t+1} = r_{t+1} - E - (1+\rho)/\alpha$.

In equilibrium, by [3]-[4], $\sigma_{t+1} \geq 0$ and $\sigma_{t+1} + \dots + \sigma_{t+n} = B$, $\forall t$.

Solutions must cycle: $\sigma_{t+n} = B - (\sigma_{t+1} + \dots + \sigma_{t+n-1}) = \sigma_t$ and $r_{t+n} = r_t \forall t$.

With [2], $(1+\rho)/\alpha + E \leq r_{t+1} \leq (1+\rho)/\alpha + E + B < (1+\rho - \gamma - \eta)/\beta$...so bad is bad.

If succeed to t+1, investors reinvest $h_t(1+\rho)/\alpha$, get $\sigma_{t+1}h_t = [r_{t+1} - E - (1+\rho)/\alpha]h_t$.

If succeed to n, banker gets $h_{n-1}B$, investors get final dividend $[r_n - E - B]h_{n-1}$.

With $\alpha > \beta$ and [2], we have $B < B\alpha/(\alpha - \beta) < (1+\rho)/\alpha$.

By [3], the dividends are all nonnegative. **No external funding is needed after h_0 .**

At time s, future dividends have expected value $h_s[\sum_{t>s} \sigma_t + (1+\rho)/\alpha - B]\alpha/(1+\rho)$;

by [4] this $=h_0$ at $s=0$; but $<h_s$ at $s>0$, so **investors must be committed to reinvest.**

(or let $h_s - h_s[\sum_{t>s} \sigma_t + (1+\rho)/\alpha - B]\alpha/(1+\rho) = h_s[\sum_{t \leq s} \sigma_t]\alpha/(1+\rho)$ be banker's capital)

Returns r_{t+1} always satisfy $r^* = (1+\rho)/\alpha + E \leq r_{t+1} \leq (1+\rho)/\alpha + E + B$.

When $r_{t+1} = r^*$, good investments yield no banking surplus over risk-free ρ -bonds.

In this case, bankers could also invest in bonds; then to fulfill the contract, their investors would re-invest all risk-free returns $(1+\rho)h_t$ if $t+1 < n$, which still yields the expected final payoff $\alpha B h_0 (1+\rho)^{n-1}$ that the banker requires whenever $r_{t+1} > r^*$.

Implementing the equilibrium without long-term commitments by investors

In these n-period equilibria, a consortium would suffer expected losses if it recruited a banker without capital who would serve strictly fewer than n periods.

So we find a kind of illiquidity: although all investments are short (1-period), in equilibrium, investors need a long n-period relationship with the bankers.

But with regulation, these equilibria can be also implemented by a system where bankers accumulate capital and invest under age-dependent leverage constraints.

The young banker starts her investment career at time 0 with no capital, $k_0 = 0$.

In period t, to invest h_t , a banker with capital k_t must borrow $h_t - k_t$.

The banker must promise to repay investors $(h_t - k_t)(1 + \rho) / \alpha$ at t+1, if success.

Success at time t+1 will yield banker's capital $k_{t+1} = \sigma_{t+1} h_t + k_t(1 + \rho) / \alpha$.

With $h_t = h_0 [(1 + \rho) / \alpha]^t$, induction implies: $k_t = h_t (\sigma_1 + \dots + \sigma_t) \alpha / (1 + \rho)$, $\forall t$.

So the banker can implement the investment h_t in any period $t = 1, \dots, n - 1$

when her maximal leverage ratio is $h_t / k_t = (1 + \rho) / (\alpha \sigma_1 + \dots + \alpha \sigma_t)$.

Regulation is needed to ensure that bankers will not leverage investment over legitimate capital earnings more than this, where *legitimate earnings* exclude diverted investment and kickbacks $[(\gamma + \eta + E)h_t]$. (prohibit money laundering)

The incentive constraint at t is, with $v_{t+1} = B / (\sigma_1 + \dots + \sigma_{t+1}) = [\alpha / (1 + \rho)]^{n - (t+1)} k_n / k_{t+1}$,
 $\alpha [\sigma_{t+1} h_t + k_t (1 + \rho) / \alpha] v_{t+1} \geq \beta [\sigma_{t+1} h_t + k_t (1 + \rho) / \alpha] v_{t+1} + (\eta + \gamma + \beta E) h_t$.

Investment-demand constraints on equilibrium credit cycles

Now consider any cycle of returns that repeats (r_1, \dots, r_n) every n time periods.

