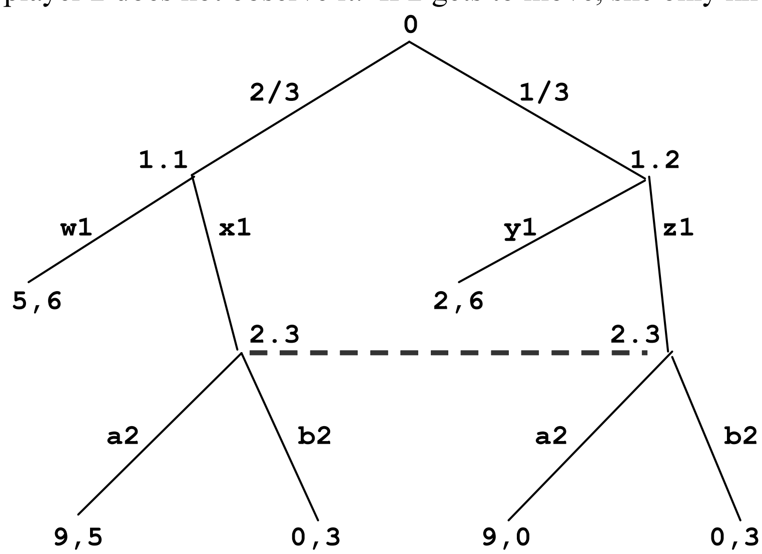
Econ 20700. Assignment 6.

<u>1.</u> Consider the three-player game in Osborne's Figure 331.2 (exercise 331.1).
(a) Show its normal representation in strategic form.
(b) Show that this strategic game has a pure-strategy Nash equilibrium that corresponds to a sequential equilibrium of the given extensive-form game. Be sure to indicate what beliefs would make this a sequential equilibrium.
(c) Show that this game also has a pure-strategy Nash equilibrium that does not correspond to a sequential equilibrium of the given extensive-form game.

<u>2.</u> Consider the following extensive-form game, where player 1 observes the chance move, but player 2 does not observe it. If 2 gets to move, she only knows that 1 chose either x1 or z1.



(a) Find a sequential equilibrium in which the (prior) probability of player 2 getting to move is 1.
(b) Find a sequential equilibrium in which the probability of player 2 getting to move is 0. (You must describe what player 2 would believe and do if she got to move.)
(c) Find a sequential equilibrium in which the probability of player 2 getting to move is strictly between 0 and 1.
(d) Show the normal representation of this game in strategic form.

<u>3.</u> Consider a version of the example in section 4.6. Player 1 is the expert, and 1's type (what he knows about 2's problem) is either <u>major</u> or <u>minor</u>. Player 2 initially believes that 1's types have probabilities P(major) = R, P(minor) = 1−R. Knowing his type, 1 can offer to do 2's job for either a <u>high price</u> or a <u>low price</u>. After learning 1's offer, 2 may either <u>accept</u> it or <u>reject</u> it (to go to another expert). Their payoffs (u_1, u_2) depend on type t_1 and actions (a_1,a_2) as follows.

		a_2 = Accept	a_2 = Reject
When t_1 = Major:	a_1 = High	H−D, −H	0, −(H+Δ)
	a_1 = Low	L−D, −L	0, −(H+Δ)
When t_1 = Minor:	a_1 = High	H−C, −H	0, −(L+Δ)
	a_1 = Low	L−C, −L	0, −(L+Δ)

The given parameters here satisfy $1 > R > 0$, $H > D > L > C > 0$, $H-L > \Delta > 0$.
(a) Given the parameters H,L,D,C,Δ, what is the smallest value of R such that there exists a sequential equilibrium in which 1 would always choose High and 2 would always Accept?
(b) Now suppose that the parameter R is less than the bound that you found in part (a). Find a sequential equilibrium where player 1 chooses High if t_1=Major, but 1 randomizes if t_1=Minor, and player 2 Accepts if a_1=Low, but 2 randomizes if a_1=High.
As a part of this sequential equilibrium, you must compute what player 2 would believe about the probability of 1's type being Major if player 1 chose High.

4. (Akerlof's lemon) Player 1 is trying to sell a car, and player 2 is the only possible buyer. Player 2 is uncertain about the quality of the car, which player 1 knows.
Player 1's value of keeping the car depends on its quality. Player 2's belief about 1's value of keeping the car is described by a Uniform probability distribution on the interval $0 to $1000.
The actual value of the car to player 2 would be 50% more than its value to player 1.
That is, when the car is worth t_1 to player 1, it would worth $V_2(t_1) = 1.5t_1$ to player 2, but this value t_1 is only known by player 1 now. (We may say that t_1 is player 1's type.)
(a) Suppose that player 2 offers to buy the car for $600.
What is the probability that player 1 will accept this offer.
What would be the conditionally expected value of $V_2(t_1)$ if player 1 accepted this offer?
What is player 2's expected profit from bidding $600 for this car? (Notice the sign!)
(b) Find the bid for player 2 that maximizes her expected profit.
(c) Now consider the game in which player 1 can offer to sell the car at any price and then player 2 can either accept this offer or not buy the car. What is the highest number R such that we can construct an equilibrium in which player 2 would surely agree to buy the car for R but not for any higher price?

