‘Rules versus Discretion’ after Twenty-Five Years

Nancy L. Stokey*

University of Chicago

June 3, 2002

Abstract

Two models of government policy are presented. In the first the choice of an instrument for conducting monetary policy is analyzed. The ease of observing policy under an exchange rate regime is shown to confer an advantage on it compared with a regime that targets the money growth rate.

In the second a discretionary fiscal regime is compared with one that mandates a simple Policy Rule restricting capital taxation. The discretionary regime is preferred under a Ramsey government, but the Rule confers an advantage if the type of government is uncertain and the probability of a myopic administration is high enough.

From time immemorial citizens have complained about their governments. When the government is a greedy despot or the society is composed of private agents with conflicting goals, it is easy to see why complaints arise. Twenty-five years ago Kydland and Prescott (1977) showed something more surprising: even in a society with identical households (with identical tastes and opportunities, and the same choices to make) and a perfectly benevolent government (one that wants to maximize the utility of this representative household), in some circumstances ‘bad’ outcomes may occur. These situations seem to involve no conflict of interest, either among different groups of households or between the private sector and the government, and the outcomes are 'bad' in the sense that better alternatives are obviously available and seem to be—almost—within reach. Settings where this paradox arises include patent protection, capital levies, default on debt, disaster relief, and monetary policy.

Two elements are needed to create such a situation. First, anticipations about future government policy must be important in shaping current decisions in the private sector. Second, there must be a public good aspect—an 'external effect'—from the private sector choices that are influenced by anticipated policy.

In a setting with these two features, even a benevolent government typically has an incentive to mislead the private sector about the policies that will be implemented in the future, in order to manipulate their current decisions and enhance the external effect. After the private sector choices have been made, the government's incentives put weight on only the direct (contemporaneous) effect of the policy. Thus, it has an incentive to implement a different policy from the one announced. If private sector agents are rational, however, they foresee that the government's incentives will change and refuse to be misled in the first place. The resulting outcome seems 'bad' if the enhanced external effect, which must be foregone, is large. All agents would all be better off if all could be fooled, but rational behavior precludes this possibility: in equilibrium the private sector must anticipate correctly the policy maker's incentives and choices. Thus, the time consistency problem offers an explanation for what seem to be paradoxically 'bad' policy outcomes.[1]

A key issue in settings where the time consistency problem arises is the ability or inability of the government to make binding commitments about future policy: 'Rules' imply commitment while 'Discretion' implies its absence. Commitment is important if anticipations about future government actions influence the current choices of the private agents in the economy. With the ability to commit the government can tie the hands of its successors in a way that may improve outcomes. Without that ability the private sector fears—with good reason—that today's government will make promises that its successors will refuse to honor.

If commitment is lacking, a framework that incorporates game theoretic elements is needed to model the policy maker's incentives.[2] And as Barro and Gorden (1983) showed early on, such a formulation also points the way to a resolution of the problem: within a game-theoretic framework it is easy to show that if the game is repeated and agents are not too impatient, there are reputation equilibria in which the 'good' outcome prevails along the equilibrium path. That is, a policy maker can be disciplined by reputation considerations even if he has discretion.

The time consistency issue has been intensively studied over the last twenty-five years, and many of the main theoretical issues have been resolved. Interesting substantive applications are, of course, still being developed. But rather than review the theoretical literature again[3] or attempt to survey the applications (which are too numerous even to list), we will look at two issues that remain. Both deal with choices about the policy regime to be used.

The first issue is reputation building. A policy instrument that can be monitored more closely implies less frequent breakdowns in a reputation equilibrium. Thus, ease of observability is one criterion involved in the choice among discretionary instruments. Here we will look at a Central Bank deciding whether to peg an exchange rate and or to set a rate of money growth. The model focuses on the tradeoff between observability—the accuracy with which the private sector can monitor the Central Bank's actions, and tightness—how closely the instrument is linked to the object of ultimate interest, the inflation rate. We will show that the ease of monitoring an exchange rate policy may outweigh other costs it imposes relative to a money growth policy.

The second issue is the robustness of a policy mechanism against mismanagement. One reason to prefer rules over discretion is that governments are not always as intelligent, benevolent, and farsighted as the Ramsey government found in theoretical discussions of policy. Policy makers who are misguided, greedy, or myopic sometimes hold office. Rules that are hard to change may offer protection against these less-than-ideal types of government officials. The robustness argument is one of the motivations in Friedman's (1948) recommendations on aggregative policy, and it is one that seems worth reviving. Many of the biggest policy blunders seem to arise from incompetence or special interest group pressures, rather than the classic time consistency issue.[4]

Here we will look at robustness using a model in which the type of government in power, Ramsey or 'other,' changes randomly from period to period. In this setting the (farsighted) Ramsey government faces an especially difficult task, since the possibility of the 'other' type adversely affects private sector behavior. Hence when the Ramsey government is in power it must distort its own policy in a way that offsets the policy of the 'other' type. If the probability of the 'other' type is high enough, a simple Policy Rule can be advantageous. A well designed Rule places an important restriction on the policy of the 'other' type, while leading to only a mild change in the policy of the Ramsey type.