If these satisfy [3] $\sigma_{t+1} = r_{t+1} - E - (1+\rho)/\alpha \geq 0$ and [4] $\sigma_{t+1} + \dots + \sigma_{t+n} = B, \forall t$, then, in each period, young bankers without capital can start n -period careers.

Let J_t denote the total investment handled by young age=0 bankers at time t .

At time $t+s$, for $1 \leq s < n$, this generation of bankers handle total investment $J_t(1+\rho)^s$, because each period a fraction α succeed and multiply investments by $(1+\rho)/\alpha$.

So the total investment at time t must be $I_t = \sum_{s \in \{0, \dots, n-1\}} J_{t-s}(1+\rho)^s, \forall t$.

This system of equations has a cyclical solution that repeats (J_1, \dots, J_n) iff

$$J_t = [(1+\rho)I_{t-1} - I_t] / [(1+\rho)^n - 1], \forall t.$$

Total investment at time t determines the rate of return r_{t+1} by an investment-demand function $R(I_t)$.

When $r_{t+1} = r^* = (1+\rho)/\alpha + E$, ρ -bonds can replace good investment projects.

So we get the adjusted investment demand $r_{t+1} = R^*(I_t) = \max\{R(I_t), r^*\}, \forall t$.

Aggregate investment of young bankers at any time must be nonnegative, $J_t \geq 0$, so

$$\text{[5] } I_t \leq (1+\rho)I_{t-1} \text{ and } R^*(I_t) = r_{t+1}, \forall t.$$

An n -period equilibrium credit cycle is any returns sequence (r_1, \dots, r_n) that satisfies [3]-[5] above with an aggregate investment sequence (I_0, \dots, I_{n-1}) .

A simple model of investment demand and labor supply

For an example, let us construct a simple linear investment-demand function R , with two parameters ψ and π .

Suppose that any investment of size h will, if it succeeds, return ψh units of grain in the next period, but then harvesting this grain will require h units of labor.

Aggregate labor demand at time $t+1$ is αI_t , when I_t was invested at t .

Suppose that the wage rate is determined by a linear labor-supply curve: $w = \pi \alpha I_t$. (Workers who supply labor h at wage rate w get utility $wh - 0.5\pi h^2$. So $h = w/\pi$.)

So $w_{t+1} = \pi \alpha I_t$, and the investment-demand function is $R(I_t) = \psi - w_{t+1} = \psi - \pi \alpha I_t$.

Workers' aggregate wages at $t+1$ are $W_{t+1} = \pi(\alpha I_t)^2$, their net utility is $0.5W_{t+1}$.

Investment is always positive in equilibrium if $\psi > (1+\rho)/\alpha + E + B$.

Largest investment compatible with $R(I_t) > r^* = (1+\rho)/\alpha + E$ is $I^* = (\psi - r^*)/(\pi \alpha)$.

When bankers are promised investments over I^* , excess is diverted to ρ -bonds.

So we get adjusted investment demand $r_{t+1} = R^*(I_t) = \max\{\psi - \pi \alpha I_t, r^*\}$, $\forall t$.

But if (r_1, \dots, r_n) satisfies [5] with the aggregate investments (I_0, \dots, I_{n-1}) , then it satisfies [5] also with the alternative investment sequence $\check{I}_t = \min\{I_t, I^*\}$, $\forall t$.

Steady-state solutions

A steady-state n-period equilibrium has $r_{t+1} = (1+\rho)/\alpha + E + B/n, \forall t$.

Bankers' ability to have longer relationships with investors increases production.

Example: Let $\rho=0.1, \alpha=0.95, \beta=0.6, \gamma=0.05, \eta=0, \pi=0.233, \psi=1.74$.

Then $E=0.143, B=0.388, r^* = 1.301$, and $r^* + B = 1.689$.

In a steady-state n-period equilibrium credit cycle, at each time t,

investment returns are $r_{t+1} = (1+\rho)/\alpha + E + B/n = 1.301 + 0.388/n$,

aggregate investment is $I_t = (\psi - r_{t+1})/(\pi\alpha) = 1.983 - 1.751/n$,

and total net output is $Y_{t+1} = (\alpha\psi - (1+\rho))I_t = 1.097 - 0.968/n$,

of which workers get $W_{t+1} = \pi(\alpha I_t)^2 = 0.828 - 1.461/n + 0.645/n^2$,

entrepreneurs get $E\alpha I_t = 0.269 - 0.238/n$,

and profit for bankers is $[r_{t+1} - E - (1+\rho)/\alpha]\alpha I_t = (B/n)\alpha I_t = 0.731/n - 0.645/n^2$.

