Electoral Politics  
with Policy Adjustment Costs  

Peter Evangelakis\(^1\)  
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**Abstract:** Politicians and voters sometimes make decisions that conflict with their ideological position. I show that the presence of policy adjustment costs can generate this type of behavior in an electoral setting. Specifically, I study a two period model in which an incumbent sets policy, the median voter chooses between the incumbent and an opposite party challenger, and the winner of the election can change policy. A small change is costless but a larger change induces a strictly positive adjustment cost. If the politicians are highly polarized, then the incumbent can choose her preferred policy in both periods and guarantee her re-election, even if she is somewhat more extreme than the challenger. Otherwise, she has to decide between guaranteeing that the election winner chooses her preferred policy during period 2 and guaranteeing her re-election. If she is not highly office-motivated, then she accomplishes the former by choosing a policy that is slightly more extreme than her preferred policy. If she is highly office-motivated and political polarization is sufficiently low, then she accomplishes the latter by choosing the most moderate policy on the other side of the political spectrum. The median voter is best off in this case.

\(^1\)Ph.D. Candidate, Department of Economics, University of Chicago. All questions or comments should be directed to the author at peterevang@uchicago.edu. I would like to acknowledge the advice of Richard Van Weelden, Scott Ashworth, Ethan Bueno de Mesquita, Pablo Montagnes, Woia Dziuda, Peter Buisseret, John Patty, Mary Li, Nuno Paixão, Elisa Giannone, Heung Jin Kwon, Chanont Banterghansa, Maria Anderson, and participants in the University of Chicago Economic Theory Reading Group and Political Economy Lunch.
Section 1: Introduction

“Should any political party attempt to abolish social security, unemployment insurance, and eliminate labor laws and farm programs, you would not hear of that party again in our political history.”

-President Dwight D. Eisenhower

Politicians and voters sometimes make decisions that conflict with their ideological position. For example, President Eisenhower largely maintained New Deal policies despite being the first Republican president to serve after their passage. In this paper, I explore an important driver of this kind of behavior: policy adjustment costs. Simply put, this means that significant policy changes, like eliminating unemployment benefits, are disproportionately costlier to implement than more incremental policy changes, like adjusting how long individuals can receive unemployment benefits. These costs may be directly financial, but they may also arise indirectly as individuals modify their behavior to adapt to a new policy landscape. This gives an incumbent politician some de facto influence over which policies are more attractive to future politicians and voters, given that her current policy choice determines which future policies will be less costly to implement. The epigraph shows that President Eisenhower understood this dynamic and that it significantly shaped his decision-making. I build a general theory that allows me to precisely characterize three different mechanisms that can drive this behavior and evaluate the implications of each for the well-being of voters.

I study a simple two period policy-making game. First, a leftist incumbent sets policy during period 1. Then, she faces an election against a rightist challenger that is decided by the median voter. Finally, the winner of the election can change policy during period 2. A small change is costless but a larger change induces a strictly positive policy adjustment cost (henceforth, adjustment cost) for the median voter and both politicians. This captures the notion that incremental policy change is easier to accommodate than larger scale policy change. It is a key assumption that the politicians incur the adjustment cost as well. This may occur through several different possible channels, including a direct effect of the policy, empathy for the median voter, or pressure from political advocacy groups or members of Congress. As I argue above, the adjustment cost gives the incumbent some influence over the future decisions of the median voter and both politicians. The incumbent’s degree of influence and how she chooses to leverage it depend on the degree of political polarization (henceforth, polarization) and her level of office motivation. Three key findings result, each of which illustrates a distinct mechanism through which the presence of an adjustment cost can lead the median voter or either politician to make a decision that conflicts with her ideological position. I am able to assess the welfare consequences of each for the median voter.

First, if polarization is high relative to the adjustment cost, then the incumbent can guarantee her re-election.

2The particular ideological affiliations of the politicians are not important, as long as each is the opposite of the other.
and choose her ideal point in both periods. In this case, the challenger would choose her own ideal point during period 2 because moving far away from the incumbent’s would not be very costly relative to the ideological gain. However, choosing the incumbent’s ideal point during period 2 would require no adjustment cost and choosing the challenger’s would, so the median voter prefers that the incumbent remain in power. Importantly, this occurs even if the incumbent is somewhat more ideologically extreme than the challenger. In that case, the median voter elects the politician with whom she is less aligned. Thus, the presence of an adjustment cost allows the incumbent to manipulate the median voter’s induced policy preferences to her advantage.

Second, if polarization is low relative to the adjustment cost, then the incumbent’s de facto influence over the challenger’s behavior is strong enough that she has to decide between choosing a favorable policy and guaranteeing her own re-election. The incumbent’s influence is stronger because the ideological gain to the challenger of choosing her own ideal point during period 2 is no longer high enough to justify moving policy far away from the period 1 policy. Consequently, the incumbent’s period 1 policy choice depends on her level of office motivation. If her office motivation is low, then she chooses a period 1 policy slightly more extreme than her ideal point, and she thereby guarantees that the winner of the election would choose her ideal point during period 2. However, since the politicians would choose the same period 2 policy, the median voter would be indifferent between them and therefore re-elect the incumbent with a probability of less than one. This offers a compelling explanation for why opposite party successors sometimes largely maintain the incumbent’s policy choice, especially if that policy choice is closely tied to the incumbent’s political legacy. Since the benefits of a successful legacy accrue over a much longer timeframe than the direct benefits of holding office, it is plausible that the incumbent’s desire to protect that legacy will outweigh her office motivation. One such policy is the Affordable Care Act, which President Obama would certainly like to claim as part of his legacy. While the changes that Donald Trump will make to it have yet to be determined, it is clear that major reform or repeal would impose a large adjustment cost on the U.S. population, not least because tens of millions of people could become uninsured. He has made strong statements about repealing the legislation and has the backing of a Republican-controlled Congress, but achieving consensus about a replacement plan has still proven very difficult. How healthcare reform plays out during the current administration will be a very interesting and consequential test of this result.

Finally, if the incumbent is highly office-motivated and polarization is sufficiently low relative to the adjustment cost, then she caters to the median voter by choosing a policy slightly to the right of the median voter’s ideal point during period 1. The adjustment cost is too high for the incumbent to choose her ideal point during period 2, so the best she can do is to choose the median voter’s ideal point during period 2. However, she has made it easier for the challenger to choose a period 2 policy that is closer to her own ideal point than the median voter’s. This creates a wedge between the politicians that induces the median voter to prefer re-electing the incumbent because the latter would choose a more moderate period 2 policy than the challenger. One example of this type of behavior occurred
when President Bush signed Medicare Part D into law in 2003. It was an unhappy compromise between Republicans, who disapproved of the large increase in entitlement spending that it produced, and Democrats, who felt that it did not go far enough to help senior citizens. While President Bush’s 2004 challenger, Senator John Kerry, wanted to expand the reform, President Bush preferred a more conservative path forward. In fact, after his re-election, President Bush signed the Deficit Reduction Act of 2005, which included a piece intended to slow spending growth in Medicare Parts A-C and thereby move Medicare policy back to the right. As such, President Bush attempted to position himself as the moderate candidate on this issue in his successful bid for re-election.

These results show that the median voter is best off if the incumbent is highly office-motivated and polarization is sufficiently low relative to the adjustment cost. In fact, this is the only case for which the presence of an adjustment cost makes the median voter strictly better off, as it incentivizes the incumbent to choose strictly more moderate policies than she would otherwise in order to guarantee her re-election. Essentially, she uses the adjustment cost to commit herself to choosing the median voter’s ideal point during period 2. Instead, the other two mechanisms encourage either the incumbent (directly) or the median voter (indirectly through her vote) to choose weakly more extreme policies than they would in the absence of an adjustment cost. In the former, the incumbent again uses the adjustment cost as a commitment tool, but she chooses a more extreme period 1 policy in order to commit the challenger to choosing the incumbent’s ideal point during period 2. Thus, the policy commitment generated by the adjustment cost can either help or hurt the median voter. In the latter case, the incumbent remains in power by exploiting the threat of incurring the adjustment cost.

Another important consequence of the above results is that the presence of an adjustment cost magnifies the costs of polarization for the median voter. If polarization is sufficiently low relative to the adjustment cost, then a highly office-motivated incumbent chooses strictly more moderate policies than she would in the absence of an adjustment cost. However, if polarization is not sufficiently low relative to the adjustment cost or if the incumbent is not highly office-motivated, then the equilibrium policies are weakly more extreme than they would be in the absence of an adjustment cost. Moreover, even if polarization is sufficiently low relative to the adjustment cost, a more extreme incumbent (which increases polarization \textit{ceteris paribus}) is less likely to be highly office-motivated because she has to make bigger ideological concessions - by choosing moderate policies - in order to guarantee her re-election.

The rest of the paper is structured as follows. Section 2 discusses the relationship of this paper to the related literature. Section 3 describes the model. Section 4 characterizes equilibrium behavior in two baseline cases. Section 5 characterizes equilibrium behavior in the full model. Section 6 discusses the welfare properties of the equilibria. Section 7 concludes.
**Section 2: Related Literature**

My paper contributes to the dynamic policy-making literature, which examines how current policy choices affect future policy choices. Generally speaking, there are three ways in which these dynamic linkages can arise. First, as Callander & Hummel (2014) show, when policy outcomes are uncertain the incumbent’s current policy choice can transmit payoff-relevant information to her successor about future policy. In this paper, there is no uncertainty about payoffs, so that channel is not active. Second, a number of papers examine how institutional features generate dynamic policy linkages. In Alesina & Tabellini (1990), the incumbent can exploit a long-run balanced budget requirement in order to constrain her opposite party successor’s policy behavior. In my low polarization results, the incumbent influences the challenger’s period 2 strategy in equilibrium by making it too costly for her to choose her own ideal point during period 2. Of course, this behavior arises not because the challenger has no choice, but rather because the incumbent has altered her incentives.

Other papers study the effect of an endogenous status quo, wherein the previous period’s policy becomes the status quo policy for the current period. Bernhardt & Buisseret (2016), for example, note that this institutional setting can sometimes induce a politician to propose reform with which she disagrees in order to forestall an even larger reform in the future by an opposite party agenda setter. I obtain a similar result in a baseline version of my model in which the incumbent’s re-election probability is exogenous (as it is in Bernhardt & Buisseret’s (2016) paper). Specifically, I find that if polarization is high relative to the adjustment cost and if the opposite party challenger is sufficiently likely to win the election, then the incumbent will compromise just enough to induce the challenger to compromise, should the latter prevail.

My paper is most closely related to the strand of the literature that analyzes dynamic policy linkages that are driven by the technology of policy-making instead of by explicit institutional constraints. Coate & Morris (1999) show that an adjustment cost can induce policy persistence because people (or in their model, firms) make investments contingent on the presence of a policy that increases its future value to them. In turn, that increased valuation incentivizes them to reward politicians for continuing the policy. This notion is captured very well by President Eisenhower’s epigraph, which argues that voters would likely punish any politician or political party who significantly rolls back New Deal social programs that have shaped their work and savings decisions. Their result is closest in spirit to my high polarization result, in which the incumbent chooses her ideal point during period 1. This makes the incumbent’s ideal point less costly than the challenger’s ideal point during period 2. Then, because each politician would choose her own ideal point during period 2 if elected, the median voter re-elects the incumbent in order to avoid a costly policy change.

Glazer, Gradstein & Konrad (1998) analyze a model very similar to mine. In theirs, they assume that the fixed adjustment cost is associated with any policy change and that polarization is low relative to the adjustment
cost. They find that the incumbent chooses her ideal point during period 1 if her office motivation is low, but she chooses a more extreme policy during period 1 if her office motivation is high. I obtain very similar results when polarization is low, although I de-emphasize the analogue of their high office motivation result because it relies heavily on the assumption that the adjustment cost is (relatively) unresponsive to the size of the policy change. My main contribution relative to their paper is the result that when office motivation is high and polarization is sufficiently low relative to the adjustment cost, the incumbent chooses a moderate opposite party period 1 policy in order to guarantee her re-election. This arises from my assumption about the form of the adjustment cost, which allows small policy changes to be costless. This means that policy may move slightly to the left or right during period 2. Thus, when the incumbent chooses a period 1 policy that is slightly to the right of the median voter’s ideal point, both politicians offer a costless policy change during period 2, but they do so in opposite directions. This creates the wedge that guarantees re-election for the incumbent - a phenomenon that Glazer, Gradstein & Konrad (1998) cannot achieve. This difference is important because the corresponding results have very different welfare implications for the median voter. In their paper, strong office motivation always makes the median voter worse off because it generates extreme policies, whereas in my paper, it makes the median voter better off if polarization is sufficiently low relative to the adjustment cost because it generates moderate policies.

Gersbach, Muller & Tejada (2015) also study an environment in which policy is costly to adjust. They find that a highly office-motivated incumbent always chooses a period 1 policy that is more moderate than her ideal point in order to make herself relatively more attractive to the median voter. I come to a similar conclusion when polarization is sufficiently low relative to the adjustment cost, although the mechanisms of our results are qualitatively different. In their paper, the incumbent moderates her period 1 policy choice just enough so that the adjustment cost associated with the challenger’s period 2 policy choice discourages the median voter from electing the challenger. However, in my result, the adjustment costs associated with the politicians’ period 2 policy choices are the same (i.e., zero). Instead, the incumbent maximizes her re-election probability by choosing a period 1 policy that incentivizes her to choose a more moderate period 2 policy than the challenger. In fact, their mechanism is most reminiscent of my high polarization result, in which the incumbent guarantees her re-election because her period 2 policy choice is less costly than the challenger’s. Nevertheless, each has similar welfare implications. In both results, the introduction of an adjustment cost is a crucial force that pushes the incumbent to moderate policy, which improves voter welfare. My paper distinguishes itself because it is also able to address what happens when the incumbent is not highly office-motivated. In that case, the incumbent exploits her first mover status not to gain an electoral advantage but to force the challenger, if she is elected, to implement the incumbent’s ideal point rather than her own.

Callander & Raiha (2014) analyze a model in which politicians, before implementing a single policy in each period, have a fixed intertemporal budget to invest in the quality of multiple policies. They show that if polarization is high, then the incumbent can efficiently invest the entire budget in her ideal point, implement it, and still guarantee
re-election. I obtain a very similar result, where high investment in their paper is analogous to a low adjustment cost in my paper, and vice versa. They also show that if office motivation is high and polarization is sufficiently low, then the incumbent implements a policy that is more extreme than her ideal point and strategically diverts just enough investment to the challenger’s ideal point to guarantee her re-election. Specifically, while she invests most of the budget in her implemented policy, she also invests just enough in the challenger’s ideal point to induce the challenger to implement it if elected; this comes at the cost of implementing a lower quality period 1 policy. While I also find that the incumbent manipulates the challenger’s period 2 policy in order to get re-elected, our papers come to fundamentally different conclusions about the incumbent’s strategy. In my paper, the incumbent implements a moderate policy. In their paper, the incumbent implements an extreme policy and compromises the quality of that policy by wasting investment on the challenger’s ideal point. Thus, while in my paper a sufficiently low degree of polarization (combined with high office motivation) makes the median voter better off, in their paper it may actually make the median voter worse off if the wasted investment outweighs the benefit of a more moderate policy. My paper also distinguishes itself because it is able to address what happens when the incumbent is not highly office-motivated.

Section 3: Model

There are 3 players \( i \in \{L, M, R\} \) who play a game over two periods \( t \in \{1, 2\} \). Players \( L \) and \( R \) are politicians and player \( M \) is the median voter. In each period \( t \), one of the politicians \( i \in \{L, R\} \) unilaterally chooses a policy \( x_{it} \in \mathbb{Z} \) during period 1, the incumbent \( L \) chooses policy. Then, there is an election between \( L \) and the challenger \( R \), which I discuss shortly. during period 2, the election winner, whom I denote \( w \in \{L, R\} \), chooses policy.

I examine two different assumptions about the election process. Under the first assumption, which I call an endogenous election, the median voter chooses the election winner, re-electing \( L \) with probability \( \pi \in (0, 1) \) if she is indifferent. This is the workhorse assumption of the paper. Under the second assumption, which I call an exogenous election, the median voter does not choose the election winner. Instead, \( L \) is re-elected with exogenous probability \( p \in [0, 1] \), and \( R \) is elected with probability \( 1 - p \). This is a baseline assumption, which I examine in a clearly demarcated part of Section 4 in order to better understand the equilibrium dynamics associated with an endogenous election. Denote the probability that \( L \) gets re-elected given \( x_{L1} \) by \( p(x_{L1}) \).

All players \( i \in \{L, M, R\} \) derive policy utility \( \hat{U}_i(x_{L1}, x_{w2}) \), which is characterized as follows. Each player \( i \) has an ideal point \( x^*_i \in \mathbb{Z} \), where \( x^*_L < 0 \), \( x^*_M = 0 \) and \( x^*_R > 0 \). Then, the utility of a single policy \( x \in \mathbb{Z} \) is \( u_i(x) = -|x - x^*_i| \). If \( |x_{w2} - x_{L1}| > 1 \), then there is an adjustment cost \( \alpha \geq 0 \) to changing policy, so \( \hat{U}_i(x_{L1}, x_{w2}) = u_i(x_{L1}) + u_i(x_{w2}) - \alpha \).
However, if $|x_w - x_L| \leq 1$, then there is no adjustment cost, so $\tilde{U}_i(x_{L1}, x_{w2}) = u_i(x_{L1}) + \eta_i$. For most of the paper I assume that $\alpha > 0$. I examine the baseline assumption that $\alpha = 0$ in a clearly demarcated part of Section 4 in order to better understand the equilibrium dynamics associated with a strictly positive adjustment cost.

For $x, x' \in \mathbb{Z}$ such that $|x| < |x'|$, I say that policy $x$ is more moderate than policy $x'$, or equivalently, that policy $x'$ is more extreme than policy $x$. If $|x| = |x'|$, then I say that policies $x$ and $x'$ are equally moderate. Also, for $i \neq j \in \{L, R\}$ such that $|x^*_i| < |x^*_j|$, I say that politician $i$ is more moderate than politician $j$, or that politician $j$ is more extreme than politician $i$. If $|x^*_i| = |x^*_j|$, then I say that the politicians are equally moderate. I define the degree of (political) polarization in this environment to be $\Delta = x^*_R - x^*_L = x^*_R + |x^*_L|$.

In addition to policy utility, politician $i \in \{L, R\}$ receives an office rent of $\eta_{it} > 0$ if she is in power during period $t$. I call $\eta_{L2} \equiv \eta$ the incumbent’s level of office motivation. Then, letting $\lambda_{it} \equiv 1$ ($i$ is in power in period $t$), total utility for $i \in \{L, R\}$ is given by

$$U_i(x_{L1}, x_{w2}, \lambda_{i1}, \lambda_{i2}) = \tilde{U}_i(x_{L1}, x_{w2}) + \lambda_{i1}\eta_{i1} + \lambda_{i2}\eta_{i2}.$$ 

The median voter $M$’s utility from electing candidate $i \in \{L, R\}$ is given by $U_M(x_{L1}) = \tilde{U}_M(x_{L1}, x_{i2})$. Let $\tilde{U}_i(\alpha, \eta, x^*_R, x^*_L)$ denote the utility of player $i \in \{L, M, R\}$ in the equilibrium defined by $(\alpha, \eta, x^*_R, x^*_L)$. I consider subgame perfect equilibria. All proofs are relegated to the Appendix.

**Section 4: Baseline Results**

In Section 5 I show that the combination of a strictly positive adjustment cost and an endogenous election gives the incumbent’s period 1 policy choice influence over both the election winner’s period 2 policy choice and the election outcome. In order to fully understand how these dynamic effects shape the incumbent’s behavior, I first restrict the model and characterize equilibria under two baseline assumptions. First, if the adjustment cost is zero, then neither dynamic effect is present. In this case, each politician always chooses her ideal point and the more moderate politician is always elected. Second, if the adjustment cost is strictly positive but the election is exogenous, then the incumbent’s period 1 policy choice can influence the election winner’s period 2 policy choice but not the election outcome. In that case, the incumbent’s period 1 policy choice depends on both the degree of polarization and the re-election probability. If polarization is high relative to the adjustment cost and the incumbent is likely to get re-elected, then each politician again always chooses her ideal point. If polarization is high relative to the adjustment cost and the incumbent is unlikely to get re-elected, then she forges a compromise with the challenger in order to avoid
a huge policy swing if she loses the election. If polarization is low relative to the adjustment cost, then the incumbent
chooses her ideal point and the challenger chooses the policy just to the right of the incumbent’s ideal point if elected.

First, suppose that the adjustment cost is zero. Then, the period 1 policy choice cannot influence the period 2
policy preferences of any player because no period 2 policy choice induces an adjustment cost. Thus, the winner of
the election chooses her ideal point regardless of the period 1 policy. Knowing this, the median voter elects the more
moderate politician knowing that she will choose a period 2 policy that is closer to the median voter’s ideal point.
Proposition 1 formalizes this intuition.

Proposition 1: Suppose that $\alpha = 0$. Then, there exists a unique equilibrium in which $x_{it} = x^*_i$ for all
$(i, t) \in \{L, R\} \times \{1, 2\}$ and $p(x_{L1}) = \begin{cases} 1, & \text{if } |x^*_L| < x^*_R \\
\pi, & \text{if } |x^*_L| = x^*_R \\
0, & \text{if } |x^*_L| > x^*_R \end{cases}$

For the remainder of the paper, I assume that $\alpha > 0$. For the remainder of this section, suppose that the election
is exogenous. Here, the period 1 policy does influence each player’s period 2 policy preferences by making the period
1 policy and its adjacent policies less costly to choose during period 2. This affects the winner’s period 2 policy, but
the exogenous re-election probability eliminates the electoral impact of the period 1 policy. The impact of allowing
the incumbent to have this single channel of dynamic influence depends on both the degree of polarization and the
re-election probability.

First, suppose that polarization is high relative to the adjustment cost - specifically $\Delta > \alpha + 1$. If the incumbent
chooses her ideal point during period 1, then she also chooses her ideal point during period 2 if elected because it is
costless for her to do so. How does the challenger respond if she gets elected? If she avoids the adjustment cost, then
she “compromises” and chooses the period 2 policy that is one step to the right of the period 1 policy, towards her
ideal point. Otherwise, she simply chooses her ideal point. If she does compromise, she saves the adjustment cost $\alpha$
but suffers an even bigger policy utility loss of $\Delta - 1 > \alpha$. As a result, she is willing to incur the adjustment cost in
order to move the period 2 policy to her own ideal point.

If the re-election probability is sufficiently high, then for the incumbent it is worthwhile risking the big policy
swing and adjustment cost associated with an (unlikely) loss and choose her ideal point during period 1. Thus,
the equilibrium behavior of the politicians remains the same as in the zero adjustment cost case. However, if the
re-election probability is low, then the incumbent moves the period 1 policy just far enough towards the challenger’s
ideal point that the challenger is willing to compromise. While this hurts the incumbent during period 1, and hurts
her during period 2 if she gets re-elected, it helps her during period 2 if she does not get re-elected by making the
period 2 policy more attractive and eliminating the adjustment cost. Thus, the incumbent uses her first mover advantage to minimize her expected losses if she loses the election. Note that by construction, the relative extremity of the politicians’ induced period 2 policy choices does not affect the election outcome. Thus, the incumbent does not need to take that into account with her period 1 policy choice. Proposition 2 formalizes this intuition.

Proposition 2: Suppose that $\Delta > \alpha + 1$ and $L$ is re-elected with exogenous probability $p \in [0, 1]$. Then, there exists $p(\alpha) < 1$ such that the following unique equilibria hold. If $p > p(\alpha)$, then $x_{L1} = x^*_L$, $x_{L2}(x_{L1}) = x^*_L$ and $x_{R2}(x_{L1}) = x^*_R$. If $p < p(\alpha)$, then $x_{L1} = x^*_R - [\alpha]$, $x_{L2}(x_{L1}) \in \{x^*_L, x_{L1} - 1\}$ and $x_{R2}(x_{L1}) = x_{L1} + 1$.

Next, suppose that polarization is low relative to the adjustment cost - specifically $\Delta < \alpha + 1$. If the incumbent chooses her ideal point during period 1, then the challenger is willing to compromise if she is elected. This means that the incumbent induces the best possible policy outcome for herself if she wins the election and if she loses the election. Here, the challenger would definitely choose a more moderate period 2 policy than the incumbent if elected, but, again, that does not affect the election outcome. Thus, the incumbent has no incentive to choose any other period 1 policy. In this case, her behavior does not deviate from the zero adjustment cost equilibrium, but the presence of the adjustment cost changes the challenger’s behavior. Proposition 3 formalizes this intuition.

Proposition 3: Suppose that $\Delta < \alpha + 1$ and $L$ is re-elected with exogenous probability $p \in [0, 1]$. Then, there exists a unique equilibrium in which $x_{L1} = x^*_L$, $x_{L2}(x_{L1}) = x^*_L$ and $x_{R2}(x_{L1}) = x^*_L + 1$.

Section 5: Full Model Results

For the remainder of the paper, I assume that the adjustment cost is strictly positive and that the election is endogenous. I show that the incumbent’s period 1 policy choice can influence both the election winner’s period 2 policy choice and the election outcome. The incumbent’s period 1 policy choice and election outcome depend on both the degree of polarization and her level of office motivation. If polarization is high relative to the adjustment cost, then she can choose her ideal point and guarantee her re-election, even if she is somewhat more extreme than the challenger. On the other hand, if polarization is low relative to the adjustment cost, then the incumbent can either guarantee that the election winner chooses her ideal point during period 2 or guarantee her re-election, but she cannot do both. If her office motivation is low, then she chooses a period 1 policy that accomplishes the former
but makes the median voter indifferent between the politicians. If her office motivation is high and polarization is sufficiently low relative to the adjustment cost, then she chooses a moderate opposite party period 1 policy that accomplishes the latter but locates the period 2 policy at the median voter’s ideal point instead of her own.

Suppose first that polarization is high relative to the adjustment cost - specifically $\Delta \in (\alpha + 1, \alpha + 2x_R^*)$. Below I discuss the necessity of the upper bound.\(^3\) As noted in Section 4, if the incumbent chooses her ideal point during period 1, then each politician would choose her ideal point during period 2 if elected. If the incumbent is more moderate than the challenger, then the median voter strictly prefers the incumbent’s ideal point to the challenger’s ideal point during period 2 for two reasons. First, the incumbent’s ideal point is closer to the median voter’s ideal point. Second, choosing the incumbent’s ideal point during period 2 would not require incurring the adjustment cost whereas choosing the challenger’s would. If the incumbent is weakly more extreme than the challenger but not excessively so - specifically $|x_L^*| - x_R^* < \alpha$ - then the median voter still strictly prefers the incumbent’s ideal point, although only for the second reason. This requisite inequality condition is equivalent to $\Delta < \alpha + 2x_R^*$. While the median voter weakly prefers the challenger’s period 2 policy in a vacuum, she does not feel strongly enough to incur the adjustment cost that is associated with the policy improvement.

This result is driven entirely by the influence that the incumbent’s period 1 policy exerts over the median voter’s period 2 policy preferences. Recall that in the zero adjustment cost baseline, each politician always chooses her ideal point and the median voter always prefers the more moderate politician. On the other hand, when the adjustment cost becomes positive, the median voter develops a strict preference for the incumbent when the latter chooses her own ideal point during period 1. When the election is exogenous, the median voter cannot act on this preference, and so it is possible for the incumbent to lose the election. However, when the median voter is allowed to vote, the incumbent’s (endogenous) re-election probability becomes one. As noted in Section 4, this implies that each politician always chooses her ideal point, so the politicians’ policy choices remain the same as in the zero adjustment cost baseline. Thus, if polarization is high relative to the adjustment cost, then the presence of an adjustment cost does not change the politicians’ behavior, but it does encourage the median voter to re-elect a (somewhat) more extreme incumbent in order to avoid the adjustment cost of switching from the incumbent’s ideal point to the challenger’s. Theorem 1 formalizes this result.

**Theorem 1:** Suppose that $\Delta \in (\alpha + 1, \alpha + 2x_R^*)$. Then, there exists a unique equilibrium in which $x_{L1} = x_L^*$, $p(x_{L1}) = 1$, $x_{L2}(x_{L1}) = x_L^*$ and $x_{R2}(x_{L1}) = x_R^*$.

\(^3\)I impose the upper bound condition throughout the rest of the paper in order to focus on the most interesting equilibria.
Now, suppose that polarization is low relative to the adjustment cost - specifically $\Delta \leq \alpha$. As noted in Section 4, if the incumbent chooses her ideal point during period 1, then the challenger is willing to compromise if elected. In fact, this condition is even stronger because the challenger is still willing to compromise even if the period 1 policy is one step more extreme than the incumbent’s ideal point. Under this condition, both politicians if elected would choose the incumbent’s ideal point during period 2. As a result, the incumbent has to decide between guaranteeing that her ideal point is chosen during period 2 and guaranteeing her re-election. To accomplish the former, her period 1 policy choice must induce both politicians to choose the same period 2 policy. However, this makes the median voter indifferent between them, and so the incumbent is no longer guaranteed re-election. Thus, the incumbent chooses this option if she is not highly office-motivated.