These two issues are examined in next two sections. The concluding section discusses some of the results.

I. REPUTATION BUILDING

The ability of a government policy maker to establish and maintain a reputation for reliable conduct depends on how well the public can observe his actions. A policy instrument that is more easily monitored, one that allows the private sector to detect deviations from announced policy rules more easily, has an obvious advantage in allowing the policy maker to build and keep a reputation. Hence observability is often a key issue. But a policy instrument that is more observable may be less tightly connected to the ultimate target, and consequently there is a tension between observability and tightness.[5]

In a recent paper Atkeson and Kehoe (2001) look at the problem of a Central Bank choosing between two instruments for conducting monetary policy. The Bank's options are to peg an exchange rate or to target the rate of money growth. If it pegs an exchange rate, realized inflation is equal to the rate of depreciation in the exchange rate plus an exogenous shock term that represents the foreign rate of inflation. If the Bank targets money growth, realized inflation is equal to the rate of money growth plus an exogenous shock term that represents a domestic velocity shock. The Central Bank chooses its instrument period by period and may switch instruments at any time. If any 'reversions' occur along the equilibrium path, the most severe punishment is implemented.

Notice that with either instrument the object of interest—the inflation rate—is imperfectly related to the Bank's action. The exogenous shock terms, the foreign rate of inflation in the first case and the domestic velocity shock in the second, are beyond the Bank's control and are unknown when the Bank is making its policy decision. In general the two shocks will have different variances, and those values are important inputs into the Bank's decision.

The two instruments also differ along a second dimension. The public is assumed to observe the exchange rate directly, so any deviation is immediately detected. That is, the exchange rate is assumed to be a perfectly observable instrument. Consequently, with the exchange rate as the instrument there exist equilibria in which the *threat* of reversion disciplines Central Bank behavior, but no reversions actually occur along the equilibrium path.

The money growth rate, on the other hand, is not directly observed. Thus, if the Central Bank uses the money growth rate for its instrument, the private sector can only infer something about its behavior by looking at the realized rate of inflation. Hence under a money growth regime (accidental) reversions cannot be avoided. As in Green and Porter's (1984) cartel model, the imperfect monitoring technology is the source of these reversions.

Atkeson and Kehoe show that if the Central Bank can commit to a policy, then it chooses the instrument with the smaller variance for its shock. That is, with commitment only tightness is valued. They also show that if the Central Bank cannot commit, then it prefers to use the perfectly observable instrument, the exchange rate, even if the variance of the foreign inflation shock is somewhat larger than the variance of the domestic velocity shock.[6]

In this section we will look at a slightly modified version of Atkeson and Kehoe's model that highlights the main conclusions. First, we will require the government to make a one-time decision about which instrument to use, instead of choosing the instrument period by period. Second, we will use reversions to the one-shot Nash equilibrium instead of the most severe punishment path. Third, we will formulate the model in the classic Ramsey tradition, as one in which the government's objective is to maximize the utility of the representative household. Finally, we will allow an alternative version of the inflation process under a money growth rule.

1. The economy

Consider a Central Bank choosing between money growth and an exchange rate as the instrument for conducting monetary policy. Suppose

$$\mu + v \quad = \quad \pi \quad = \quad e + \zeta,$$

where μ is the money growth rate, v is a velocity shock, π is the inflation rate, e is the rate of depreciation in the exchange rate, and ζ is the foreign rate of inflation, all in logs. Assume that the shocks v, ζ are i.i.d. and independent of each other, with means of zero and variances $\sigma_v^2, \sigma_\zeta^2 > 0$. Assume that π and e are observed.[7]

Under a money growth policy the instrument is μ, the (noisy) signal is π, and the velocity shock v affects the realized inflation rate. Although $e = \pi - \zeta$ is observed, it is not useful in assessing the Central Bank's performance: e is a noisy signal about π, and π is observed directly. Under an exchange rate policy the instrument is e, the (noiseless) signal is e, and the foreign inflation rate ζ affects the realized inflation rate.

Figure 1 illustrates the tradeoff. Figure 1a displays the realized rate of inflation under a money growth rule. Since the actual rate of money growth (which is always zero in equilibrium) is not observed by the private sector, the reputation equilibrium involves reversions when the realized inflation rate exceeds some (optimally chosen) threshold. The small circles depict situations where a reversion is triggered.

Figure 1b displays the situation under an exchange rate rule. The horizontal line is the actual rate of depreciation in the exchange rate, and the fluctuations around it depict the realized rate of inflation. The variance of realized inflation is larger than under a money growth rule, but since the exchange rate is observed directly, no reversions occur. Thus, the optimal choice trades off the higher ongoing cost of larger fluctuations under the exchange rate regime against the cost of occasional reversions under a money growth regime.