With $n=1$, the steady-state has $r = 1.689, I=0.233, Y = 0.129$,

including 0.011 for workers, 0.032 for entrepreneurs, and 0.086 for bankers.

With $n=10$, the steady-state has $r = 1.340, I=1.808, Y = 1$ (π chosen for 1 here),

including $W=0.688$ for workers, 0.245 for entrepreneurs, and 0.067 for bankers.

New bankers each period start investing $J=0.113$, and $J[1+(1+\rho)+\dots+(1+\rho)^{n-1}] = I$.

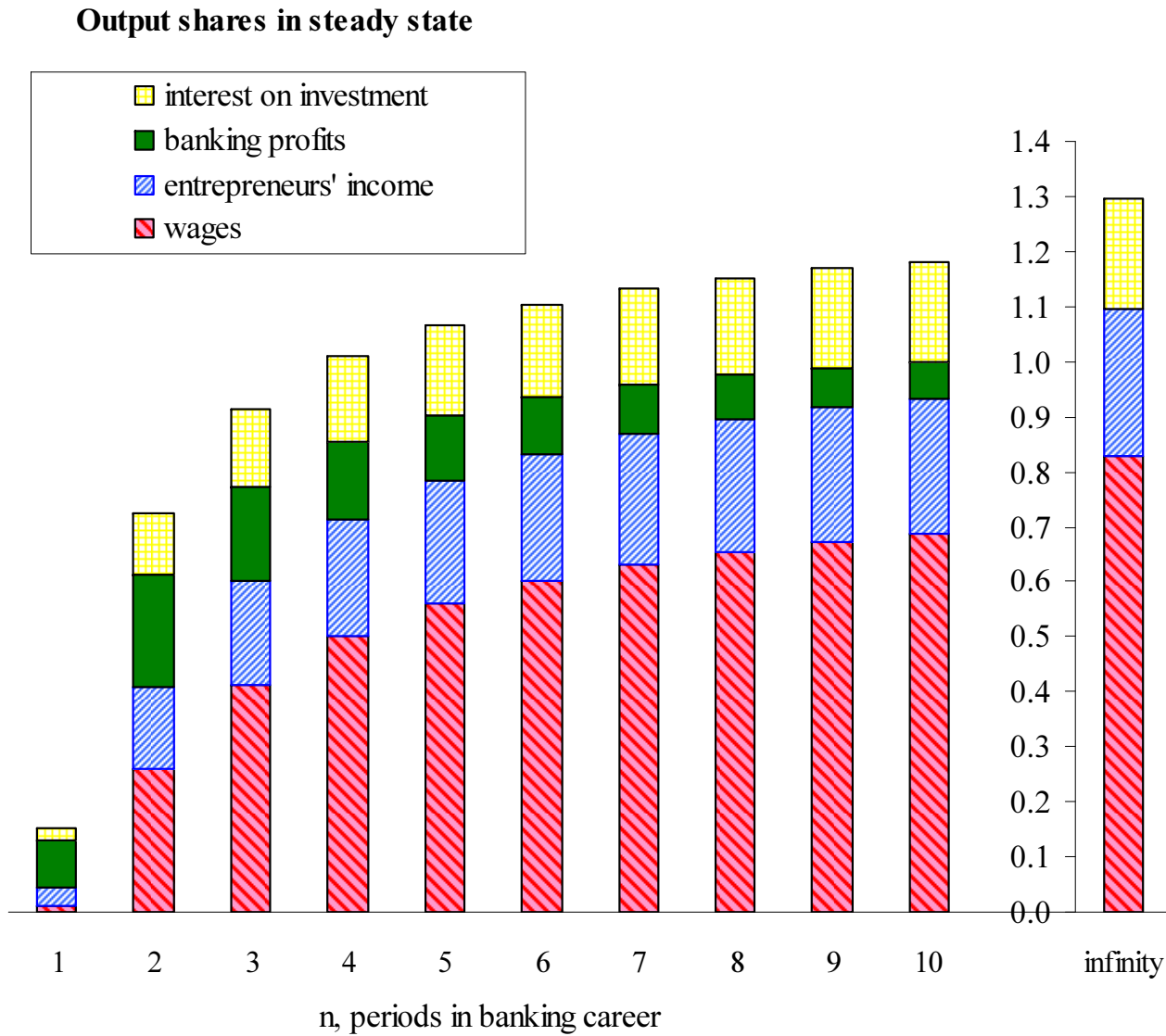


Figure 0. Output shares in steady state, with parameters: $\rho=0.1$, $\alpha=0.95$, $\beta=0.6$, $\gamma=0.05$, $\eta=0$, $\psi=1.74$, $\pi = 0.233$. (Differences resemble those of development.)

