

THE UNIVERSITY OF CHICAGO
Department of Economics
Econ 303 Problem Set 4

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Due Friday June 5

All questions below refer to the single indivisible good model with private information about values developed in class.

1. Find, for each agent, i , a cost function, $c_i(\cdot)$, depending only upon i 's type t_i , and independent of the other agents' types t_{-i} , such that these cost functions together with the ex-post efficient probability assignment functions constitute an incentive compatible mechanism, when each bidder's type is drawn independently and uniformly from $[0, 1]$.
2. We showed in class that a mechanism is incentive compatible only if conditions (i) and (ii) held. Show that these conditions are sufficient for incentive compatibility.
3. Prove that a second price auction implements an ex-post efficient outcome in weakly dominant strategies.
4. In class we defined $U_i(t_i) = \max_{r_i \in [0,1]} (\bar{p}_i(r_i)t_i - \bar{c}_i(r_i))$. Prove that $U_i(t_i)$ is convex in t_i on $[0, 1]$.
5. In a first-price all pay auction, bidders submit sealed bids, the high-bid wins and every bidder – not just the winner – pays their bid. Suppose all n bidders are ex-ante symmetric, i.e., $f_i = f$ for all i . Find a symmetric equilibrium of the all-pay first-price auction and show in two ways that it raises the same expected revenue as a standard first-price auction in which the high-bid wins and only the winner pays his bid.
6. When solving the revenue maximization problem of a seller with a single indivisible good, I claimed that

$$E \left(\sum_{i=1}^n \left[\bar{p}_i(t_i)t_i - \int_0^{t_i} \bar{p}_i(s)ds \right] \right) = E \left(\sum_{i=1}^n p_i(t_1, \dots, t_n) \left[t_i - \frac{1 - F_i(t_i)}{f_i(t_i)} \right] \right),$$

where both expectations are over (t_1, \dots, t_n) according to $\prod_i f_i(t_i)$. Show that this equality holds.