For the most part in this chapter, we've concentrated on the disease and its symptoms, only occasionally hinting at a potential cure. We end this chapter by noting that very often these information problems can be mitigated if not surmounted. If adverse selection is the problem, signaling or screening can help. If moral hazard is the problem, contracts can be designed so that the agents' incentives lead them nearer to Pareto-efficient outcomes.

The analysis of markets with asymmetric information raises new questions and offers important challenges to economists. It is an area that offers few simple and broadly applicable answers, but it is an area where all the analyst's creativity, insight, and logical rigor can pay handsome dividends.

8.4 EXERCISES

8.1 Consider the insurance model of section 8.1, but treat each insurance company as if it were a risk-neutral consumer with wealth endowment $\bar{w} \geq L$ in every state, where L is the size of the loss should one of the m risk-averse consumers have an accident. Also assume that the number of risk-neutral consumers exceeds the number of risk-averse ones. Show that the competitive equilibrium derived in section 8.1 is a competitive equilibrium in this exchange economy.

8.2 Suppose that in the insurance model with asymmetric information, a consumer's accident probability is a function of his wealth. That is, $\pi = f(w)$. Also suppose that different consumers have different wealth levels, and that $f' > 0$. Does adverse selection necessarily occur here?

8.3 In our insurance model of section 8.1, many consumers may have the same accident probability. We allowed policy prices to be person specific. Show that, with symmetric information, equilibrium policy prices depend only on probabilities, not on the particular individuals purchasing them.

8.4 Answer the following questions related to the insurance model with adverse selection.

(a) When there are finitely many consumers, F, the distribution of consumer accident probabilities is a step function. Show that $g : [0, \bar{\pi}L] \to [0, \bar{\pi}L]$ then is also a step function and that it is nondecreasing.

(b) Show that g must therefore possess a fixed point.

(c) More generally, show that a nondecreasing function mapping the unit interval into itself must have a fixed point. (Note that the function need not be continuous! This is a special case of a fixed-point theorem due to Tarski (1955)).

8.5 Suppose there are two states, 1 and 2. State 1 occurs with probability π, and w_i denotes a consumer's wealth in state i.

(a) If the consumer is strictly risk-averse and $w_1 \neq w_2$, show that an insurance company can provide her with insurance rendering her wealth constant across the two states so that she is better off and so that the insurance company earns positive expected profits.

(b) Suppose there are many consumers and many insurance companies and that a feasible allocation is such that each consumer's wealth is constant across states. Suppose also that in this allocation, some consumers are insuring others. Show that the same wealth levels for consumers and expected profits for insurance companies can be achieved by a feasible allocation in which no consumer insures any other.

8.6 (Akerlof) Consider the following market for used cars. There are many sellers of used cars. Each seller has exactly one used car to sell and is characterized by the quality of the used car he wishes to sell. Let $\theta \in [0, 1]$ index the quality of a used car and assume that θ is uniformly distributed on $[0, 1]$. If a seller of type θ sells his car (of quality θ) for a price of p, his utility is $u_s(p, \theta)$. If he does not sell his car, then his utility is 0. Buyers of used cars receive utility $\theta - p$ if they buy a car of quality θ at price p and receive utility 0 if they do not purchase a car. There is asymmetric information regarding the quality of used cars. Sellers know the quality of the car they are selling, but buyers do not know its quality. Assume that there are not enough cars to supply all potential buyers.

(a) Argue that in a competitive equilibrium under asymmetric information, we must have $E(\theta|p) = p$.

(b) Show that if $u_s(p, \theta) = p - \theta/2$, then every $p \in (0, 1/2]$ is an equilibrium price.

(c) Find the equilibrium price when $u_s(p, \theta) = p - \sqrt{\theta}$. Describe the equilibrium in words. In particular, which cars are traded in equilibrium?

(d) Find an equilibrium price when $u_s(p, \theta) = p - \theta^3$. How many equilibria are there in this case?

(e) Are any of the preceding outcomes Pareto efficient? Describe Pareto improvements whenever possible.