In this result, the electoral process distorts the incumbent’s behavior in the face of an adjustment cost because her level of office motivation is low but positive. When the election is exogenous and polarization is low relative to the adjustment cost, recall that she chooses her ideal point whenever she is in power. If the challenger wins the election, then she chooses a period 2 policy that is one step more moderate than the incumbent’s ideal point. Thus, the median voter prefers the challenger, but because she cannot vote, that preference does not affect the incumbent’s election outcome. However, when the median voter can vote, the influence that the incumbent’s period 1 policy has on the challenger’s period 2 policy does affect the election outcome. In particular, if the incumbent chooses her ideal point during period 1, then she will lose the election. To improve her election prospects, she chooses a policy that is one step more extreme than her ideal point. Since both politicians then choose the same period 2 policy, this tactic improves her re-election probability from zero to $\pi$. Note that the incumbent is indifferent between the policy outcomes of these two strategies. In both, her ideal point is chosen during one period and an adjacent policy is chosen during the other period. Thus, she chooses a more extreme policy solely to manipulate the electoral process in her favor. Also, neither politician’s period 2 policy incurs the adjustment cost. Consequently, the median voter always chooses the politician with the more moderate period 2 policy, as she would in the absence of the adjustment cost. In contrast to the high polarization case, the incumbent manipulates the election outcome entirely via her influence over the challenger’s period 2 policy. Theorem 2 formalizes this result.

**Theorem 2:** Suppose that $\Delta \leq \alpha$. Then, there exists $\eta(\Delta) \in (0, +\infty)$ such that for all $\eta \in (0, \eta(\Delta))$, the unique equilibrium is characterized by $x_{L1} = x^*_L - 1$, $p(x_{L1}) = \pi$ and $x_{L2}(x_{L1}) = x_{R2}(x_{L1}) = x^*_L$.

If the incumbent is highly office-motivated, then she willingly accepts less desirable policies in order to improve her re-election probability from zero to one (rather than from zero to $\pi$). If polarization is not sufficiently low relative

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4The equilibrium in which $\Delta \in (\alpha, \alpha + 1]$ is a unique case. I refrain from discussing it in the body of the paper and relegate its formal characterization to Proposition 4, which appears in the Appendix directly after the proof of Theorem 1.
to the adjustment cost - specifically $\Delta \in (\alpha - |x_L^*| + 1, \alpha]$ - then she chooses a period 1 policy that is strictly more extreme than the period 1 policy she would choose if she were not highly office-motivated. The logic is similar to the high polarization case. She chooses a period 1 policy just extreme enough that the challenger would be willing to incur the adjustment cost to choose her own ideal point during period 2 if elected. However, the period 1 policy is not extreme enough for the incumbent or the median voter to prefer the same. Thus, the incumbent would compromise during period 2 in order to avoid incurring the adjustment cost. As a result, the median voter strictly prefers the incumbent’s period 2 policy choice to the challenger’s, and so she re-elects the incumbent. I de-emphasize this result relative to my other results because it relies heavily on the assumption that the adjustment cost is constant with respect to the size of the policy change in the case of policy changes that are larger than two steps. For example, if the adjustment cost were quadratic in the size of the policy change instead of a step function, then the incumbent and the challenger would move policy by the same amount, which would thereby incur the same adjustment cost. This would eliminate for the incumbent the electoral benefit of choosing an extreme period 1 policy. However, for the sake of completeness, Proposition 5 formalizes this result.

**Proposition 5:** Suppose that $\Delta \in (\alpha - |x_L^*| + 1, \alpha]$. Then, for all $\eta > \eta(\Delta)$, the unique equilibrium is characterized by $x_{L1} < x_L^* - 1, p(x_{L1}) = 1, x_{L2}(x_{L1}) = x_{L1} + 1$ and $x_{R2}(x_{L1}) = x_R^*$.

If polarization is sufficiently low relative to the adjustment cost - specifically $\Delta \leq \alpha - |x_L^*|$ - then the incumbent chooses the period 1 policy one step to the right of the median voter’s ideal point.\(^5\) As a result, the incumbent would locate the period 2 policy at the median voter’s ideal point instead of at her own ideal point because the adjustment cost is large relative to the distance between them. However, the challenger would choose a weakly more extreme period 2 policy. If the period 1 policy is located at her ideal point, then she would keep the period 2 policy the same. If the period 1 policy is more moderate than her ideal point, then she would choose the period 2 policy one step to the right of the period 1 policy. In either case, the median voter prefers the incumbent’s period 2 policy choice to the challenger’s, and so she votes to re-elect the incumbent. Here, the incumbent eschews the extreme period 1 policy choice in favor of this particular very moderate period 1 policy choice because the latter requires a smaller policy sacrifice to guarantee the incumbent’s re-election.

Once again, the electoral process has a fundamental impact on the incumbent’s behavior, but in this case it generates more moderate policy instead of more extreme policy. Rather than maximizing her re-election probability subject to the constraint that she accept no policy sacrifice (as she does when her office motivation is low), the incumbent minimizes the policy sacrifice required to maximize her re-election probability. Interestingly, when the

\(^5\)If $\Delta \in (\alpha - |x_L^*|, \alpha - |x_L^*| + 1]$, then the incumbent randomizes between this period 1 policy choice and the extreme period 1 policy choice. For the remainder of the paper, I omit that case from consideration in order to focus on unique equilibria.
election is exogenous, the incumbent chooses a period 1 policy to the right of her ideal point only when polarization is high relative to the adjustment cost and her re-election probability is low. There, she seeks to moderate the challenger’s period 2 policy choice lest the latter win the election. However, when the median voter can vote, high polarization helps the incumbent because she can choose her ideal point and guarantee her re-election. Instead, she chooses a period 1 policy to the right of her ideal point only when she is highly office-motivated and polarization is low relative to the adjustment cost - in order to encourage the challenger to choose a period 2 policy just extreme enough to guarantee the incumbent’s re-election. Theorem 3 formalizes this result.

**Theorem 3:** Suppose that $\Delta \leq \alpha - |x_L^1|$. Then, for all $\eta > \eta(\Delta)$, the unique equilibrium is characterized by $x_{L1} = 1$, $p(x_{L1}) = 1$, $x_{L2}(x_{L1}) = 0$ and $x_{R2}(x_{L1}) \in \{1, 2\}$.

**Section 6: Welfare**

In this section, I examine the welfare consequences of an adjustment cost. Specifically, I show that the median voter is best off if the incumbent is highly office-motivated and polarization is sufficiently low relative to the adjustment cost. In fact, that is the only case in which the presence of an adjustment cost makes the median voter strictly better off. A key consequence of these results is that the presence of an adjustment cost amplifies the effect of polarization on the median voter’s welfare.

If the incumbent is highly office-motivated and polarization is sufficiently low relative to the adjustment cost, then Theorem 3 shows that the incumbent chooses very moderate policies in equilibrium. In all other cases, Theorems 1 and 2 (and Propositions 4 and 5) show that the incumbent chooses more extreme policies that get increasingly extreme as her ideal point moves farther to the left. If polarization is high relative to the adjustment cost, then she simply chooses her ideal point during both periods. If polarization is low relative to the adjustment cost but she is not highly office-motivated, then she chooses the period 1 policy one step more extreme than her ideal point, and the winner of the election chooses the incumbent’s ideal point. Thus, the median voter’s equilibrium utility is strictly lower in all these other cases because the policies that arise in equilibrium are farther away from her ideal point. Theorem 4 formalizes this result.

**Theorem 4:** $U_M(\alpha, \eta, x_R^1, x_L^1)$ is maximized if $\Delta \leq \alpha - |x_L^1|$ and $\eta > \eta(\Delta)$. 

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In fact, a stronger result than Theorem 4 holds. The presence of an adjustment cost makes the median voter strictly better off if and only if the incumbent is highly office-motivated and polarization is sufficiently low relative to the adjustment cost. In that case, the incumbent chooses strictly more moderate policies than she would otherwise in order to guarantee her re-election. She uses the adjustment cost as a tool to commit herself to choosing the median voter’s ideal point during period 2. The other two cases of interest encourage either the incumbent (directly) or the median voter (indirectly through her vote) to choose weakly more extreme policies than they would in the absence of an adjustment cost. In the former, the incumbent again uses the adjustment cost as a commitment tool, but she chooses a more extreme period 1 policy in order to commit the challenger to choosing the incumbent’s ideal point during period 2. Thus, the policy commitment generated by the adjustment cost can either help or hurt the median voter. In the latter case, the incumbent exploits the threat of incurring the adjustment cost to remain in power. Theorem 5 formalizes this result.

**Theorem 5:** If \( \alpha > 0 \), then \( \overline{U}_M (\alpha, \eta, x^*_R, x^*_L) > \overline{U}_M (0, \eta, x^*_R, x^*_L) \) if and only if \( \Delta \leq \alpha - |x^*_L| \) and \( \eta > \bar{\eta}(\Delta) \).

An important implication of Theorem 5 is that the presence of an adjustment cost magnifies the costs of polarization to the median voter. If polarization is sufficiently low relative to the adjustment cost, then a highly office-motivated incumbent chooses strictly more moderate policies than she would in the absence of an adjustment cost. However, if polarization is not sufficiently low relative to the adjustment cost or if the incumbent is not highly office-motivated, then the equilibrium policies are weakly more extreme than they would be in the absence of an adjustment cost. Also, even if polarization is sufficiently low relative to the adjustment cost, a more extreme incumbent - which increases polarization *ceteris paribus* - is less likely to be highly office-motivated because she has to make bigger ideological concessions (i.e., choose moderate policies) in order to guarantee her re-election.

**Discussion:** Before concluding, I discuss in more detail the importance and robustness of some of the model’s assumptions. First, I assume that the incumbent’s re-election probability is strictly interior if her period 1 policy choice makes the median voter indifferent between the two politicians’ period 2 policy choices. When polarization is low relative to the adjustment cost, that assumption generates a substantive tradeoff for the incumbent between choosing a period 1 policy close to her ideal point and choosing a period 1 policy that maximizes her re-election probability. This tradeoff, I believe, exists in many real-world political situations. It does not hold under a common alternative assumption, which is that the incumbent is re-elected with probability one unless the median voter *strictly* prefers the challenger’s period 2 policy choice. In that case, \( x_{L1} = x^*_L - 1 \) would be the incumbent’s uniquely optimal period 1 policy choice if polarization is low relative to the adjustment cost, which would maximize both her policy utility and her re-election probability.
Nevertheless, my assumption can be micro-founded in an environment where the median voter also judges each politician on the basis of non-policy factors, such as her character and charisma (i.e., her valence), which seems eminently plausible. Specifically, suppose that each politician publicly draws a one-time “valence shock” from some distribution with full support on the real line (e.g., Uniform, Normal, etc.) when she first runs for office, which captures all non-policy factors in a reduced form. This is a fairly common assumption in models of electoral competition and in economic models of discrete choice more generally (where the random draw takes on a different interpretation). Then, the incumbent’s re-election probability is strictly interior if her period 1 policy choice makes the median voter indifferent between the two politicians’ period 2 policy choices. This is so because there is always some chance that the challenger will draw the higher (or lower) valence shock. My assumption arises in the limit as the variance of the valence shock goes to zero. Thus, while the incumbent’s re-election probability is smoother as a function of her period 1 policy choice in that environment relative to mine, there is no other substantive difference, so the same qualitative results will obtain. In other words, my assumption is the simplified version of a reasonable model of electoral competition.

Second, I assume a discrete policy space (i.e., integers) instead of a continuous policy space (e.g., reals). I show that if the incumbent is highly office-motivated and polarization is sufficiently low relative to the adjustment cost, then during period 1 the incumbent would like to choose the most moderate policy on the opposite side of the political spectrum. Mathematically, this means that she would like to choose the lowest number in the policy space that is strictly greater than zero, which is the median voter’s ideal point. If the policy space is continuous, then no such number exists. As a result, her choice converges to the median voter’s ideal point and the politicians’ induced period 2 policy choices become equally appealing to the median voter (just on different sides), which eliminates the strategy’s appeal. The presence of a discrete policy space eliminates this complication, allowing the result to hold mathematically.

Finally, I assume that the adjustment cost is zero if policy does not change by more than one step; otherwise it is positive but fixed. The first part of this assumption captures a convex adjustment cost (at least in the neighborhood of the original policy), wherein the (non-ideological) cost of changing policy by two steps is significantly (in this case, infinitely) larger than the cost of changing policy by one step. The second part of the assumption simplifies the analysis. However, it is arguably less realistic, given that one might expect the adjustment cost to continue to grow as the magnitude of the policy change increases. Nevertheless, the three main positive results (i.e., Theorems 1-3) remain qualitatively unchanged if I replace my adjustment cost function with a quadratic adjustment cost function. Thus, none of them fundamentally depends on the assumption that the adjustment cost is flat for large policy changes.
Section 7: Conclusion

I have shown that the presence of a policy adjustment cost can explain why politicians and voters sometimes take actions that conflict with their ideological position. This behavior arises through three distinct mechanisms. If political polarization is high relative to the adjustment cost, then the incumbent induces the median voter to re-elect her even if she is somewhat more extreme than the challenger. The median voter does this in order to avoid the adjustment cost associated with changing policy to the challenger’s ideal point. If the incumbent is not highly office-motivated and polarization is low relative to the adjustment cost, then she chooses a period 1 policy that leaves open the possibility of losing the election to the opposite party challenger. However, in that event, the challenger is compelled to choose the incumbent’s ideal point during period 2 because in this case even the challenger is no longer willing to incur the adjustment cost. If the incumbent is highly office-motivated and polarization is sufficiently low relative to the adjustment cost, then, to guarantee her re-election, she chooses the most moderate policy on the opposite side of the political spectrum during period 1. In essence, she uses the presence of the adjustment cost to commit herself to choosing a more moderate period 2 policy than the challenger. The median voter is best off in this case because the policies that arise in equilibrium are very moderate. In fact, this is the only case for which the presence of an adjustment cost makes the median voter strictly better off; it generates strictly more moderate policies instead of weakly more extreme policies in equilibrium. In an important corollary of these welfare results, the presence of an adjustment cost amplifies the costs of polarization to the median voter.

Note that if polarization is low relative to the adjustment cost, then the period 2 policy is more moderate than the period 1 policy, whether or not the incumbent is highly office-motivated. This is at least partly driven by the model’s two period time horizon because the period 2 policy choice has no dynamic effect. Thus, the period 1 policy is never located at the winner’s ideal point, and so she moves policy one step closer to her ideal point during period 2. It would be interesting in future work to consider an infinite horizon version of the model in order to shed more light on the long-term dynamic consequences of policy adjustment costs.
References


Appendix

Proof of Proposition 1: Suppose that $\alpha = 0$. I characterize the equilibrium and prove that it is unique by solving the model backwards in the standard way. During period 2, the election winner $w \in \{L, R\}$ solves

$$V_{w2} = \max_{x_{w2} \in \mathbb{Z}} - |x_{L1} - x_{w2}^*| - |x_{w2} - x_{w2}^*| - 1 (|x_{L1} - x_{w2}| > 1) \cdot \alpha + (\lambda_{w1} + 1) \cdot \eta$$

$$= \max_{x_{w2} \in \mathbb{Z}} - |x_{L1} - x_{w2}^*| - |x_{w2} - x_{w2}^*| - 1 (|x_{L1} - x_{w2}| > 1) \cdot 0 + (\lambda_{w1} + 1) \cdot \eta$$

$$= \max_{x_{w2} \in \mathbb{Z}} - |x_{L1} - x_{w2}^*| - |x_{w2} - x_{w2}^*| + (\lambda_{w1} + 1) \cdot \eta$$

$$= \max_{x_{w2} \in \mathbb{Z}} - |x_{w2} - x_{w2}^*|,$$  

where the final equality comes because the first term and last term in the maximization are constant with respect to $x_{w2}$. The maximand is strictly negative if $x_{w2} \neq x_{w2}^*$ and equal to zero if $x_{w2} = x_{w2}^*$, so the unique maximizer is $x_{w2} = x_{w2}^*$.

Thus, when the median voter elects $w$, she receives utility

$$U_{M} (x_{L1}) = \tilde{U}_{M} (x_{L1}, x_{w}^*)$$

$$= - |x_{L1} - x_{M}^*| - |x_{w}^* - x_{M}^*| - 1 (|x_{L1} - x_{w}^*| > 1) \cdot \alpha$$

$$= - |x_{L1} - x_{w}^*| - 1 (|x_{L1} - x_{w}^*| > 1) \cdot 0$$

$$= - |x_{L1} - x_{w}^*|.$$

She re-elects $L$ with probability one if she receives strictly higher utility from re-electing $L$ than from electing $R$. This occurs if and only if

$$U_{M}^L (x_{L1}) > U_{M}^R (x_{L1})$$

$$- |x_{L1}| - |x_{L}^*| > - |x_{L1}^* - x_{R}^*|$$

$$- |x_{L}^*| > - |x_{R}^*|$$

$$|x_{L}^*| < |x_{R}^*|$$

$$|x_{L}^*| < x_{R}.$$

She elects $R$ with probability one if she receives strictly higher utility from electing $R$ than from re-electing $L$. This occurs if and only if

$$U_{M}^L (x_{L1}) < U_{M}^R (x_{L1})$$

$$- |x_{L1}| - |x_{L}^*| < - |x_{L1}^* - x_{R}^*|$$

$$- |x_{L}^*| < - |x_{R}^*|$$

$$|x_{L}^*| > x_{R}^*$$

$$|x_{L}^*| > x_{R}.$$

She re-elects $L$ with probability $\pi$ if she receives equal utility from re-electing $L$ and from electing $R$. This occurs if and only if

$$U_{M}^L (x_{L1}) = U_{M}^R (x_{L1})$$

$$- |x_{L1}| - |x_{L}^*| = - |x_{L1}^* - x_{R}^*|$$

$$- |x_{L}^*| = - |x_{R}^*|$$

$$|x_{L}^*| = x_{R}^*$$

$$|x_{L}^*| = x_{R}.$$

Note that neither $L$’s re-election probability nor the election winner’s period 2 policy choice depends on $x_{L1}$. In
particular, let \( p(x_{L1}) \equiv \rho \). Thus, during period 1, \( L \) solves
\[
V_{L1} = \max_{x_{L1} \in \mathbb{Z}} -|x_{L1} - x_1^*| - \rho |x_{L1}^* - x_1^*| - (1 - \rho) |x_R^* - x_1^*| - \rho \cdot 1(|x_{L1} - x_1^*| > 1) + (1 - \rho) \cdot 1(|x_{L1} - x_1^*| > 1) \cdot \alpha + (1 + \rho) \cdot \eta \\
= \max_{x_{L1} \in \mathbb{Z}} -|x_{L1} - x_1^*| - \rho |x_{L1}^* - x_1^*| - (1 - \rho) |x_R^* - x_1^*| - \rho \cdot 1(|x_{L1} - x_1^*| > 1) + (1 - \rho) \cdot 1(|x_{L1} - x_1^*| > 1) \cdot 0 + (1 + \rho) \cdot \eta \\
= \max_{x_{L1} \in \mathbb{Z}} -|x_{L1} - x_1^*| - \rho |x_{L1}^* - x_1^*| - (1 - \rho) |x_R^* - x_1^*| + (1 + \rho) \cdot \eta \\
= \max_{x_{L1} \in \mathbb{Z}} -|x_{L1} - x_1^*|,
\]
where the final equality comes because the last three terms are constant with respect to \( x_{L1} \). The maximand is strictly negative if \( x_{L1} \neq x_1^* \) and equal to zero if \( x_{L1} = x_1^* \), so the unique maximizer is \( x_{L1} = x_1^* \).

Thus, the unique sequentially rational strategy for each politician is to choose \( x_{it} = x_i^* \) for all \( (i,t) \in \{L,R\} \times \{1,2\} \). As a result, the unique sequentially rational strategy for the median voter is to choose \( \rho = 1 \) if \( |x_1^*| < x_2^* \), \( \rho = 0 \) if \( |x_1^*| > x_2^* \) and \( \rho = \pi \) if \( |x_1^*| = x_2^* \). The final result does require the assumption that the median voter re-elects \( L \) with probability \( \pi \) if indifferent between the politicians. Thus, the equilibrium stated in Proposition 1 exists and is unique.

**Proof of Proposition 2:** Suppose that \( \Delta > \alpha + 1 \) and \( L \) is re-elected with exogenous probability \( p \in [0,1] \). There are four main steps to this proof. First, I show that if \( x_{L1} = x_1^* \), then \( x_{t2} = x_t^* \) for all \( i \in \{L,R\} \). Second, I calculate \( L \)'s expected utility from choosing \( x_{L1} = x_1^* \). Third, I show that \( x_{L1} = x_1^* \) and \( x_{L1} = x_2^* + \lceil \alpha \rceil \) strictly dominate all other period 1 policy choices. Finally, in two different possible cases, I show that there exists \( p(\alpha) < 1 \) such that \( x_{L1} = x_1^* \) if \( p > p(\alpha) \) and \( x_{L1} = x_2^* - \lceil \alpha \rceil \) if \( p < p(\alpha) \).

**Step 1:** Suppose that \( x_{L1} = x_1^* \). During period 2, \( L \) would solve
\[
V_{L2} = \max_{x_{L2} \in \mathbb{Z}} -|x_{L1} - x_1^*| - |x_{L2} - x_1^*| - 1(|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2\eta \\
= \max_{x_{L2} \in \mathbb{Z}} -|x_1^* - x_1^*| - |x_{L2} - x_1^*| - 1(|x_1^* - x_{L2}| > 1) \cdot \alpha + 2\eta \\
= \max_{x_{L2} \in \mathbb{Z}} -|x_{L2} - x_1^*| - 1(|x_1^* - x_{L2}| > 1) \cdot \alpha + 2\eta \\
= \max_{x_{L2} \in \mathbb{Z}} -|x_{L2} - x_1^*| - 1(|x_1^* - x_{L2}| > 1) \cdot \alpha,
\]
where the final equality comes because the last term is constant with respect to \( x_{L2} \). The first term of the maximand is strictly negative if \( x_{L2} \neq x_1^* \) and equal to zero if \( x_{L2} = x_1^* \). The second term of the maximand is strictly negative if \( |x_1^* - x_{L2}| > 1 \) and equal to zero if \( |x_1^* - x_{L2}| \leq 1 \). Note that \( x_{L2} = x_1^* \) satisfies the latter condition. Thus, the maximand is strictly negative if \( x_{L2} \neq x_1^* \) and equal to zero if \( x_{L2} = x_1^* \), so its unique maximizer is \( x_{L2} = x_1^* \).

During period 2, \( R \) would solve
\[
V_{R2} = \max_{x_{R2} \in \mathbb{Z}} -|x_{L1} - x_1^*| - |x_{R2} - x_1^*| - 1(|x_{L1} - x_{R2}| > 1) \cdot \alpha + \eta \\
= \max_{x_{R2} \in \mathbb{Z}} -|x_1^* - x_1^*| - |x_{R2} - x_1^*| - 1(|x_1^* - x_{R2}| > 1) \cdot \alpha + \eta \\
= \max_{x_{R2} \in \mathbb{Z}} -|x_{R2} - x_1^*| - 1(|x_1^* - x_{R2}| > 1) \cdot \alpha,
\]
where the final equality comes because the first and last terms are constant with respect to \( x_{R2} \). Recall that \( x_1^* \leq -1 < -1 + 2 = 1 \leq x_1^* \), so \( |x_1^* - x_1^*| \geq 2 > 1 \). Thus, if \( x_{R2} = x_1^* \), then \( |x_1^* - x_{R2}| > 1 \). In that case, the maximand is equal to \(-\alpha \). Clearly, any other \( x_{R2} \) that satisfies \( |x_1^* - x_{R2}| > 1 \) makes \( R \) strictly worse off. Thus, either \( x_{R2} = x_1^* \) or \( x_{R2} = x_1^* \). The set of \( x_{R2} \) that satisfies the latter condition is \( \{x_1^* - 1, x_1^*, x_1^* + 1\} \). Of that set, \( x_{R2} = x_1^* + 1 \) maximizes \( R \)'s utility because it is closest to \( x_1^* \), and it yields a utility of \( -|x_1^* + 1 - x_1^*| = - (\Delta - 1) = -\Delta + 1 \). By assumption, \( \Delta > \alpha + 1 \), which holds if and only if \( -\alpha > -\Delta + 1 \). Thus, of the two possible choices left \( R \), she uniquely maximizes her utility by choosing \( x_{R2} = x_1^* + 1 \).

**Step 2:** Let \( U_i(x_{L1}) \equiv \mathbb{E}_p(x_{L1})[U_i(x_{L1}, x_{w2}, \lambda_{i1}, \lambda_{i2})] \) denote \( i \)'s expected utility from the choice of \( x_{L1} \). Given
the responses to $x_{L1} = x^*_L$ calculated in Step 1, $L$’s expected utility from making that choice is

$$
\hat{U}_L (x^*_L) = -|x^*_L - x^*_L| - p|x^*_L - x^*_L| - (1 - p)|x^*_R - x^*_L| - [p \cdot 1 (|x^*_L - x^*_L| > 1) + (1 - p) \cdot 1 (|x^*_L - x^*_R| > 1)] \cdot \alpha + (1 + p) \cdot \eta
$$

Step 3: Suppose first that $x_{L1} \neq x^*_L$ and $x_{L1} < x^*_R - [\alpha]$. Note that since $\alpha < \Delta - 1$, it follows that $[\alpha] \leq x^*_R - x^*_L - 1$, so $x^*_R - [\alpha] \geq x^*_L + 1 > x^*_L$. By an analogous argument as Step 1, $L$ either chooses $x_{L2} = x^*_L$, $x_{L2} = x_{L1} + 1$ or $x_{L2} = x_{L1} - 1$. Also, $R$ chooses between $x_{R2} = x^*_R$ and $x_{R2} = x_{L1} + 1 \leq x^*_R - [\alpha] - 1 < x^*_R$. Her utility from the former choice is $-\alpha$. Her utility from the latter choice is $-\alpha - 1 - x^*_R \leq x^*_R - [\alpha] - 1 + 1 - x^*_R = -\alpha$. Since $-\alpha < -\alpha$, $R$ chooses $x_{R2} = x^*_R$. Thus, $L$’s expected utility is

$$
\hat{U}_L (x_{L1}) = -|x_{L1} - x^*_L| - p|x_{L2} - x^*_L| - (1 - p)|x^*_R - x^*_L| - [p \cdot 1 (|x_{L1} - x_{L2}| > 1) + (1 - p) \cdot 1 (|x_{L1} - x^*_R| > 1)] \cdot \alpha + (1 + p) \cdot \eta
$$

Next, suppose that $x_{L1} = x^*_R - [\alpha]$. Then, $L$ chooses between $x_{L2} = x^*_L$ and $x_{L2} = x^*_R - [\alpha] - 1$. If $\alpha \in (\Delta - 2, \Delta - 1)$, then those choices are the same. Suppose that $\alpha < \Delta - 2$. Then, her utility from the former choice is $-\alpha$. Her utility from the latter choice is $-\alpha - 1 - x^*_R = -\Delta + [\alpha] + 1$. Thus, $L$ chooses $x_{L2} = x^*_L$ if $\alpha > -\Delta + [\alpha] + 1$, i.e., $[\alpha] + \alpha < \Delta - 1$, and she chooses $x_{L2} = x^*_R - [\alpha] - 1$ if $[\alpha] + \alpha > \Delta - 1$.

Also, $R$ chooses between $x_{R2} = x^*_R$ and $x_{R2} = x^*_R - [\alpha] + 1$. If $\alpha \in (0, 1)$, then those choices are the same. Suppose that $\alpha > 1$. Then, her utility from the former choice is $-\alpha$. Her utility from the latter choice is $-\alpha - 1 - x^*_R = -\alpha + 1$. Since $-\alpha + 1 > -\alpha$, $R$ chooses $x_{R2} = x^*_R - [\alpha] + 1$.