5. (Modified lemon) Now change the Lemon problem by supposing instead that player 2's belief about 1's value of the car is described by a Uniform distribution on the interval $250 to $1000.
As above 2's value of getting the car would always be 50% more than 1's value of keeping it.
(a),(b),(c) For this modified problem, answer the questions in parts (a), (b), and (c) above.
(d) In the seller-offer game (described in part (c)), find an equilibrium in which player 2 would surely accept a price of $500, would have some probability more than 0 but less than 1 of accepting a price of $1100, but would reject any other price over $500.
In this equilibrium, what is the probability of trade?

6. (Education signaling) Each worker's talent is measured by his type t_1 that is independently drawn from a Uniform distribution on the interval from 0 to 1.
Each worker knows his own talent-type, but nobody else can observe it.
Hiring a worker of type t_1 would have productive value $10t_1$ to any firm.
Because of competition among firms, each worker will be offered a wage equal to his conditionally expected productive value, given what is publicly observable about him.
A worker's education does not affect his productivity, but his educational achievement is publicly observable, and the effective cost of getting education is higher for less talented workers.
Suppose that the effective cost of getting d years of education is d/t_1 for a worker of type t_1, and a worker's objective is to maximize his wage-offer minus the effective cost of his education.
(a) Find an equilibrium in which each type-t_1 worker gets 0 years of education if $0 \leq t_1 < 0.5$ but gets some positive number D years of education if $0.5 < t_1 \leq 1$.
(So the competitive wage offered to worker with 0 years of education is $E(10t_1 | t_1<0.5) = 2.5$, and the competitive wage offered to workers with D years of education is $E(10t_1 | t_1>0.5) = 7.5$.)
What must D be to make this an equilibrium?
(b) Find an equilibrium in which workers with types above some number θ get d=4 years of education and are offered a high wage H, and all lower-type workers get d=0 years of education and are offered a low wage L. What must θ, H, and L be to make this an equilibrium?

Econ 20700. Assignment 5.

1. Consider a repeated game where 1 and 2 repeatedly play the game below infinitely often.

	a_2	b_2
a_1	8, 8	1, 2
b_1	2, 1	0, 0

The players want to maximize their δ-discount average value of payoffs, for some $0<\delta<1$.
Consider the following state-dependent strategies: The possible states are state 1 and state 2.
In state 1, we anticipate that player 1 will play b_1 and player 2 will play a_2.
In state 2, we anticipate that player 1 will play a_1 and player 2 will play b_2.
The game begins at period 1 in state 1. The state of the game would change after any period where the outcome of play was (a_1,a_2), but otherwise the state always stays the same.
What is the lowest value of δ such that these strategies form an equilibrium?

2. Consider a repeated game where they repeatedly play the game below infinitely often.

	a_2	b_2
a_1	3, 3	0, 5
b_1	5, 0	−4,−4

The players want to maximize their δ-discount average value of payoffs, for some $0<\delta<1$.
(a) Find the lowest value of δ such that you can construct an equilibrium in which the players will actually choose (a_1,a_2) forever, but if any player i ever chose b_i at any period then they would play the symmetric randomized equilibrium of the one-stage game forever afterwards.
(b) What is the lowest value of δ such that you can construct an equilibrium in which the players will actually choose (a_1,a_2) forever, but if some player i unilaterally deviated to b_i at any period then that player i would get payoff 0 at every round thereafter? Be sure to precisely describe state-dependent strategies that form this equilibrium.

3. In Chapter 9, do exercise 282.1.

4. Consider a two-person game where player 1 chooses T or B, and player 2 chooses L or R.
When they play this game, player 2 also knows whether her type is A or B.
The players' utility payoffs (u_1,u_2) depend on their actions and on player 2's type as follows:

2'sType = A	L	R
T	4,0	0,2
B	0,4	2,0

2'sType = B	L	R
T	4,0	0,4
B	0,2	2,0

(a) Suppose first that, in this game, player 1 thinks that 2's type is equally likely to be A or B.
Find a Bayesian equilibrium.
(b) How would the Bayesian equilibrium change in a game where player 1 thinks that player 2 has probability 1/6 of being type A, and has probability 5/6 of being type B?