8.7 Show that in the insurance signaling game, if the consumers have finitely many policies from which to choose, then an assessment is consistent if and only if it satisfies Bayes' rule. Conclude that a sequential equilibrium is then simply an assessment that satisfies Bayes' rule and is sequentially rational.

8.8 Analyze the insurance signaling game when benefit B is restricted to being equal to L.

(a) Show that there is a unique sequential equilibrium when attention is restricted to those in which the insurance company earns zero profits.

(b) Show that among all sequential equilibria, there are no separating equilibria. Is this intuitive?

(c) Show that there are pooling equilibria in which the insurance company earns positive profits.

8.9 Consider the insurance signaling game.

(a) Show that there are separating equilibria in which the low-risk consumer's policy proposal is rejected in equilibrium if and only if $MRS_l(0, 0) \leq \bar{\pi}$.

(b) Given a separating equilibrium in which the low-risk consumer's policy proposal is rejected, construct a separating equilibrium in which it is accepted without changing any player's equilibrium payoff.

(c) Continue to consider this setting with one insurance company and two types of consumers. Show that when α (the probability that the consumer is low-risk) is high enough, the only competitive equilibrium under asymmetric information gives the low-risk consumer no insurance and the high-risk consumer full insurance.

(d) Returning to the general insurance signaling game, show that every separating equilibrium Pareto dominates the competitive equilibrium described in part (c).

8.10 Consider the insurance screening game. Suppose that the insurance companies had only finitely many policies from which to construct their lists of policies. Show that a joint strategy is a subgame perfect equilibrium if and only if there are beliefs that would render the resulting assessment a sequential equilibrium.

8.11 Consider the moral hazard insurance model where the consumer has the option of exerting either high or low accident avoidance effort (i.e., $e = 0$ or 1). Recall that $\pi_l(e) > 0$ denotes the probability that a loss of l dollars is incurred due to an accident. Show that if the monotone likelihood ratio property holds so that $\pi_l(0)/\pi_l(1)$ is strictly increasing in l, then $\sum_{l=0}^{L} \pi_l(0)x_l > \sum_{l=0}^{L} \pi_l(1)x_l$ for every increasing sequence of real numbers $x_1 < x_2 < \cdots < x_L$.

8.12 Consider the moral hazard insurance model.

(a) Show that when information is symmetric, the profit-maximizing policy price is higher when low effort is induced compared to high effort.

(b) Let the consumer's reservation utility, $\bar{u}$, be the highest she can achieve by exerting the utility-maximizing effort level when no insurance is available. Suppose that when information is asymmetric, it is impossible for the insurance company to earn nonnegative profits by inducing the consumer to exert high effort. Show then that were no insurance available at all, the consumer would exert low effort.

8.13 Consider once again the moral hazard insurance model. Let the consumer's von Neumann-Morgenstern utility of wealth be $u(w) = \sqrt{w}$, let her initial wealth be $w_0 = \$100$, and suppose that there are but two loss levels, $l = 0$ and $l = \$51$. As usual, there are two effort levels, $e = 0$ and $e = 1$. The consumer's disutility of effort is given by the function $d(e)$, where $d(0) = 0$ and $d(1) = 1/3$. Finally, suppose that the loss probabilities are given by the following entries, where the rows correspond to effort and the columns to loss levels.

e \ l	$l = 0$	$l = 51$
$e = 0$	1/3	2/3
$e = 1$	2/3	1/3

So, for example, the probability that a loss of \$51 occurs when the consumer exerts high effort is 1/3.

(a) Verify that the probabilities given in the table satisfy the monotone likelihood ratio property.

(b) Find the consumer's reservation utility assuming that there is only one insurance company and that the consumer's only other option is to self-insure.

(c) What effort level will the consumer exert if no insurance is available?

(d) Show that if information is symmetric, then it is optimal for the insurance company to offer a policy that induces high effort.

(e) Show that the policy in part (d) will not induce high effort if information is asymmetric.

(f) Find the optimal policy when information is asymmetric.

(g) Compare the insurance company's profits in the symmetric and asymmetric information cases. Also, compare the consumer's utility in the two cases. Argue that the symmetric information solution Pareto dominates that with asymmetric information.