ECONOMICS 30200b ASSIGNMENT 4

1. The new widget production process that firm 1 is developing is equally likely to have high cost or low cost. Firm 1 will learn whether its production cost is high or low at the beginning of next year. Then firm 1 can choose whether to build a new factory or not. Firm 2 will not be able to observe firm 1's production cost, but firm 2 will be able to observe whether firm 1 builds a new factory or not. Firm 2 will subsequently decide whether to enter the widget market or not. Firm 2 will earn \$2 million (in present discounted value of long-run profits) from entering the widget market if firm 1's production cost is high, but firm 2 will lose \$4 million from entering if firm 1's production cost is low. (These payoffs are relative to a payoff of \$0 to firm 2 if it does not enter.) Let a payoff of 0 to firm 1 denote its profit if new cost is high, firm 1 does not build, and firm 2 does not enter. Lower costs in the new process will increase firm 1's profit by \$4 million (ceteris paribus). Building a new factory would add \$2 million more to firm 1's profit if the new process has low cost (because conversion to the new process would be easier in a new factory), but building a new factory would subtract \$4 million from firm 1's profit if the new process has high cost. In any event, firm 2's entry into the widget market would reduce firm 1's profit by \$6 million. Both firms are risk neutral.
(a) Describe this game in extensive form.
(b) Construct the normal representation of this game in strategic form (the normal form).
(c) Analyze this strategic-form game by iterative elimination of weakly dominated strategies.
(d) Find two different pure-strategy equilibria of this strategic-form game. For each, show the beliefs (if any) that would make it a sequential equilibrium of the extensive-form game.

2. Firms 1 and 2 are competing in the same market. Each firm i must choose a quantity q_i to supply, and the market price p will depend on their choices according to the inverse demand formula $p(q_1,q_2) = \max\{A-(q_1+q_2), 0\}$. The total cost of production for each firm i is $(q_i)^2$, and so the total profit for firm i will be $u_i(q_1,q_2) = p(q_1,q_2)\, q_i - (q_i)^2$.
(a) For any given q_2, what would be firm 1's best response q_1 to maximize u_1?
(b) Find the Nash equilibrium of this game when the two firms choose their supply quantities simultaneously and independently. Compute each firm's expected profit in this equilibrium.
(c) Now suppose that firm 1 chooses q_1 first, and then firm 2 chooses its q_2 after observing q_1. Find the subgame-perfect (sequential) equilibrium of this game with perfect information, and compute each firm's expected profit in this equilibrium.

3. Players 1 and 2 are in a sequential all-pay-own-bid auction for a prize worth \$3. First, player 1 must pay \$1 or pass. When anyone passes, the other player gets the \$3 prize (and game ends). Otherwise, the other player can bid next, and must either pay \$2 (if he has it) or pass.
(a) Find a subgame-perfect (sequential) equilibrium if each player has \$4 available to spend.
(b) Find a subgame-perfect (sequential) equilibrium if each player has \$5 available to spend.

4. Player 2 is a mechanic, and player 1 has arrived at her shop with a car that has either a big problem or a small problem. Player 1 cannot distinguish these problems, and he thinks that there is probability 1/2 of the problem being big. Player 2 can observe the true nature of 1's problem, but she may lie about it. After observing the problem, player 2 will quote a price for the repair, which will be either \$300 or \$700. If player 2 quotes the \$300 price, then player 1 will accept it, but if player 2 quotes the \$700 price then player 1 can either accept it or can reject it and pay \$50 to have his car towed to an honest mechanic whom he knows will charge him \$300 if the problem

is actually small but will charge him $700 if his problem is big. The cost to player 2 of repairing the car is $100 if the problem is (actually) small, but is $500 if the problem is big.
Find a sequential equilibrium in which player 1 randomizes between accepting and rejecting if player 2 quotes the $700 price. Be sure to completely describe each player's strategy, and show player 1's belief about the probability of his problem being big if player 2 quotes the $700 price. (Hint: player 2 would never quote the $300 price if the problem were big, as she'd lose money.)