Thus, $L$’s expected utility is

$$
\hat{U}_L (x^*_R - [\alpha]) = -|x^*_R - [\alpha] - x^*_L| - p|x_{L2} - x^*_L| - (1 - p)|x^*_R - [\alpha] + 1 - x^*_L|
$$

If $[\alpha] + \alpha < \Delta - 1$, then $x_{L2} = x^*_L$, so

$$
\hat{U}_L (x^*_R - [\alpha]) = -|x^*_R - [\alpha] - x^*_L| - p|x^*_R - [\alpha] + 1 - x^*_L| - p \cdot 1 (|x^*_R - [\alpha] - x^*_L| + 1) \cdot \alpha + (1 + p) \cdot \eta
$$

If $[\alpha] + \alpha > \Delta - 1$, then $x_{L2} = x^*_R - [\alpha] - 1$, so

$$
\hat{U}_L (x^*_R - [\alpha]) = -|x^*_R - [\alpha] - x^*_L| - p \cdot 1 (|x^*_R - [\alpha] - x^*_L|) + 1) \cdot \alpha + (1 + p) \cdot \eta
$$

Finally, suppose that $x_{L1} > x^*_R - [\alpha]$. Then, $L$ chooses between $x_{L2} = x^*_L$ and $x_{L2} = x_{L1} - 1$, and $R$ either
chooses \( x_{R2} = x_R^* \), \( x_{R2} = x_{L1} + 1 \) or \( x_{R2} = x_{L1} - 1 \). Thus, \( L \)'s expected utility is

\[
\hat{U}_L (x_{L1}) = -|x_{L1} - x_L^*| - p|x_{L2} - x_L^*| - (1 - p) |x_{R2} - x_R^*| - |p \cdot 1 (|x_{L1} - x_{L2}| > 1) + (1 - p) \cdot 1 (|x_{L1} - x_{R2}| > 1)| \cdot \alpha + (1 + p) \cdot \eta
\]

\[
< - |x_R^* - [\alpha] - x_L^*| - p \cdot |x_{L2} - x_L^*| - (1 - p) |x_R^* - [\alpha] + 1 - x_L^*|
\]

\[
- |p \cdot 1 (|x_{L1} - x_{L2}| > 1) + (1 - p) \cdot 1 (|x_{L1} - x_{R2}| > 1)| \cdot \alpha + (1 + p) \cdot \eta
\]

\[
= - (2 - p) (\Delta - [\alpha]) - (1 - p) - p \cdot |x_{L2} - x_L^*| + 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha - (1 - p) \cdot 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha + (1 + p) \cdot \eta
\]

\[
\leq - (2 - p) (\Delta - [\alpha]) - (1 - p) - p \cdot |x_{L2} - x_L^*| + 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha + (1 + p) \cdot \eta.
\]

Recall that if \( [\alpha] + \alpha < \Delta - 1 \), then \( x_{L2} = x_L^* \) when \( x_{L1} = x_R^* - [\alpha] \) because \( -\alpha > -|x_{L1} - 1 - x_L^*| = -x_R^* - [\alpha] - 1 - x_L^* \). Since in this case \( x_{L1} > x_R^* - [\alpha] \), it follows that \( -\alpha > -|x_R^* - [\alpha] - 1 - x_L^*| > -|x_{L1} - 1 - x_L^*| \). Thus, it will remain the case that \( x_{L2} = x_L^* \), so

\[
\hat{U}_L (x_{L1}) < - (2 - p) (\Delta - [\alpha]) - (1 - p) - p \cdot 1 (|x_{L1} - x_L^*| > 1) \cdot \alpha + (1 + p) \cdot \eta.
\]

Note that \( |x_{L1} - x_L^*| > |x_R^* - [\alpha] - x_L^*| \geq 1 \), so \( |x_{L1} - x_L^*| > 1 \). Thus,

\[
\hat{U}_L (x_{L1}) < - (2 - p) (\Delta - [\alpha]) - (1 - p) - p \cdot \alpha + (1 + p) \cdot \eta
\]

\[
= \hat{U}_L (x_R^* - [\alpha]).
\]

On the other hand, if \( [\alpha] + \alpha > \Delta - 1 \), then \( x_{L2} = x_{L1} - 1 \) when \( x_{L1} = x_R^* - [\alpha] \). It is possible in this case for \( x_{L2} = x_{L1} - 1 \) or \( x_{L2} = x_L^* \). If \( L \) chooses the former, then

\[
\hat{U}_L (x_{L1}) < - (2 - p) (\Delta - [\alpha]) - (1 - p) - p \cdot |x_{L1} - 1 - x_L^*| + (1 + p) \cdot \eta
\]

\[
< - (2 - p) (\Delta - [\alpha]) - (1 - p) - p \cdot x_R^* - [\alpha] - 1 - x_L^* + (1 + p) \cdot \eta
\]

\[
= -2(\Delta - [\alpha]) - (1 - p) + p + (1 + p) \cdot \eta
\]

\[
= -2(\Delta - [\alpha]) + (2p - 1) + (1 + p) \cdot \eta
\]

\[
= \hat{U}_L (x_R^* - [\alpha]).
\]

If \( L \) chooses the latter, then

\[
\hat{U}_L (x_{L1}) < - (2 - p) (\Delta - [\alpha]) - (1 - p) - p \cdot 1 (|x_{L1} - x_L^*| > 1) \cdot \alpha + (1 + p) \cdot \eta
\]

\[
= - (2 - p) (\Delta - [\alpha]) - (1 - p) - p \cdot \alpha + (1 + p) \cdot \eta
\]

\[
< - (2 - p) (\Delta - [\alpha]) - (1 - p) - p (\Delta - [\alpha] - 1) + (1 + p) \cdot \eta
\]

\[
= -2(\Delta - [\alpha]) - (1 - p) + p + (1 + p) \cdot \eta
\]

\[
= -2(\Delta - [\alpha]) + (2p - 1) + (1 + p) \cdot \eta
\]

\[
= \hat{U}_L (x_R^* - [\alpha]),
\]

where the second inequality holds because \( \alpha > \Delta - [\alpha] - 1 \). Thus, \( x_{L1} = x_L^* \) and \( x_{L1} = x_R^* - [\alpha] \) strictly dominate all other period 1 policy choices.

**Step 4:** I consider two cases: \( [\alpha] + \alpha < \Delta - 1 \) and \( [\alpha] + \alpha > \Delta - 1 \).
Case 1: \([\alpha] + \alpha < \Delta - 1\), so \(x_{L2} = x^*_L\) if \(x_{L1} = x^*_R - [\alpha]\). Thus, \(L\) chooses \(x_{L1} = x^*_L\) if

\[
\hat{U}_L(x^*_L) > \hat{U}_L(x^*_R - [\alpha])
\]

\[
- (1 - p)(\Delta + \alpha) + (1 + p) \cdot \eta > - (2 - p)(\Delta - [\alpha]) - (1 - p) - p \cdot \alpha + (1 + p) \cdot \eta
\]

\[
- (1 - p)(\Delta + \alpha) > - (2 - p)(\Delta - [\alpha]) - (1 - p) - p \cdot \alpha
\]

\[
(\Delta + \alpha)p - \Delta - \alpha > (\Delta - [\alpha])p - 2\Delta + 2 [\alpha] - 1 + p - p \cdot \alpha
\]

\[
(2\alpha + [\alpha] - 1)p > \alpha + 2 [\alpha] - \Delta - 1
\]

\[
p > \frac{\alpha + 2 [\alpha] - \Delta - 1}{2\alpha + [\alpha] - 1}
\]

\[
= \frac{(\alpha + [\alpha] - 1) + [\alpha] - \Delta}{(\alpha + [\alpha] - 1) + \alpha}
\]

\[
= \bar{p}(\alpha).
\]

Note that

\[
\bar{p}(\alpha) = \frac{(\alpha + [\alpha] - 1) + [\alpha] - \Delta}{(\alpha + [\alpha] - 1) + \alpha}
\]

\[
< 1
\]

because \([\alpha] - \Delta \leq [\alpha] - 2 < [\alpha] - 1 < \alpha\). Conversely, \(L\) chooses \(x_{L1} = x^*_R - [\alpha]\) if \(p < \bar{p}(\alpha)\).

Case 2: \([\alpha] + \alpha > \Delta - 1\), so \(x_{L2} = x^*_R - [\alpha] - 1\) if \(x_{L1} = x^*_R - [\alpha]\). Thus, \(L\) chooses \(x_{L1} = x^*_L\) if

\[
\hat{U}_L(x^*_L) > \hat{U}_L(x^*_R - [\alpha])
\]

\[
- (1 - p)(\Delta + \alpha) + (1 + p) \cdot \eta > - 2(\Delta - [\alpha]) + (2p - 1) + (1 + p) \cdot \eta
\]

\[
- (1 - p)(\Delta + \alpha) > - 2(\Delta - [\alpha]) + (2p - 1)
\]

\[
(\Delta + \alpha)p - \Delta - \alpha > - 2\Delta + 2 [\alpha] + 2p - 1
\]

\[
(\Delta + \alpha - 2)p > \alpha + 2 [\alpha] - \Delta - 1
\]

\[
p > \frac{\alpha + 2 [\alpha] - \Delta - 1}{\Delta + \alpha - 2}
\]

\[
= \bar{p}(\alpha).
\]

Note that

\[
\bar{p}(\alpha) = \frac{\alpha + 2 [\alpha] - \Delta - 1}{\Delta + \alpha - 2}
\]

\[
\leq \frac{\alpha + 2(\Delta - 1) - \Delta - 1}{\Delta + \alpha - 2}
\]

\[
= \frac{\alpha + 2\Delta - 2 - \Delta - 1}{\Delta + \alpha - 2}
\]

\[
= \frac{\Delta + \alpha - 3}{\Delta + \alpha - 2}
\]

\[
< 1,
\]

where the weak inequality comes from the fact that \(\alpha < \Delta - 1\), so \([\alpha] \leq \Delta - 1\). Conversely, \(L\) chooses \(x_{L1} = x^*_R - [\alpha]\) if \(p < \bar{p}(\alpha)\).

If \(p > \bar{p}(\alpha)\), then by Step 4, \(x_{L1} = x^*_L\). Thus, by Step 1, \(x_{i2} = x^*_i\) for \(i \in \{L,R\}\). If \(p < \bar{p}(\alpha)\), then by Step 4, \(x_{L1} = x^*_R - [\alpha]\). Thus, by Step 3, \(x_{L2} \in \{x^*_L, x_{L1} - 1\}\) and \(x_{R2} = x_{L1} + 1\). This is the unique sequentially rational behavior, so this completes the existence and uniqueness proof of the equilibrium characterized in Proposition 2. □

Proof of Proposition 3: Suppose that \(\Delta < \alpha + 1\) and \(L\) is re-elected with exogenous probability \(p \in [0,1]\).
Suppose that $x_{L1} = x_{L1}^*$. During period 2, $L$ would solve

$$V_{L2} = \max_{x_{L2} \in \mathbb{Z}} -|x_{L1} - x_{L1}^*| - |x_{L2} - x_{L1}^*| - 1\,(|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2\eta$$

$$= \max_{x_{L2} \in \mathbb{Z}} -|x_{L2}^* - x_{L1}^*| - |x_{L2} - x_{L1}^*| - 1\,(|x_{L1}^* - x_{L2}| > 1) \cdot \alpha + 2\eta$$

$$= \max_{x_{L2} \in \mathbb{Z}} -|x_{L2} - x_{L2}^*| - 1\,(|x_{L2}^* - x_{L2}| > 1) \cdot \alpha + 2\eta$$

$$= \max_{x_{L2} \in \mathbb{Z}} -|x_{L2} - x_{L2}^*| - 1\,(|x_{L2}^* - x_{L2}| > 1) \cdot \alpha,$$

where the final equality comes because the last term is constant with respect to $x_{L2}$. The first term of the maximand is strictly negative if $x_{L2} \neq x_{L1}^*$ and equal to zero if $x_{L2} = x_{L1}^*$. The second term of the maximand is strictly negative if $|x_{L2} - x_{L1}| > 1$ and equal to zero if $|x_{L2}^* - x_{L2}| \leq 1$. Note that $x_{L2} = x_{L2}^*$ satisfies the latter condition. Thus, the maximand is strictly negative if $x_{L2} \neq x_{L1}^*$ and equal to zero if $x_{L2} = x_{L1}^*$, so its unique maximizer is $x_{L2} = x_{L1}^*$.

During period 2, $R$ would solve

$$V_{R2} = \max_{x_{R2} \in \mathbb{Z}} -|x_{L1} - x_{R1}| - |x_{R2} - x_{R1}^*| - 1\,(|x_{L1} - x_{R2}| > 1) \cdot \alpha + \eta$$

$$= \max_{x_{R2} \in \mathbb{Z}} -|x_{R2}^* - x_{R1}| - |x_{R2} - x_{R1}^*| - 1\,(|x_{R1}^* - x_{R2}| > 1) \cdot \alpha + \eta$$

$$= \max_{x_{R2} \in \mathbb{Z}} -|x_{R2} - x_{R1}^*| - 1\,(|x_{R1}^* - x_{R2}| > 1) \cdot \alpha,$$

where the final equality comes because the first and last terms are constant with respect to $x_{R2}$. Recall that $x_{R1}^* \leq -1 < -1 + 2 = 1 \leq x_{R1}^*$, so $|x_{R2}^* - x_{R1}| \geq 2 > 1$. Thus, if $x_{R2} = x_{R1}^*$, then $|x_{R2} - x_{R2}| > 1$. In that case, the maximand is equal to $-\alpha$. Clearly, any other $x_{R2}$ that satisfies $|x_{R1}^* - x_{R2}| > 1$ makes $R$ strictly worse off. Thus, either $x_{R2} = x_{R1}^*$ or $|x_{R1}^* - x_{R2}| \leq 1$. The set of $x_{R2}$ that satisfies the latter condition is $\{x_{R1}^* - 1, x_{R1}^* + 1\}$. Of that set, $x_{R2} = x_{R1}^* + 1$ maximizes $R$’s utility because it is closest to $x_{R1}^*$, and it yields a utility of $-|x_{R1}^* + 1 - x_{R1}^*| = -\Delta = -\Delta + 1$. By assumption, $\Delta < \alpha + 1$, which holds if and only if $-\alpha < -\Delta + 1$. Thus, of the two possible choices left $R$, she uniquely maximizes her utility by choosing $x_{R2} = x_{R1}^* + 1$.

It remains to show that $x_{L1} = x_{L1}^*$ is sequentially rational for $L$. Let $\hat{U}_i(x_{L1}) = \mathbb{E}_{p(x_{L1})}[U_i(x_{L1}, x_{w2}, \lambda_1, \lambda_2)]$ denote $i$’s expected utility from the choice of $x_{L1}$. Given the responses to $x_{L1} = x_{L1}^*$ calculated above, $L$’s expected utility from making that choice is

$$\hat{U}_L(x_{L1}^*) = -|x_{L2}^* - x_{L1}^*| - p |x_{L2}^* - x_{L1}^*| - (1 - p) |x_{L2}^* - x_{L1}^*| - 1\,(|x_{L2}^* - x_{L1}| > 1) \cdot \alpha + (1 + p) \cdot \eta$$

$$= -|x_{L2} - x_{L1}^*| - p |x_{L2} - x_{L1}^*| - (1 - p) |x_{L2} - x_{L1}^*| - 1\,(|x_{L2} - x_{L1}| > 1) \cdot \alpha + (1 + p) \cdot \eta.$$

Suppose that $x_{L1} \neq x_{L1}^*$ instead. Then, $L$’s expected utility is

$$\hat{U}_L(x_{L1}) = -|x_{L1} - x_{L1}^*| - p |x_{L2} - x_{L1}^*| - (1 - p) |x_{L2} - x_{L1}^*| - 1\,(|x_{L1} - x_{L2}| > 1) \cdot \alpha + (1 + p) \cdot \eta$$

$$\leq -1 - p |x_{L2} - x_{L1}^*| - (1 - p) |x_{L2} - x_{L1}^*| - 1\,(|x_{L1} - x_{L2}| > 1) \cdot \alpha + (1 + p) \cdot \eta$$

$$\leq -1 + (1 + p) \cdot \eta$$

$$< - (1 - p) + (1 + p) \cdot \eta$$

$$= \hat{U}_L(x_{L1}^*),$$

so $x_{L1} = x_{L1}^*$ is sequentially rational. Thus, $x_{L1} = x_{L1}^*$, $x_{L2} = x_{L1}^*$ and $x_{R2} = x_{R1}^* + 1$ is the unique equilibrium. □

**Proof of Theorem 1:** Suppose that $\Delta \in (\alpha + 1, \alpha + 2x_{R1}^*)$. Also, suppose that $x_{L1} = x_{L1}^*$. During period 2, $L$
would solve

\[ V_{L2} = \max_{x_{L2} \in \mathbb{Z}} \ - |x_{L1} - x^*_L| - |x_{L2} - x^*_L| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2\eta \]
\[ = \max_{x_{L2} \in \mathbb{Z}} \ - |x^*_L - x^*_L| - |x_{L2} - x^*_L| - 1 (|x^*_L - x_{L2}| > 1) \cdot \alpha + 2\eta \]
\[ = \max_{x_{L2} \in \mathbb{Z}} \ - |x_{L2} - x^*_L| - 1 (|x^*_L - x_{L2}| > 1) \cdot \alpha + 2\eta \]
\[ = \max_{x_{L2} \in \mathbb{Z}} \ - |x_{L2} - x^*_L| - 1 (|x^*_L - x_{L2}| > 1) \cdot \alpha, \]

where the final equality comes because the last term is constant with respect to \( x_{L2} \). The first term of the maximand is strictly negative if \( x_{L2} \neq x^*_L \) and equal to zero if \( x_{L2} = x^*_L \). The second term of the maximand is strictly negative if \( |x^*_L - x_{L2}| > 1 \) and equal to zero if \( |x^*_L - x_{L2}| \leq 1 \). Note that \( x_{L2} = x^*_L \) satisfies the latter condition. Thus, the maximand is strictly negative if \( x_{L2} \neq x^*_L \) and equal to zero if \( x_{L2} = x^*_L \), so its unique maximizer is \( x_{L2} = x^*_L \).

During period 2, \( R \) would solve

\[ V_{R2} = \max_{x_{R2} \in \mathbb{Z}} \ - |x_{L1} - x^*_R| - |x_{R2} - x^*_R| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha + \eta \]
\[ = \max_{x_{R2} \in \mathbb{Z}} \ - |x^*_R - x^*_R| - |x_{R2} - x^*_R| - 1 (|x^*_L - x_{R2}| > 1) \cdot \alpha + \eta \]
\[ = \max_{x_{R2} \in \mathbb{Z}} \ - |x_{R2} - x^*_R| - 1 (|x^*_L - x_{R2}| > 1) \cdot \alpha, \]

where the final equality comes because the first and last terms are constant with respect to \( x_{R2} \). Recall that \( x^*_L \leq -1 < -1 + 2 = 1 \leq x^*_R \), so \( |x^*_L - x^*_R| \geq 2 > 1 \). Thus, if \( x_{R2} \neq x^*_R \), then \( |x^*_L - x_{R2}| > 1 \). In that case, the maximand is equal to \( -\alpha \). Clearly, any other \( x_{R2} \) that satisfies \( |x^*_L - x_{R2}| > 1 \) makes \( R \) strictly worse off. Thus, either \( x_{R2} = x^*_R \) or \( |x^*_L - x_{R2}| \leq 1 \). The set of \( x_{R2} \) that satisfies the latter condition is \( \{x^*_L - 1, x^*_L, x^*_L + 1\} \). Of that set, \( x_{R2} = x^*_L + 1 \) maximizes \( R \)'s utility because it is closest to \( x^*_R \), and it yields a utility of \( -|x^*_L + 1 - x^*_R| = -(\Delta - 1) = -\Delta + 1 \). By assumption, \( \Delta > \alpha + 1 \), which holds if and only if \( -\alpha > -\Delta + 1 \). Thus, of the two possible choices left \( R \), she uniquely maximizes her utility by choosing \( x_{R2} = x^*_R \).

Thus, the median voter’s utility from re-electing \( L \) is

\[ U^L_M (x^*_L) = -|x^*_L - x^*_M| - |x^*_L - x^*_M| - 1 (|x^*_L - x^*_L| > 1) \cdot \alpha \]
\[ = -|x^*_L| - |x^*_L| \cdot \alpha \cdot \alpha. \]

The median voter’s utility from electing \( R \) is

\[ U^R_M (x^*_L) = -|x^*_L - x^*_M| - |x^*_R - x^*_M| - 1 (|x^*_L - x^*_R| > 1) \cdot \alpha \]
\[ = -|x^*_L| - |x^*_R| - \alpha \]
\[ = -|x^*_L - x^*_R| - \alpha. \]

Thus, the median voter re-elects \( L \) with probability one if and only if

\[ \frac{U^L_M (x^*_L)}{U^R_M (x^*_L)} > \frac{U^R_M (x^*_L)}{U^R_M (x^*_L)} \]
\[ - |x^*_L| - |x^*_L| > - |x^*_L| - |x^*_L| - \alpha \]
\[ - |x^*_L| > - |x^*_L| - \alpha \]
\[ - |x^*_L| > - |x^*_L| - \alpha \]
\[ \alpha > |x^*_L| - |x^*_L| \]
\[ \alpha > - |x^*_L| - x^*_R \]
\[ \alpha > |x^*_L| - |x^*_R| - x^*_L - x^*_R \]
\[ \alpha > |x^*_L| - |x^*_R| - x^*_L - x^*_R \]
\[ \alpha > \Delta - 2x^*_R \]
\[ \alpha + 2x^*_R > \Delta, \]

which holds by assumption. Thus, \( p(x^*_L) = 1 \).
Finally, I show that \( x_{L1} = x^*_L \) is optimal for \( L \). During period 1, she solves

\[
V_{L1} \equiv \max_{x_{L1} \in \mathbb{Z}} -|x_{L1} - x^*_L| - \mathbb{E}_{p(x_{L1})} [ |x_{u2}(x_{L1}) - x^*_L| - \mathbb{E}_{p(x_{L1})} [1 (|x_{L1} - x_{u2}(x_{L1})| > 1) ] \cdot \alpha + (1 + p(x_{L1})) \cdot \eta.
\]

Note that \( p(x^*_L) = 1 \), so \( w = L \) with probability one in that case. The first term of the maximand is strictly negative if \( x_{L1} \neq x^*_L \) and equal to zero if \( x_{L1} = x^*_L \). Since \( x_{u2}(x^*_L) = x_{L2}(x^*_L) = x^*_L \), the same holds for the second term of the maximand as well. The third term of the maximand is strictly negative if \( |x_{L1} - x^*_L| > 1 \) and equal to zero if \( |x_{L1} - x^*_L| \leq 1 \). Note that \( x_{L1} = x^*_L \) satisfies the latter condition. Thus, the sum of the first three terms of the maximand is strictly negative if \( x_{L1} \neq x^*_L \) and equal to zero if \( x_{L1} = x^*_L \), so its unique maximizer is \( x_{L1} = x^*_L \). Furthermore, \( x_{L1} = x^*_L \) maximizes \( p(x_{L1}) \) and thus the fourth term of the maximand as well. Therefore, \( x_{L1} = x^*_L \) is the unique optimum.

Thus, the unique sequentially rational behavior is given by \( x_{L1} = x^*_L \), \( p(x_{L1}) = 1 \), \( x_{L2}(x_{L1}) = x^*_L \) and \( x_{R2}(x_{L1}) = x^*_R \), so this is the unique equilibrium. \( \square \)

**Proposition 4:** Suppose that \( \Delta \in (\alpha, \alpha + 1) \). Then, the unique equilibrium is characterized by \( x_{L1} = x^*_L - 1 \), \( p(x_{L1}) = 1 \), \( x_{L2}(x_{L1}) = x^*_L \) and \( x_{R2}(x_{L1}) = x^*_R \).

**Proof of Proposition 4:** Suppose that \( \Delta \in (\alpha, \alpha + 1) \). Then, during period 1, \( L \) solves

\[
V_{L1} \equiv \max_{x_{L1} \in \mathbb{Z}} -|x_{L1} - x^*_L| - \mathbb{E}_{p(x_{L1})} [ |x_{u2}(x_{L1}) - x^*_L| - \mathbb{E}_{p(x_{L1})} [1 (|x_{L1} - x_{u2}(x_{L1})| > 1) ] \cdot \alpha + (1 + p(x_{L1})) \cdot \eta.
\]

First, suppose that \( x_{L1} = x^*_L - 1 \). During period 2, \( L \) would solve

\[
V_{L2} \equiv \max_{x_{L2} \in \mathbb{Z}} -|x_{L1} - x^*_L| - |x_{L2} - x^*_L| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2 \eta
\]

\[
= \max_{x_{L2} \in \mathbb{Z}} -|x_{L2} - 1 - x^*_L| - |x_{L2} - x^*_L| - 1 (|x^*_L - 1 - x_{L2}| > 1) \cdot \alpha + 2 \eta
\]

\[
= \max_{x_{L2} \in \mathbb{Z}} -|x_{L2} - x^*_L - 1| - 1 (|x^*_L - 1 - x_{L2}| > 1) \cdot \alpha,
\]

where the final equality comes because the first and last terms are constant with respect to \( x_{L2} \). The first term of the maximand is strictly negative if \( x_{L2} \neq x^*_L \) and equal to zero if \( x_{L2} = x^*_L \). The second term of the maximand is strictly negative if \( |x^*_L - 1 - x_{L2}| > 1 \) and equal to zero if \( |x^*_L - 1 - x_{L2}| \leq 1 \). Note that \( x_{L2} = x^*_L \) satisfies the latter condition. Thus, the maximand is strictly negative if \( x_{L2} \neq x^*_L \) and equal to zero if \( x_{L2} = x^*_L \), so its unique maximizer is \( x_{L2} = x^*_L \).

During period 2, \( R \) would solve

\[
V_{R2} \equiv \max_{x_{R2} \in \mathbb{Z}} -|x_{L1} - x^*_R| - |x_{R2} - x^*_R| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha + \eta
\]

\[
= \max_{x_{R2} \in \mathbb{Z}} -|x_{R2} - 1 - x^*_R| - |x_{R2} - x^*_R| - 1 (|x^*_L - 1 - x_{R2}| > 1) \cdot \alpha + \eta
\]

\[
= \max_{x_{R2} \in \mathbb{Z}} -|x_{R2} - x^*_R - 1| - 1 (|x^*_L - 1 - x_{R2}| > 1) \cdot \alpha,
\]

where the final equality comes because the first and last terms are constant with respect to \( x_{R2} \). Recall that \( x^*_L - 1 < (x^*_L - 1) + 1 = x^*_L \leq -1 < -1 + 2 = 1 \leq x^*_R \), so \( |x^*_L - 1 - x^*_R| \geq 3 > 1 \). Thus, if \( x_{R2} = x^*_R \), then \( |x^*_L - 1 - x_{R2}| > 1 \). In that case, the maximand is equal to \( -\alpha \). Clearly, any other \( x_{R2} \) that satisfies \( |x^*_L - 1 - x_{R2}| > 1 \) makes \( R \) strictly worse off. Thus, either \( x_{R2} = x^*_R \) or \( |x^*_L - 1 - x_{R2}| \leq 3 \). The set of \( x_{R2} \) that satisfies the latter condition is \( \{x^*_L - 2, x^*_L - 1, x^*_L \} \). Of that set, \( x_{R2} = x^*_L \) maximizes \( R \)'s utility because it is closest to \( x^*_R \), and it yields a utility of \( -|x^*_L - x^*_R| = -\Delta \). By assumption, \( \Delta > \alpha \), so \( -\Delta > -\Delta \). Thus, of the two possible choices left \( R \), she uniquely maximizes her utility by choosing \( x_{R2} = x^*_R \).

Thus, the median voter’s utility from re-electing \( L \) is

\[
U^*_M (x^*_L - 1) = -|x^*_L - 1 - x^*_M| - |x^*_L - x^*_M| - 1 (|x^*_L - 1 - x^*_L| > 1) \cdot \alpha
\]

\[
= -|x^*_L - 1 - x^*_L|.
\]
Thus, the median voter re-elects

\[ U^R_M (x^*_L - 1) = -|x^*_L - 1 - x^*_M| - |x^*_R - x^*_M| - 1 (|x^*_L - 1 - x^*_R| > 1) \cdot \alpha \]

\[ = -|x^*_L - 1| - |x^*_R| + |x^*_L| - |x^*_R - \alpha| \]

\[ = -|x^*_L - 1| - x^*_R - \alpha. \]

Thus, the median voter re-elects \( L \) with probability one if and only if

\[ U^L_M (x^*_L - 1) > U^R_M (x^*_L - 1) \]

\[ - |x^*_L - 1| - |x^*_R| > - |x^*_L - 1| - x^*_R - \alpha \]

\[ \alpha > |x^*_L| - x^*_R \]

\[ \alpha > -x^*_L - x^*_R \]

\[ \alpha > x^*_R - x^*_L - x^*_R - x^*_R \]

\[ \alpha > \Delta - 2x^*_R \]

\[ \alpha + 2x^*_R > \Delta, \]

which holds because \( \Delta \leq \alpha + 1 < \alpha + 2 \leq \alpha + 2x^*_R \). Thus, \( p(x^*_L - 1) = 1 \).