A slightly more complicated model of money growth incorporates output growth. Suppose

$$\pi = (\hat{\mu} - g) + u,$$

where $\hat{\mu}$ is money growth, g is real GDP growth over the period, and u is a velocity shock. Let

$$g^e = g + \varepsilon$$

be the Central Bank's (imperfect) forecast of real growth, where ε is the forecast error. Assume that the shocks ε, u, ζ are i.i.d. and independent of each other, with means of zero and variances $\sigma_\varepsilon^2, \sigma_u^2, \sigma_\zeta^2 > 0$. Under a money growth policy the Bank can be viewed as choosing

$$\mu = \hat{\mu} - g^e,$$

the excess of money growth over expected real growth. For simplicity we will continue to call μ the rate of money growth. Assume that the private sector cannot observe g^e, but does observe $\hat{\mu}$ and g. Then

$$\hat{\mu} - g = \mu + \varepsilon$$

is its signal about the Bank's action, and

$$\pi = \mu + \varepsilon + u$$

is the realized inflation rate.[8]

Thus, in both models of money growth the signal about the Central Bank's action is noisy, so reputation equilibria involve reversions ('punishments') along equilibrium outcome paths. In the first model the realized inflation rate is itself the signal, while in the second the inflation rate is the signal plus additional noise. To capture both models of money growth the framework analyzed here allows two shocks. For the first interpretation one shock is set identically to zero.

In the next two sections we will characterize a certain class of reputation equilibria and calculate expected payoffs along the equilibrium outcome paths. These equilibria are then compared with those for the exchange rate model. Since the signal is noiseless under an exchange rate regime, no reversions occur along the equilibrium outcome path.

2. Household behavior

Under a money growth rule the timing of events within each period is:

(i) the government sets the money growth rate μ;

(ii) each household chooses w, interpreted as a rate of wage growth, in anticipation of the current inflation rate;

(iii) the signal

$$s = \mu + \varepsilon,$$

and the inflation rate

$$\pi = \mu + \varepsilon + u,$$

are observed, where ε and u are the exogenous shocks. In the simple model $\varepsilon \equiv v$ and $u \equiv 0$.

Let $\overline{w}$ denote the average rate of wage growth in the economy. The one-period loss for a household that sets the wage w is

$$L(w, \overline{w}, \pi) = \frac{a}{2}\pi^2 + \frac{b}{2}\left(\overline{w} - \pi\right)^2 + \frac{d}{2}\left(w - \alpha - \pi\right)^2,$$

where $\alpha > 0$, and where $a, b, d > 0$ with $\left(a + b + d\right)/2 = 1$ are relative weights.

The household's loss function has a "new Keynesian" interpretation.[9] Suppose each household is the monopolistic supplier of a differentiated commodity produced with labor as the only input. Since households set wages before the current inflation rate is known, wages are sticky for one period.

Suppose each household's target wage is $W = \left(1 + \hat{\alpha}\right) P$, where P is the average price level in the economy and $\hat{\alpha}$ is the desired markup. It is convenient to renormalize units each period so $P_{-1} = 1$, and let $w = \ln W$, $\pi = \ln P$, and $\alpha = \ln\left(1 + \hat{\alpha}\right)$.

The first term in the loss function represents the "shoe leather" cost of inflation. It depends only on the actual rate of inflation P/P_{-1}, and with the chosen normalization it is proportional to $\left[\ln\left(P/P_{-1}\right)\right]^2 = \pi^2$.

The second term represents the household's interests as a consumer. Its surplus is maximized if other producers set wages at $\overline{W} = P$, and its relative loss is proportional to $\left[\ln\left(\overline{W}/P\right)\right]^2 = \left(\overline{w} - \pi\right)^2$.

The last term represents the household's interests as a producer. Its surplus is maximized if its wage equals the target value, and its relative loss is proportional to $[\ln (W/ (1+\alpha) P)]^2 = (w-\alpha-\pi)^2$.

Notice that $\mu = \mathrm{E}[\pi \,|\, \mu]$ is the expected rate of inflation, conditional on the value μ for money growth. Let μ^a denote the rate of money growth anticipated by households. Then μ^a is also the inflation rate expected by households, where the word 'expected' encompasses uncertainty about the Central Bank's action as well as uncertainty about the shock.

Consider the expected value of the current period loss if μ^a is anticipated and μ is carried out. Households set wages at $w = \mu^a + \alpha$, so the expected loss is

$$\begin{aligned} \Lambda(\mu^a, \mu) &= \mathrm{E}\left[L(\mu^a+\alpha,\, \mu^a+\alpha,\, \mu+\varepsilon+u)\right] \\ &= \frac{a}{2}\mu^2 - b\alpha\left(\mu-\mu^a\right) + \frac{b+d}{2}\left(\mu-\mu^a\right)^2 + M, \end{aligned} \tag{1}$$

where

$$M \equiv \sigma_\varepsilon^2 + \sigma_u^2 + \frac{b}{2}\alpha^2$$

is an unavoidable part of the expected loss. The first two terms in Λ, which are exactly as in Barro-Gordon (1983), are important for the incentive constraints for the Central Bank. The third term and its derivative vanish when households correctly anticipate the action of the Central Bank, $\mu^a = \mu$, as they do in equilibrium. The last term, M, is important for cost comparisons across instruments.

