

PROBABILITY MODELS FOR ECONOMIC DECISIONS

by Roger B. Myerson

PREFACE

This book is an introduction to the use of probability models for analyzing risks and economic decisions. Some prior study of probability may be desirable but is not assumed here, as the basic ideas of probability are introduced in the book itself. Throughout this book, the focus is on showing the reader how to use probability in complex realistic situations. All the analytical work in this book is done in Microsoft Excel spreadsheets, because the spreadsheet medium enables students to handle much more complex problems than they could handle with traditional ways of representing mathematical formulas. As a result of this emphasis on spreadsheet modeling, many readers may also learn sophisticated spreadsheet skills from this book. But the main goal of the book is to make the practical power of probability analysis accessible to students at the undergraduate or MBA level.

The ultimate goal of any quantitative-analysis course should be to teach students the art of making quantitative models that can give practical insights into real decision problems. But students usually have difficulty with complex models that involve more than two variables, and so quantitative methods are often taught only in the context of simple applications that lack any realistic complexity. The typical result is that even good students who show a mastery of mathematical concepts on their final exams often cannot see how to apply these concepts in the real world.

This problem was particularly frustrating to me during the 1980s, as I did theoretical research in information economics while I taught probability to MBA students at the Northwestern University's Kellogg School of Management. New advances in information economics were teaching economic theorists that analysis of uncertainty has vital importance for understanding the competitive behavior of economic agents, and probability theory is the foundation for all analysis of uncertainty. But it seemed difficult or impossible to communicate this practical importance of probability analysis to my MBA students, even though their career goal was to become the competitively successful economic agents that we theorists were

studying. So in the late 1980s, I began searching for radically new ways to teach probability analysis so that students, after a ten-week MBA-level course, should be able to apply probability analysis to really interesting economic problems and cases. An important motivation in this search was my perception that the powerful new spreadsheet programs had great unexplored potential to transform the way that quantitative analysis is taught. This book is the fulfillment of this long process of experimentation and pedagogical development.

It has now become commonplace for probability and statistics books to do calculations in Excel spreadsheets. In most cases, however, these books have simply taken the old material that professors used to teach on blackboards and have moved it into a spreadsheet. Only rarely have authors asked harder questions about how the new spreadsheet medium should change the content of an introductory quantitative-analysis course. But if we really want to maximize the practical value of the skills that a student can take away from the course, then such questions need to be asked. Analytical methods that seemed too difficult or complex for an audience of applied business and economics students, when we worked on paper and blackboards, now may be straightforward for the same students to master in spreadsheets. On the other hand, there is less need for students to memorize basic computational formulas, now that computer programs with built-in functions and help screens are so universally available.

The most important advantage of teaching spreadsheets is that they make multi-variable models much easier for students to visually understand. Most students lose the big picture when they are asked to think about more than two variables that are represented by letters like "x" and "alpha" on a blackboard, but the same students can visually understand a spreadsheet model with many variables that are represented as cells on a spreadsheet page. Working in spreadsheets brings down the barrier that prevents most students from becoming sophisticated quantitative modelers. So throughout this book, I have tried to show how the methods of probability analysis can be applied to examples of a realistic complexity.

The emphasis on "Monte Carlo" randomized simulation of probability models grew gradually in the development of this book. At first I emphasized other methods of computing probabilities and expected values, but I realized that other computational methods generally require special assumptions that limit the scope of applications that we can consider. Monte

Carlo simulation is the most versatile general framework for modeling any kind of situation that involves uncertainty, and it has the advantage of letting us really see the uncertainty in our models, as our unknown quantities change at every recalculation. With Monte Carlo simulation, we have a general framework that can be used to illustrate and motivate any advanced concept in probability and decision analysis. We can even give new and effective illustrations of conventional statistical topics such as confidence intervals (see Section 2.6) and regression (see Sections 6.7 and 6.8).