Let \( \hat{U}_i (x_{L1}) \equiv E(p(x_{L1}) | U_i (x_{L1}, x_{w2}, \lambda_1, \lambda_2)) \) denote \( i \)'s expected utility from the choice of \( x_{L1} \). Then,

\[ \hat{U}_L (x^*_L - 1) = -|x^*_L - 1 - x^*_L| - E_p(x_{L1}) [ |x_{w2} (x^*_L - 1) - x^*_L | - E_p(x^*_L) [ 1 (|x^*_L - 1 - x_{w2} (x^*_L - 1)| > 1) ] - \alpha \]

\[ = -|1 - |x_{L2} (x^*_L - 1) - x^*_L| - 1 (|x^*_L - 1 - x_{L2} (x^*_L - 1)| > 1) \cdot \alpha + (1 + 1) \cdot \eta \]

\[ = -1 - |x^*_L - x^*_L| - 1 (|x^*_L - 1 - x^*_L| > 1) \cdot \alpha + 2\eta \]

\[ = -1 + 2\eta, \]

where the second equality comes from the fact that \( p(x^*_L - 1) = 1 \) so \( w = L \) with probability one, and the third equality comes from the fact that \( x_{L2} (x^*_L - 1) = x^*_L \).

Next, suppose that \( x_{L1} = x^*_L \). During period 2, \( L \) would solve

\[ V_{L2} = \max_{x_{L2} \in \mathbb{Z}} - |x_{L1} - x^*_L| - |x_{L2} - x^*_L| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2\eta \]

\[ = \max_{x_{L2} \in \mathbb{Z}} - |x^*_L - x^*_L| - |x_{L2} - x^*_L| - 1 (|x^*_L - x_{L2}| > 1) \cdot \alpha + 2\eta \]

\[ = \max_{x_{L2} \in \mathbb{Z}} - |x_{L2} - x^*_L| - 1 (|x^*_L - x_{L2}| > 1) \cdot \alpha, \]

where the final equality comes because the last term is constant with respect to \( x_{L2} \). The first term of the maximand is strictly negative if \( x_{L2} \neq x^*_L \) and equal to zero if \( x_{L2} = x^*_L \). The second term of the maximand is strictly negative if \( |x^*_L - x_{L2}| > 1 \) and equal to zero if \( |x^*_L - x_{L2}| \leq 1 \). Note that \( x_{L2} = x^*_L \) satisfies the latter condition. Thus, the maximand is strictly negative if \( x_{L2} \neq x^*_L \) and equal to zero if \( x_{L2} = x^*_L \), so its unique maximizer is \( x_{L2} = x^*_L \).

During period 2, \( R \) would solve

\[ V_{R2} = \max_{x_{R2} \in \mathbb{Z}} - |x_{L1} - x^*_R| - |x_{R2} - x^*_R| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha + \eta \]

\[ = \max_{x_{R2} \in \mathbb{Z}} - |x^*_L - x^*_R| - |x_{R2} - x^*_R| - 1 (|x^*_L - x_{R2}| > 1) \cdot \alpha + \eta \]

\[ = \max_{x_{R2} \in \mathbb{Z}} - |x_{R2} - x^*_R| - 1 (|x^*_L - x_{R2}| > 1) \cdot \alpha, \]

where the final equality comes because the first and last terms are constant with respect to \( x_{R2} \). Recall that \( x^*_L \leq -1 < -1 + 2 = 1 \leq x^*_R \), so \( |x^*_L - x^*_R| \geq 2 > 1 \). Thus, if \( x_{R2} = x^*_R \), then \( |x^*_L - x_{R2}| > 1 \). In that case, the maximand is equal to \(-\alpha \). Clearly, any other \( x_{R2} \) that satisfies \( |x^*_L - x_{R2}| > 1 \) makes \( R \) strictly worse off. Thus, either \( x_{R2} = x^*_R \) or \( |x^*_L - x_{R2}| \leq 1 \). The set of \( x_{R2} \) that satisfies the latter condition is \( \{x^*_L - 1, x^*_L, x^*_L + 1\} \). Of that set, \( x_{R2} = x^*_L + 1 \)
maximizes $R$’s utility because it is closest to $x^*_L$, and it yields a utility of $-|x^*_L + 1 - x^*_R| = -(\Delta - 1) = -\Delta + 1$. By assumption, $\Delta \leq \alpha + 1$, so $-\alpha \leq -\Delta + 1$. First, suppose that $-\alpha < -\Delta + 1$. Then, of the two possible choices left $R$, she uniquely maximizes her utility by choosing $x_{R2} = x^*_L + 1$. Now, suppose that $-\alpha = -\Delta + 1$, so $R$ is indifferent between $x_{R2} = x^*_L + 1$ and $x_{R2} = x^*_R$. I assume that $R$ breaks the indifference by choosing the $x_{R2}$ that maximizes $\hat{U}_R(x^*_L)$.

First, suppose that $x_{R2} = x^*_L + 1$. Then, the median voter’s utility from re-electing $L$ is

$$U^L_M(x^*_L) = -|x^*_L - x^*_M| - |x^*_L - x^*_L| - 1 \cdot (|x^*_L - x^*_L| > 1) \cdot \alpha$$

$$= -|x^*_L| - |x^*_L|$$

$$= -2|x^*_L|.$$

The median voter’s utility from electing $R$ is

$$U^R_M(x^*_L) = -|x^*_L - x^*_M| - |x^*_L + 1 - x^*_M| - 1 \cdot (|x^*_L - (x^*_L + 1)| > 1) \cdot \alpha$$

$$= -|x^*_L| - |x^*_L + 1|$$

$$= -2|x^*_L| + 1.$$

Thus, the median voter re-elects $L$ with probability zero if and only if

$$U^L_M(x^*_L) < U^R_M(x^*_L)$$

$$-2|x^*_L| < -2|x^*_L| + 1$$

$$0 < 1,$$

which holds. Thus, $p(x^*_L) = 0$. Then,

$$\hat{U}_R(x^*_L) = -|x^*_L - x^*_R| - \mathbb{E}_{p(x^*_L)}[|x_{w2}(x^*_L) - x^*_R|] - \mathbb{E}_{p(x^*_L)}[1 \cdot (|x^*_L - x_{w2}(x^*_L)| > 1) \cdot \alpha + (1 - p(x^*_L)) \cdot \eta$$

$$= -\Delta - |x_{R2}(x^*_L) - x^*_R| - 1 \cdot (|x^*_L - x_{R2}(x^*_L)| > 1) \cdot \alpha + (1 - 0) \cdot \eta$$

$$= -\Delta - |x^*_L + 1 - x^*_R| - 1 \cdot (|x^*_L - (x^*_L + 1)| > 1) \cdot \alpha + \eta$$

$$= -2\Delta + 1 + \eta,$$

where the second equality comes from the fact that $p(x^*_L) = 0$, so $w = R$ with probability one.

Now, suppose that $x_{R2} = x^*_R$. Then, the median voter’s utility from re-electing $L$ is

$$U^L_M(x^*_L) = -|x^*_L - x^*_M| - |x^*_L - x^*_M| - 1 \cdot (|x^*_L - x^*_L| > 1) \cdot \alpha$$

$$= -|x^*_L| - |x^*_L|.$$

The median voter’s utility from electing $R$ is

$$U^R_M(x^*_L) = -|x^*_L - x^*_M| - |x^*_R - x^*_M| - 1 \cdot (|x^*_L - x^*_R| > 1) \cdot \alpha$$

$$= -|x^*_L| - |x^*_R| - \alpha$$

$$= -|x^*_L| - x^*_R - \alpha.$$

Thus, the median voter re-elects $L$ with probability one if and only if

$$U^L_M(x^*_L) > U^R_M(x^*_L)$$

$$-|x^*_L| - |x^*_L| > -|x^*_L| - x^*_R - \alpha$$

$$-|x^*_L| > -x^*_R - \alpha$$

$$\alpha > |x^*_L| - x^*_R.$$
which holds. Thus, \( p(x^*_L) = 1 \). Then,

\[
\bar{U}_R(x^*_L) = -|x^*_L - x^*_R| - \mathbb{E}_{p(x_L)} [ |x_{w2}(x^*_L) - x^*_R| ] - \mathbb{E}_{p(x_L)} [ 1 (|x^*_L - x_{w2}(x^*_L)| > 1) \cdot \alpha + (1 - p(x^*_L)) \cdot \eta \\
= -\Delta - |x_{L2}(x^*_L) - x^*_R| - 1 (|x^*_L - x_{L2}(x^*_L)| > 1) \cdot \alpha + (1 - 1) \cdot \eta \\
= -\Delta - |x^*_L - x^*_R| - 1 (|x^*_L - x^*_L| > 1) \cdot \alpha \\
= -2\Delta \\
< -2\Delta + 1 + \eta,
\]

where the second equality comes from the fact that \( p(x^*_L) = 1 \), so \( w = L \) with probability one. Thus, \( R \) chooses \( x_{R2} = x^*_L + 1 \) because it maximizes \( \bar{U}_R(x^*_L) \), so \( x_{R2} = x^*_L + 1 \) in all cases. This implies that \( p(x^*_L) = 0 \).

Then,

\[
\bar{U}_L(x^*_L) = -|x^*_L - x^*_L| - \mathbb{E}_{p(x_L)} [ |x_{w2}(x^*_L) - x^*_L| ] - \mathbb{E}_{p(x_L)} [ 1 (|x^*_L - x_{w2}(x^*_L)| > 1) \cdot \alpha + (1 + p(x^*_L)) \cdot \eta \\
= -|x_{R2}(x^*_L) - x^*_L| - 1 (|x^*_L - x_{R2}(x^*_L)| > 1) \cdot \alpha + (1 + 0) \cdot \eta \\
= -|x^*_L + 1 - x^*_L| - 1 (|x^*_L + 1 - x_{L2}| > 1) \cdot \alpha + \eta \\
= -1 + \eta \\
= -1 + \Delta + 2\eta \\
= \bar{U}_L(x^*_L - 1),
\]

where the second equality comes from the fact that \( p(x^*_L) = 0 \), so \( w = R \) with probability one.

Next, suppose that \( x_{L1} = x^*_L + 1 \). During period 2, \( L \) would solve

\[
V_{L2} \equiv \max_{x_{L2} \in \mathbb{Z}} -|x_{L1} - x^*_L| - |x_{L2} - x^*_L| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2\eta \\
= \max_{x_{L2} \in \mathbb{Z}} -|x^*_L + 1 - x^*_L| - |x_{L2} - x^*_L| - 1 (|x^*_L + 1 - x_{L2}| > 1) \cdot \alpha + 2\eta \\
= \max_{x_{L2} \in \mathbb{Z}} -|x_{L2} - x^*_L| - 1 (|x^*_L + 1 - x_{L2}| > 1) \cdot \alpha,
\]

where the final equality comes because the first and last terms are constant with respect to \( x_{L2} \). The first term of the maximand is strictly negative if \( x_{L2} \neq x^*_L \) and equal to zero if \( x_{L2} = x^*_L \). The second term of the maximand is strictly negative if \( |x^*_L + 1 - x_{L2}| > 1 \) and equal to zero if \( |x^*_L + 1 - x_{L2}| \leq 1 \). Note that \( x_{L2} = x^*_L \) satisfies the latter condition. Thus, the maximand is strictly negative if \( x_{L2} \neq x^*_L \) and equal to zero if \( x_{L2} = x^*_L \), so its unique maximizer is \( x_{L2} = x^*_L \).

During period 2, \( R \) would solve

\[
V_{R2} \equiv \max_{x_{R2} \in \mathbb{Z}} -|x_{L1} - x^*_R| - |x_{R2} - x^*_R| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha + \eta \\
= \max_{x_{R2} \in \mathbb{Z}} -|x^*_L + 1 - x^*_R| - |x_{R2} - x^*_R| - 1 (|x^*_L + 1 - x_{R2}| > 1) \cdot \alpha + \eta \\
= \max_{x_{R2} \in \mathbb{Z}} -|x_{R2} - x^*_R| - 1 (|x^*_L + 1 - x_{R2}| > 1) \cdot \alpha,
\]

where the final equality comes because the first and last terms are constant with respect to \( x_{R2} \). Recall that \( x^*_L + 1 \leq 0 < 0 + 1 = 1 \leq x^*_R \), so \( x^*_L + 1 - x^*_R \geq 1 \). First, suppose that \( x^*_L + 1 - x^*_R = 1 \). Then, the first term of the maximand is strictly negative if \( x_{R2} \neq x^*_R \) and equal to zero if \( x_{R2} = x^*_R \). The second term of the maximand is strictly negative if \( |x^*_L + 1 - x_{R2}| > 1 \) and equal to zero if \( |x^*_L + 1 - x_{R2}| \leq 1 \). Note that \( x_{R2} = x^*_R \) satisfies the latter condition. Thus, the maximand is strictly negative if \( x_{R2} \neq x^*_R \) and equal to zero if \( x_{R2} = x^*_R \), so its unique maximizer is \( x_{R2} = x^*_R = x^*_L + 2 \).

Now, suppose that \( |x^*_L + 1 - x^*_R| > 1 \). Thus, if \( x_{R2} = x^*_R \), then \( |x^*_L + 1 - x_{R2}| > 1 \). In that case, the maximand is equal to \(-\alpha \). Clearly, any other \( x_{R2} \) that satisfies \( |x^*_L + 1 - x_{R2}| > 1 \) makes \( R \) strictly worse off. Thus, either \( x_{R2} = x^*_R \) or \( |x^*_L + 1 - x_{R2}| \leq 1 \). The set of \( x_{R2} \) that satisfies the latter condition is \( \{x^*_L, x^*_L + 1, x^*_L + 2\} \). Of that set, \( x_{R2} = x^*_L + 2 \) maximizes \( R \)'s utility because it is closest to \( x^*_R \), and it yields a utility of \(-|x^*_L + 2 - x^*_R| = -\Delta + 2 \). By assumption, \( \Delta \leq \alpha + 1 < \alpha + 2 \), so \(-\alpha < -\Delta + 2 \). Thus, of the two possible choices left \( R \), she uniquely maximizes her utility by choosing \( x_{R2} = x^*_L + 2 \). Thus, in either case, \( x_{R2} = x^*_L + 2 \).
Thus, the median voter’s utility from re-electing $L$ is

$$U_M^L (x_L^* + 1) = -|x_L^* + x_M^*| - |x_L^* - x_M^*| - 1 \cdot (|x_L^* + 1 - x_M^*| > 1) \cdot \alpha$$

$$= -|x_L^* + 1| - |x_L^*|$$

$$= -2 |x_L^*| + 1.$$

The median voter’s utility from electing $R$ is

$$U_M^R (x_L^* + 1) = -|x_L^* + 1 - x_M^*| - |x_L^* + 2 - x_M^*| - 1 \cdot (|x_L^* + 1 - (x_L^* + 2)| > 1) \cdot \alpha$$

$$= -|x_L^* + 1| - |x_L^* + 2|$$

$$= \begin{cases} -2 |x_L^*| + 1, & \text{if } |x_L^*| = 1 \\ -2 |x_L^*| + 3, & \text{otherwise.} \end{cases}$$

Thus, the median voter re-elects $L$ with probability $p(x_L^* + 1) \leq \pi$ if and only if

$$U_M^L (x_L^* + 1) \leq U_M^R (x_L^* + 1),$$

which holds for all $|x_L^*| \in \mathbb{Z}_{++}$. Thus, $p(x_L^* + 1) \leq \pi$.

Then,

$$\hat{U}_L (x_L^* + 1) = -|x_L^* + x_M^*| - E_{p(x_L^* + 1)} [ |x_{w2} (x_L^* + x_M^*)| ] - E_{p(x_L^* + 1)} [ \mathbf{1} (|x_L^* + 1 - x_{w2} (x_L^* + 1)| > 1) ] \cdot \alpha$$

$$+ (1 + p(x_L^* + 1)) \cdot \eta$$

$$\leq -|x_L^*| \cdot \pi \cdot |x_{w2} (x_L^* + 1) - x_M^*| - (1 \cdot \pi) \cdot |x_{R2} (x_L^* + 1) - x_M^*| - \pi \cdot \mathbf{1} (|x_L^* + 1 - x_{R2} (x_L^* + 1)| > 1) \cdot \alpha$$

$$- (1 - \pi) \cdot \mathbf{1} (|x_L^* + 1 - (x_L^* + 2)| > 1) \cdot \alpha + (1 + \pi) \cdot \eta$$

$$= -1 \cdot \pi \cdot |x_L^* - x_M^*| - (1 \cdot \pi) \cdot |x_{L2} + 2 - x_M^*| - \pi \cdot \mathbf{1} (|x_L^* + 1 - x_M^*| > 1) \cdot \alpha$$

$$- (1 - \pi) \cdot \mathbf{1} (|x_L^* + 1 - (x_L^* + 2)| > 1) \cdot \alpha + (1 + \pi) \cdot \eta$$

$$= -1 \cdot (1 - \pi) \cdot 2 + (1 + \pi) \cdot \eta$$

$$< -1 + 2 \eta$$

$$= \hat{U}_L (x_L^* - 1).$$

Finally, suppose that $x_{L1} \notin \{x_L^* - 1, x_L^*, x_L^* + 1\}$. Then,

$$\hat{U}_L (x_{L1}) = -|x_{L1} - x_L^*| - E_{p(x_{L1})} [ |x_{w2} (x_{L1}) - x_L^*| ] - E_{p(x_{L1})} [ \mathbf{1} (|x_L^* - x_{w2} (x_{L1})| > 1) ] \cdot \alpha + (1 + p(x_{L1})) \cdot \eta$$

$$\leq -|x_{L1} - x_L^*| + 2 \eta$$

$$\leq -2 \eta$$

$$< -1 + 2 \eta$$

$$= \hat{U}_L (x_L^* - 1).$$

Thus, $x_{L1} = x_L^* - 1$ is the unique optimum for $L$.

Therefore, the unique sequentially rational behavior is $x_{L1} = x_L^* - 1$, $p(x_{L1}) = 1$, $x_{L2} (x_{L1}) = x_L^*$ and $x_{R2} (x_{L1}) = x_R^*$, so this is the unique equilibrium. □

**Proof of Theorem 2:** Suppose that $\Delta \leq \alpha$. Specifically, suppose that $\Delta \in (\alpha - z, \alpha - z + 1]$ for $z \in \mathbb{Z}_{++}$. I first state $L$’s period 1 optimization problem. Then, I examine twelve cases for the value of $x_{L1} \in \mathbb{Z}$. Cases 1-3 correspond to the $x_{L1}$ choices that can be equilibria under three different parameter assumptions. Specifically, Case 1 corresponds to the equilibrium $x_{L1}$ choice that arises in Theorem 2, Case 2 corresponds to the equilibrium $x_{L1}$ choice that arises in Proposition 5, and Case 3 corresponds to the equilibrium $x_{L1}$ choice that arises in Theorem 3. In the course of analyzing Cases 1-3, I also derive the corresponding equilibrium values of $p(x_{L1})$, $x_{L2} (x_{L1})$ and $x_{R2} (x_{L1})$. In Cases 4-12, I show that all other $x_{L1}$ choices are strictly dominated by at least one of the $x_{L1}$ choices from Cases 1-3. Then, after analyzing all twelve cases, I show that the parameter values assumed in Theorem 2 lead to the stated equilibrium. In the Proofs of Proposition 5 and Theorem 3, I use the results from this proof and show...
that their assumed parameter values lead to their stated equilibria.

During period 1, $L$ solves

$$V_{L1} \equiv \max_{x_{L1} \in Z} -|x_{L1} - x^*_L| - \mathbb{E}_{p(x_{L1})} \left[ |x_{u2} (x_{L1}) - x^*_L| \right] - \mathbb{E}_{p(x_{L1})} \left[ 1 \left( |x_{L1} - x_{u2} (x_{L1})| > 1 \right) \right] \cdot \alpha + (1 + p(x_{L1})) \cdot \eta.$$

Case 1: First, suppose that $x_{L1} = x^*_L - 1$. During period 2, $L$ would solve

$$V_{L2} \equiv \max_{x_{L2} \in Z} -|x_{L1} - x^*_L| - |x_{L2} - x^*_L| - 1 \left( |x_{L1} - x_{L2}| > 1 \right) \cdot \alpha + 2 \eta$$

$$= \max_{x_{L2} \in Z} -x^*_L - x^*_L - |x_{L2} - x^*_L| - 1 \left( |x^*_L - 1 - x_{L2}| > 1 \right) \cdot \alpha + 2 \eta$$

$$= \max_{x_{L2} \in Z} -|x_{L2} - x^*_L| - 1 \left( |x^*_L - 1 - x_{L2}| > 1 \right) \cdot \alpha,$$

where the final equality comes because the first and last terms are constant with respect to $x_{L2}$. The first term of the maximand is strictly negative if $x_{L2} \neq x^*_L$ and equal to zero if $x_{L2} = x^*_L$. The second term of the maximand is strictly negative if $|x^*_L - 1 - x_{L2}| > 1$ and equal to zero if $|x^*_L - 1 - x_{L2}| \leq 1$. Note that $x_{L2} = x^*_L$ satisfies the latter condition. Thus, the maximand is strictly negative if $x_{L2} \neq x^*_L$ and equal to zero if $x_{L2} = x^*_L$, so its unique maximizer is $x_{L2} = x^*_L$.

During period 2, $R$ would solve

$$V_{R2} \equiv \max_{x_{R2} \in Z} -|x_{L1} - x^*_R| - |x_{R2} - x^*_R| - 1 \left( |x_{L1} - x_{R2}| > 1 \right) \cdot \alpha + \eta$$

$$= \max_{x_{R2} \in Z} -x^*_L - x^*_R - |x_{R2} - x^*_R| - 1 \left( |x^*_R - 1 - x_{R2}| > 1 \right) \cdot \alpha + \eta$$

$$= \max_{x_{R2} \in Z} -|x_{R2} - x^*_R| - 1 \left( |x^*_R - 1 - x_{R2}| > 1 \right) \cdot \alpha,$$

where the final equality comes because the first and last terms are constant with respect to $x_{R2}$. Recall that $x^*_L - 1 < (x^*_L - 1) + 1 = x^*_L \leq -1 < -1 + 2 = 1 \leq x^*_R$, so $|x^*_L - 1 - x_{R2}| \geq 3 > 1$. Thus, if $x_{R2} = x^*_R$, then $|x^*_L - 1 - x_{R2}| > 1$. In that case, the maximand is equal to $-\alpha$. Clearly, any other $x_{R2}$ that satisfies $|x^*_L - 1 - x_{R2}| > 1$ makes $R$ strictly worse off. Thus, either $x_{R2} = x^*_R$ or $|x^*_L - 1 - x_{R2}| \leq 1$. The set of $x_{R2}$ that satisfies the latter condition is $\{x^*_L - 2, x^*_L - 1, x^*_R\}$. Of that set, $x_{R2} = x^*_R$ maximizes $R$'s utility because it is closest to $x^*_R$, and it yields a utility of $-|x^*_L - x^*_R| = -\Delta$. By assumption, $\Delta \leq \alpha$, so $-\alpha \leq -\Delta$. First, suppose that $-\alpha < -\Delta$. Then, of the two possible choices left, $R$, she uniquely maximizes her utility by choosing $x_{R2} = x^*_R$. Now, suppose that $-\alpha = -\Delta$. Then, $R$ is indifferent between $x_{R2} = x^*_R$ and $x_{R2} = x^*_L$. I assume that $R$ breaks the indifference by choosing the $x_{R2}$ that maximizes $U_R(x^*_L - 1)$, where $U_i(x_{L1}) \equiv \mathbb{E}_{p(x_{L1})} \left[ U_i(x_{L1}, x_{u2}, \lambda_{i1}, \lambda_{i2}) \right]$ denotes $i$’s expected utility from the choice of $x_{L1}$.

First, suppose that $x_{R2} = x^*_R$. Then, the median voter’s utility from re-electing $L$ is

$$U^L_M(x^*_L - 1) = - |x^*_L - 1 - x^*_M| - |x^*_L - x^*_M| - 1 \left( |x^*_L - 1 - x^*_M| > 1 \right) \cdot \alpha$$

$$= - |x^*_L - 1| - |x^*_L|$$

$$= -2 |x^*_L| - 1.$$

The median voter’s utility from electing $R$ is

$$U^R_M(x^*_L - 1) = - |x^*_L - 1 - x^*_M| - |x^*_L - x^*_M| - 1 \left( |x^*_L - 1 - x^*_M| > 1 \right) \cdot \alpha$$

$$= - |x^*_L - 1| - |x^*_L|$$

$$= -2 |x^*_L| - 1.$$

Thus, the median voter re-elects $L$ with probability $\pi$ if and only if

$$U^L_M(x^*_L - 1) = U^R_M(x^*_L - 1)$$

$$-2 |x^*_L| - 1 = -2 |x^*_L| - 1,$$

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which holds. Thus, \( p(x^*_L - 1) = \pi \). Then,
\[
\hat{U}_R(x^*_L - 1) = -|x^*_L - 1 - x^*_R| - E_{p(x^*_L - 1)}[|x_{w2}(x^*_L - 1) - x^*_R|] - E_{p(x^*_L - 1)}[1(|x^*_L - 1 - x_{w2}(x^*_L - 1)| > 1)] \cdot \alpha + (1 - p(x^*_L - 1)) \cdot \eta
\]
\[
= -\Delta - 1 - |x^*_L - x^*_R| - 1 (|x^*_L - 1 - x^*_L| > 1) \cdot \alpha + (1 - \pi) \cdot \eta
\]
\[
= -2\Delta - 1 + (1 - \pi) \cdot \eta,
\]
where the second equality comes from the fact that \( p(x^*_L - 1) = \pi \) and \( x_{w2}(x^*_L - 1) = x^*_L \) for all \( w \in \{L, R\} \).

Now, suppose that \( x_{R2} = x^*_R \). Then, the median voter’s utility from re-electing \( L \) is
\[
U^L_M(x^*_L - 1) = -|x^*_L - 1 - x^*_M| - |x^*_L - 1 - x^*_M| - 1 (|x^*_L - 1 - x^*_L| > 1) \cdot \alpha
\]
\[
= -|x^*_L - 1| - 1 \cdot \alpha
\]
\[
= -|x^*_L - 1| - x^*_R - \alpha
\]

The median voter’s utility from electing \( R \) is
\[
U^R_M(x^*_L - 1) = -|x^*_L - 1 - x^*_M| - |x^*_L - x^*_M| - 1 (|x^*_L - 1 - x^*_R| > 1) \cdot \alpha
\]
\[
= -|x^*_L - 1| - x^*_R - \alpha
\]

Thus, the median voter re-elects \( L \) with probability one if and only if
\[
U^L_M(x^*_L - 1) > U^R_M(x^*_L - 1)
\]
\[
-|x^*_L - 1| - |x^*_L| > -|x^*_L - 1| - x^*_R - \alpha
\]
\[
-|x^*_L| > -x^*_R - \alpha
\]
\[
\alpha > |x^*_L| - x^*_R,
\]
which holds. Thus, \( p(x^*_L - 1) = 1 \). Then,
\[
\hat{U}_R(x^*_L - 1) = -|x^*_L - 1 - x^*_R| - E_{p(x^*_L - 1)}[|x_{w2}(x^*_L - 1) - x^*_R|] - E_{p(x^*_L - 1)}[1(|x^*_L - 1 - x_{w2}(x^*_L - 1)| > 1)] \cdot \alpha + (1 - p(x^*_L - 1)) \cdot \eta
\]
\[
= -\Delta - 1 - |x_{L2}(x^*_L - 1) - x^*_R| - 1 (|x^*_L - 1 - x_{L2}(x^*_L - 1)| > 1) \cdot \alpha + (1 - \pi) \cdot \eta
\]
\[
= -\Delta - 1 - |x^*_L - x^*_R| - 1 (|x^*_L - 1 - x^*_L| > 1) \cdot \alpha
\]
\[
= -2\Delta - 1
\]
\[
< -2\Delta - 1 + (1 - \pi) \cdot \eta,
\]
where the second equality comes from the fact that \( p(x^*_L - 1) = 1 \), so \( w = L \) with probability one. Thus, \( R \) chooses \( x_{R2} = x^*_L \) because it maximizes \( \hat{U}_R(x^*_L - 1) \), so \( x_{R2} = x^*_L \) in all cases. This implies that \( p(x^*_L - 1) = \pi \).

Then,
\[
\hat{U}_L(x^*_L - 1) = -|x^*_L - 1 - x^*_L| - E_{p(x^*_L - 1)}[|x_{w2}(x^*_L - 1) - x^*_L|] - E_{p(x^*_L - 1)}[1(|x^*_L - 1 - x_{w2}(x^*_L - 1)| > 1)] \cdot \alpha + (1 + p(x^*_L - 1)) \cdot \eta
\]
\[
= -|1 - |x^*_L - x^*_L| - 1 (|x^*_L - 1 - x^*_L| > 1) \cdot \alpha + (1 + \pi) \cdot \eta
\]
\[
= -1 + (1 + \pi) \cdot \eta.
\]
where the second equality comes from the facts that \( p(x^*_L - 1) = \pi \) and \( x_{L2}(x^*_L - 1) = x_{R2}(x^*_L - 1) = x^*_L \).