The second term in Λ can be interpreted as a Phillips curve coefficient. If households anticipate an average rate of inflation μ^a, then the Central Bank can reduce this part of the expected loss by setting the money growth rate a little higher, $\mu > \mu^a$. Of course, a higher value for μ increases the first and third terms in Λ, putting a bound on the net gain from unanticipated inflation.

In equilibrium households correctly anticipate the action of the Central Bank,

$\mu^a = \mu$, so the expected loss is

$$\Lambda(\mu, \mu) = \frac{a}{2}\mu^2 + M.$$

Consequently, if the Central Bank could precommit it would set $\mu = 0$ to minimize this loss. Call $\mu = 0$ the Ramsey rate of money growth. For the reasons noted above, if $\mu^a = 0$ is anticipated, short-run considerations tempt the Central Bank to set $\mu > 0$.

Define μ^N to be the unique rate of money growth with the property that if μ^N is anticipated by households, so they set wages at $w = \mu^N + \alpha$, then the Central Bank has no short-run temptation to deviate. The latter requires $\Lambda_2(\mu^N, \mu^N) = 0$, so

$$\mu^N = \frac{b\alpha}{a}.$$

Call μ^N the Nash rate of money growth.

Let

$$\delta \equiv \Lambda(\mu^N, \mu^N) - \Lambda(0, 0) = \frac{a}{2}\left(\mu^N\right)^2 = \frac{(b\alpha)^2}{2a}.$$

denote the difference between the expected losses (over one period) under the Nash and Ramsey money growth rates.

3. Markov equilibria

The game described above is infinitely repeated, and future losses are discounted by the constant factor $\beta \in (0, 1)$ per period. If β is close to one, as we will assume here, the repeated game has many subgame perfect equilibria. We will focus on a particular subset: Markov equilibria in which there are two states, 'good' and 'bad,' that also satisfy some other restrictions. In the rest of this section we will briefly describe this set of equilibria and sketch the argument for characterizing the subset that minimize expected discounted losses. A more detailed discussion is provided in Appendix A.

Each equilibrium in the class we are considering is characterized by rates of money growth $\left(\mu^g, \mu^b\right)$ for the Central Bank and rates of wage growth $\left(w^g, w^b\right)$ for the representative household for each state, and rules for updating the state at the end of each period. These must satisfy the usual equilibrium conditions. The additional restrictions are twofold.

First, we will focus on equilibria in which the Central Bank chooses the Ramsey rate of money growth in the good state, $\mu^g = 0$, and the Nash rate in the bad state, $\mu^b = \mu^N$. It then follows immediately that the rates of wage growth chosen by households are $w^g = 0 + \alpha$ and $w^b = \mu^N + \alpha$.

Second, we will restrict the class of rules for updating the state. We will assume that only the current signal s is used and that it is used in a particular way in each state. Specifically, if the economy is currently in the 'good' state, households compare the signal with a one-sided threshold S^g, and the state remains 'good' in the next period if and only if $s \leq S^g$. If the economy is currently in the 'bad' state, households check whether the signal lies in a symmetric interval around μ^N, and the state reverts to 'good' in the next period if and only if $s \in \left[\mu^N - \varepsilon^b, \mu^N + \varepsilon^b\right]$. The simple structure of these equilibria makes them appealing candidates for attention.

The pair of thresholds $\left(S^g, \varepsilon^b\right)$ must also satisfy incentive compatibility (IC) constraints for the Central Bank in each state. These constraints ensure that any deviation from the equilibrium rate of money growth, 0 or μ^N, is unattractive to the Bank.

Definition: **Simple two-state Markov equilibria** are characterized by money growth rates $\mu^g = 0$ and $\mu^b = \mu^N$, rates of wage growth $w^g = 0 + \alpha$ and $w^b = \mu^N + \alpha$, and updating rules that use only the current signal. Depending on the current state, the state next period is 'good' if and only if $s \leq S^g$ or $s \in \left[\mu^N - \varepsilon^b, \mu^N + \varepsilon^b\right]$, where the critical values $S^g, \varepsilon^b \geq 0$ satisfy the IC constraints for the Central Bank.

The symmetric form of the test in the 'bad' state ensures that the IC constraint holds in that state. The IC constraint in the good state imposes an additional restriction on the pair $\left(S^g, \varepsilon^b\right)$. We turn next to a brief discussion of that constraint.

Instead of using S^g and ε^b, it is convenient to analyze the model in terms of the corresponding probabilities p of a reversion from the good state to the bad and q of a return in the other direction. It is also useful to place a mild restriction on the distribution of the shock ε.

Assumption 1: ε has a continuous, symmetric, unimodal density $f(\varepsilon)$ with mean zero, whose support is all of $\mathbf{R}$.