Example: dynamics of a financial crisis

At any time t , the aggregate investments $\theta_s(t) = J_{t-s}(1+\rho)^s$ handled by bankers of ages $s=1, \dots, n-1$ define inherited the state of the economy.

Consider the above example with $n=10$, with bankers of ages 1 to 9 handling time-0 investments θ_s that are **80% of the steady-state values Θ_s** :

$$(\theta_1, \dots, \theta_9) = (0.100, 0.110, 0.121, 0.133, 0.146, 0.161, 0.177, 0.195, 0.214).$$

The resulting 10-period equilibrium credit cycle is shown in Figure 1 below.

A large cohort of young (age=0) bankers enter to handle investment $J_0 = \mathbf{0.318}$, but **time-1 output is 7.5% below steady state**, with $r_1=1.37$ and $\sigma_1 = 0.069$.

In the shadow of J_0 , **subsequent cohorts of young bankers are smaller: $J_t = 0.091$. The economy gradually grows, just reaching steady-state output at time 6.**

Thereafter, growing investments of the large generation-0 bankers put the economy into a boom with investment and output greater than in the steady state, reaching **peak at time 10, with output 9.6% above steady state**, returns $r_{10}=1.30$.

Workers get $(W_1, \dots, W_{10}) = (.589, .605, .623, .643, .666, .691, .719, .751, .786, .826)$

At time 10, the generation-0 bankers retire and consume their accumulated profits, thus creating a scarcity of investment intermediaries and a recession as at time 0.

In steady state: $r=1.34$, $\sigma = B/n = 0.039$, $J=0.113$, $W=0.688$,

$$(\Theta_1, \dots, \Theta_9) = (0.125, 0.137, 0.151, 0.166, 0.183, 0.201, 0.221, 0.243, 0.268).$$

