THE UNIVERSITY OF CHICAGO
Department of Economics
Econ 303: Price Theory
Problem Set 3

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Due Friday May 29

1. Prove that in our insurance screening model there is no pure strategy subgame perfect equilibrium if α is sufficiently high.

2. In the context of our insurance screening model, consider the following optimization problem.

max $\alpha(p_l - \underline{\pi} B_l) + (1-\alpha)(p_h - \bar{\pi} B_h)$ subject to

$U_l(B_l, p_l) \geq \max[U_l(B_h, p_h), U_l(\bar{\psi}_l)]$ and

$U_h(B_h, p_h) \geq \max[U_h(B_l, p_l), U_h(\psi_h^c)]$,

where the maximization is over all feasible policies (B_l, p_l) and (B_h, p_h). Let $V(\alpha)$ denote the optimal value of this problem as a function of α, the fraction of low-risk consumers.

(a) Show that $V(0) = 0$, $V(1) > 0$, $V'(\alpha) \geq 0$.

(b) Prove that a pure strategy subgame perfect equilibrium of the screening game exists if and only if $V(\alpha) = 0$.

3. Again, in the context of our insurance screening model, let

$$U_{hp}^c = \frac{\partial U_h(\psi_h^c)}{\partial p},$$

and for $i = l, h$, let

$$\bar{U}_{ip} = \frac{\partial U_i(\bar{\psi}_l)}{\partial p} \text{ and } \bar{U}_{iB} = \frac{\partial U_i(\bar{\psi}_l)}{\partial B}.$$

Note that $U_{hp}^c, \bar{U}_{ip} < 0$ and $\bar{U}_{iB} > 0$.

Improve upon the bound for α that is implicit in your solution to problem 1 above by proving that a pure strategy subgame perfect equilibrium of the screening game does not exist if

$$\frac{(-U_{hp}^{c})(\bar{U}_{lB} + \underline{\pi}\bar{U}_{lp})}{\bar{U}_{hp}\bar{U}_{lB} - \bar{U}_{hB}\bar{U}_{lp}} > \frac{1-\alpha}{\alpha}.$$

Hint: Show that for $\varepsilon > 0$ sufficiently small, there exists a pair of policies, $(B_h^\varepsilon, p_h^\varepsilon)$ and $(B_l^\varepsilon, p_l^\varepsilon)$, that earn positive expected profits when employed as a deviation, where $(B_h^\varepsilon, p_h^\varepsilon) = (L, \bar{\pi}L - \varepsilon)$.

4-6. Complete problems 8.11-8.13 in JR, p. 371-372.