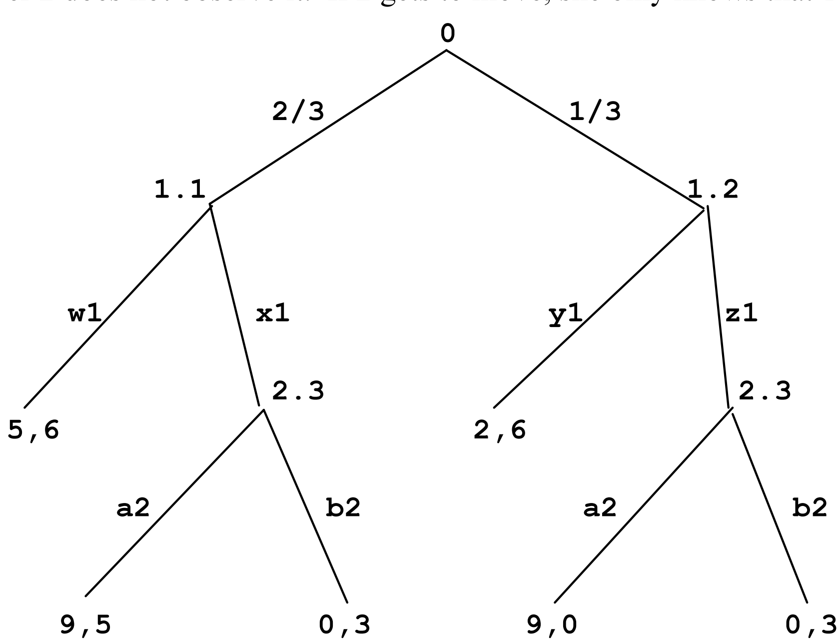
5. Consider a game where player 1 must choose T or B, player 2 must choose L or R, and their payoffs depend on their choices as follows.

Player 1 \ Player 2:	L	R
T	3, 2	1, 1
B	4, 3	2, 4

Suppose that player 1 moves first, and then player 2 makers her choice after observing 1's move.
(a) Show the extensive-form game with perfect information that describes this situation.
(b) Show the normal representation in strategic form for the extensive-form game in part (a).
(c) Identify the unique sequential or subgame-perfect equilibrium of this game.
(d) Find a Nash equilibrium (of the game in (b)) that is not subgame-perfect.

6. Consider again the game in exercise 5 where player 1 moves first. But now suppose that, whatever 1 chooses, the probability that player 2 will correctly observe 1's action is 0.9, and there is probability 0.1 that player 2 will mistakenly observe the other action (which 1 did not choose). The payoffs depend on the players' **actual** choices according to the previous table (so, for example, if 1 chose T but 2 mistakenly observed B and chose R then 2's payoff would be 1).
(a) Show the extensive-form game that describes this situation.
(b) Show the normal representation in strategic form for the extensive-form game in part (a).
(c) Find a sequential equilibrium in which player 2 would choose [L] for sure if she observed T.
*(d) Characterize the other sequential equilibria of this game.

7. Consider the following extensive-form game, where player 1 observes the chance move, but player 2 does not observe it. If 2 gets to move, she only knows that 1 chose either x1 or z1.



(a) Find a sequential equilibrium in which the (prior) probability of player 2 getting to move is 1.
(b) Find a sequential equilibrium in which the probability of player 2 getting to move is 0.
(c) Find a sequential equilibrium in which the probability of player 2 getting to move is strictly between 0 and 1.

ECONOMICS 30200b ASSIGNMENT 3

1. Find all Nash equilibria of the following 2×3 game:

Player 1: \ Player 2:	L	M	R
T	0, 4	5, 6	8, 7
B	2, 9	6, 5	5, 1

2. Consider the following 3×3 games that depend on a parameter α:

Player 1: \ Player 2:	L	M	R
T	α, α	$-1, 1$	$1, -1$
C	$1, -1$	α, α	$-1, 1$
B	$-1, 1$	$1, -1$	α, α

(a) Suppose we are given $\alpha>1$. Show that there are equilibria where the support includes two pure strategies for each player. Show also that there are pure-strategy equilibria, and show that there is an equilibrium where the support includes all three pure strategies for both player.

(b) For the support sets that you found in part (a), which of them also can be the support of an equilibrium when $\alpha<1$?

(c) Suppose $\alpha=0$, but now change the game by eliminating player 1's option to choose B. Find all equilibria of this 2×3 game.

3. Consider an all-pay-own-bid auction among n bidders. Each bidder i independently chooses a nonnegative bid c_i, which he will pay in the auction regardless of whether he wins or not, but if he is the high bidder then he will win a prize worth V. (So $u_i = V-c_i$ if i wins, else $u_i = -c_i$.) Find a symmetric equilibrium in which each bidder randomizes over the interval from 0 to V.

4. Players 1 and 2 each must decide whether to fight for a valuable prize.
If both players decide to fight, then they both lose \$1, and nobody gets the prize (it is destroyed).
If one player decides to fight but the other does not, then the player who is willing to fight gets the prize. A player who does not fight is guaranteed a payoff of 0.
Everybody knows that the prize is worth $V_2 = \$2$ to player 2.
But the prize may be worth more to player 1. Let V_1 denote the value of the prize to player 1.
In terms of V_1, the players' payoffs (u_1,u_2) will depend on their actions as follows:

Player 1: \ Player 2:	NotFight	Fight
NotFight	0, 0	0, 2
Fight	V_1, 0	$-1, -1$

Let us explore some different assumptions about this value V_1.