**Case 2:** Next, suppose that \( x_{L1} = x^*_L - z - 1 \). During period 2, \( L \) would solve
\[
V_{L2} = \max_{x_{L2} \in Z} -|x_{L1} - x^*_L| - |x_{L2} - x^*_L| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2 \eta
\]
\[
= \max_{x_{L2} \in Z} -|x^*_L - z - 1 - x^*_L| - |x_{L2} - x^*_L| - 1 (|x^*_L - z - 1 - x_{L2}| > 1) \cdot \alpha + 2 \eta
\]
\[
= \max_{x_{L2} \in Z} -|x_{L2} - x^*_L| - 1 (|x^*_L - z - 1 - x_{L2}| > 1) \cdot \alpha,
\]

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where the final equality comes because the first and last terms are constant with respect to $x_{L2}$. Note that if $x_{L2} = x^*_L$, then $|x^*_L - z - 1 - x_{L2}| > 1$ because $z \geq 1$. In that case, the maximand is equal to $-\alpha$. Clearly, any other $x_{L2}$ that satisfies $|x^*_L - z - 1 - x_{L2}| > 1$ makes $L$ strictly worse off. Thus, either $x_{L2} = x^*_L$ or $|x^*_L - z - 1 - x_{L2}| \leq 1$.

The set of $x_{L2}$ that satisfies the latter condition is $\{x^*_L - z - 2, x^*_L - z - 1, x^*_L - z\}$. Of that set, $x_{L2} = x^*_L - z$ maximizes $L$’s utility because it is closest to $x^*_L$, and it yields a utility of $-|x^*_L - z - x^*_L| = -z$. By assumption, $\Delta \leq \alpha - z + 1$, so $-\alpha \leq -\Delta - z + 1 \leq -2 - z + 1 < -z$. Thus, of the two possible choices left $L$, she uniquely maximizes her utility by choosing $x_{L2} = x^*_L - z$.

During period 2, $R$ would solve

\[
V_{R2} = \max_{x_{R2} \in \mathbb{Z}} -|x_{L1} - x^*_R| - |x_{R2} - x^*_R| - 1(|x_{L1} - x_{R2}| > 1) \cdot \alpha + \eta
\]

\[
= \max_{x_{R2} \in \mathbb{Z}} -|x^*_L - z - 1 - x^*_R| - |x_{R2} - x^*_R| - 1(|x^*_L - z - 1 - x_{R2}| > 1) \cdot \alpha + \eta
\]

\[
= \max_{x_{R2} \in \mathbb{Z}} -|x_{R2} - x^*_R| - 1(|x^*_L - z - 1 - x_{R2}| > 1) \cdot \alpha,
\]

where the final equality comes because the first and last terms are constant with respect to $x_{R2}$. Recall that $x^*_L - z - 1 \leq x^*_L - 2 < x^*_L \leq -1 < -1 + 2 = 1 = x^*_R$, so $|x^*_L - z - 1 - x^*_R| \geq 4 > 1$. Thus, if $x_{R2} = x^*_R$, then $|x^*_L - z - 1 - x_{R2}| > 1$. In that case, the maximand is equal to $-\alpha$. Clearly, any other $x_{R2}$ that satisfies $|x^*_L - z - 1 - x_{R2}| > 1$ makes $R$ strictly worse off. Thus, either $x_{R2} = x^*_R$ or $|x^*_L - z - 1 - x_{R2}| \leq 1$. The set of $x_{R2}$ that satisfies the latter condition is $\{x^*_L - z - 2, x^*_L - z - 1, x^*_L - z\}$. Of that set, $x_{R2} = x^*_L - z$ maximizes $R$’s utility because it is closest to $x^*_L$, and it yields a utility of $-|x^*_L - z - x^*_R| = -\Delta - z$. By assumption, $\Delta > \alpha - z$, so $-\alpha > -\Delta - z$. Thus, of the two possible choices left $R$, she uniquely maximizes her utility by choosing $x_{R2} = x^*_R$.

Thus, the median voter’s utility from re-electing $L$ is

\[
U^L_M(x^*_L - z - 1) = -|x^*_L - z - 1 - x^*_M| - |x^*_L - z - x^*_M| - 1(|x^*_L - z - 1 - (x^*_L - z)| > 1) \cdot \alpha
\]

\[
= -|x^*_L - z - 1| - |x^*_L - z| - 1
\]

\[
= -|x^*_L - z - 1| - |x^*_L - z|.
\]

The median voter’s utility from electing $R$ is

\[
U^R_M(x^*_L - z - 1) = -|x^*_L - z - 1 - x^*_M| - |x^*_L - x^*_M| - 1(|x^*_L - z - 1 - x^*_R| > 1) \cdot \alpha
\]

\[
= -|x^*_L - z - 1| - |x^*_L - x^*_R| - \alpha
\]

\[
= -|x^*_L - z - 1| - x^*_R - \alpha.
\]

Thus, the median voter re-elects $L$ with probability one if and only if

\[
U^L_M(x^*_L - z - 1) > U^R_M(x^*_L - z - 1)
\]

\[
-|x^*_L - z - 1| - |x^*_L - z| > -|x^*_L - z - 1| - x^*_R - \alpha
\]

\[
-|x^*_L - z - 1| - x^*_R - \alpha > |x^*_L - x^*_R| + \alpha
\]

\[
\alpha > x^*_R - x^*_L - x^*_R + z
\]

\[
\alpha > x^*_R - x^*_L - x^*_R + z
\]

\[
\alpha > \Delta - 2x^*_R + z
\]

\[
\Delta < \alpha + 2x^*_R - z,
\]

which holds because $\Delta < \alpha - z + 1 < \alpha + 2x^*_R - z$. Thus, $p(x^*_L - z - 1) = 1$. 33
Then,
\[
\dot{U}_L(x_L^* - z - 1) = -|x_L^* - z - 1 - x_L^*| - \mathbb{E}_p(x_L^* - z - 1) \left[ |x_{w2}(x_L^* - z - 1) - x_L^*| \right] \\
- \mathbb{E}_p(x_L^* - z - 1) \left[ |1(x_L^* - z - 1 - x_{w2}(x_L^* - z - 1)| > 1 \right] \alpha + (1 + p(x_L^* - z - 1)) \cdot \eta \\
= -(|x_L^* - z - 1 - x_{L2}(x_L^* - z - 1)) - 1 \left( |x_L^* - z - 1 - x_{L2}(x_L^* - z - 1)| > 1 \right) \cdot \alpha + (1 + 1) \cdot \eta \\
= -z - 1 - |x_L^* - z - x_L^*| - 1 \left( |x_L^* - z - 1 - (x_L^* - z)| > 1 \right) \cdot \alpha + 2 \eta \\
= -z - 1 - |z - 2| + 2 \eta \\
= -2z - 1 + 2 \eta.
\]

where the second equality comes from the fact that \( p(x_L^* - z - 1) = 1 \), so \( w = L \) with probability one.

Case 3: Next, suppose that \( x_{L1} = 1 \). During period 2, \( L \) would solve
\[
\dot{V}_{L2} = \max_{x_{L2} \in \mathbb{Z}} - |x_{L1} - x_{L2}^*| - |x_{L2} - x_{L2}^*| - 1 \left( |x_{L1} - x_{L2}| > 1 \right) \cdot \alpha + 2 \eta \\
= \max_{x_{L2} \in \mathbb{Z}} - |1 - x_{L2}^*| - |x_{L2} - x_{L2}^*| - 1 \left( |1 - x_{L2}| > 1 \right) \cdot \alpha + 2 \eta \\
= \max_{x_{L2} \in \mathbb{Z}} - |x_{L2} - x_{L2}^*| - 1 \left( |1 - x_{L2}| > 1 \right) \cdot \alpha,
\]

where the final equality comes because the first and last terms are constant with respect to \( x_{L2} \). Note that since \( x_L^* \leq -1 < -1 + 2 = 1 \), it follows that if \( x_{L2} = x_L^* \), then \( |1 - x_{L2}| > 1 \). In that case, the maximand is equal to \(-\alpha\). Clearly, any other \( x_{L2} \) that satisfies \( |1 - x_{L2}| > 1 \) makes \( L \) strictly worse off. Thus, either \( x_{L2} = x_L^* \) or \( |1 - x_{L2}| \leq 1 \). The set of \( x_{L2} \) that satisfies the latter condition is \( \{0, 1, 2\} \). Of that set, \( x_{L2} = 0 \) maximizes \( L \)'s utility because it is closest to \( x_L^* \) and it yields a utility of \(-|0 - x_L^*| = -|x_L^*|\). By assumption, \( \Delta > \alpha - z \), so \(-\alpha < -\Delta - z < -\Delta < -x_L^* \). Thus, of the two possible choices left \( L \), she uniquely maximizes her utility by choosing \( x_{L2} = 0 \).

During period 2, \( R \) would solve
\[
\dot{V}_{R2} = \max_{x_{R2} \in \mathbb{Z}} - |x_{L1} - x_{R2}^*| - |x_{R2} - x_{R2}^*| - 1 \left( |x_{L1} - x_{R2}| > 1 \right) \cdot \alpha + \eta \\
= \max_{x_{R2} \in \mathbb{Z}} - |1 - x_{R2}^*| - |x_{R2} - x_{R2}^*| - 1 \left( |1 - x_{R2}| > 1 \right) \cdot \alpha + \eta \\
= \max_{x_{R2} \in \mathbb{Z}} - |x_{R2} - x_{R2}^*| - 1 \left( |1 - x_{R2}| > 1 \right) \cdot \alpha,
\]

where the final equality comes because the first and last terms are constant with respect to \( x_{R2} \). First, suppose that \( x_R^* = 1 \). The first term of the maximand is strictly negative if \( x_{R2} \neq x_R^* \) and equal to zero if \( x_{R2} = x_R^* \). The second term of the maximand is strictly negative if \( |1 - x_{R2}| > 1 \) and equal to zero if \( |1 - x_{R2}| \leq 1 \). Note that \( x_{R2} = x_R^* \) satisfies the latter condition. Thus, the maximand is strictly negative if \( x_{R2} \neq x_R^* \) and equal to zero if \( x_{R2} = x_R^* \), so its unique maximizer is \( x_{R2} = x_R^* = 1 \).

Now, suppose that \( x_R^* = 2 \). The first term of the maximand is strictly negative if \( x_{R2} \neq x_R^* \) and equal to zero if \( x_{R2} = x_R^* \). The second term of the maximand is strictly negative if \( |1 - x_{R2}| > 1 \) and equal to zero if \( |1 - x_{R2}| \leq 1 \). Note that \( x_{R2} = x_R^* \) satisfies the latter condition. Thus, the maximand is strictly negative if \( x_{R2} \neq x_R^* \) and equal to zero if \( x_{R2} = x_R^* \), so its unique maximizer is \( x_{R2} = x_R^* = 2 \).

Now, suppose that \( x_R^* > 2 \). Then, \(|1 - x_R^*| \geq 2 > 1 \). Thus, if \( x_{R2} = x_R^* \), then \(|1 - x_{R2}| > 1 \). In that case, the maximand is equal to \(-\alpha \). Clearly, any other \( x_{R2} \) that satisfies \(|1 - x_{R2}| > 1 \) makes \( R \) strictly worse off. Thus, either \( x_{R2} = x_R^* \) or \(|1 - x_{R2}| \leq 1 \). The set of \( x_{R2} \) that satisfies the latter condition is \( \{0, 1, 2\} \). Of that set, \( x_{R2} = 2 \) maximizes \( R \)'s utility because it is closest to \( x_R^* \) and it yields a utility of \(-|2 - x_R^*| = 2 - x_R^* \). By assumption, \( \Delta \leq \alpha - z + 1 \), so \(-\alpha \leq -\Delta - z + 1 \leq -\Delta < -x_R^* < 2 - x_R^* \). Thus, of the two possible choices left \( R \), she uniquely maximizes her utility by choosing \( x_{R2} = 2 \). Thus, in any case, \( x_{R2} \in \{1, 2\} \).
Thus, the median voter’s utility from re-electing $L$ is

$$U_M(1) = -|1 - x^*| - |0 - x^*| - 1(|1 - 0| > 1) \cdot \alpha = -|1| - |0| = -1.$$  

The median voter’s utility from electing $R$ is

$$U_M(1) = -|1 - x^*| - |x_{R2} - x^*| - 1(|1 - x_{R2}| > 1) \cdot \alpha = -|1| - |x_{R2}| \leq -1 - 1 = -2,$$

where the second equality comes from the fact that $|1 - x_{R2}| \leq 1$, the third equality comes from the fact that $x_{R2} > 0$ and the inequality comes from the fact that $x_{R2} \geq 1$. Thus, the median voter re-elects $L$ with probability one if and only if

$$U_M(1) > U_M^R(1),$$

which holds because $U_M^R(1) \leq -2 < -1$. Thus, $p(1) = 1$.

Then,

$$\bar{U}_L(1) = -|1 - x_L^*| - \mathbb{E}_{p(1)}[|x_{w2}(1) - x_L^*|] - \mathbb{E}_{p(1)}[1(|1 - x_{w2}(1)| > 1)] \cdot \alpha + (1 + p(1)) \cdot \eta = -|x_L^* - 1 - x_{L2}^*| - 1(|1 - x_{L2}(1)| > 1) \cdot \alpha + (1 + 1) \cdot \eta = -|x_L^* - 1 - 0 - x_L^*| - 1(|1 - 0| > 1) \cdot \alpha + 2\eta = -|x_L^* - 1| - |x_L^*| + 2\eta = -2|x_L^*| - 1 + 2\eta,$$

where the second equality comes from the fact that $p(1) = 1$, so $w = L$ with probability one.

**Case 4:** Next, suppose that $x_{L1} \in \{\ldots, x_L^* - z - \Delta - 3, x_L^* - z - \Delta - 1\}$. During period 2, $L$ would solve

$$V_{L2} = \max_{x_{L2} \in \mathbb{E}} -|x_{L1} - x_L^*| - |x_{L2} - x_L^*| - 1(|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2\eta = \max_{x_{L2} \in \mathbb{E}} -|x_{L2} - x_L^*| - 1(|x_{L1} - x_{L2}| > 1) \cdot \alpha,$$

where the final equality comes because the first and last terms are constant with respect to $x_{L2}$. Note that $x_{L1} \leq x_L^* - z - \Delta - 1 \leq x_L^* - z - \Delta - 4 < x_L^*$. Thus, if $x_{L2} = x_L^*$, then $|x_{L1} - x_{L2}| \geq 4 > 1$. In that case, the maximand is equal to $-\alpha$. Clearly, any other $x_{L2}$ that satisfies $|x_{L1} - x_{L2}| > 1$ makes $L$ strictly worse off. Thus, either $x_{L2} = x_L^*$ or $|x_{L1} - x_{L2}| \leq 1$. The set of $x_{L2}$ that satisfies the latter condition is $\{x_{L1} - 1, x_{L1}, x_{L1} + 1\}$. Of that set, $x_{L2} = x_{L1} + 1$ maximizes $L$’s utility because it is closest to $x_L^*$, and it yields a utility of

$$-|x_{L1} + 1 - x_L^*| = -|x_{L1}| + 1 + |x_L^*| \leq -|x_L^* - z - \Delta - 1| + 1 + |x_L^*| = -|x_L^*| - z - \Delta - 1 + 1 + |x_L^*| = -\Delta - z.$$  

By assumption, $\Delta > \alpha - z$, so $-\alpha > -\Delta - z \geq -|x_{L1} + 1 - x_L^*|$. Thus, of the two possible choices left $L$, she uniquely maximizes her utility by choosing $x_{L2} = x_L^*$. 

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During period 2, $R$ would solve

$$V_{R_2} \equiv \max_{x_{R_2} \in \mathbb{R}} -|x_{L_1} - x_{R_2}^*| - |x_{R_2} - x_{R}^*| - \mathbf{1}(|x_{L_1} - x_{R_2}| > 1) \cdot \alpha + \eta$$

$$= \max_{x_{R_2} \in \mathbb{R}} -|x_{R_2} - x_{R}^*| - \mathbf{1}(|x_{L_1} - x_{R_2}| > 1) \cdot \alpha,$$

where the final equality comes because the first and last terms are constant with respect to $x_{R_2}$. Note that $x_{L_1} \leq x_{L}^* - z - \Delta - 1 \leq x_{L}^* - 4 < z_{L}^* \leq -1 < -1 + 2 \leq 1 \leq x_{R}^*$. Thus, if $x_{R_2} = x_{R}^*$, then $|x_{L_1} - x_{R_2}| \geq 6 > 1$. In that case, the maximand is equal to $-\alpha$. Clearly, any other $x_{R_2}$ that satisfies $|x_{L_1} - x_{R_2}| > 1$ makes $R$ strictly worse off. Thus, either $x_{R_2} = x_{R}^*$ or $|x_{L_1} - x_{R_2}| \leq 1$. The set of $x_{R_2}$ that satisfies the latter condition is $\{x_{L_1} - 1, x_{L_1}, x_{L_1} + 1\}$. Of that set, $x_{R_2} = x_{L_1} + 1$ maximizes $R$’s utility because it is closest to $x_{R}^*$, and it yields a utility of $-|x_{L_1} + 1 - x_{R}^*| < -|x_{L_1} + 1 - x_{L}^*| \leq -\Delta - z$. By assumption, $\Delta > \alpha - z$, so $-\alpha > -\Delta - z > -|x_{L_1} + 1 - x_{R}^*|$. Thus, of the two possible choices left $R$, she uniquely maximizes her utility by choosing $x_{R_2} = x_{R}^*$.

Thus, the median voter’s utility from re-electing $L$ is

$$U^L_M (x_{L_1}) = -|x_{L_1} - x_{M}^*| - |x_{L}^* - x_{M}^*| - \mathbf{1}(|x_{L_1} - x_{L}^*| > 1) \cdot \alpha$$

$$= -|x_{L_1} - x_{L}^*| - \alpha.$$ 

The median voter’s utility from electing $R$ is

$$U^R_M (x_{L_1}) = -|x_{L_1} - x_{M}^*| - |x_{R}^* - x_{M}^*| - \mathbf{1}(|x_{L_1} - x_{R}^*| > 1) \cdot \alpha$$

$$= -|x_{L_1} - x_{R}^*| - \alpha$$

$$= -|x_{L_1} - x_{R}^*| - \alpha.$$

Note that

$$U^L_M (x_{L_1}) \geq U^R_M (x_{L_1}) \iff -|x_{L_1} - x_{L}^*| - \alpha \geq -|x_{L_1} - x_{R}^*| - \alpha$$

$$-|x_{L}^*| \geq -x_{R}^*$$

$$|x_{L}^*| \leq x_{R}^*.$$ 

Thus, depending on the values of $x_{R}^*$ and $|x_{L}^*|$, the median voter re-elects $L$ with probability $p(x_{L_1}) \leq 1$.

Then,

$$\hat{U}_L (x_{L_1}) = -|x_{L_1} - x_{L}^*| - \mathbb{E}_p(x_{L_1}) \left[ |x_{w_2} (x_{L_1}) - x_{L}^*| \right] - \mathbb{E}_p(x_{L_1}) \left[ \mathbf{1}(|x_{L_1} - x_{w_2} (x_{L_1})| > 1) \right] \cdot \alpha + (1 + p(x_{L_1})) \cdot \eta$$

$$\leq -|x_{L_1} - x_{L}^*| - |x_{L_2} (x_{L_1}) - x_{L}^*| - \mathbf{1}(|x_{L_1} - x_{L_2} (x_{L_1})| > 1) \cdot \alpha + (1 + 1) \cdot \eta$$

$$= -|x_{L_1} - x_{L}^*| - |x_{L}^* - x_{L}^*| - \mathbf{1}(|x_{L_1} - x_{L}^*| > 1) \cdot \alpha + 2\eta$$

$$= -|x_{L_1} - x_{L}^*| - \alpha + 2\eta$$

$$\leq -|x_{L}^* - z - \Delta - 1 - x_{L}^*| - \alpha + 2\eta$$

$$= -|z - \Delta - 1 - x_{L}^*| - \alpha + 2\eta$$

$$< -z - \Delta - 1 - z - \Delta + 1 + 2\eta$$

$$= -2\Delta - 2z + 2\eta$$

$$< -2\Delta - 1 + 2\eta$$

$$< -2|x_{L}^*| - 1 + 2\eta$$

$$= \hat{U}_L (1),$$

where the first strict inequality comes from the fact that $\Delta < \alpha - z + 1$. 

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Case 5: Next, suppose that \( x_{L1} \in \{x^*_L - z - \Delta, \ldots, x^*_L - z - 2 \} \). During period 2, \( L \) would solve

\[
V_{L2} = \max_{x_{L2} \in \mathbb{Z}} -|x_{L1} - x^*_L| - |x_{L2} - x^*_L| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2 \eta \\
= \max_{x_{L2} \in \mathbb{Z}} -|x_{L2} - x^*_L| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha,
\]

where the final equality comes because the first and last terms are constant with respect to \( x_{L2} \). Note that \( x_{L1} \leq x^*_L - z - 2 \leq x^*_L - 3 < x^*_L \) since \( z \geq 1 \). Thus, if \( x_{L2} = x^*_L \), then \( |x_{L1} - x_{L2}| \geq 3 > 1 \). In that case, the maximand is equal to \(-\alpha\). Clearly, any other \( x_{L2} \) that satisfies \( |x_{L1} - x_{L2}| > 1 \) makes \( L \) strictly worse off. Thus, either \( x_{L2} = x^*_L \) or \( |x_{L1} - x_{L2}| \leq 1 \). The set of \( x_{L2} \) that satisfies the latter condition is \( \{x_{L1} - 1, x_{L1}, x_{L1} + 1\} \). Of that set, \( x_{L2} = x_{L1} + 1 \) maximizes \( L \)'s utility because it is closest to \( x^*_L \), and it yields a utility of

\[
-|x_{L1} + 1 - x^*_L| = -|-x_{L1}| + 1 + |x^*_L| \\
\geq -|x^*_L - z - \Delta| + 1 + |x^*_L| \\
= -|-x^*_L| - z - \Delta + 1 + |x^*_L| \\
= -\Delta - z + 1.
\]

By assumption, \( \Delta \leq \alpha - z + 1 \), so \( -\alpha \leq -\Delta - z + 1 \leq -|x_{L1} + 1 - x^*_L| \). First, suppose that \( -\alpha < -|x_{L1} + 1 - x^*_L| \). Then, of the two possible choices left \( L \), she uniquely maximizes her utility by choosing \( x_{L2} = x_{L1} + 1 \). Now, suppose that \( -\alpha = -|x_{L1} + 1 - x^*_L| \). Note that this requires \( -\alpha = -\Delta - z + 1 \) and \( x_{L1} = x^*_L - z - \Delta \). Then, \( L \) is indifferent between \( x_{L2} = x_{L1} + 1 = x^*_L - z - \Delta + 1 = x^*_L - \alpha \) and \( x_{L2} = x^*_L \). I assume that \( L \) breaks the indifference by choosing the \( x_{L2} \) that maximizes \( U_L (x^*_L - \alpha - 1) \). I evaluate the two options after determining \( x_{R2} \).

During period 2, \( R \) would solve

\[
V_{R2} = \max_{x_{R2} \in \mathbb{Z}} -|x_{L1} - x^*_R| - |x_{R2} - x^*_R| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha + \eta \\
= \max_{x_{R2} \in \mathbb{Z}} -|x_{R2} - x^*_R| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha,
\]

where the final equality comes because the first and last terms are constant with respect to \( x_{R2} \). Note that \( x_{L1} \leq x^*_L - z - 2 \leq x^*_L - 3 < x^*_L \leq -1 < -1 + 2 \leq 1 \leq x^*_R \). Thus, if \( x_{R2} = x^*_R \), then \( |x_{L1} - x_{R2}| \geq 3 > 1 \). In that case, the maximand is equal to \(-\alpha\). Clearly, any other \( x_{R2} \) that satisfies \( |x_{L1} - x_{R2}| > 1 \) makes \( R \) strictly worse off. Thus, either \( x_{R2} = x^*_R \) or \( |x_{L1} - x_{R2}| \leq 1 \). The set of \( x_{R2} \) that satisfies the latter condition is \( \{x_{L1} - 1, x_{L1}, x_{L1} + 1\} \). Of that set, \( x_{R2} = x_{L1} + 1 \) maximizes \( R \)'s utility because it is closest to \( x^*_R \), and it yields a utility of

\[
-|x_{L1} + 1 - x^*_R| = -|-x_{L1}| + 1 + |x^*_R| \\
\leq -|x^*_R - z - 2| + 1 + x^*_R \\
= -|-x^*_L| - z - 2 + 1 - x^*_R \\
= -|x^*_L| - z - 1 + x^*_R \\
= -\Delta - z - 1 \\
< -\Delta - z.
\]

By assumption, \( \Delta > \alpha - z \), so \( -\alpha > -\Delta - z > -|x_{L1} + 1 - x^*_R| \). Thus, of the two possible choices left \( R \), she uniquely maximizes her utility by choosing \( x_{R2} = x^*_R \).

Now, I determine \( x_{L2} \) when \( -\alpha = -|x_{L1} + 1 - x^*_L| \). First, suppose that \( x_{L2} = x^*_L - \alpha \). Then, the median voter's
utility from re-electing \( L \) is
\[
U_M^L (x_L^* - \alpha - 1) = - |x_L^* - \alpha - 1 - x_M^*| - |x_L^* - \alpha - x_M^*| - 1 (|x_L^* - \alpha - 1 - (x_L^* - \alpha)| > 1) \cdot \alpha \\
= - |x_L^* - \alpha - 1| - |x_L^* - \alpha| \\
= - |x_L^* - \alpha - 1| - |x_L^* - \alpha|.
\]

The median voter’s utility from electing \( R \) is
\[
U_M^R (x_L^* - \alpha - 1) = - |x_L^* - \alpha - 1 - x_M^*| - |x_R^* - x_M^*| - 1 (|x_L^* - \alpha - 1 - x_R^*| > 1) \cdot \alpha \\
= - |x_L^* - \alpha - 1| - |x_R^* - \alpha| \\
= - |x_L^* - \alpha - 1| - x_R^* - \alpha.
\]

Note that
\[
U_M^L (x_L^* - \alpha - 1) \geq U_M^R (x_L^* - \alpha - 1) \\
\iff \\
- |x_L^* - \alpha - 1| - |x_L^* - \alpha| \geq - |x_L^* - \alpha - 1| - x_R^* - \alpha \\
- |x_L^* - \alpha| \geq -x_R^* \\
|x_L^*| \leq x_R^*.
\]

If \( |x_L^*| < x_R^* \), then \( p(x_L^* - \alpha - 1) = 1 \). If \( |x_L^*| = x_R^* \), then \( p(x_L^* - \alpha - 1) = \pi \). If \( |x_L^*| > x_R^* \), then \( p(x_L^* - \alpha - 1) = 0 \). Then,
\[
\hat{U}_L (x_L^* - \alpha - 1) = - |x_L^* - \alpha - 1 - x_M^*| - E_{p(x_L^* - \alpha)} |x_L^* - \alpha - 1 - x_M^*| - E_{p(x_L^* - \alpha - 1)} \left[ 1 (|x_L^* - \alpha - 1 - x_L^* - \alpha - 1| > 1) \right] \cdot \alpha + (1 + p(x_L^* - \alpha - 1)) \cdot \eta.
\]

Now, suppose that \( x_L^* = x_M^* \). Then, the median voter’s utility from re-electing \( L \) is
\[
U_M^L (x_L^* - \alpha - 1) = - |x_L^* - \alpha - 1 - x_M^*| - |x_M^* - x_M^*| - 1 (|x_L^* - \alpha - 1 - x_M^*| > 1) \cdot \alpha \\
= - |x_L^* - \alpha - 1| - |x_L^* - \alpha|.
\]

The median voter’s utility from electing \( R \) is
\[
U_M^R (x_L^* - \alpha - 1) = - |x_L^* - \alpha - 1 - x_M^*| - |x_R^* - x_M^*| - 1 (|x_L^* - \alpha - 1 - x_R^*| > 1) \cdot \alpha \\
= - |x_L^* - \alpha - 1| - |x_R^* - \alpha| \\
= - |x_L^* - \alpha - 1| - x_R^* - \alpha.
\]

Note that
\[
U_M^L (x_L^* - \alpha - 1) \geq U_M^R (x_L^* - \alpha - 1) \\
\iff \\
- |x_L^* - \alpha - 1| - |x_L^* - \alpha| \geq - |x_L^* - \alpha - 1| - x_R^* - \alpha \\
- |x_L^* - \alpha| \geq -x_R^* \\
|x_L^*| \leq x_R^*.
\]

If \( |x_L^*| < x_R^* \), then \( p(x_L^* - \alpha - 1) = 1 \). If \( |x_L^*| = x_R^* \), then \( p(x_L^* - \alpha - 1) = \pi \). If \( |x_L^*| > x_R^* \), then \( p(x_L^* - \alpha - 1) = 0 \). Then,
\[
\hat{U}_L (x_L^* - \alpha - 1) = - |x_L^* - \alpha - 1 - x_M^*| - E_{p(x_L^* - \alpha)} |x_L^* - \alpha - 1 - x_M^*| - E_{p(x_L^* - \alpha - 1)} \left[ 1 (|x_L^* - \alpha - 1 - x_L^* - \alpha - 1| > 1) \right] \cdot \alpha + (1 + p(x_L^* - \alpha - 1)) \cdot \eta.
\]

Note that this expected utility is same as if \( x_L^* = x_L^* - \alpha \). The first term is clearly the same as if \( x_L^* = x_L^* - \alpha \). The last term is the same as if \( x_L^* = x_L^* - \alpha \) because \( p(x_L^* - \alpha - 1) \) is the same as if \( x_L^* = x_L^* - \alpha \). The sum of
the middle two terms is the same as if \(x_{L2} = x_L^* - \alpha\) by the following logic. If \(p(x_L^* - \alpha - 1) = 0\), then only \(x_{R2}\) is realized in equilibrium and it does not depend on \(x_{L2}\). If \(p(x_L^* - \alpha - 1) = 1\), then only \(x_{L2}\) is realized in equilibrium, so the sum of the middle two terms is the same as if \(x_{L2} = x_L^* - \alpha\) because \(L\) is indifferent between \(x_{L2} = x_L^* - \alpha\) and \(x_{L2} = x_L^*\). If \(p(x_L^* - \alpha - 1) = \pi\), then the sum of the middle two terms is the convex combination of two sums that are the same as if \(x_{L2} = x_L^* - \alpha\), so the sum of the middle two terms is the same as if \(x_{L2} = x_L^* - \alpha\).