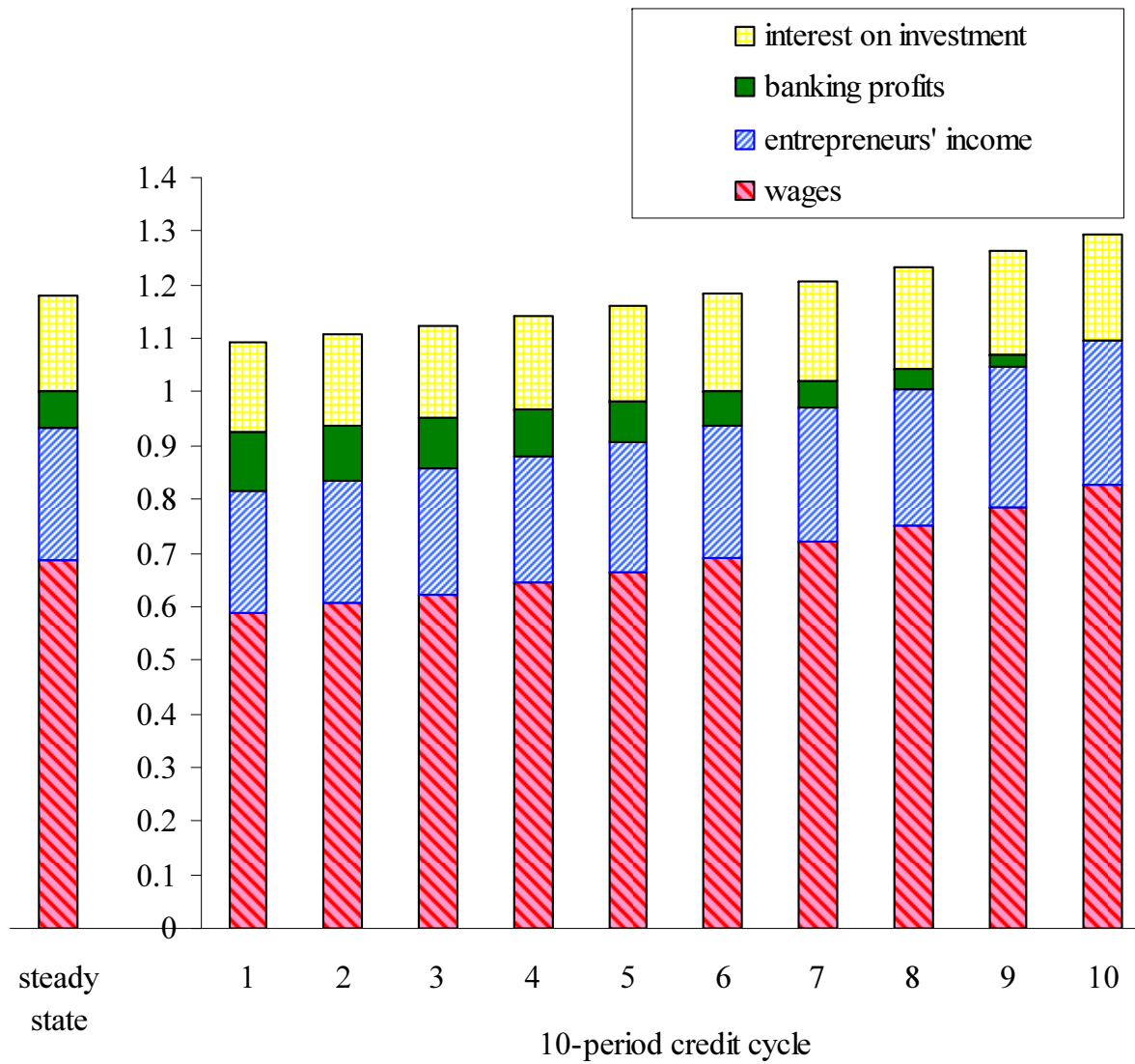


Figure 1. Net product in a 10-period credit cycle, with non-young bankers' time-0 investments being 80% of steady state (parameters: $\rho=0.1$, $n=10$, $\alpha=0.95$, $\beta=0.6$, $\gamma=0.05$, $\eta=0$, $\psi=1.74$, $\pi = 0.233$).