(a) Suppose first that the value of the prize to player 1 is $V_1 = \$3$, and everybody knows this. Find all equilibria of this game, including a mixed-strategy equilibrium in which both players have a positive probability of fighting.

(b) Suppose next that the value of the prize to player 1 is either $V_1 = \$2$ or $V_1 = \$3$. Player 1 knows his actual value, but player 2 thinks each of these possibilities has probability 1/2. Find a Bayesian equilibrium of this game where player 2 randomizes between fighting and not fighting.

(c) Finally, suppose that the value of the prize to player 1 is $V_1 = \$2+\tilde{t}_1$ where $\tilde{t}_1$ can be any number between 0 and 1. Player 1 knows its actual value, but player 2 thinks of $\tilde{t}_1$ as a uniform random variable on the interval from 0 to 1. Find a Bayesian equilibrium of this game where player 2 randomizes between fighting and not fighting.

5. Consider a two-person game where player 1 chooses T or B, and player 2 chooses L or R. When they play this game, player 2 also knows whether her type is A or B.
The players' utility payoffs (u_1,u_2) depend on their actions and on player 2's type as follows:

2'sType = A	L	R
T	4,0	0,2
B	0,4	2,0

2'sType = B	L	R
T	4,0	0,4
B	0,2	2,0

(a) Suppose first that, in this game, player 1 thinks that 2's type is equally likely to be A or B. Find a Bayesian equilibrium.
(b) How would the Bayesian equilibrium change in a game where player 1 thinks that player 2 has probability 1/6 of being type A, and has probability 5/6 of being type B?

6. Consider a first-price sealed-bid auction with n bidders to buy an object for which they have independent private values drawn from a Uniform distribution on the interval [L,M], given $n\geq 2$ and $M>L\geq 0$. Let $\tilde{t}_i$ denote i's private value of the object, which is i's private information (his type). (So the payoff to bidder i is $u_i = \tilde{t}_i - c_i$ if i's bid c_i is (uniquely) highest, but is $u_i=0$ if some other bid c_j is greater than c_i.)
(a) If the other n bidders are expected to bid according to a linear bidding strategy of the form $b_i(\tilde{t}_i) = \beta+\alpha\tilde{t}_i$ where $\beta+\alpha L \leq L$ and $\beta+\alpha M \leq M$ (so that no type bids more than its value), then what is the optimal bid for type t_1 of bidder 1, to maximize his expected profit?
(b) Find a symmetric equilibrium in which each bidder uses such a linear bidding strategy.
(c) In this symmetric equilibrium, what is the conditional expected profit for bidder 1 given his private value t_1?

7. Consider an all-pay-own-bid auction (as in problem 3) with n=2 bidders, where 1 has better information. The object being sold here has a common monetary value $\tilde{V}$ that has been drawn from a Uniform distribution on the interval [L,M], where $0\leq L<M$ are given. [So for any v between L and M, $\text{Prob}(\tilde{V}<v) = (v-L)/(M-L)$, and $E(\tilde{V}) = (M+L)/2$.] When they bid, bidder 1 knows the actual value $\tilde{V}$ (so $\tilde{t}_1=\tilde{V}$), but bidder 2 only knows its probability distribution.
Find an equilibrium in which bidder 2 randomizes according to some probability density $f_2(c_2)$ over bids c_2 in the interval [0, (M+L)/2], while bidder 1 chooses a bid in the same interval according to some strictly increasing strategy $b_1(v)$. Show $b_1(v)$ and $f_2(c_2)$ in terms of L and M.

8. Consider the model where each bidder i in {1,2} privately observes an independent random variable $\tilde{t}_i$ (i's type) that is drawn from a Uniform [0,1] probability distribution, and then they bid against each other to buy an object that has a common value $\tilde{V} = A_1\tilde{t}_1+A_2\tilde{t}_2$, but now suppose that it is a second-price sealed-bid auction. (So, letting c_i denote the bid of each bidder i, the payoff to bidder 1 is $u_1 = A_1\tilde{t}_1+A_2\tilde{t}_2-c_2$ if $c_1>c_2$, but is $u_1=0$ if $c_1<c_2$.)
(a) Show that there is an equilibrium in which each bidder i bids a linear function of his type, of the form $b_i(t_i) = \alpha t_i$ for some $\alpha>0$.
(b) In this symmetric equilibrium, what is the conditional expected profit for bidder 1 given his private information t_1?