Thus, \(\hat{U}_L (x_L^* - \alpha - 1)\) and \(p(x_L^* - \alpha - 1)\) do not depend on whether \(x_{L2} = x_L^* - \alpha\) or \(x_{L2} = x_L^*\). Thus, for the sake of simplicity in calculation, I can without loss of generality assume that \(x_{L2} = x_L^* - \alpha\). Thus, \(x_{L2} = x_L + 1\) in all cases.

Thus, the median voter’s utility from re-electing \(L\) is

\[
U^L_M (x_{L1}) = - |x_{L1} - x_L^*| - |x_{L1} + 1 - x_L^*| - 1 (|x_{L1} - (x_{L1} + 1)| > 1) \cdot \alpha \\
= - |x_{L1}| - |x_{L1} + 1| \\
= - |x_{L1}| - |x_{L1} + 1| \\
= - |x_{L1}| + x_{L1} + 1.
\]

The median voter’s utility from electing \(R\) is

\[
U^R_M (x_{L1}) = - |x_{L1} - x_L^*| - |x_R^* - x_L^*| - 1 (|x_{L1} - x_R^*| > 1) \cdot \alpha \\
= - |x_{L1}| - |x_R^* - \alpha| \\
= - |x_{L1}| - x_R^* - \alpha.
\]

Note that

\[
U^L_M (x_{L1}) \geq U^R_M (x_{L1}) \\
\iff - |x_{L1}| + x_{L1} + 1 \geq - |x_{L1}| - x_R^* - \alpha \\
x_{L1} + 1 \geq -x_R^* - \alpha \\
x_{L1} \geq -x_R^* - \alpha - 1.
\]

Thus, depending on the value of \(x_R^*\), the median voter re-elects \(L\) with probability \(p(x_{L1}) \leq 1\).

Then,

\[
\hat{U}_L (x_{L1}) = - |x_{L1} - x_L^*| - \mathbb{E}_{p(x_{L1})} [ |x_{w2}(x_{L1}) - x_L^*|] - \mathbb{E}_{p(x_{L1})} [ 1 (|x_{L1} - x_{w2}(x_{L1})| > 1) \cdot \alpha + (1 + p(x_{L1})) \cdot \eta ] \\
\leq - |x_{L1} - x_L^*| - |x_{L2}(x_{L1}) - x_L^*| - 1 (|x_{L1} - x_{L2}(x_{L1})| > 1) \cdot \alpha + (1 + 1) \cdot \eta \\
= - |x_{L1} - x_L^*| - |x_{L1} + 1 - x_L^*| - 1 (|x_{L1} - (x_{L1} + 1)| > 1) \cdot \alpha + 2\eta \\
= - |x_{L1} - x_L^*| - |x_{L1} + 1 - x_L^*| - 2\eta \\
\leq - |x_L^* - z - 2 - x_L^*| - |x_L^* - z - 2 - 1 - x_L^*| + 2\eta \\
= - |z - 2| - |z - 1| + 2\eta \\
= -z - 2 - z - 1 + 2\eta \\
= -2z - 3 + 2\eta \\
< -2z - 1 + 2\eta \\
= \hat{U}_L (x_L^* - z - 1).
\]

**Case 6:** Next, suppose that \(x_{L1} \in \{x_L^* - z, \ldots, x_L^* - 2\}\). During period 2, \(L\) would solve

\[
V_{L2} = \max_{x_{L2} \in \mathbb{Z}} [ - |x_{L1} - x_L^*| - |x_{L2} - x_L^*| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2\eta ] \\
= \max_{x_{L2} \in \mathbb{Z}} [ - |x_{L2} - x_L^*| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha ]
\]

where the final equality comes because the first and last terms are constant with respect to \(x_{L2}\). Note that \(x_{L1} \leq x_L^* - 2 < x_L^*\). Thus, if \(x_{L2} = x_L^*\), then \(|x_{L1} - x_{L2}| \geq 2 > 1\). In that case, the maximand is equal to \(-\alpha\). Clearly,
any other $x_{L2}$ that satisfies $|x_{L1} - x_{L2}| > 1$ makes $L$ strictly worse off. Thus, either $x_{L2} = x^*_L$ or $|x_{L1} - x_{L2}| \leq 1$. The set of $x_{L2}$ that satisfies the latter condition is $\{x_{L1} - 1, x_{L1}, x_{L1} + 1\}$. Of that set, $x_{L2} = x_{L1} + 1$ maximizes $L$’s utility because it is closest to $x^*_L$, and it yields a utility of

$$- |x_{L1} + 1 - x^*_L| = - |x_{L1} + 1 + |x^*_L|| = - |x_{L1} + 1 + |x^*_L| \
= - |x_{L1} - z + 1 + |x^*_L| \
= - |x^*_L| - z + 1 + |x^*_L| \
= - z + 1.$$  

By assumption, $\Delta \leq \alpha - z + 1$, so $-\alpha \leq -\Delta - z + 1 \leq -z + 1 \leq - |x_{L1} + 1 - x^*_L|$. Thus, of the two possible choices left $L$, she uniquely maximizes her utility by choosing $x_{L2} = x_{L1} + 1$.

During period 2, $R$ would solve

$$V_{R2} = \max_{x_{R2} \in Z} - |x_{L1} - x^*_L| - |x_{R2} - x^*_R| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha + \eta \
= \max_{x_{R2} \in Z} - |x_{R2} - x^*_R| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha,$$

where the final equality comes because the first and last terms are constant with respect to $x_{R2}$. Note that $x_{L1} \leq x^*_L - 2 < x^*_L \leq -1 < -1 + 2 \leq x^*_R$. Thus, if $x_{R2} = x^*_R$, then $|x_{L1} - x_{R2}| \geq 4 > 1$. In that case, the maximand is equal to $-\alpha$. Clearly, any other $x_{R2}$ that satisfies $|x_{L1} - x_{R2}| > 1$ makes $R$ strictly worse off. Thus, either $x_{R2} = x^*_R$ or $|x_{L1} - x_{R2}| \leq 1$. The set of $x_{R2}$ that satisfies the latter condition is $\{x_{L1} - 1, x_{L1}, x_{L1} + 1\}$. Of that set, $x_{R2} = x_{L1} + 1$ maximizes $R$’s utility because it is closest to $x^*_R$, and it yields a utility of

$$- |x_{L1} + 1 - x^*_R| = - |x_{L1} + 1 + |x^*_R| = - |x_{L1} + 1 + |x^*_R| \
\geq - |x^*_L - z + 1 + |x^*_R| = - |x^*_L| - z + 1 + |x^*_R| = - |x^*_L| - z + 1 + |x^*_R| = - \Delta - z + 1.$$  

By assumption, $\Delta \leq \alpha - z + 1$, so $-\alpha \leq -\Delta - z + 1 \leq - |x_{L1} + 1 - x^*_R|$. First, suppose that $-\alpha < - |x_{L1} + 1 - x^*_R|$. Then, of the two possible choices left $R$, she uniquely maximizes her utility by choosing $x_{R2} = x_{L1} + 1$. Now, suppose that $-\alpha = - |x_{L1} + 1 - x^*_R|$. Note that this requires $-\alpha = -\Delta - z + 1$ and $x_{L1} = x^*_L - z$. Then, $R$ is indifferent between $x_{R2} = x_{L1} + 1 = x^*_L - z + 1 = x^*_L + \Delta - \alpha$ and $x_{R2} = x^*_R$. I assume that $R$ breaks the indifference by choosing the $x_{R2}$ that maximizes $U_R (x^*_L + \Delta - \alpha - 1)$.

First, suppose that $x_{R2} = x^*_L + \Delta - \alpha$. Then, the median voter’s utility from re-electing $L$ is

$$U^L_M (x^*_L + \Delta - \alpha - 1) = - |x^*_L + \Delta - \alpha - 1 - x^*_M| - |x^*_L + \Delta - \alpha - x^*_M| - 1 (|x^*_L + \Delta - \alpha - 1 - (x^*_L + \Delta - \alpha)| > 1) \cdot \alpha \
= - |x^*_L + \Delta - \alpha - 1 - x^*_L + \Delta - \alpha|.$$  

The median voter’s utility from electing $R$ is

$$U^R_M (x^*_L + \Delta - \alpha - 1) = - |x^*_L + \Delta - \alpha - 1 - x^*_M| - |x^*_L + \Delta - \alpha - x^*_M| - 1 (|x^*_L + \Delta - \alpha - 1 - (x^*_L + \Delta - \alpha)| > 1) \cdot \alpha 
\cdot \ = - |x^*_L + \Delta - \alpha - 1 - x^*_L + \Delta - \alpha|.$$  

Thus, the median voter re-elects $L$ with probability $\pi$ if and only if

$$U^L_M (x^*_L - 1) = U^R_M (x^*_L - 1) - |x^*_L + \Delta - \alpha - 1 - x^*_L + \Delta - \alpha| = - |x^*_L + \Delta - \alpha - 1 - x^*_L + \Delta - \alpha|,$$
which holds. Thus, \( p(x_L^* + \Delta - \alpha - 1) = \pi \). Then,

\[
\bar{U}_R(x_L^* + \Delta - \alpha - 1) = -|x_L^* + \Delta - \alpha - 1 - x_R^*| - \mathbb{E}_p(x_L^* + \Delta - \alpha - 1, 1) |x_{v,2}(x_L^* + \Delta - \alpha - 1) - x_R^*| \\
- \mathbb{E}_p(x_L^* + \Delta - \alpha - 1, 1) [1 (|x_L^* + \Delta - \alpha - 1 - x_{v,2}(x_L^* + \Delta - \alpha - 1)| > 1)] \cdot \alpha + (1 - p(x_L^* + \Delta - \alpha - 1)) \cdot \eta \\
= -|\alpha - 1 - |x_L^* + \Delta - \alpha - x_R^*| - 1 (|x_L^* + \Delta - \alpha - 1 - (x_L^* + \Delta - \alpha)| > 1) \cdot \alpha + (1 - \pi) \cdot \eta \\
= -2 \alpha - 1 + (1 - \pi) \cdot \eta,
\]

where the second equality comes from the fact that \( p(x_L^* + \Delta - \alpha - 1) = \pi \) and \( x_{v,2}(x_L^* + \Delta - \alpha - 1) = x_L^* \) for all \( w \in \{L, R\} \).

Now, suppose that \( x_{R,2} = x_R^* \). Then, the median voter’s utility from re-electing \( L \) is

\[
U^L_M(x_L^* + \Delta - \alpha - 1) = -|x_L^* + \Delta - \alpha - 1 - x_M^*| - |x_L^* + \Delta - \alpha - x_M^*| - 1 (|x_L^* + \Delta - \alpha - 1 - (x_L^* + \Delta - \alpha)| > 1) \cdot \alpha \\
= -|x_L^* + \Delta - \alpha - 1| - |x_L^* + \Delta - \alpha| \\
= -2 \alpha - 1 - |\alpha - 1| - (1 - \pi) \cdot \eta \\

The median voter’s utility from electing \( R \) is

\[
U^R_M(x_L^* + \Delta - \alpha - 1) = -|x_L^* + \Delta - \alpha - 1 - x_M^*| - |x_R^* - x_M^*| - 1 (|x_L^* + \Delta - \alpha - 1 - x_R^*| > 1) \cdot \alpha \\
= -|x_L^* + \Delta - \alpha - 1| - |x_R^* - \alpha| \\
= -|x_L^* + \Delta - \alpha - 1| - x_R^* - \alpha.
\]

Thus, the median voter re-elects \( L \) with probability one if and only if

\[
U^L_M(x_L^* + \Delta - \alpha - 1) > U^R_M(x_L^* + \Delta - \alpha - 1) \\
- |x_L^* + \Delta - \alpha - 1| + x_R^* - \alpha > - |x_L^* + \Delta - \alpha - 1| - x_R^* - \alpha \\
x_R^* > -x_R^*.
\]

which holds. Thus, \( p(x_L^* + \Delta - \alpha - 1) = 1 \). Then,

\[
\bar{U}_R(x_L^* + \Delta - \alpha - 1) = -|x_L^* + \Delta - \alpha - 1 - x_R^*| - \mathbb{E}_p(x_L^* + \Delta - \alpha - 1, 1) |x_{v,2}(x_L^* + \Delta - \alpha - 1) - x_R^*| \\
- \mathbb{E}_p(x_L^* + \Delta - \alpha - 1, 1) [1 (|x_L^* + \Delta - \alpha - 1 - x_{v,2}(x_L^* + \Delta - \alpha - 1)| > 1)] \cdot \alpha + (1 - p(x_L^* + \Delta - \alpha - 1)) \cdot \eta \\
= -|\alpha - 1 - |x_L^* + \Delta - \alpha - x_R^*| - 1 (|x_L^* + \Delta - \alpha - 1 - (x_L^* + \Delta - \alpha)| > 1) \cdot \alpha + (1 - 1) \cdot \eta \\
= -|\alpha - 1 - |x_L^* + \Delta - \alpha - x_R^*| - 1 (|x_L^* + \Delta - \alpha - 1 - (x_L^* + \Delta - \alpha)| > 1) \cdot \alpha \\
= -|\alpha - 1 - |\alpha - 1| - (1 - \pi) \cdot \eta,
\]

where the second equality comes from the fact that \( p(x_L^* + \Delta - \alpha - 1) = 1 \), so \( w = L \) with probability one. Thus, \( R \) chooses \( x_{R,2} = x_L^* + \Delta - \alpha = x_{L,1} + 1 \) because it maximizes \( \bar{U}_R(x_L^* + \Delta - \alpha - 1) \), so \( x_{R,2} = x_{L,1} + 1 \) in all cases. This implies that \( p(x_L^* + \Delta - \alpha - 1) = \pi \).

Thus, the median voter’s utility from re-electing \( L \) is

\[
U^L_M(x_{L,1}) = -|x_L - x_M^*| - |x_{L,1} + 1 - x_M^*| - 1 (|x_{L,1} - (x_{L,1} + 1)| > 1) \cdot \alpha \\
= -|x_L - x_{L,1}| - |x_{L,1} + 1| \\
= -|x_L - x_{L,1}| + 1 \\
= -2 |x_{L,1}| + 1.
\]
The median voter’s utility from electing $R$ is

$$U_M^R (x_{L1}) = -|x_{L1} - x_M^*| - |x_{L1} + 1 - x_M^*| - 1 (|x_{L1} - (x_{L1} + 1)| > 1) \cdot \alpha$$

$$= -|x_{L1} - |x_{L1} + 1|$$

$$= -|x_{L1} - |x_{L1} + 1$$

$$= -2|x_{L1} + 1.$$

Thus, the median voter re-elects $L$ with probability $\pi$ if and only if

$$U_M^L (x_{L1}) = U_M^R (x_{L1})$$

$$-2|x_{L1} + 1 = -2|x_{L1} + 1,$$

which holds. Thus, $p (x_{L1}) = \pi$.

Then,

$$U_L (x_{L1}) = -|x_{L1} - x_L^*| - \mathbb{E}_{p(x_{L1})} (|x_{w2} (x_{L1}) - x_L^*|) - \mathbb{E}_{p(x_{L1})} (1 (|x_{L1} - x_{w2} (x_{L1})| > 1) \cdot \alpha + (1 + p(x_{L1})) \cdot \eta$$

$$= -|x_{L1} - x_L^*| - |x_{L1} + 1 - x_L^*| - 1 (|x_{L1} - (x_{L1} + 1)| > 1) \cdot \alpha + (1 + \pi) \cdot \eta$$

$$= -|x_{L1} - x_L^*| - |x_{L1} + 1 - x_L^*| + (1 + \pi) \cdot \eta$$

$$\leq -|x_L^* - 2 - x_L^*| - |x_L^* - 2 - 1 + x_L^*| + (1 + \pi) \cdot \eta$$

$$= -|-2| - |-1| + (1 + \pi) \cdot \eta$$

$$= -2 - 1 + (1 + \pi) \cdot \eta$$

$$= -3 + (1 + \pi) \cdot \eta$$

$$< -1 + (1 + \pi) \cdot \eta$$

$$= \hat{U}_L (x_{L1} - 1),$$

where the second equality comes from the fact that $x_{w2} (x_{L1}) = x_{L1} + 1$ for all $w \in \{L, R\}$.

**Case 7:** Next, suppose that $x_{L1} \in \{x_L^*, \ldots, -1\}$. During period 2, $L$ would solve

$$V_{L2} = \max_{x_{L2} \in \mathbb{Z}} -|x_{L1} - x_L^*| - |x_{L2} - x_L^*| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2\eta$$

$$= \max_{x_{L2} \in \mathbb{Z}} -|x_{L2} - x_L^*| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha,$$

where the final equality comes because the first and last terms are constant with respect to $x_{L2}$. First, suppose that $x_{L1} \in \{x_L^*, x_L^* + 1\}$. The first term of the maximand is strictly negative if $x_{L2} \neq x_L^*$ and equal to zero if $x_{L2} = x_L^*$. The second term of the maximand is strictly negative if $|x_{L1} - x_{L2}| > 1$ and equal to zero if $|x_{L1} - x_{L2}| \leq 1$. Note that $x_{L2} = x_L^*$ satisfies the latter condition. Thus, the maximand is strictly negative if $x_{L2} \neq x_L^*$ and equal to zero if $x_{L2} = x_L^*$, so its unique maximizer is $x_{L2} = x_L^* \leq x_{L1}$.

Now, suppose that $x_{L1} \in \{x_L^* + 2, \ldots, -1\}$. Then, $|x_{L1} - x_L^*| \geq 2 > 1$. Thus, if $x_{L2} = x_L^*$, then $|x_{L1} - x_{L2}| > 1$. In that case, the maximand is equal to $-\alpha$. Clearly, any other $x_{L2}$ that satisfies $|x_{L1} - x_{L2}| > 1$ makes $L$ strictly worse off. Thus, either $x_{L2} = x_L^*$ or $|x_{L1} - x_{L2}| \leq 1$. The set of $x_{L2}$ that satisfies the latter condition is $\{x_{L1} - 1, x_{L1}, x_{L1} + 1\}$. Of that set, $x_{L2} = x_{L1} - 1$ maximizes $L$’s utility because it is closest to $x_L^*$. Thus, $x_{L2} \in \{x_{L1} - 1, x_L^*\}$. In all cases, $x_{L2} \leq x_{L1}$.

During period 2, $R$ would solve

$$V_{R2} = \max_{x_{R2} \in \mathbb{Z}} -|x_{L1} - x_R^*| - |x_{R2} - x_R^*| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha + \eta$$

$$= \max_{x_{R2} \in \mathbb{Z}} -|x_{R2} - x_R^*| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha,$$

where the final equality comes because the first and last terms are constant with respect to $x_{R2}$. Note that $x_{L1} \leq -1 < -1 + 2 \leq x_R^*$. Thus, if $x_{R2} = x_R^*$, then $|x_{L1} - x_{R2}| \geq 2 > 1$. In that case, the maximand is equal to $-\alpha$. Clearly, any other $x_{R2}$ that satisfies $|x_{L1} - x_{R2}| > 1$ makes $R$ strictly worse off. Thus, either $x_{R2} = x_R^*$ or $|x_{L1} - x_{R2}| \leq 1$. The set of $x_{R2}$ that satisfies the latter condition is $\{x_{L1} - 1, x_{L1}, x_{L1} + 1\}$. Of that set, $x_{R2} = x_{L1} + 1$
maximizes $R$'s utility because it is closest to $x^*_R$, and it yields a utility of

$$-|x_{L1} + 1 - x^*_R| = -|x_{L1} + 1 - x^*_R|$$

$$= -|x_{L1}| + 1 - x^*_R$$

$$\geq -|x^*_L| + 1 - x^*_R$$

$$= -\Delta + 1.$$

By assumption, $\Delta \leq \alpha - z + 1$, so $-\alpha \leq -\Delta - z + 1 < -\Delta + 1 \leq -|x_{L1} + 1 - x^*_R|$. Thus, of the two possible choices left $R$, she uniquely maximizes her utility by choosing $x_{R2} = x_{L1} + 1 \leq 0$.

Thus, the median voter’s utility from re-electing $L$ is

$$U^L_M (x_{L1}) = -|x_{L1} - x^*_M| - |x_{L2} - x^*_M| - 1(|x_{L1} - x_{L2}| > 1) \cdot \alpha$$

$$= -|x_{L1} - |x_{L2}| - 1(|x_{L1} - x_{L2}| > 1) \cdot \alpha$$

$$\leq -|x_{L1}| - |x_{L1}|$$

$$= -2|x_{L1}|.$$

The median voter’s utility from electing $R$ is

$$U^R_M (x_{L1}) = -|x_{L1} - x^*_M| - |x_{L1} + 1 - x^*_M| - 1(|x_{L1} - (x_{L1} + 1)| > 1) \cdot \alpha$$

$$= -|x_{L1} - |x_{L1} + 1|$$

$$\geq -2|x_{L1}|$$

$$\geq U^L_M (x_{L1}),$$

where the strict inequality comes from the fact that $x_{L1} + 1 \leq 0$, so $|x_{L1} + 1| < |x_{L1}|$. The median voter re-elects $L$ with probability zero if and only if

$$U^L_M (x_{L1}) < U^R_M (x_{L1}),$$

which holds. Thus, $p(x_{L1}) = 0$.

Then,

$$\hat{U}_L (x_{L1}) = -|x_{L1} - x^*_L| - \mathbb{E}_{p(x_{L1})} [w_2 (x_{L1}) - x^*_L| - \mathbb{E}_{p(x_{L1})} [1(|x_{L1} - w_2 (x_{L1})| > 1) \cdot \alpha + (1 + p(x_{L1})) \cdot \eta$$

$$= -|x_{L1} - x^*_L| - |x_{R2} (x_{L1}) - x^*_L| - 1(|x_{L1} - x_{R2} (x_{L1})| > 1) \cdot \alpha + (1 + 0) \cdot \eta$$

$$= -|x_{L1} - x^*_L| - |x_{L1} + 1 - x^*_L| - 1(|x_{L1} - (x_{L1} + 1)| > 1) \cdot \alpha - \eta$$

$$= -|x_{L1} - x^*_L| - |x_{L1} + 1 - x^*_L| + \eta$$

$$\leq -|x^*_L - x^*_L| - |x_L^* + 1 - x^*_L| + \eta$$

$$= -|1| + \eta$$

$$= -1 + \eta$$

$$< -1 + (1 + \pi) \cdot \eta$$

$$= \hat{U}_L (x_{L1} - 1),$$

where the second equality comes from the fact that $p(x_{L1}) = 0$, so $w = R$ with probability one.

**Case 8:** Next, suppose that $x_{L1} = 0$. During period 2, $L$ would solve

$$V_L = \max_{x_{L2} \in \mathbb{Z}} -|x_{L1} - x^*_L| - |x_{L2} - x^*_L| - 1(|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2\eta$$

$$= \max_{x_{L2} \in \mathbb{Z}} -|0 - x^*_L| - |x_{L2} - x^*_L| - 1(0 - x_{L2}| > 1) \cdot \alpha + 2\eta$$

$$= \max_{x_{L2} \in \mathbb{Z}} -|x^*_L| - |x_{L2} - x^*_L| - 1(|x_{L2}| > 1) \cdot \alpha + 2\eta$$

$$= \max_{x_{L2} \in \mathbb{Z}} -|x_{L2} - x^*_L| - 1(|x_{L2}| > 1) \cdot \alpha.$$
where the final equality comes because the first and last terms are constant with respect to \( x_{L2} \). First, suppose that 
\[ x^*_L = -1. \]
The first term of the maximand is strictly negative if \( x_{L2} \neq x^*_L \) and equal to zero if \( x_{L2} = x^*_L \). The second term of the maximand is strictly negative if \( |x_{L2}| > 1 \) and equal to zero if \( |x_{L2}| \leq 1 \). Note that \( x_{L2} = x^*_L = -1 \) satisfies the latter condition. Thus, the maximand is strictly negative if \( x_{L2} \neq x^*_L \) and equal to zero if \( x_{L2} = x^*_L \), so its unique maximizer is \( x_{L2} = x^*_L = -1 \).

Now, suppose that \( x^*_L \leq -2 \). Then, \( |x^*_L| \geq 2 > 1 \). Thus, if \( x_{L2} = x^*_L \), then \( |x_{L2}| > 1 \). In that case, the maximand is equal to \(-\alpha\). Clearly, any other \( x_{L2} \) that satisfies \( |x_{L2}| > 1 \) makes \( L \) strictly worse off. Thus, either \( x_{L2} = \pm x^*_L \) or \( |x_{L2}| \leq 1 \). The set of \( x_{L2} \) that satisfies the latter condition is \( \{-1, 0, 1\} \). Of that set, \( x_{L2} = -1 \) maximizes \( L \)'s utility because it is closest to \( x^*_L \), and yields a utility of \(-\Delta + 1 = -1 + |x^*_L| = -|x^*_L| + 1 \). By assumption, \( \Delta \leq \alpha - z + 1 \), so \( -\alpha \leq -\Delta - z + 1 < -\Delta + 1 < -|x^*_L| + 1 \). Thus, of the two possible choices left \( L \), she uniquely maximizes her utility by choosing \( x_{L2} = -1 \). Thus, \( x_{L2} = -1 \) in either case.

During period 2, \( R \) would solve
\[
V_{R2} = \max_{x_{R2} \in \mathbb{Z}} -|x_{L1} - x^*_R| - |x_{R2} - x^*_R| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha + \eta \\
= \max_{x_{R2} \in \mathbb{Z}} -|0 - x^*_R| - |x_{R2} - x^*_R| - 1 (|0 - x_{R2}| > 1) \cdot \alpha + \eta \\
= \max_{x_{R2} \in \mathbb{Z}} -x^*_R - |x_{R2} - x^*_R| - 1 (|x_{R2}| > 1) \cdot \alpha + \eta \\
= \max_{x_{R2} \in \mathbb{Z}} -|x_{R2} - x^*_R| - 1 (|x_{R2}| > 1) \cdot \alpha.
\]

where the final equality comes because the first and last terms are constant with respect to \( x_{R2} \). First, suppose that \( x^*_R = 1 \). The first term of the maximand is strictly negative if \( x_{R2} \neq x^*_R \) and equal to zero if \( x_{R2} = x^*_R \). The second term of the maximand is strictly negative if \( |x_{R2}| > 1 \) and equal to zero if \( |x_{R2}| \leq 1 \). Note that \( x_{R2} = x^*_R = 1 \) satisfies the latter condition. Thus, the maximand is strictly negative if \( x_{R2} \neq x^*_R \) and equal to zero if \( x_{R2} = x^*_R \), so its unique maximizer is \( x_{R2} = x^*_R = 1 \).

