

ECON 202: Macroeconomics I

Summer 2011 Problem Set 8 - The Last One

(Due: July 21, Thursday, in class.)

1 The Solow Model

Consider an economy which has the production technology:

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$$

1.1 Labor and Capital Share

For simplicity, assume that there is one big competitive firm that operates this production function and that hires workers and capital for wage w_t and rental rate r_t . Formulate the profit maximization problem of this firm and find the equilibrium wage and rental rate. Verify that the labor share ($w_t L_t / Y_t$) and capital share ($r_t K_t / Y_t$) are constant in this economy. (Note: This is the primary reason why the Cobb-Douglas production function is so popular: the fraction of output paid to workers, i.e. the labor share, and the capital share have historically been pretty constant in the U.S.)

1.2 The K-Change Rule

Now assume that productivity and population are constant ($A_t = A$ and $L_t = L$), that capital depreciates at rate δ , where $0 < \delta < 1$, and fraction s of output is invested in new capital each period. The law of motion for capital, K , is given by:

$$K_{t+1} = (1 - \delta)K_t + sY_t$$

Note that the timing here is different than the one we discussed in class. Find an expression for the initial growth rate of output:

$$\frac{Y_1 - Y_0}{Y_0}$$

as a function of the initial capital K_0 . Verify that the growth rate is decreasing in K_0 .

1.3 Steady State K^*

Derive the steady-state level of capital K^* as a function of the parameters. Use your solution for K^* to derive expressions for output Y^* and consumption C^* in the steady state. (Hint: recall that $Y_t = C_t + I_t$, where $I_t = sY_t$).

1.4 Solow Model with Population Growth

Contrary to the initial assumption, assume that population grows at the constant rate n : $L_{t+1} = (1 + n)L_t$, while A_t continues to be constant. Rewrite the law of motion in terms of capital per worker $k_t \equiv K_t/L_t$, and find the steady state level of k^* , y^* , c^* and i^* as a function of the model's parameters.

What is the growth rate of output Y_t at steady state?

What happens when n increases? Use a Solow model graph to illustrate your answer.