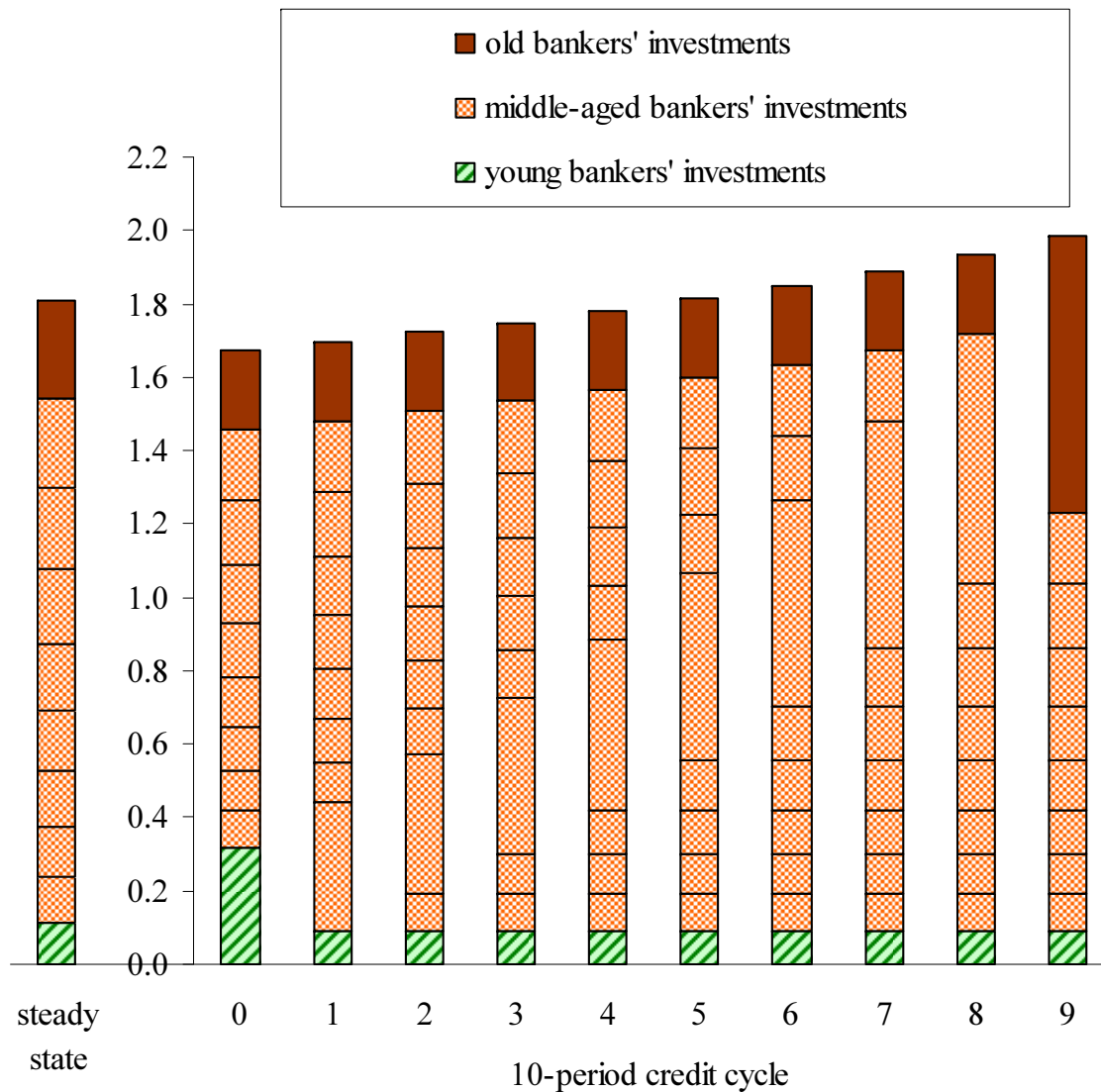


Figure 2. Investment amounts handled by different generations of bankers over a 10-period credit cycle, with non-young bankers' investments at time 0 being 80% of steady state. (parameters: $\rho=0.1$, $n=10$, $\alpha=0.95$, $\beta=0.6$, $\gamma=0.05$, $\eta=0$, $\psi=1.74$, $\pi = 0.233$) Older higher.

Benefits of bank bailouts for macroeconomic stabilization in our example

The figure above was constructed from an economy in the steady state by reducing the investments θ of age ≥ 1 bankers by 20% at time 0 (as if the probability of success at time 0 was reduced from $\alpha=0.95$ to $\alpha'=0.76$ in a one-time shock).

Aggregate investment at time 0 is $I_0=1.674$, compared with $I=1.808$ in steady state.

To restore steady-state stability, the government could subsidize new investment consortia that **hire enough older bankers to restore steady-state investments Θ** .

But new investments for age= s bankers need a subsidy to cover their expected loss $(\Theta_s - \theta_s)(sB/n)\alpha/(1+\rho)$. Total stabilization subsidy required at time 0 is 0.064, which could be financed by bonds that are repaid by 0.070 in taxes at time 1.

This tax bill is less than time-1 wage gains $0.688 - 0.589 = 0.099$ from stabilization.

Utility gains are only half of wage gains, but wage gains continue 5 periods.

At time 1, the discounted values of future utility gains from stabilization for workers who have 1 to 10 periods of employment remaining in their careers are respectively: (0.049, 0.087, 0.114, 0.130, 0.138, 0.137, 0.128, 0.112, 0.089, 0.060).

Middle-aged workers gain the most here. Old workers have less future time to gain, and stabilization eliminates benefits of a future boom for young workers.

Aggregating, we find that **time-1 workers' average long-run utility gains from stabilization (0.105) exceed its cost here**.

In an equilibrium cycle with time-0 investment I_0 , J_0 handled by young, consider **an unanticipated short-term balanced stimulus** at time 0: adding δ investment by old bankers who serve 1 term, and subsidizing young bankers to keep J_0 unchanged.

While $R(I_0+\delta) \geq r^* = (1+\rho)/\alpha+E$, the costs at time 1 of required time-0 subsidies are $\delta\alpha[B+E+(1+\rho)/\alpha-R(I_0+\delta)]$ for the one-term bankers who invest δ [*fiscal*], and $J_0\alpha[R(I_0)-R(I_0+\delta)]$ to keep the young n-term bankers investing J_0 [*monetary*], but workers' utility from wages at time 1 becomes $0.5\alpha(I_0+\delta)[\psi-R(I_0+\delta)]$.

When the subsidies are paid by lump-sum taxes on workers, with linear investment demand $R(I_0+\delta) = \psi - \alpha\pi(I_0+\delta)$, the workers' net benefit is $0.5\alpha^2\pi(I_0+\delta)^2 - \delta\alpha[B+E+(1+\rho)/\alpha + \alpha\pi(I_0+\delta) - \psi] - J_0\alpha^2\pi\delta$.

This quadratic in δ is maximized when $J_0+\delta = [\psi - (1+\rho)/\alpha - E - B]/(\pi\alpha)$.

Thus, workers can benefit from a small short-term balanced stimulus when young bankers invest less than what total investment would be in the steady state with $n=1$.

In our example, the steady state with $n=1$ had total investment $I=0.233$, but the steady state with $n=10$ has young bankers investing $J=0.113$.

So with $n=10$ here, a short-term balanced stimulus would actually increase workers' welfare when the economy is not too far from the steady state.

Note: the stimulus reduces profits for reinvestment by established bankers, and this analysis assumes that the stimulus would not be anticipated by financial markets.