Now, suppose that \( x^*_R \geq 2 > 1 \). Thus, if \( x_{R2} = x^*_R \), then \( |x_{R2}| = |x^*_R| = x^*_R > 1 \). In that case, the maximand is equal to \(-\alpha\). Clearly, any other \( x_{R2} \) that satisfies \( |x_{R2}| > 1 \) makes \( R \) strictly worse off. Thus, either \( x_{R2} = x^*_R \) or \( |x_{R2}| \leq 1 \). The set of \( x_{R2} \) that satisfies the latter condition is \( \{-1, 0, 1\} \). Of that set, \( x_{R2} = 1 \) maximizes \( R \)'s utility because it is closest to \( x^*_R \), and yields a utility of \(-1 - x^*_R = -x^*_R + 1 \). By assumption, \( \Delta \leq \alpha - z + 1 \), so \(-\alpha \leq -\Delta - z + 1 < -\Delta + 1 < -x^*_R + 1 \). Thus, of the two possible choices left \( R \), she uniquely maximizes her utility by choosing \( x_{R2} = 1 \). Thus, \( x_{R2} = 1 \) in either case.

Thus, the median voter’s utility from re-electing \( L \) is
\[
U^L_M (0) = -|0 - x^*_M| - |1 - x^*_M| - 1 (|0 - (-1)| > 1) \cdot \alpha \\
= -|1| \\
= -1.
\]

The median voter’s utility from electing \( R \) is
\[
U^R_M (0) = -|0 - x^*_M| - |1 - x^*_M| - 1 (|0 - 1| > 1) \cdot \alpha \\
= -|1| \\
= -1.
\]

The median voter re-elects \( L \) with probability \( \pi \) if and only if
\[
U^L_M (0) = U^R_M (0),
\]
which holds. Thus, \( p(0) = \pi \).
Then,
\[ \hat{U}_L(0) = -|0 - x_L^*| - \mathbb{E}_{p(0)} [|x_{w2}(0) - x_L^*| - \mathbb{E}_{p(0)} [1 (|x_{w2}(0)| > 1)] \cdot \alpha + (1 + p(0)) \cdot \eta \]
\[ = -|x_L^*| - \pi |x_{L2}(0) - x_L^*| - (1 - \pi) |x_{R2}(0) - x_L^*| + (1 + \pi) \cdot \eta \]
\[ < -|x_L^*| - |x_{L2}(0) - x_L^*| + (1 + \pi) \cdot \eta \]
\[ = -|x_L^*| - |-1 - x_L^*| + (1 + \pi) \cdot \eta \]
\[ = -|x_L^*| - |x_L^*| + 1 + (1 + \pi) \cdot \eta \]
\[ \leq -2 + 1 + (1 + \pi) \cdot \eta \]
\[ = -1 + (1 + \pi) \cdot \eta \]
\[ = \hat{U}_L(x_L^* - 1), \]
where the second equality comes from the fact that $|x_{w2}(0)| \leq 1$ for $w \in \{L, R\}$.

Case 9: Next, suppose that $x_{L1} \in \{2, \ldots, x_R^* \}$. Note that this assumes $x_R^* \geq 2$. During period 2, $L$ would solve
\[ V_{L2} = \max_{x_{L2} \in \mathbb{Z}} -|x_{L1} - x_L^*| - |x_{L2} - x_L^*| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2\eta \]
\[ = \max_{x_{L2} \in \mathbb{Z}} -|x_{L2} - x_L^*| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha, \]
where the final equality comes because the first and last terms are constant with respect to $x_{L2}$. Note that $x_{L1} \geq 2 > 2 - 3 = -1 \geq x_L^*$, so $|x_{L1} - x_L^*| \geq 3 > 1$. Thus, if $x_{L2} = x_L^*$, then $|x_{L1} - x_{L2}| > 1$. In that case, the maximand is equal to $-\alpha$. Clearly, any other $x_{L2}$ that satisfies $|x_{L1} - x_{L2}| > 1$ makes $L$ strictly worse off. Thus, either $x_{L2} = x_L^*$ or $x_{L1} - x_{L2} \leq 1$. The set of $x_{L2}$ that satisfies the latter condition is $\{x_{L1} - 1, x_{L1}, x_{L1} + 1\}$. Of that set, $x_{L2} = x_{L1} - 1$ maximizes $L$’s utility because it is closest to $x_L^*$, and yields a utility of $-|x_{L1} - 1 - x_L^*| = -|x_{L1} - 1 - x_L^*| = -x_{L1} + 1 - |x_L^*| \geq -x_R^* + 1 - |x_L^*| = -\Delta + 1$. By assumption, $\Delta \leq \alpha - z + 1$, so $-\alpha \leq -\Delta - z + 1 < -\Delta + 1 \leq -|x_{L1} - 1 - x_L^*|$. Thus, of the two possible choices left $L$, she uniquely maximizes her utility by choosing $x_{L2} = x_{L1} - 1 \geq 2 - 1 > 0$.

During period 2, $R$ would solve
\[ V_{R2} = \max_{x_{R2} \in \mathbb{Z}} -|x_{L1} - x_R^*| - |x_{R2} - x_R^*| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha + \eta \]
\[ = \max_{x_{R2} \in \mathbb{Z}} -|x_{R2} - x_R^*| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha, \]
where the final equality comes because the first and last terms are constant with respect to $x_{R2}$. First, suppose that $x_{L1} \in \{x_R^*, x_R^* - 1\}$. The first term of the maximand is strictly negative if $x_{R2} \neq x_R^*$ and equal to zero if $x_{R2} = x_R^*$. The second term of the maximand is strictly negative if $|x_{L1} - x_{R2}| > 1$ and equal to zero if $|x_{L1} - x_{R2}| \leq 1$. Note that $x_{R2} = x_R^*$ satisfies the latter condition. Thus, the maximand is strictly negative if $x_{R2} \neq x_R^*$ and equal to zero if $x_{R2} = x_R^*$, so its unique maximizer is $x_{R2} = x_R^* \geq x_{L1}$.

Now, suppose that $x_{L1} \in \{x_R^* - 2, \ldots, 2\}$. Then, $|x_{L1} - x_R^*| \geq 2 > 1$. Thus, if $x_{R2} = x_R^*$, then $|x_{L1} - x_{R2}| > 1$. In that case, the maximand is equal to $-\alpha$. Clearly, any other $x_{R2}$ that satisfies $|x_{L1} - x_{R2}| > 1$ makes $R$ strictly worse off. Thus, either $x_{R2} = x_R^*$ or $|x_{L1} - x_{R2}| \leq 1$. The set of $x_{R2}$ that satisfies the latter condition is $\{x_{L1} - 1, x_{L1}, x_{L1} + 1\}$. Of that set, $x_{R2} = x_{L1} + 1$ maximizes $R$’s utility because it is closest to $x_R^*$. Thus, $x_{R2} \in \{x_{L1} + 1, x_R^* \}$. In all cases, $x_{R2} \geq x_{L1}$.

Thus, the median voter’s utility from re-electing $L$ is
\[ U_{M}^{L}(x_{L1}) = -|x_{L1} - x_M^*| - |x_{L1} - 1 - x_M^*| - 1 (|x_{L1} - (x_{L1} - 1)| > 1) \cdot \alpha \]
\[ = -|x_{L1} - x_{L1} - 1| \]
\[ = -x_{L1} - x_{L1} + 1 \]
\[ = -2x_{L1} + 1, \]
where the second equality comes from the fact that $x_{L1} - 1 > 0$. The median voter’s utility from electing $R$ is

$$U^R_M (x_{L1}) = - |x_{L1} - x_M^*| - |x_{R2} - x_M^*| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha$$

$$= - |x_{L1} - x_{R2}| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha$$

$$= - x_{L1} - x_{R2} - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha$$

$$\leq - x_{L1} - x_{L1}$$

$$= - 2x_{L1}$$

$$< - 2x_{L1} + 1$$

$$= U^L_M (x_{L1}).$$

The median voter re-elects $L$ with probability one if and only if

$$U^L_M (x_{L1}) > U^R_M (x_{L1}),$$

which holds. Thus, $p(x_{L1}) = 1$.

Then,

$$U_L (x_{L1}) = - |x_{L1} - x_L^*| - \mathbb{E}_p (x_{L1}) [ |x_{w2} (x_{L1}) - x_L^*|] - \mathbb{E}_p (x_{L1}) [1 (|x_{L1} - x_{w2} (x_{L1})| > 1)] \cdot \alpha + (1 + p(x_{L1})) \cdot \eta$$

$$= - |x_{L1} - x_L^*| - |x_{L2} (x_{L1}) - x_L^*| - 1 (|x_{L1} - x_{L2} (x_{L1})| > 1) \cdot \alpha + (1 + 1) \cdot \eta$$

$$= - |x_{L1} - x_L^*| - |x_{L1} - 1 - x_L^*| - 1 (|x_{L1} - (x_{L1} - 1)| > 1) \cdot \alpha + 2 \eta$$

$$= - |x_{L1} - x_L^*| - |x_{L1} - 1 - x_L^*| + 2 \eta$$

$$\leq - |x_R^* - x_L^*| - |x_R^* - 1 - x_L^*| + 2 \eta$$

$$= - \Delta - \Delta + 1 + 2 \eta$$

$$= - 2 \Delta + 1 + 2 \eta$$

$$= - 2 (x_R^* + |x_L^*|) + 1 + 2 \eta$$

$$= - 2 |x_L^*| - 2x_R^* + 1 + 2 \eta$$

$$\leq - 2 |x_L^*| - 4 + 1 + 2 \eta$$

$$= - 2 |x_L^*| - 1 + 2 \eta$$

$$= - \hat{U}_L (1),$$

where the second equality comes from the fact that $p(x_{L1}) = 1$, so $w = L$ with probability one.

**Case 10:** Next, suppose that $x_{L1} \in \{x_R^* + 1, \ldots, x_R^* + z\}$. During period 2, $L$ would solve

$$V_{L2} = \max_{x_{L2} \in \mathbb{Z}} - |x_{L1} - x_L^*| - |x_{L2} - x_L^*| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2 \eta$$

$$= \max_{x_{L2} \in \mathbb{Z}} - |x_{L2} - x_L^*| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha,$$

where the final equality comes because the first and last terms are constant with respect to $x_{L2}$. Note that $x_{L1} \geq x_R^* + 1 > x_R^* \geq 1 > 1 - 2 = -1 \geq x_L^*$. Thus, if $x_{L2} = x_L^*$, then $|x_{L1} - x_{L2}| \geq 3 > 1$. In that case, the maximand is equal to $- \alpha$. Clearly, any other $x_{L2}$ that satisfies $|x_{L1} - x_{L2}| > 1$ makes $L$ strictly worse off. Thus, either $x_{L2} = x_L^*$ or $|x_{L1} - x_{L2}| \leq 1$. The set of $x_{L2}$ that satisfies the latter condition is $\{x_{L1} - 1, x_{L1}, x_{L1} + 1\}$. Of that set, $x_{L2} = x_{L1} - 1$ maximizes $L$’s utility because it is closest to $x_L^*$, and it yields a utility of

$$- |x_{L1} - 1 - x_L^*| = - |x_{L1} - 1 + |x_L^*| |$$

$$= - x_{L1} + 1 - |x_L^*|$$

$$\geq - x_R^* - z + 1 - |x_L^*|$$

$$= - \Delta - z + 1.$$

By assumption, $\Delta \leq \alpha - z + 1$, so $- \alpha \leq - \Delta - z + 1 \leq - |x_{L1} - 1 - x_L^*|$. First, suppose that $- \alpha < - |x_{L1} - 1 - x_L^*|$. Then, of the two possible choices left $L$, she uniquely maximizes her utility by choosing $x_{R2} = x_{L1} - 1$. Now, suppose
that $-\alpha = -|x_{L1} - 1 - x_{L1}^*|$. Note that this requires $-\alpha = -\Delta - z + 1$ and $x_{L1} = x_{R1}^* + z$. Then, $L$ is indifferent between $x_{R2} = x_{L1} = x_{R1}^* + z - 1 = x_{R1}^* + \alpha - \Delta$ and $x_{R2} = x_{L1}^*$. I assume that $L$ breaks the indifference by choosing the $x_{L2}$ that maximizes $U_L(x_{R1}^* + \alpha - \Delta + 1)$. I evaluate the two options after determining $x_{R2}$.

During period 2, $R$ would solve

$$V_{R2} = \max_{x_{R2} \in \mathbb{Z}} -|x_{L1} - x_{R1}^*| - |x_{R2} - x_{R1}^*| - 1(|x_{L1} - x_{R2}| > 1) \cdot \alpha + \eta$$

$$= \max_{x_{R2} \in \mathbb{Z}} -|x_{R2} - x_{R1}^*| - 1(|x_{L1} - x_{R2}| > 1) \cdot \alpha,$$

where the final equality comes because the first and last terms are constant with respect to $x_{R2}$. First, suppose that $x_{L1} = x_{R1}^* + 1$. The first term of the maximand is strictly negative if $x_{R2} \neq x_{R1}^*$ and equal to zero if $x_{R2} = x_{R1}^*$. The second term of the maximand is strictly negative if $|x_{L1} - x_{R2}| = |x_{R1}^* + 1 - x_{R2}| > 1$ and equal to zero if $|x_{R1}^* + 1 - x_{R2}| \leq 1$. Note that $x_{R2} = x_{R1}^*$ satisfies the latter condition. Thus, the maximand is strictly negative if $x_{R2} \neq x_{R1}^*$ and equal to zero if $x_{R2} = x_{R1}^*$, so its unique maximizer is $x_{R2} = x_{R1}^* = x_{L1} - 1$.

Now, suppose that $x_{L1} \geq x_{R1}^* + 2$. Thus, if $x_{R2} = x_{R1}^*$, then $|x_{L1} - x_{R2}| \geq 2 > 1$. In that case, the maximand is equal to $-\alpha$. Clearly, any other $x_{R2}$ that satisfies $|x_{L1} - x_{R2}| > 1$ makes $R$ strictly worse off. Thus, either $x_{R2} = x_{R1}^*$ or $|x_{L1} - x_{R2}| \leq 1$. The set of $x_{R2}$ that satisfies the latter condition is $\{x_{L1} - 1, x_{L1}, x_{L1} + 1\}$. Of that set, $x_{R2} = x_{L1} - 1$ maximizes $R$’s utility because it is closest to $x_{R1}^*$, and yields a utility of $-|x_{L1} - 1 - x_{R1}^*| = -x_{L1} + 1 + x_{R1}^* \geq -x_{R1}^* - z + 1 + x_{R1}^* = -z + 1$. By assumption, $\Delta \leq \alpha - z + 1$, so $-\alpha \leq -\Delta - z + 1 \leq -x_{L1} + 1 - x_{R1}^*$. Thus, of the two possible choices left $R$, she uniquely maximizes her utility by choosing $x_{R2} = x_{L1} - 1$. Thus, $x_{R2} = x_{L1} - 1$ in both cases.

Now, I determine $x_{L2}$ when $-\alpha = -|x_{L1} - 1 - x_{L1}^*|$. First, suppose that $x_{L2} = x_{R1}^* + \alpha - \Delta$. Then, the median voter’s utility from re-electing $L$ is

$$U_M^L (x_{R1}^* + \alpha - \Delta + 1) = -|x_{R1}^* + \alpha - \Delta + 1 - x_{M1}^*| - |x_{R1}^* + \alpha - \Delta - x_{M1}^*| - 1(|x_{R1}^* + \alpha - \Delta + 1 - (x_{R1}^* + \alpha - \Delta)| > 1) \cdot \alpha$$

$$= -|x_{R1}^* + \alpha - \Delta + 1| - |x_{R1}^* + \alpha - \Delta|.$$

The median voter’s utility from electing $R$ is

$$U_M^R (x_{R1}^* + \alpha - \Delta + 1) = -|x_{R1}^* + \alpha - \Delta + 1 - x_{M1}^*| - |x_{R1}^* + \alpha - \Delta - x_{M1}^*| - 1(|x_{R1}^* + \alpha - \Delta + 1 - (x_{R1}^* + \alpha - \Delta)| > 1) \cdot \alpha$$

$$= -|x_{R1}^* + \alpha - \Delta + 1| - |x_{R1}^* + \alpha - \Delta|.$$

The median voter re-elects $L$ with probability $\pi$ if and only if

$$U_M^L (x_{R1}^* + \alpha - \Delta + 1) = U_M^R (x_{R1}^* + \alpha - \Delta + 1)$$

$$= -|x_{R1}^* + \alpha - \Delta + 1| - |x_{R1}^* + \alpha - \Delta|,$$

which holds. Thus, $p(x_{R1}^* + \alpha - \Delta + 1) = \pi$. Then,

$$\hat{U}_L (x_{R1}^* + \alpha - \Delta + 1) = -|x_{R1}^* + \alpha - \Delta + 1 - x_{L}^*| - \mathbb{E}_p(x_{R1}^* + \alpha - \Delta + 1) \cdot \mathbb{E}_p(x_{w2}^* (x_{R1}^* + \alpha - \Delta + 1) - x_{L}^*)$$

$$= -\mathbb{E}_p(x_{R1}^* + \alpha - \Delta + 1) \cdot \mathbb{E}_p(x_{w2}^* (x_{R1}^* + \alpha - \Delta + 1) - x_{L}^*) + (1 + p(x_{R1}^* + \alpha - \Delta + 1) \cdot \eta$$

$$= -|x_{R1}^* + \alpha - \Delta - x_{L}^*| - 1(|x_{R1}^* + \alpha - \Delta + 1 - (x_{R1}^* + \alpha - \Delta)| > 1) \cdot \alpha + (1 + \pi) \cdot \eta$$

$$= -\alpha - 1 - \alpha \cdot (1 + \pi) \cdot \eta$$

$$= -2 \alpha - 1 + (1 + \pi) \cdot \eta,$$

where the second equality comes from the facts that $p(x_{R1}^* + \alpha - \Delta + 1) = \pi$ and $x_{w2}^* (x_{R1}^* + \alpha - \Delta + 1) = x_{R1}^* + \alpha - \Delta$ for all $w \in \{L, R\}$.

Now, suppose that $x_{L2} = x_{L1}^*$. Then, the median voter’s utility from re-electing $L$ is

$$U_M^L (x_{R1}^* + \alpha - \Delta + 1) = -|x_{R1}^* + \alpha - \Delta + 1 - x_{M1}^*| - |x_{L1}^* - x_{M1}^*| - 1(|x_{R1}^* + \alpha - \Delta + 1 - x_{L1}^*| > 1) \cdot \alpha$$

$$= -|x_{R1}^* + \alpha - \Delta + 1| - |x_{L1}^*| - \alpha.$$
The median voter’s utility from electing $R$ is
\[
U_M^R (x_R^* + \alpha - \Delta + 1) = \left( - |x_R^* + \alpha - \Delta + 1 - x_M^*| - |x_R^* + \alpha - \Delta - x_M^*| - 1 (|x_R^* + \alpha - \Delta + 1 - (x_R^* + \alpha - \Delta)| > 1) \right) \cdot \alpha \\
= - |x_R^* + \alpha - \Delta + 1| - |x_R^* + \alpha - \Delta| \\
= - |x_R^* + \alpha - \Delta + 1| - x_R^* - \alpha + \Delta \\
= - |x_R^* + \alpha - \Delta + 1| + x_R^* - \alpha.
\]

The median voter re-elects $L$ with probability zero if and only if
\[
U_M^L (x_L^* + \alpha - \Delta + 1) < U_M^R (x_R^* + \alpha - \Delta + 1) \\
- |x_R^* + \alpha - \Delta + 1| - x_L^* - \alpha = - |x_R^* + \alpha - \Delta + 1| + x_L^* - \alpha,
\]
which holds. Thus, $p(x_R^* + \alpha - \Delta + 1) = 0$. Then,
\[
\bar{U}_L (x_R^* + \alpha - \Delta + 1) = - |x_R^* + \alpha - \Delta + 1 - x_L^*| - \mathbb{E}_{p(x_R^* + \alpha - \Delta + 1)} \left[ |x_{w2} (x_R^* + \alpha - \Delta + 1) - x_L^*| \right] \\
= - |x_R^* + \alpha - \Delta + 1 - x_{w2} (x_R^* + \alpha - \Delta + 1)| > 1 \right) \cdot \alpha + (1 + 0) \cdot \eta \\
= - |\alpha + 1| - x_{w2} (x_R^* + \alpha - \Delta + 1) - x_L^*| \\
= - |x_R^* + \alpha - \Delta + 1 - x_{w2} (x_R^* + \alpha - \Delta + 1)| > 1 \right) \cdot \alpha + \eta \\
= - |x_R^* + \alpha - \Delta - x_L^*| - 1 (|x_R^* + \alpha - \Delta + 1 - (x_R^* + \alpha - \Delta)| > 1) \cdot \alpha + \eta \\
= - |\alpha + 1| - \alpha + \eta \\
= - 2\alpha - 1 + \eta \\
< - 2\alpha - 1 + (1 + \pi) \cdot \eta,
\]
where the second equality comes from the fact that $p(x_R^* + \alpha - \Delta + 1) = 0$, so $w = R$ with probability one. Thus, $L$ chooses $x_{L2} = x_R^* + \alpha - \Delta = x_{L1} - 1$ because it maximizes $\bar{U}_L (x_R^* + \alpha - \Delta + 1)$. Thus, $x_{L2} = x_{L1} - 1$ in all cases. This implies that $p(x_R^* + \alpha - \Delta + 1) = \pi$.

Thus, the median voter’s utility from re-electing $L$ is
\[
U_M^L (x_{L1}) = - |x_{L1} - x_M^*| - |x_{L1} - 1 - x_M^*| - 1 (|x_{L1} - (x_{L1} - 1)| > 1) \cdot \alpha \\
= - |x_{L1} - x_{L1} - 1| \\
= -x_{L1} - x_{L1} + 1 \\
= -2x_{L1} + 1.
\]

The median voter’s utility from electing $R$ is
\[
U_M^R (x_{L1}) = - |x_{L1} - x_M^*| - |x_{L1} - 1 - x_M^*| - 1 (|x_{L1} - (x_{L1} - 1)| > 1) \cdot \alpha \\
= - |x_{L1} - x_{L1} + 1| \\
= -x_{L1} - x_{L1} + 1 \\
= -2x_{L1} + 1.
\]

Thus, the median voter re-elects $L$ with probability $\pi$ if and only if
\[
U_M^L (x_{L1}) = U_M^R (x_{L1}) \\
-2x_{L1} + 1 = -2x_{L1} + 1,
\]
which holds. Thus, $p(x_{L1}) = \pi$. 

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By assumption, \( \Delta \) either is equal to 
\[
- |x_{L1} - x_L^*| = E_{p(x_{L1})} [ |x_{w2} (x_{L1}) - x_L^*| - E_{p(x_{L1})} [1 (|x_{L1} - x_{w2} (x_{L1})| > 1)] \cdot \alpha + (1 + p (x_{L1})) \cdot \eta
\]
\[
= - |x_{L1} - x_L^*| - |x_{L1} - 1 - x_L^*| - 1 (|x_{L1} - (x_{L1} - 1)| > 1) \cdot \alpha + (1 + \pi) \cdot \eta
\]
\[
= - |x_{L1} - x_L^*| - |x_{L1} - 1 - x_L^*| + (1 + \pi) \cdot \eta
\]
\[
\leq - |x_{L1}^* + 1 - x_L^*| - |x_{L1}^* + 1 - 1 - x_L^*| + (1 + \pi) \cdot \eta
\]
\[
= - \Delta - 1 - \Delta + (1 + \pi) \cdot \eta
\]
\[
= - 2 \Delta - (1 + (1 + \pi)) \cdot \eta
\]
\[
< -1 + (1 + \pi) \cdot \eta
\]
\[
= \hat{U}_L (x_L^* - 1),
\]
where the second equality comes from the fact that \( x_{w2} (x_{L1}) = x_{L1} - 1 \) for all \( w \in \{L, R\} \).

Case II: Next, suppose that \( x_{L1} \in \{x_R^* + z + 1, \ldots, x_R^* + z + \Delta\} \). During period 2, \( L \) would solve
\[
V_{L2} = \max_{x_{L2} \in \mathbb{Z}} - |x_{L1} - x_L^*| - |x_{L2} - x_L^*| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2 \eta
\]
\[
= \max_{x_{L2} \in \mathbb{Z}} - |x_{L2} - x_L^*| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha,
\]
where the final equality comes because the first and last terms are constant with respect to \( x_{L2} \). Note that \( x_{L1} \geq x_R^* + z + 1 \geq x_R^* + 2 > x_R^* \geq 1 \geq 2 - 1 = -1 \geq x_L^* \). Thus, if \( x_{L2} = x_L^* \), then \( |x_{L1} - x_{L2}| \geq 4 > 1 \). In that case, the maximand is equal to \(-\alpha\). Clearly, any other \( x_{L2} \) that satisfies \(|x_{L1} - x_{L2}| > 1\) makes \( L \) strictly worse off. Thus, either \( x_{L2} = x_L^* \) or \( |x_{L1} - x_{L2}| \leq 1 \). The set of \( x_{L2} \) that satisfies the latter condition is \( \{x_{L1} - 1, x_{L1}, x_{L1} + 1\} \). Of that set, \( x_{L2} = x_{L1} - 1 \) maximizes \( L \)'s utility because it is closest to \( x_L^* \), and it yields a utility of
\[
- |x_{L1} - 1 - x_L^*| = -x_{L1} + 1 + x_L^*
\]
\[
\leq -x_R^* - z + 1 + x_L^*
\]
\[
= - \Delta - z.
\]
By assumption, \( \Delta > \alpha - z \), so \(-\alpha > -\Delta - z \geq -|x_{L1} - 1 - x_L^*| \). Thus, of the two possible choices left \( L \), she uniquely maximizes her utility by choosing \( x_{L2} = x_L^* \).

During period 2, \( R \) would solve
\[
V_{R2} = \max_{x_{R2} \in \mathbb{Z}} - |x_{L1} - x_R^*| - |x_{R2} - x_R^*| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha + \eta
\]
\[
= \max_{x_{R2} \in \mathbb{Z}} - |x_{R2} - x_R^*| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha,
\]
where the final equality comes because the first and last terms are constant with respect to \( x_{R2} \). Note that \( x_{L1} \geq x_R^* + z + 1 \geq x_R^* + 2 > x_R^* \) since \( z \geq 1 \). Thus, if \( x_{R2} = x_R^* \), then \( |x_{L1} - x_{R2}| \geq 2 > 1 \). In that case, the maximand is equal to \(-\alpha\). Clearly, any other \( x_{R2} \) that satisfies \(|x_{L1} - x_{R2}| > 1\) makes \( R \) strictly worse off. Thus, either \( x_{R2} = x_R^* \) or \( |x_{L1} - x_{R2}| \leq 1 \). The set of \( x_{R2} \) that satisfies the latter condition is \( \{x_{L1} - 1, x_{L1}, x_{L1} + 1\} \). Of that set, \( x_{R2} = x_{L1} - 1 \) maximizes \( R \)'s utility because it is closest to \( x_R^* \), and it yields a utility of
\[
- |x_{L1} - 1 - x_R^*| = -x_{L1} + 1 + x_R^*
\]
\[
\geq -x_R^* - z - \Delta + 1 + x_R^*
\]
\[
= - \Delta - z + 1.
\]
By assumption, \( \Delta \leq \alpha - z + 1 \), so \(-\alpha \leq -\Delta - z + 1 \leq -|x_{L1} - 1 - x_R^*| \). First, suppose that \(-\alpha < -|x_{L1} - 1 - x_R^*| \). Then, of the two possible choices left \( R \), she uniquely maximizes her utility by choosing \( x_{R2} = x_{L1} - 1 \). Now, suppose that \(-\alpha = -|x_{L1} - 1 - x_R^*| \). Note that this requires \(-\alpha = -\Delta - z + 1 \) and \( x_{L1} = x_R^* + z + \Delta \). Then, \( R \) is indifferent between \( x_{R2} = x_R^* + z + \Delta - 1 = x_R^* + \alpha \) and \( x_{R2} = x_R^* \). I assume that \( R \) breaks the indifference by choosing the \( x_{R2} \) that maximizes \( \hat{U}_R (x_R^* + \alpha)\).
First, suppose that \( x_{R2} = x_R^* + \alpha \). Then, the median voter’s utility from re-electing \( L \) is
\[
U^L_M (x_R^* + \alpha + 1) = -|x_R^* + \alpha + 1 - x_M^*| - |x_L^* - x_M^*| - 1 (|x_R^* + \alpha + 1 - x_R^*| > 1) \cdot \alpha \\
= -|x_R^* + \alpha + 1| - |x_L^*| - \alpha.
\]

The median voter’s utility from electing \( R \) is
\[
U^R_M (x_R^* + \alpha + 1) = -|x_R^* + \alpha + 1 - x_M^*| - |x_R^* + \alpha - x_M^*| - 1 (|x_R^* + \alpha + 1 - (x_R^* + \alpha)| > 1) \cdot \alpha \\
= -|x_R^* + \alpha + 1| - |x_R^* + \alpha|.
\]

Note that
\[
U^L_M (x_R^* + \alpha + 1) \geq U^R_M (x_R^* + \alpha + 1) \\
- |x_R^* + \alpha + 1| - |x_L^*| - \alpha \geq - |x_R^* + \alpha + 1| - x_R^* - \alpha \\
- |x_L^*| \geq -x_R^* \\
|x_L^*| \leq x_R^*.
\]

If \( |x_L^*| < x_R^* \), then \( p(x_R^* + \alpha + 1) = 1 \). If \( |x_L^*| = x_R^* \), then \( p(x_R^* + \alpha + 1) = \pi \). If \( |x_L^*| > x_R^* \), then \( p(x_R^* + \alpha + 1) = 0 \).