5. Players 1 and 2 each must decide whether to fight for a valuable prize.
If both players decide to fight, then they both lose \$1, and nobody gets the prize (it is destroyed).
If one player decides to fight but the other does not, then the player who is willing to fight gets the prize. A player who does not fight is guaranteed a payoff of 0.
Everybody knows that the prize is worth V_2 = \$2 to player 2.
But the prize may be worth more to player 1. Let V_1 denote the value of the prize to player 1.
In terms of V_1, the players' payoffs (u_1,u_2) will depend on their actions as follows:

	Player 2	
Player 1	NotFight	Fight
NotFight	0, 0	0, 2
Fight	V_1, 0	−1, −1

Let us explore some different assumptions about this value V_1.

(a) Suppose first that the value of the prize to player 1 is V_1 = \$3, and everybody knows this. Find all equilibria of this game, including a mixed-strategy equilibrium in which both players have a positive probability of fighting.

(b) Suppose next that the value of the prize to player 1 is either V_1 = \$2 or V_1 = \$3. Player 1 knows his actual value, but player 2 thinks each of these possibilities has probability 1/2. Find a Bayesian equilibrium of this game where player 2 randomizes between fighting and not fighting.

(c) Finally, suppose that the value of the prize to player 1 is V_1 = \$2+$\tilde{t}_1$ where $\tilde{t}_1$ can be any number between 0 and 1. Player 1 knows its actual value, but player 2 thinks of $\tilde{t}_1$ as a uniform random variable on the interval from 0 to 1.
Find a Bayesian equilibrium of this game where player 2 randomizes between fighting and not fighting.

6. Consider the following Bayesian game (where player 2's type is her vulnerability at (T,L)).
Player 2 knows the amount t_2 that she would have to pay player 1 if they play (T,L), but player 1 only knows that t_2 was drawn from a Uniform distribution on the interval from 0 to 1.

	Player 2	
Player 1	L	R
T	$t_2,-t_2$	0,0
B	0,0	0.5,−0.5

(a) A student said, "player 2 should choose R if $t_2>0.5$, but 2 should choose L if $t_2<0.5$".
Show that this student's analysis is not compatible with any equilibrium, by first computing player 1's best response to the strategy that the student has recommended for 2, and then computing player 2's best response to this best-response of player 1.

(b) Find an equilibrium of this game. Be sure to fully specify the strategies for both players.

Econ 20700. Assignment 4. (for after the midterm)
In Chapters 3, 4, and 6 do exercises 59.1, 145.1, and 189.1.
Also, find all Nash equilibria (pure and mixed) of the following 2×3 game:

	Player 2		
Player 1	L	M	R
T	0, 4	5, 6	8, 7
B	2, 9	6, 5	5, 1

Econ 20700. Assignment 3.

In Chapter 5, do exercises:

156.2ac,

173.2 and show the strategic form that represents Figure 173.1,

176.1 simplify the problem by assuming that a player who does not pass must bid exactly 1 more than the other player's most recent bid, analyze the cases (v,w)=(2,3) and (v,w)=(3,4) and (v,w)=(3,5),

177.1.

Econ 20700. Assignment 2.
In Osborne chapter 4, do exercises:
114.1, 114.3, 120.2 (only consider mixed strategies that randomize among M and B),
128.1, 130.2, 141.2b, 142.1 (look for a symmetric randomized equilibrium).

Econ 20700. Assignment 1.
In Osborne chapter 2, do exercises:
42.1,
42.2 (notice that each player gets half of $f(x_1,x_2)$, and each x_i satisfies $0 \le x_i \le 1$),
47.1,
52.2,
34.3 (just find one equilibrium for each of the two games),
and the following small attrition game:

Small Attrition game. There are two players numbered 1 and 2. Each player i must choose a number c_i in the set $\{0, 1, 2\}$, which represents the number of days that player i is prepared to fight for a prize that has value V=$9. A player wins the prize only if he is prepared to fight strictly longer than the other player. They will fight for as many days as both are prepared to fight, and each day of fighting costs each player $1. Thus, the payoffs for players 1 and 2 are as follows:
Player 1's payoff is $u_1(c_1,c_2) = 9-c_2$ if $c_1 > c_2$, but $u_1(c_1,c_2) = -c_1$ if $c_1 \le c_2$.
Player 2's payoff is $u_2(c_1,c_2) = 9-c_1$ if $c_2 > c_1$, but $u_2(c_1,c_2) = -c_2$ if $c_2 \le c_1$.
Show a 3×3 matrix that represents this game.
What dominated strategies can you find for each player in this game?
What pure-strategy equilibria can you find for this game?