Under Assumption 1 the reversion probability p can be adjusted continuously from 0 to 1/2 by adjusting S^g from 0 to $+\infty$; and the return probability q can be adjusted continuously from 0 to 1 by adjusting ε^b from 0 to $+\infty$. Normal distributions with mean zero satisfy this assumption and will be used in the examples.[10]

It is useful to define the function

$$\gamma(p) \equiv f(F^{-1}(1-p)), \quad p \in (0,1),$$

where F is the c.d.f. for f. Then $\varepsilon = F^{-1}(1-p)$ is the value for the shock that leaves probability p in the upper tail, and $f(\varepsilon)$ is the height of the density function at this point. Thus, $\gamma(\cdot)$ maps probabilities in the upper tail into levels for the density function. We will also use the hazard function, $h(p) = \gamma(p)/p$.

Fix β and define the function

$$\psi(p,q;\beta) = \frac{1}{1-\beta(1-p-q)}.$$

Recall that δ is the incremental expected loss from being in the bad state rather than the good in the current period. If the switching probabilities are (p,q), then $\delta\psi(p,q)$ is the expected discounted value of the (current and future) incremental losses from being (currently) in the bad state. That is, $\psi(p,q)$ accounts for all future switches back and forth between states, discounting and weighting them appropriately. Note that ψ is decreasing in p and q: higher switching probabilities reduce the difference between the states.

Fix the parameters (a,b,α) and the density f; let $\mu^N = b\alpha/a$ be the Nash inflation rate and let γ, h be the functions defined above. The set of probabilities $(p,q) \in (0,1/2] \times [0,1]$ that satisfy the Central Bank's IC constraint in the 'good' state are those for which

$$\gamma(p)\beta\delta\psi(p,q) \geq b\alpha.$$

The interpretation is as follows: increasing the money growth rate above $\mu^g = 0$ leads to a marginal gain of $b\alpha$ in the current period and a marginal increase of $\gamma(p)$ in the probability of reversion to the bad state. The latter is multiplied by $\beta\delta\psi(p,q)$, the expected discounted loss if a reversion occurs. Using h instead of γ and rearranging terms, we can rewrite this constraint as

$$\beta\delta p\psi(p,q) \geq \frac{b\alpha}{h(p)}. \tag{2}$$

Suppose the pair (p,q) satisfies (2). If the economy is currently in the good state, the expected discounted cost of (future) reversions is $\beta\delta p\psi(p,q)$. Hence the equilibria that minimize expected discounted losses are those that solve

$$\min_{p,q\in(0,1/2]\times[0,1]} \beta\delta p\psi(p,q) \quad \text{s.t.} \quad (2). \tag{3}$$

Proposition 1 characterizes the set of equilibria that attain the minimum value for expected discounted losses among all simple two-state Markov equilibria.

Proposition: Let $f(\varepsilon)$ satisfy Assumption 1. Then

(i) any pair $(p,q) \in (0,1/2] \times [0,1]$ satisfying (2) characterizes a simple two-state Markov equilibrium;

(ii) the set of such equilibria is nonempty if and only if (2) holds for $q = 0$, for some $p \in (0, 1/2]$;

(iii) a pair (p^*, q^*) attains the minimum expected cost if and only if it solves (3), and a solution exists if the set of equilibria is nonempty;

(iv) if (p^*, q^*) is a cost-minimizing pair and $q^* < 1$, then the expected cost per period, conditional on starting in the good state, is

$$C = \Lambda(0, 0) + \frac{b\alpha}{h(p^*)}; \tag{4}$$

(v) if f is a normal density then the solution (p^*, q^*) is unique (if one exists) and $q^* = 0$.

The first and third claims summarize the discussion of (2) and (3). The second claim follows from the fact that ψ is decreasing in q. The fourth follows from the fact that if (p^*, q^*) is cost-minimizing and $q^* < 1$, then (2) holds with equality. If it did not, q^* could be increased, shortening reversions and further reducing expected costs. To illustrate the last claim we turn to an example.

If $f_i(\cdot)$ is a normal $(0, \sigma_i^2)$ density, the associated hazard function is $h_i(p) = H(p)/\sigma_i$, where H is the hazard function for a normal $(0,1)$. The function H is decreasing for $p \leq 1/2$. (Recall that p is the probability in the *upper* tail).

Figure 2 displays the function $\Psi(p, q; \beta) = \beta p \psi(p, q; \beta)$, for $\beta = 0.99$ and $q = 0.0,\ 0.03$; and the function $b\alpha/\delta h_i(p) = 2/\mu^N h_i(p)$, for $\mu^N = 10\%$ and $\sigma_i = 0.8,\ 1.4$. Suppose $\sigma = \sigma_1 = 0.8$. The points E_1^* and F_1^* occur where the $2/\mu^N h_1(p)$ curve crosses the $\Psi(p, q)$ curves, for $q = 0$ and 0.03. Call the $x-$coordinates of these points $p_1^{\min}(q)$. For each q, the IC constraint (2) holds to the right of this point, so there are equilibria for $p \geq p_1^{\min}(q)$. Reducing q extends the feasible range for p downward, reflecting the fact that the Central Bank's IC constraint involves a tradeoff: longer punishments (lower q) permit less frequent punishments (lower p).