As I developed this course, I simultaneously worked to develop my own add-in program (Simtools.xla) to extend Excel's capabilities for probability analysis. I did not use one of the commercially produced add-ins for statistical risk analysis (like Crystal Ball or @Risk) because I wanted to go farther in decision-analysis and economic modeling with less alteration of Excel's structure. But I found that there was a synergy between the development of the course and the development of the software. Students would regularly ask me to add a function to simplify a difficult part of the course; and as the new software simplified some topics, it became possible to teach other important topics that had previously seemed too advanced. As a result of this process, there are a number of new functions for facilitating decision analysis which are currently unique to Simtools (see GENLINV, CORAND, and CE, for example) but may eventually be imitated in other simulation add-ins.

I have used this book with MBA and undergraduate students at Northwestern University and the University of Chicago. Most of my students had some prior exposure to probability theory, but they often had difficulty recalling it when it was applied in our course. So this book includes an introductory discussion of all the basic probability concepts that we use. Chapter 1 introduces the basic ideas of probability and conditional probability. Chapter 2 covers discrete random variables, expected values, and standard deviations. In a short digression from probability theory, Chapter 3 introduces risk aversion and utility analysis with constant risk tolerance. Then Chapter 4 introduces continuous random variables, with an emphasis on Normal and Lognormal probability distributions. Chapter 5 covers joint distributions and correlation, and Chapter 6 discusses conditional expectation and regression models.

The later chapters of the book are organized around types of models. Simple decision

models with one decision variable are introduced in Chapter 7, with an emphasis on decisions about production quantities (the "newsboy" problem) and decisions about prices (bidding in auctions). Chapter 8 focuses on risk sharing in partnerships and financial markets. Finally, Chapter 9 introduces a wide range of dynamic models for forecasting how quantities may change over time.

There is more than enough material here for a full semester course. In a recent ten-week course where many students needed a review of basic probability, I covered Sections 1.1-1.4 in Chapter 1, all of Chapter 2, Sections 3.1-3.2 in Chapter 3, Sections 4.1-4.7 in Chapter 4, Sections 5.1-5.8 in Chapter 5, Sections 6.1-6.2 and 6.6 in Chapter 6, Sections 7.1-7.2 and 7.4-7.6 in Chapter 7, and Sections 8.1 and 8.3 in Chapter 8. With students who have a stronger background in probability, I would try to spend less time in Chapters 2 and 4, and devote more time to Chapters 8 and 9. The heart of this book is in Chapter 5 (for relationships among random variables) and Chapter 7 (for decisions variables), and these two chapters should get careful treatment in any course. I have also tried to make this book accessible for self study or professional enrichment outside of the classroom. When you open up the spreadsheet file for each chapter, you have everything that I would show you in my classroom.

Acknowledgements must begin with Howard Raiffa, who was my teacher and whose impact may be found throughout this book. Sid Deshmukh and Sam Savage have given me good collegial suggestions and encouragement in this project over many years. It was Donald Jacobs who first told me to use more spreadsheets in my teaching, and Robert Weber first showed me how to use them. Last but not least is my debt to Rebecca Myerson, who carefully read every chapter of this book to find errors in both English and Excel. At the points where my MBA students had typically expressed some confusion, she kept asking questions until I found a way to rewrite it more clearly. I know that not every high school student would have been willing to proof-read a long technical book by her father, and I am very grateful to Rebecca for helping to make this a better book.

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BIBLIOGRAPHIC NOTE

For a more sophisticated development of the mathematical ideas of probability and utility that are used in this book, the reader may consult any advanced text on probability models and financial mathematics. I would particularly recommend De Groot's Optimal Statistical Decisions and Luenberger's Investment Science.

Within the field of decision analysis, I have focused here almost exclusively on simulation analysis, as the most versatile tool for handling realistic complexity. For a broader introduction to other methods of decision analysis, the reader may consult Robert Clemen's current textbook Making Hard Decisions and Howard Raiffa's classic Decision Analysis. Other authors who have led the way in exploring the use of spreadsheets for decision analysis include Cliff Ragsdale, Sam Savage, David Vose, Wayne Winston and Christian Albright. David Vose's Risk Analysis may be particularly comparable to this book in its presentation of sophisticated probability models for applied decision analysis.

There are many good general references on how to use Microsoft Excel. In particular, I would highly recommend John Walkenbach's Excel Bible, in any of its versions.

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