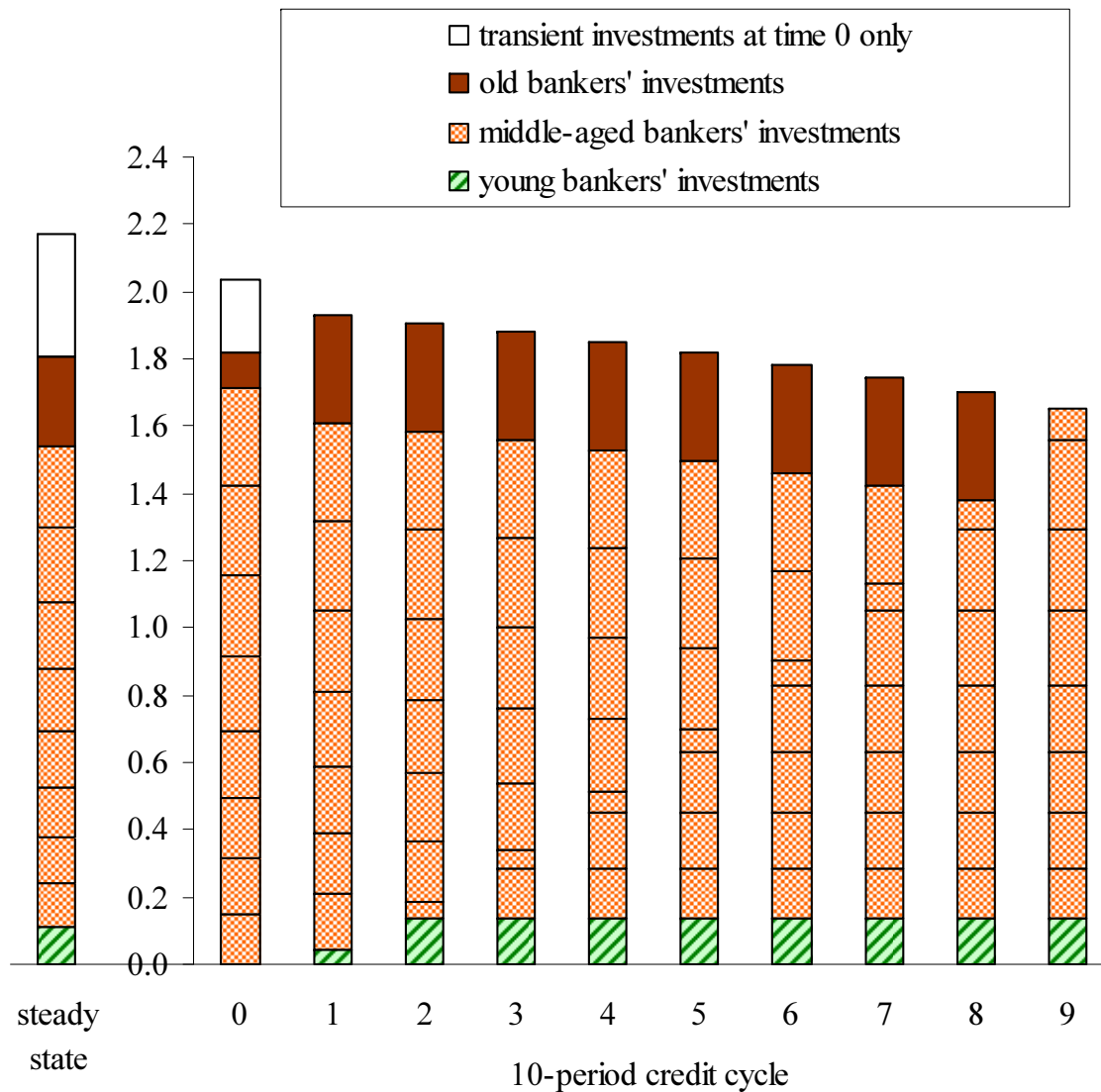


Figure 3. Investment amounts handled by different cohorts of bankers over a 10-period credit cycle, with continuing bankers' investments at time 0 being 120% of steady state (zombie banks). Same parameters as in Figure 1. (Older cohorts are higher here.)

Conclusions

Because of financial moral hazard, bankers need long-term relationships with investors, and these relationships can create complex macroeconomic dynamics.

In the recessions of our model, productive investment is reduced by a scarcity of trusted financial intermediaries.

Competitive recruitment of new bankers cannot fully remedy such undersupply, because bankers can be efficiently hired only with long-term contracts in which their responsibilities are expected to grow during their careers.