For each fixed q, the pair $\left(p_1^{\min}(q), q\right)$ minimizes expected discounted costs. And since a pair of this form satisfies the IC constraint with equality, the expected discounted cost of future reversions is proportional to the quantity on the vertical axis. Hence the minimum expected loss overall is attained at $E_1^* = (p_1^*, q_1^*) = \left(p_1^{\min}(0), 0\right)$.

The figure is qualitatively the same for any parameter values, provided the shock ε has a normal distribution, establishing claim (v). For $\sigma = \sigma_2 = 1.4$ the (unique) minimum cost equilibrium occurs at the crossing point E_2^*, again with $q^* = 0$. Notice that increasing σ raises the minimum expected cost: a less informative signal requires a higher reversion probability p^*.

Figure 3a displays the optimal reversion probability p^* as a function of the standard deviation σ, for Nash inflation rates of 3%, 5%, 10%, and 20%. Looking along each curve we see that increasing the standard deviation of the shock—reducing the accuracy of the signal—leads to more frequent reversions. Looking across curves we see that increasing the Nash inflation rate—raising the cost of reversions—reduces the frequency of reversions. Figure 3b displays the corresponding thresholds for the inflation rate.

The conclusion that $q^* = 0$ is a direct consequence of the fact that the hazard function $h(p)$ for a normal density is a decreasing function. It holds for other distributions with that property, but not in general. For example, suppose $f(\varepsilon)$ has an exponential distribution in the relevant range,

$$f(\varepsilon) = \frac{1}{2}\eta e^{-\eta\varepsilon}, \quad \varepsilon \geq 0.$$

Then the hazard rate is constant in the region of interest: $h(p) = \eta$, $p \leq 1/2$. For this distribution the curve $2/\mu^N h(p)$ in Figure 2a is a horizontal line. Hence if there are any equilibria at all, there are many that attain the minimum expected cost, each with the form $\left(p^{\min}(q), q\right)$, $q \in Q^*$. These equilibria have switching probabilities that rise and fall together.

Alternatively, if f has an increasing hazard rate in the relevant range, then the cost-minimizing equilibrium is again unique and has $q = 1$.

4. Observability and tightness

With the characterization of the least-cost equilibria in hand, we can return to the Central Bank's problem of choosing between the two potential policy instruments, money growth and the exchange rate. Recall from (1) that $\Lambda(0,0) = M = \sigma_\varepsilon^2 + \sigma_u^2 + b\alpha^2/2$ is the expected loss per period, ignoring reversions. Since no reversions occur under the exchange rate regime and there is only one shock, the expected loss per period is simply

$$C^{ex} = \sigma_\zeta^2 + \frac{1}{2}b\alpha^2.$$

Under a money growth regime the expected cost of reversions must also be included. Consider first the simple model of money growth. The velocity shock intervenes between the money growth rate and the signal, $s = \pi = \mu + v$, and it is the only shock. Hence $\varepsilon = v$ and $u = 0$, and Assumption 1 must hold for the velocity shock. Let h_v^* denote the hazard rate in a cost-minimizing equilibrium. Then the expected cost per period is

$$C^{mg} = \sigma_v^2 + \frac{1}{2}b\alpha^2 + \frac{b\alpha}{h_v^*}.$$

Comparing the two costs we find that the exchange rate is preferred to money growth as an instrument if and only if

$$\sigma_\zeta^2 \leq \sigma_v^2 + \frac{b\alpha}{h_v^*}.$$

If $\sigma_\zeta^2 \leq \sigma_v^2$, then the exchange rate is obviously preferred: it is both tighter and more observable. If $\sigma_\zeta^2 > \sigma_v^2$, then the exchange rate is the preferred instrument if and only if the higher cost from its 'looser' relationship with the target (the higher variance of its shock) is more than offset by the cost of the 'opaqueness' of the signal under a money growth policy.

Figure 4 displays the tradeoff for $\beta = 0.99$, a Nash inflation rate of $\mu^N = b\alpha/a =$

10% and the four values $b\alpha = 1, 2, 4, 8$ for the Phillips curve coefficient. (The corresponding weights on the 'shoe leather' cost of inflation, the first term in (1) are $a = 0.1, 0.2, 0.4, 0.8$.) The exchange rate is the preferred instrument along and below the 45^0 line, where the standard deviation of the foreign inflation shock is no greater than that of the domestic velocity shock. In addition it is preferred if the former is somewhat larger than the latter, with the exact position of the separating curve depending on the parameters.

A higher value for the Phillips curve coefficient $b\alpha$ increases the Central Bank's incentive to deviate under a money growth regime, increasing the size of the region where the exchange rate is preferred instrument. That coefficient measures the gain from surprise inflation, which in the model here is interpreted as arising because of monopolistic (rather than perfect) competition among producers (households). But it can have other interpretations as well. For example, it might represent the value of additional seinorage revenue, or the benefit from devaluing outstanding (nominal) debt.