Then,
\[
\hat{U}_R (x_R^* + \alpha + 1) = -|x_R^* + \alpha + 1 - x_R^*| - E_p(x_R^* + \alpha + 1) [ |x_{w2} (x_R^* + \alpha + 1) - x_R^*| ] \\
- E_p (x_R^* + \alpha + 1) [ 1 (|x_R^* + \alpha + 1 - x_{w2} (x_R^* + \alpha + 1)| > 1) ] \cdot \alpha + (1 - p(x_R^* + \alpha + 1)) \cdot \eta
\]
and
\[
\hat{U}_L (x_R^* + \alpha + 1) = -|x_R^* + \alpha + 1 - x_L^*| - E_p (x_R^* + \alpha + 1) [ |x_{w2} (x_R^* + \alpha + 1) - x_L^*| ] \\
- E_p (x_R^* + \alpha + 1) [ 1 (|x_R^* + \alpha + 1 - x_{w2} (x_R^* + \alpha + 1)| > 1) ] \cdot \alpha + (1 + p(x_R^* + \alpha + 1)) \cdot \eta.
\]

Now, suppose that \( x_{R2} = x_R^* \). Then, the median voter’s utility from re-electing \( L \) is
\[
U^L_M (x_R^* + \alpha + 1) = -|x_R^* + \alpha + 1 - x_M^*| - |x_L^* - x_M^*| - 1 (|x_R^* + \alpha + 1 - x_L^*| > 1) \cdot \alpha \\
= -|x_R^* + \alpha + 1| - |x_L^*| - \alpha.
\]

The median voter’s utility from electing \( R \) is
\[
U^R_M (x_R^* + \alpha + 1) = -|x_R^* + \alpha + 1 - x_M^*| - |x_R^* - x_M^*| - 1 (|x_R^* + \alpha + 1 - x_R^*| > 1) \cdot \alpha \\
= -|x_L^* + \Delta - \alpha| - |x_R^*| - \alpha \\
= -|x_L^* + \Delta - \alpha| - x_R^* - \alpha.
\]

Note that
\[
U^L_M (x_R^* + \alpha + 1) \geq U^R_M (x_R^* + \alpha + 1) \\
- |x_R^* + \alpha + 1| - |x_L^*| - \alpha \geq - |x_R^* + \alpha + 1| - x_R^* - \alpha \\
- |x_L^*| \geq -x_R^* \\
|x_L^*| \leq x_R^*.
\]

If \( |x_L^*| < x_R^* \), then \( p(x_R^* + \alpha + 1) = 1 \). If \( |x_L^*| = x_R^* \), then \( p(x_R^* + \alpha + 1) = \pi \). If \( |x_L^*| > x_R^* \), then \( p(x_R^* + \alpha + 1) = 0 \).
Then,

\[ \hat{U}_R(x_R^* + \alpha + 1) = -|x_R^* + \alpha + 1 - x_R^*| - \mathbb{E}_{p(x_R^* + \alpha + 1)} \left( |x_{w2} (x_R^* + \alpha + 1) - x_R^*| \right) \]

\[ \quad - \mathbb{E}_{p(x_R^* + \alpha + 1)} \left[ 1 \left( |x_R^* + \alpha + 1 - x_{w2} (x_R^* + \alpha + 1)| > 1 \right) \right] \cdot \alpha \]

\[ \quad + (1 - p(x_R^* + \alpha + 1)) \cdot \eta \]

and

\[ \hat{U}_L(x_L^* + \alpha + 1) = -|x_L^* + \alpha + 1 - x_L^*| - \mathbb{E}_{p(x_L^* + \alpha + 1)} \left( |x_{w2} (x_L^* + \alpha + 1) - x_L^*| \right) \]

\[ \quad - \mathbb{E}_{p(x_L^* + \alpha + 1)} \left[ 1 \left( |x_L^* + \alpha + 1 - x_{w2} (x_L^* + \alpha + 1)| > 1 \right) \right] \cdot \alpha \]

\[ \quad + (1 + p(x_L^* + \alpha + 1)) \cdot \eta. \]

Note that both expected utilities are same as if \( x_{R2} = x_R^* + \alpha \). The first terms are clearly the same as if \( x_{R2} = x_R^* + \alpha \). The last terms are the same as if \( x_{R2} = x_R^* + \alpha \) because \( p(x_R^* + \alpha + 1) \) is the same as if \( x_{R2} = x_R^* + \alpha \). The sums of the middle two terms are the same as if \( x_{R2} = x_R^* + \alpha \) by the following logic. If \( p(x_R^* + \alpha + 1) = 1 \), then only \( x_{L2} \) is realized in equilibrium and it does not depend on \( x_{R2} \). If \( p(x_R^* + \alpha + 1) = 0 \), then only \( x_{R2} \) is realized in equilibrium, so the sum of the middle two terms is the same for \( R \) as if \( x_{R2} = x_R^* + \alpha \) because \( R \) is indifferent between \( x_{R2} = x_R^* + \alpha \) and \( x_{R2} = x_R^* \). In this case, the sum of the middle two terms for \( L \) if \( x_{R2} = x_R^* + \alpha \) is \(-|x_R^* + \alpha - x_L^*| = -\Delta - \alpha \), and the sum of the middle two terms for \( L \) if \( x_{R2} = x_R^* \) is \(-|x_R^* - x_L^*| - \alpha = -\Delta - \alpha \), so \( L \) is also indifferent between \( x_{R2} = x_R^* + \alpha \) and \( x_{R2} = x_R^* \). If \( p(x_L - \alpha - 1) = 1 \), then the sums of the middle two terms are the convex combinations of two sums that are the same as if \( x_{R2} = x_R^* + \alpha \), so the sums of the middle two terms are the same as if \( x_{R2} = x_R^* + \alpha \).

Thus, \( \hat{U}_L(x_L^* + \alpha + 1) \) and \( \hat{U}_R(x_R^* + \alpha + 1) \) do not depend on whether \( x_{R2} = x_R^* + \alpha \) or \( x_{R2} = x_R^* \). Thus, for the sake of simplicity in calculation, I can without loss of generality assume that \( x_{R2} = x_R^* + \alpha = x_{L1} - 1 \).

Thus, \( x_{L2} = x_{L1} - 1 \) in all cases.

Thus, the median voter’s utility from re-electing \( L \) is

\[ U_M^L(x_{L1}) = -|x_{L1} - x_M^*| - |x_L^* - x_M^*| - 1 \left( |x_{L1} - x_L^*| > 1 \right) \cdot \alpha \]

\[ \quad = -|x_{L1} - x_L^*| - \alpha \]

\[ \quad = -x_{L1} - |x_L^*| - \alpha. \]

The median voter’s utility from electing \( R \) is

\[ U_M^R(x_{L1}) = -|x_{L1} - x_M^*| - |x_{L1} - 1 - x_M^*| - 1 \left( |x_{L1} - (x_{L1} - 1)| > 1 \right) \cdot \alpha \]

\[ \quad = -|x_{L1} - x_{L1} - 1| \]

\[ \quad = -x_{L1} - 1. \]

Note that

\[ U_M^L(x_{L1}) \geq U_M^R(x_{L1}) \]

\[ \iff \]

\[ -x_{L1} - |x_L^*| - \alpha \geq -x_{L1} - x_{L1} + 1 \]

\[ -|x_L^*| - \alpha \geq -x_{L1} + 1 \]

\[ x_{L1} \geq |x_L^*| + \alpha + 1. \]

Thus, depending on the value of \( |x_L^*| \), the median voter re-elects \( L \) with probability \( p(x_{L1}) \leq 1 \).
Then,
\[
\hat{U}_L(x_{L1}) = -|x_{L1} - x^*_L| - \mathbb{E}_{p(x_{L1})} [\|x_{w2}(x_{L1}) - x^*_L\|] - \mathbb{E}_{p(x_{L1})} [1 (|x_{L1} - x_{w2}(x_{L1})| > 1)] \cdot \alpha + (1 + p(x_{L1})) \cdot \eta
\]
\[
\leq -|x_{L1} - x^*_L| - |x_{L2}(x_{L1}) - x^*_L| - 1 (|x_{L1} - x_{L2}(x_{L1})| > 1) \cdot \alpha + (1 + 1) \cdot \eta
\]
\[
= -|x_{L1} - x^*_L| - |x^*_L - x^*_L| - 1 (|x_{L1} - x^*_L| > 1) \cdot \alpha + 2\eta
\]
\[
= -|x^*_L - x^*_L| - \alpha + 2\eta
\]
\[
\leq -|x^*_R + z + 1 - x^*_L| - \alpha + 2\eta
\]
\[
= -\Delta - z - 1 - \alpha + 2\eta
\]
\[
\leq -\Delta - z - 1 - \Delta - z + 1 + 2\eta
\]
\[
= -2\Delta - 2z + 2\eta
\]
\[
< -2\Delta - 1 + 2\eta
\]
\[
< -2|x^*_L| - 1 + 2\eta
\]
\[
= \hat{U}_L(1).
\]

**Case 2:** Next, suppose that \(x_{L1} \in \{x^*_R + z + \Delta + 1, x^*_R + z + \Delta + 2, \ldots\}\). During period 2, \(L\) would solve
\[
V_{L2} = \max_{x_{L2} \in \mathbb{Z}} -|x_{L1} - x^*_L| - |x_{L2} - x^*_L| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha + 2\eta
\]
\[
= \max_{x_{L2} \in \mathbb{Z}} -|x_{L2} - x^*_L| - 1 (|x_{L1} - x_{L2}| > 1) \cdot \alpha,
\]
where the final equality comes because the first and last terms are constant with respect to \(x_{L2}\). Note that \(x_{L1} \geq x^*_R + z + \Delta + 1 \geq x^*_R + 1 \geq 1 - 2 = -1 \geq x^*_L\). Thus, if \(x_{L2} = x^*_L\), then \(|x_{L1} - x_{L2}| \geq 6 > 1\). In that case, the maximand is equal to \(-\alpha\). Clearly, any other \(x_{L2}\) that satisfies \(|x_{L1} - x_{L2}| > 1\) makes \(L\) strictly worse off. Thus, either \(x_{L2} = x^*_L\) or \(|x_{L1} - x_{L2}| \leq 1\). The set of \(x_{L2}\) that satisfies the latter condition is \(|x_{L1} - 1, x_{L1}, x_{L1} + 1, \ldots\). Of that set, \(x_{L2} = x_{L1} - 1\) maximizes \(L\)'s utility because it is closest to \(x^*_L\), and it yields a utility of
\[
-|x_{L1} - 1 - x^*_L| = -x_{L1} + 1 + x^*_L
\]
\[
\leq -x^*_R - z - \Delta - 1 + 1 + x^*_L
\]
\[
= -\Delta - z - \Delta
\]
\[
< -\Delta - z.
\]
By assumption, \(\Delta > \alpha - z\), so \(-\alpha > -\Delta - z > -|x_{L1} - 1 - x^*_L|\). Thus, of the two possible choices left \(L\), she uniquely maximizes her utility by choosing \(x_{L2} = x^*_L\).

During period 2, \(R\) would solve
\[
V_{R2} = \max_{x_{R2} \in \mathbb{Z}} -|x_{L1} - x^*_R| - |x_{R2} - x^*_R| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha + \eta
\]
\[
= \max_{x_{R2} \in \mathbb{Z}} -|x_{R2} - x^*_R| - 1 (|x_{L1} - x_{R2}| > 1) \cdot \alpha,
\]
where the final equality comes because the first and last terms are constant with respect to \(x_{R2}\). Note that \(x_{L1} \geq x^*_R + z + \Delta + 1 \geq x^*_R + 4 > x^*_R \geq 1 > 1 - 2 = -1 \geq x^*_L\). Thus, if \(x_{R2} = x^*_L\), then \(|x_{L1} - x_{R2}| \geq 4 > 1\). In that case, the maximand is equal to \(-\alpha\). Clearly, any other \(x_{R2}\) that satisfies \(|x_{L1} - x_{R2}| > 1\) makes \(R\) strictly worse off. Thus, either \(x_{R2} = x^*_R\) or \(|x_{L1} - x_{R2}| \leq 1\). The set of \(x_{R2}\) that satisfies the latter condition is \(|x_{L1} - 1, x_{L1}, x_{L1} + 1, \ldots\). Of that set, \(x_{R2} = x_{L1} - 1\) maximizes \(R\)'s utility because it is closest to \(x^*_R\), and it yields a utility of
\[
-|x_{L1} - 1 - x^*_R| = -x_{L1} + 1 + x^*_R
\]
\[
\leq -x^*_R - z - \Delta - 1 + 1 + x^*_L
\]
\[
= -\Delta - z.
\]
By assumption, \(\Delta > \alpha - z\), so \(-\alpha > -\Delta - z > -|x_{L1} - 1 - x^*_R|\). Thus, of the two possible choices left \(R\), she
uniquely maximizes her utility by choosing $x_{R2} = x^*_R$.

Thus, the median voter’s utility from re-electing $L$ is

$$U^L_M (x_{L1}) = -|x_{L1} - x^*_M| - |x^*_L - x^*_M| - 1 (|x_{L1} - x^*_L| > 1) \cdot \alpha$$

$$= -|x_{L1}| - |x^*_L| - \alpha.$$

The median voter’s utility from electing $R$ is

$$U^R_M (x_{L1}) = -|x_{L1} - x^*_M| - |x^*_R - x^*_M| - 1 (|x_{L1} - x^*_R| > 1) \cdot \alpha$$

$$= -|x_{L1}| - |x^*_R| - \alpha$$

Note that

$$U^L_M (x_{L1}) \geq U^R_M (x_{L1}) \iff -|x_{L1}| - |x^*_L| - \alpha \geq -|x_{L1}| - |x^*_R| - \alpha$$

Thus, depending on the values of $x^*_R$ and $|x^*_L|$, the median voter re-elects $L$ with probability $p (x_{L1}) \leq 1$.

Then,

$$\hat{U}_L (x_{L1}) = -|x_{L1} - x^*_L| - \mathbb{E}_{p(x_{L1})} [ |x_{w2} (x_{L1}) - x^*_L| ] - \mathbb{E}_{p(x_{L1})} [ 1 (|x_{L1} - x_{w2} (x_{L1})| > 1) \cdot \alpha + (1 + p (x_{L1})) \cdot \eta$$

$$\leq -|x_{L1} - x^*_L| - |x_{L2} (x_{L1}) - x^*_L| - 1 (|x_{L1} - x_{L2} (x_{L1})| > 1) \cdot \alpha + (1 + 1 + \eta)$$

$$= -|x_{L1} - x^*_L| - |x^*_L - x^*_L| - 1 (|x_{L1} - x^*_L| > 1) \cdot \alpha + 2 \eta$$

$$= -|x_{L1} - x^*_L| - \alpha + 2 \eta$$

$$\leq -|x^*_R + z + \Delta + 1 - x^*_L| - \alpha + 2 \eta$$

$$= -|z + 2 \Delta + 1| - \alpha + 2 \eta$$

$$< -2 \Delta - 1 + 2 \eta$$

$$< -2 |x^*_L| - 1 + 2 \eta$$

$$= \hat{U}_L (1).$$

Thus, the only possible choices of $x_{L1}$ that maximize $L$’s expected utility when $\Delta \in (\alpha - z, \alpha - z + 1)$ for some $z \in \mathbb{Z}_+$ are $x_{L1} = x^*_L - 1$, $x_{L1} = x^*_L - z - 1$ and $x_{L1} = 1$. Note that the latter two choices both generate $p (x_{L1}) = 1$.

I compare them first. The choice of $x_{L1} = x^*_L - z - 1$ is strictly preferred to the choice of $x_{L1} = 1$ if and only if

$$\hat{U}_L (x^*_L - z - 1) > \hat{U}_L (1)$$

$$-2z - 1 + 2 \eta > -2 |x^*_L| - 1 + 2 \eta$$

$$-2z > -2 |x^*_L|$$

$$z < |x^*_L|,$$

which is equivalent to $z \leq |x^*_L| - 1$ since $z \in \mathbb{Z}_+$. Analogously, the choice of $x_{L1} = 1$ is strictly preferred to the choice of $x_{L1} = x^*_L - z - 1$ if and only if $z \geq |x^*_L| + 1$. When $z = |x^*_L|$, $L$ is indifferent between $x_{L1} = x^*_L - z - 1$ and $x_{L1} = 1$, so she randomizes.

Note that $x_{L1} = x^*_L - 1$ generates $p (x_{L1}) = \pi$, but gives $L$ a higher policy utility than the other two potential choices. I compare it to the better choice between $x_{L1} = x^*_L - z - 1$ and $x_{L1} = 1$. The choice of $x_{L1} = x^*_L - 1$ is
uniquely optimal if and only if

\[
\begin{align*}
    \hat{U}_L (x^*_L - 1) & > \max \left\{ \hat{U}_L (x^*_L - z - 1), \hat{U}_L (1) \right\} \\
-1 + (1 + \pi) \cdot \eta & > \max \left\{ -2z - 1 + 2\eta, -2 |x^*_L| - 1 + 2\eta \right\} \\
-1 + (1 + \pi) \cdot \eta & > -2 \cdot \min \left\{ z, |x^*_L| \right\} - 1 + 2\eta \\
(1 + \pi) \cdot \eta & > -2 \cdot \min \left\{ z, |x^*_L| \right\} + 2\eta \\
2 \cdot \min \left\{ z, |x^*_L| \right\} & > (1 - \pi) \cdot \eta \\
\eta & < \frac{2 \cdot \min \left\{ z, |x^*_L| \right\}}{1 - \pi} \\
& \equiv \eta (\Delta) \in (0, +\infty),
\end{align*}
\]

since \( z, |x^*_L| > 0 \) and \( \pi \in (0, 1) \). Analogously, at least one of the other two potential choices is optimal if \( \eta > \eta (\Delta) \). This proves Theorem 2. If \( \Delta \leq \alpha \), then there exists \( \eta (\Delta) \in (0, +\infty) \) such that for all \( \eta \in (0, \eta (\Delta)) \), the unique equilibrium period 1 policy choice is \( x_{L1} = x^*_L - 1 \). Then, by Case 1, \( p(x_{L1}) = \pi \) and \( x_{L2} (x_{L1}) = x_{R2} (x_{L1}) = x^*_L \). This is the unique sequentially rational behavior, so it is the unique equilibrium. \( \square \)

**Proof of Proposition 5:** Note that \( \Delta \in (\alpha - |x^*_L| + 1, \alpha] \) implies that \( z \leq |x^*_L| - 1 \). In turn, by the Proof of Theorem 2, \( z \leq |x^*_L| - 1 \) implies that \( x_{L1} = x^*_L - z - 1 \) is strictly preferred to \( x_{L1} = 1 \). Also, by the Proof of Theorem 2, \( \eta > \eta (\Delta) \) implies that \( x_{L1} = x^*_L - z - 1 \) is strictly preferred to \( x_{L1} = x^*_L - 1 \). Thus, \( x_{L1} = x^*_L - z - 1 < x^*_L - 1 \). Then, by Case 2 of the Proof of Theorem 2, \( p(x_{L1}) = 1 \), \( x_{L2} (x_{L1}) = x_{L1} + 1 \) and \( x_{R2} (x_{L1}) = x^*_L \). This is the unique sequentially rational behavior, so it is the unique equilibrium. \( \square \)

**Proof of Theorem 3:** Note that \( \Delta \leq \alpha - |x^*_L| \) implies that \( z \geq |x^*_L| + 1 \). In turn, by the Proof of Theorem 2, \( z \geq |x^*_L| + 1 \) implies that \( x_{L1} = 1 \) is strictly preferred to \( x_{L1} = x^*_L - z - 1 \). Also, by the Proof of Theorem 2, \( \eta > \eta (\Delta) \) implies that \( x_{L1} = 1 \) is strictly preferred to \( x_{L1} = x^*_L - 1 \). Thus, \( x_{L1} = 1 \). Then, by Case 3 of the Proof of Theorem 2, \( p(x_{L1}) = 1 \), \( x_{L2} (x_{L1}) = 0 \) and \( x_{R2} (x_{L1}) = 1, 2 \). This is the unique sequentially rational behavior, so it is the unique equilibrium. \( \square \)

**Proof of Theorem 4:** Theorem 4 follows from Theorems 1-3 and Propositions 4-5. There are two steps. First, I calculate \( \bar{U}_M (\alpha, \eta, x^*_L, x^*_L) \) in five different cases. Second, I show that \( \bar{U}_M (\alpha, \eta, x^*_L, x^*_L) \) is highest in the case where \( \Delta \leq \alpha - |x^*_L| \) and \( \eta \leq \eta (\Delta) \).

**Step 1:** Case 1: Suppose that \( \Delta \in (\alpha + 1, \alpha + 2 |x^*_L|) \). Then, by Theorem 1, \( x_{L1} = x^*_L = x^*_L \) are the policies chosen in equilibrium. Since \( |x_{L1} - x_{L2}| = |x^*_L - x^*_L| \leq 1 \), choosing the policy pair \( (x^*_L, x^*_L) \) does not require incurring the adjustment cost \( \alpha \). Thus,

\[
\bar{U}_M (\alpha, \eta, x^*_L, x^*_L) = \hat{U}_M (x^*_L, x^*_L) = - |x^*_L - x^*_L| - |x^*_L - x^*_L| = - |x^*_L| - |x^*_L| = -2 |x^*_L|.
\]

Case 2: Suppose that \( \Delta \in (\alpha, \alpha + 1] \). Then, by Proposition 4, \( x_{L1} = x^*_L - 1 \) and \( x_{L2} = x^*_L \) are the policies chosen in equilibrium. Since \( |x_{L1} - x_{L2}| = |x^*_L - 1 - x^*_L| \leq 1 \), choosing the policy pair \( (x^*_L - 1, x^*_L) \) does not require incurring the adjustment cost \( \alpha \). Thus,

\[
\bar{U}_M (\alpha, \eta, x^*_L, x^*_L) = \hat{U}_M (x^*_L - 1, x^*_L) = - |x^*_L - 1 - x^*_L| - |x^*_L - x^*_L| = - |x^*_L| - |x^*_L| = -2 |x^*_L| - 1.
\]

Case 3: Suppose that \( \Delta \leq \alpha \) and \( \eta \in (0, \eta (\Delta)) \). Then, by Theorem 2, \( x_{L1} = x^*_L - 1 \) and \( x_{R2} = x^*_L \) are the policies chosen in equilibrium. Since \( |x_{L1} - x_{R2}| = |x^*_L - 1 - x^*_L| \leq 1 \), choosing the policy pair \( (x^*_L - 1, x^*_L) \) does not
Note that calculations of Theorem 4. I proceed in two steps. First, I use Proposition 1 to calculate \( \Delta \), adjusting for \( \alpha \). Thus,

\[
\begin{align*}
U_M(\alpha, \eta, x_R^*, x_L^*) &= \hat{U}_M(x_L^*, x_R^*) - \hat{U}_M(x_L^*, x_R^* - 1) \\
&= -|x_L^* - 1 - x_M^*| - |x_L^* - x_M^*| \\
&= -|x_L^* - 1| - |x_L^*| \\
&= -2|x_L^*| - 1.
\end{align*}
\]

**Case 4:** Suppose that \( \Delta \in (\alpha - |x_R^*| + 1, \alpha) \) and \( \eta > \eta(\Delta) \). Then, by Proposition 5, \( x_{L1} < x_L^* - 1 \) and \( x_{L2} = x_{L1} + 1 \leq x_L^* - 1 \) are the policies chosen in equilibrium. Since \( |x_{L1} - x_{L2}| = |x_{L1} - (x_{L1} + 1)| \leq 1 \), choosing the policy pair \((x_{L1}, x_{L1} + 1)\) does not require incurring the adjustment cost \( \alpha \). Thus,

\[
\begin{align*}
U_M(\alpha, \eta, x_R^*, x_L^*) &= \hat{U}_M(x_{L1}, x_{L1} + 1) \\
&= -|x_{L1} - x_M^*| - |x_{L1} + 1 - x_M^*| \\
&= -|x_{L1}| - |x_{L1} + 1| \\
&< -|x_L^* - 1| - |x_L^*| \\
&= -2|x_L^*| - 1.
\end{align*}
\]

**Case 5:** Suppose that \( \Delta \leq \alpha - |x_R^*| \) and \( \eta > \eta(\Delta) \). Then, by Theorem 3, \( x_{L1} = 1 \) and \( x_{L2} = 0 \) are the policies chosen in equilibrium. Since \( |x_{L1} - x_{L2}| = |1 - 0| \leq 1 \), choosing the policy pair \((1, 0)\) does not require incurring the adjustment cost \( \alpha \). Thus,

\[
\begin{align*}
U_M(\alpha, \eta, x_R^*, x_L^*) &= \hat{U}_M(1, 0) \\
&= -|1 - x_M^*| - |0 - x_M^*| \\
&= -|1| - |0| \\
&= -1.
\end{align*}
\]

**Step 2:** Note that in Cases 1-4, \( U_M(\alpha, \eta, x_R^*, x_L^*) \leq -2|x_L^*| \leq -2 < -1 \). However, in Case 5, \( U_M(\alpha, \eta, x_R^*, x_L^*) = -1 \). This covers all the relevant cases. Thus, it follows that \( U_M(\alpha, \eta, x_R^*, x_L^*) \) is maximized if \( \Delta \leq \alpha - |x_L^*| \) and \( \eta > \eta(\Delta) \). □

**Proof of Theorem 5:** Theorem 5 follows from Proposition 1, and Step 1, Case 5 and Step 2 of the Proof of Theorem 4. I proceed in two steps. First, I use Proposition 1 to calculate \( U_M(0, \eta, x_R^*, x_L^*) \). Then, I draw upon the calculations of \( U_M(\alpha, \eta, x_R^*, x_L^*) \) in the Proof of Theorem 4 to compare to \( U_M(0, \eta, x_R^*, x_L^*) \).

**Step 1:** Suppose that \( \alpha = 0 \). By Proposition 1, \( x_{L1} = x_L^* \). If \( |x_L^*| < |x_R^*| \), then \( x_{w2} = x_{L2} = x_L^* \) with probability one. If \( |x_L^*| > |x_R^*| \), then \( x_{w2} = x_{R2} = x_R^* \) with probability one. If \( |x_L^*| = |x_R^*| \), then \( x_{w2} = x_{L2} = x_L^* \) with probability \( \pi \) and \( x_{w2} = x_{R2} = x_R^* = -x_L^* \) with probability \( 1 - \pi \). Thus,

\[
\begin{align*}
U_M(0, \eta, x_R^*, x_L^*) &= \hat{U}_M(x_L^*, x_R^*) \\
&= -|x_L^* - x_M^*| - |w_{w2} - x_M^*| \\
&= -|x_L^*| - |w_{w2}| \\
&= \begin{cases} 
-|x_L^*| - |x_L^*| = -|x_L^*| - |x_L^*| & \text{if } |x_L^*| \leq |x_R^*| \\
-|x_L^*| - |x_R^*| & \text{if } |x_L^*| > |x_R^*| 
\end{cases} \\
&= -|x_L^*| - \min\{|x_R^*, x_L^*|\}.
\end{align*}
\]

Note that

\[
\begin{align*}
U_M(0, \eta, x_R^*, x_L^*) &= -|x_L^*| - \min\{|x_R^*, x_L^*|\} \\
&\geq -|x_L^*| - |x_L^*| \\
&= -2|x_L^*|.
\end{align*}
\]
Also, note that

\[
\overline{U}_M (0, \eta, x^*_R, x^*_L) = -|x^*_L| - \min \{ |x^*_R|, |x^*_L| \} \\
\leq -1 - 1 \\
< -1.
\]

**Step 2:** Suppose that \( \alpha > 0 \). By Step 1, Case 5 of the Proof of Theorem 4, if \( \Delta \leq \alpha - |x^*_L| \) and \( \eta > \eta (\Delta) \), then \( \overline{U}_M (\alpha, \eta, x^*_R, x^*_L) = -1 > \overline{U}_M (0, \eta, x^*_R, x^*_L) \). This proves the “if” direction. By Step 2 of the Proof of Theorem 4, if \( \Delta \leq \alpha - |x^*_L| \) and \( \eta > \eta (\Delta) \) do not both hold, then \( \overline{U}_M (\alpha, \eta, x^*_R, x^*_L) \leq -2 |x^*_L| \leq \overline{U}_M (0, \eta, x^*_R, x^*_L) \) in all relevant cases. This proves the contrapositive of the “only if” direction, and therefore it proves the “only if” direction. This completes both directions of the “if and only if” proof. \( \Box \)