So a large adjustment to reach steady-state financial capacity in one period would create oversupply in future periods.

Thus, a financial recovery must move gradually uphill into the next boom, which in turn contains the seeds of the next recession.

A tax on workers to subsidize bankers may benefit workers by more than the tax, but some of the workers' gains are at the expense of past long-term investors.

Next: risk-averse bankers, uncertainty about long-term parameters that determine steady-state banking profits (which may reverse the correlation of production and banking profits)...

Appendix: recursive formulation of investors' optimal contracts with bankers

Consider a contractual relationship, at time t , between a consortium of investors and a banker who started at time 0.

Let y_t denote the value at time t of rewards that were previously promised to the banker by the consortium.

Let m_{t+1} denote the expected marginal cost to the consortium at time $t+1$ of increasing the banker's expected future rewards by one unit of value at time $t+1$. Rewards cannot be deferred at time n , so $m_n = 1$. At time 0, $y_0 = 0$.

In the contract, let h_t denote the size of their investment at time t . For the cases of success and failure, respectively, let e and f denote payments to the entrepreneur, and let b and c denote the value of rewards to the banker at time $t+1$.

The investors' optimization problem is: choose $h_t \geq 0$, $b \geq 0$, $c \geq 0$, $e \geq 0$, and $f \geq 0$ to

maximize $\alpha(r_{t+1}h_t - e - m_{t+1}b) - (1-\alpha)(f + m_{t+1}c) - (1+\rho)h_t$ subject to [Lagrange multipliers]

$$\alpha e + (1-\alpha)f - [\gamma h_t + \beta e + (1-\beta)f] \geq 0, \quad [\lambda_e]$$

$$\alpha b + (1-\alpha)c - [(\gamma+\eta)h_t + \beta(b+e) + (1-\beta)(c+f)] \geq 0, \quad [\lambda_b]$$

$$\alpha b + (1-\alpha)c - (1+\rho)y_t \geq 0. \quad [\mu_t]$$

Recall the definitions: $E = \gamma/(\alpha-\beta)$, $B = (\eta+\gamma+\beta E)/(\alpha-\beta)$, and $\sigma_{t+1} = r_{t+1} - E - (1+\rho)/\alpha$.

If $\sigma_{t+1} > Bm_{t+1}$ then infinite solutions would be feasible with $c=0$, $f=0$, $e= Eh_t$, $b=Bh_t$, $h_t \rightarrow +\infty$.

So $m_{t+1} \geq \sigma_{t+1}/B$. Then the optimal solution is $c=0$, $f=0$, $e=h_t E$, $b=h_t B$, and $h_t = y_t(1+\rho)/(\alpha B)$.

This solution satisfies the three constraints with equality, and it maximizes the Lagrangean with multipliers:

$$\lambda_e = (\alpha + \lambda_b \beta)/(\alpha - \beta), \quad \lambda_b = \alpha \sigma_{t+1}/[(\alpha - \beta)B], \quad \mu_t = m_{t+1} - \sigma_{t+1}/B.$$

These make e , h_t , and b drop out of the Lagrangean, which becomes $-\mu_t(1+\rho)y_t - c\sigma_{t+1}/B - (1+\lambda_b)f$.

A unit increase in y_t would decrease the consortium's expected profit at time $t+1$ by $(1+\rho)\mu_t$,

and so it would decrease the consortium's expected profit at time t by μ_t .

So the multiplier μ_t that we get from the above problem is the parameter m_t , marginal cost of rewards promised to the banker at time t , for the consortium's analogous investment problem at time $t-1$. So $m_t = \mu_t = m_{t+1} - \sigma_{t+1}/B$.

Thus, with $m_n=1$, we get by induction: $m_t = \mu_t = 1 - (\sigma_{t+1} + \dots + \sigma_n)/B$, $\forall t \in \{1, \dots, n-1\}$.

If successful at time $t+1$, the banker will be promised $y_{t+1} = b = h_t B$, and her next investment will be

$h_{t+1} = h_t B(1+\rho)/(\alpha B) = h_t(1+\rho)/\alpha$. Thus, success multiplies her investment by $(1+\rho)/\alpha$ each period.

At time $t=0$, we have $y_0=0$ (no prior promise), but then a solution with $e = h_0 E$, $b = h_0 B$, and $h_0 > 0$ is optimal for the consortium, with slack in the promise-keeping constraint, iff this constraint has multiplier $\mu_0 = 0$,

which is equivalent to the equilibrium equation [2] $B = \sigma_1 + \dots + \sigma_n$.