ECONOMICS 30200b ASSIGNMENT 2

1. Consider a game where players 1 chooses an action in {T,B}, player 2 simultaneously chooses an action in {L,R}, and their payoffs (u_1,u_2) depend on their actions as follows:

Player 1 \ Player 2:	L	R
T	1, 9	8, 3
B	7, 2	4, 5

Find all Nash equilibria of this game (including equilibria with randomized strategies), and compute the players' expected payoffs in each equilibrium.

2. Consider a game where players 1 chooses an action in {T,B}, player 2 simultaneously chooses an action in {L,R}, and their payoffs (u_1,u_2) depend on their actions as follows:

Player 1 \ Player 2:	L	R
T	0, 3	8, 5
B	4, 6	7, 2

(a) Find all Nash equilibria of this game (including equilibria with randomized strategies), and compute the players' expected payoffs in each equilibrium.
(b) How would your answer change if player 1's payoff from (B,R) were increased from 7 to 9?

3. Consider a game where player 1 must choose T or M or B, player 2 must choose L or R, and their utility payoffs (u_1,u_2) depend on their choices as follows:

Player 1 \ Player 2:	L	R
T	6, 1	4, 9
M	5, 7	6, 0
B	9, 7	1, 8

(a) Show a randomized strategy that strongly dominates T for player 1.
(b) Find an equilibrium in randomized strategies for this game, and compute the expected payoff for each player in this equilibrium.
(c) Assuming that player 2 will act according to her equilibrium strategy that you found in part b, what would player 1's expected payoff be if he chose the action T?

4. Players 1 and 2 are involved in a joint project, and each must decide whether to work or shirk. If both work then each gets a benefit worth 1, but each also has a private effort cost e of working. So their payoffs depend on their payoffs (u_1,u_2) depend on their actions as follows:

Player 1 \ Player 2:	2 works	2 shirks
1 works	$1-e$, $1-e$	$-e$, 0
1 shirks	0, $-e$	0, 0

Suppose that e is a known parameter between 0 and 1. Find all Nash equilibria of this game.

5. Consider the penalty kick in soccer. Player 1 is the kicker, and player 2 is the goalie. Player 1 can kick to left or right. Player 2 must simultaneously decide to jump left or right. The probability that of the kick being blocked is λ if they both go left, but is ρ if they both go right. If they choose different directions then the probability of the kick being blocked is 0. So the players' payoffs (u_1,u_2) depend on their choices as follows:

Player 1 \ Player 2:	L	R
L	$1-\lambda$, λ	1, 0
R	1, 0	$1-\rho$, ρ

(a) Find a Nash equilibrium, and compute the expected payoffs to each player.

(b) If player 2 becomes more skilled at defending left then λ would increase in this game. How would this parametric change affect 2's probability of choosing left in equilibrium?

6. Find the nonrandomized Nash equilibria of the two-player strategic game in which each player's set of actions is the nonnegative real numbers and the players' payoff functions are $u_1(c_1,c_2) = c_1(c_2-c_1)$, $u_2(c_1,c_2) = c_2(1-c_1-c_2)$.

7. Players 1 and 2 are involved in a joint project. Each player i independently chooses an effort c_i that can be any number in the interval from 0 to 1; that is, $0 \le c_1 \le 1$ and $0 \le c_2 \le 1$.
(a) Suppose that their output will depend on their efforts by the formula $y(c_1,c_2) = 3c_1c_2$, and each player will get half the output, but each player i must also pay an effort cost equal to c_i^2.
So $u_1(c_1,c_2) = 1.5c_1c_2 - c_1^2$ and So $u_2(c_1,c_2) = 1.5c_1c_2 - c_2^2$.
Find all Nash equilibria without randomization.
(b) Now suppose that their output is worth $y(c_1,c_2) = 4c_1c_2$, of which each player gets half, but each player i must also pay an effort cost equal to c_i.
So $u_1(c_1,c_2) = 2c_1c_2 - c_1$ and $u_2(c_1,c_2) = 2c_1c_2 - c_2$.
Find all Nash equilibria without randomization.