For the complex model of money growth the signal is $s = \mu + \varepsilon$, where ε is the error in the Bank's forecast of GDP growth, and Assumption 1 must hold for ε. Repeating the argument above and letting h_ε^* denote the optimal hazard rate, we find that

$$C^{mg} = \sigma_\varepsilon^2 + \sigma_u^2 + \frac{1}{2} b\alpha^2 + \frac{b\alpha}{h_\varepsilon^*},$$

so the exchange rate is the preferred instrument if and only if

$$\sigma_\zeta^2 - \sigma_u^2 \leq \sigma_\varepsilon^2 + \frac{b\alpha}{h_\varepsilon^*}.$$

If $\sigma_\zeta^2 \leq \sigma_\varepsilon^2 + \sigma_u^2$ then the exchange rate is obviously preferred. Otherwise there is, as before, a tradeoff between tightness and observability. Figure 4 still applies, with the axes relabeled: on the horizontal axis is σ_ε, and on the vertical is $\sqrt{\sigma_\zeta^2 - \sigma_u^2}$.

With a normal distribution for ε the optimal punishment length is infinite, $q^* = 0$.

Such an outcome strains the imagination: presumably a new Central Banker or a new institution altogether would be put in place in finite time. It is very easy to modify the model here to deliver that result, by adding a strictly positive lower bound, $q^0 > 0$, on the return probability. The argument above proceeds exactly as before (cf. Figure 2), and the (unique) equilibrium has $q^* = q^0$. The reversion length is random, and it is straightforward to calculate its expected value as a function of q^0. Since the additional restriction operates on the money growth instrument, the result is to enlarge the region of parameter space where the exchange rate is the preferred instrument.

II. ROBUSTNESS

Not all governments are as benevolent and clever as a Ramsey government. The possibility that the government is 'bad,' which may mean greedy, incompetent or myopic, creates difficulties for a 'good' (Ramsey) government. Some of the difficulties are unavoidable: a legacy of large outstanding debt, bad legislation, etc. can be difficult to undo. In addition, the behavior of the private sector will be predicated on a certain apprehension about the nature of the administration currently in power. In this section we will show that if a 'good' government cannot easily distinguish itself from a 'bad' one, this mistrust by the private sector makes its task more onerous. In such an environment a simple Policy Rule can be very useful, even if it cannot respond to shocks in the environment. In the model here, the fact that a Rule reduces or eliminates the potential damage done by a 'bad' government has a very useful effect on private behavior. This effect far outweighs the small additional gain that a 'good' government could attain with discretion. As will be shown, even a moderate probability that the government is 'bad' makes the Rule worthwhile.

Suppose that there are two types of governments, Ramsey and 'other.' Reputation equilibria are delicate, and there are countless ways for the 'other' type of government to deviate from the Ramsey policy. Here we will assume that the 'other' type is myopic, setting current tax rates to maximize current-period utility. The Ramsey government behaves in the usual fashion, raising revenue in a way that maximizes the expected discounted utility of the representative household. For simplicity, we will assume that the government's type is i.i.d.

The environment is adopted from Fischer's (1980) paper. Each household receives an endowment of goods that can be invested or consumed directly. Invested goods earn a return but are also subject to taxation. The household can also use labor to produce goods. The government must finance an exogenous expenditure sequence. The tension is between the government's short-run temptation to use a (nondistorting) capital levy to finance current expenditures, and the adverse effect such a policy has on the incentive to invest. The expenditure sequence is stochastic, and for simplicity is taken to be i.i.d.

First we study a setting where policy is discretionary. If the government is known to be the Ramsey type and the discount factor is sufficiently close to one, then the standard reputation argument applies. In the setting here, the Ramsey government uses a carefully calculated capital tax to finance part of spending and to provide insurance against the high expenditure shock. The capital tax varies with the expenditure level, but its expected value is low enough so that investment is worthwhile.

If the type of government is uncertain, but the probability of the myopic type is not too high, a reputation equilibrium still exists. The policy of the Ramsey government is qualitatively similar to the previous case. The main difference is that the Ramsey government must offer a high enough expected return on capital during the periods when it is in office to compensate the household for the fact that capital earns a negative expected return when the myopic government is in power. The policy adopted by the Ramsey government becomes rather odd and expected utility declines as the probability of a myopic government increases. The Ramsey government is willing to continue participating in this equilibrium because abandoning it means that households stop investing, which entails a substantial cost. (For sufficiently high probabilities reputation equilibria cease to exist, but here we will focus on probabilities that are below that threshold.)

We then consider what happens if, instead of allowing the government discretion in setting fiscal policy each period, the society adopts a Policy Rule placing an upper bound on the capital tax. If the probability of the Ramsey type is sufficiently close to one, this Rule reduces welfare, since the insurance feature of a variable capital tax is lost. But if the probability of the myopic type is high enough the Rule is welfare enhancing.