8. There are two players numbered 1 and 2. Each player i must choose a number c_i in the set $\{0,1,2\}$, which represents the number of days that player i is prepared to fight for a prize that has value V=\$9. A player wins the prize only if he is prepared to fight strictly longer than the other player. They will fight for as many days as both are prepared to fight, and each day of fighting costs each player \$1. Thus, the payoffs for players 1 and 2 are as follows:
Player 1's payoff is $u_1(c_1,c_2) = 9-c_2$ if $c_1 > c_2$, but $u_1(c_1,c_2) = -c_1$ if $c_1 \le c_2$.
Player 2's payoff is $u_2(c_1,c_2) = 9-c_1$ if $c_2 > c_1$, but $u_2(c_1,c_2) = -c_2$ if $c_2 \le c_1$.
(a) Show a 3×3 matrix that represents this game.
(b) What dominated strategies can you find for each player in this game?
(c) What pure-strategy (nonrandomized) equilibria can you find for this game?
(d) Find a symmetric equilibrium in randomized strategies.

9. Consider a symmetric three-player game where each player must choose L or R.
If all three players choose L, then each of them gets payoff 1.
If all three players choose R, then each of them gets payoff 4.
Otherwise, if the players do not all choose the same action, then they all get payoff 0.
Find a symmetric randomized equilibrium in which both actions get positive probability.

10. Players 1 and 2 are bidding to buy an object in a sealed-bid auction. The object would be worth $V_1 = 53.40$ to player 1 if he could get it, but it would be worth $V_2 = 67.90$ to player 2 if she could get it. These values are commonly known by both players. Each player i chooses a bid c_i that must be a nonnegative multiple of ε, the smallest monetary unit. ($\varepsilon>0$ is given.)
The high bidder wins the object, paying the price that he or she bid, and the loser pays nothing. If their bids are equal, then they each have probability 1/2 of buying the object for the bid price.
So $u_i(c_1,c_2) = V_i-c_i$ if $c_i > c_{-i}$, but $u_i(c_1,c_2) = 0$ if $c_i < c_{-i}$, and $u_i(c_1,c_2) = 0.5(V_i-c_i)$ if $c_1=c_2$.
(a) Show that, for each player i, bidding more than V_i is a weakly dominated action.
(b) Suppose that $\varepsilon=1$. Show that there is a unique nonrandomized equilibrium of this game after weakly dominated actions are eliminated, and compute the players' payoffs in this equilibrium.
(c) If we considered a sequence of games as $\varepsilon \to 0$, what would be a limit of undominated equilibrium strategies and payoffs in this game?

ECONOMICS 30200b ASSIGNMENT 1

1. A decision-maker has expressed the following preferences:
Getting \$1000 for sure is as good as a lottery offering 0.27 probability of \$5000 or else \$0.
Getting \$2000 for sure is as good as a lottery offering 0.50 probability of \$5000 or else \$0.
That is: [\$1000] ~ 0.27[\$5000]+0.73[\$0], [\$2000] ~ 0.50[\$5000]+0.50[\$0].
If this person is logically consistent, which should he prefer among the following:
a lottery offering a 0.5 probability of \$2000 or else \$1000 (0.5[\$2000]+0.5[\$1000]),
a lottery offering a 0.4 probability of \$5000 or else \$0 (0.4[\$5000]+0.6[\$0]).
Justify your answer as fundamentally as you can.

2. Members of a primitive tribe may own bundles of various goods, which anthropologists have numbered {1,..,m}. The tribe has various ritual exchange activities, numbered {1,...,n}.
In each activity j, there is a "host" and a "guest", and the host gives the guest some net quantity θ_{ij} of each good i (where a negative θ_{ij} denotes the guest giving $-\theta_{ij}$ units of i to the host).
Any tribesman may do each activity any number of times, as guest or host.
Prove a theorem of the following form: "Given any such matrix of parameters θ_{ij}, exactly one of the following two conditions is true: (1) There is a way to use some combination of these exchange activities to increase one's holdings of every good by at least one unit. (2) ...."
[You may assume that people can also do any activity j at a fractional level x_j, which would then yield a net transfer $\theta_{ij}x_j$ of each good i. But this assumption is not actually necessary.]

3. A decision-maker must choose between three alternative decisions {d1,d2,d3}. Her utility payoff will depend as follows on her decision and on an uncertain state of the world in {s1,s2}:

	State s1	State s2
Decision d1	15	90
Decision d2	B	75
Decision d3	55	40

Let p denote the decision-maker's subjective probability of state s2.

(a) Suppose first that B=35. For what range of values of p is decision d1 optimal? For what range is decision d2 optimal? For what range is decision d3 optimal? Is any decision strongly dominated? If so, by what randomized strategies?
(b) Suppose now the B=20. For what range of values of p is decision d1 optimal? For what range is decision d2 optimal? For what range is decision d3 optimal? Is any decision strongly dominated? If so, by what randomized strategies?
(c) For what range of values for the parameter B is decision d2 strongly dominated?