1. The environment

Each period the household receives an endowment of goods, ω, and an endowment of time. It can invest all or part of its goods endowment in a productive activity and it can 'hide' the rest. Let $\theta \in [0, 1]$ denote the fraction of the goods endowment that is invested. Investments earn a rate of return $r > 0$, but they can also be taxed. Hidden goods earn no return but cannot be taxed. Time spent working produces goods according to the linear technology $q = w\ell$, where $w > 0$ is an implicit wage rate and ℓ is labor supply.

Households value private consumption goods c and time worked ℓ according to a utility function that is additively separable and linear in labor supply:

$$U = \mathrm{E}\left\{\sum_{t=0}^{\infty} \beta^t \left[u(c(t)) - \ell(t)\right]\right\}.$$

Assume u is strictly increasing, strictly concave, and twice differentiable, and $0 < \beta < 1$. Assume $u'((1 + r)\omega) >> w$, so that the household chooses to work even if it is consuming its entire endowment, with interest, and faces a positive tax on labor income.

Government expenditure is exogenous and stochastic. For simplicity assume it takes only two values, $g_1 = 0$ and $g_2 = g > 0$, and that the realizations are i.i.d. Let $\pi_1 = \pi$ and $\pi_2 = 1 - \pi$ denote the probabilities.

In each period the government levies flat-rate taxes $\tau_k \in [0, 1 + r]$, $\tau_\ell \in [0, 1]$ on capital and on labor income. The government cannot issue debt, so its budget must be balanced each period. Assume

$$r\omega < (1 - \pi) g, \tag{5}$$

so that the required revenue cannot be raised with a capital tax that leaves the household with a positive expected return on investment. For simplicity assume in addition that $g < (1 + r)\omega$, so that the required revenue *can* be raised with a confiscatory capital tax. Finally, assume that g is small enough so that it can be financed entirely with a labor tax when the household hides its good endowment.

2. Ramsey government

First consider an economy in which it is known for sure that the government is the Ramsey type. We are interested in settings where there is a reputation equilibrium of the usual form. The tax policy in that equilibrium is the one that the Ramsey government would employ if it could commit ex ante to fixed, state-contingent tax rates. For discount factors β that are sufficiently close to unity there an equilibrium of this form, supported by the threat of a reversion to the one-shot Nash equilibrium.

Suppose the government could precommit, and consider the problem of choosing the optimal tax policy subject to the constraints imposed by household behavior. In this stationary environment with i.i.d. expenditure shocks and no state variables, the solution is a stationary tax policy $\{(\tau_{\ell i}, \tau_{ki}),\ i = 1, 2\}$ that maximizes the household's expected utility per period, where subscripts $i = 1, 2$ denote the values of the tax rates, consumption, etc. in the two states.

Suppose that the household has invested all of its endowment, and consider its problem after the state i has been realized and the current tax rates $(\tau_{ki}, \tau_{\ell i})$ are known. Its problem is

$$
\begin{aligned}
& \max_{c_i,\ell_i} \left[u(c_i) - \ell_i\right] && (6) \\
\text{s.t.} \quad c_i &= (1 + r - \tau_{ki})\,\omega + (1 - \tau_{\ell i})\,w\ell_i, \quad i = 1, 2. && (7)
\end{aligned}
$$

The equilibrium allocation must also satisfy the market clearing condition for goods:

$$c_i + g_i = w\ell_i + (1 + r)\,\omega, \quad i = 1, 2; \tag{8}$$

and the government's budget constraint (redundant, by Walras' Law) must hold:

$$g_i = \tau_{ki}\omega + \tau_{\ell i} w\ell_i, \quad i = 1, 2.$$

Finally, notice that the household's net income gain from investment in state i is $(r - \tau_{ki})\,\omega$. The household is willing to invest its endowment if and only if the associated change in expected utility is positive. Hence investment occurs if and only if the capital tax satisfies the rate of return constraint

$$\sum_i \pi_i u'(c_i)\omega\,(r - \tau_{ki}) \geq 0. \tag{9}$$

The Ramsey government's problem is

$$\max \sum_i \pi_i \left[u(c_i) - \ell_i\right]$$

subject to (8), (9), and the constraints imposed by household optimization. As shown in Appendix A, the solution $\left\{\theta^R, \left(c_i^R, \ell_i^R, \tau_{ki}^R, \tau_{\ell i}^R\right),\ i = 1, 2\right\}$, with $\theta^R = 1$, has the following features:

(i) consumption is the same in the two states, $c_1^R = c_2^R$;

(ii) the labor tax is the same in the two states, $\tau^R_{\ell 1} = \tau^R_{\ell 2}$;

(iii) labor supply is higher by g in the second state, $\ell^R_2 = \ell^R_1 + g$;

(iv) the expected capital tax is equal to the rate of return, $\Sigma_i \pi_i \tau^R_{ki} = r$;

(v) capital is subsidized when spending is low and taxed when it is high, $\tau^R_{k1} < 0 < \tau^R_{k_2}$.

Features (i) - (iii) follow from the assumption that utility is linear in labor supply. Given (i), result (iv) is an immediate consequence of the rate of return restriction in (9). Result (v) is an instance of the principle developed in Zhu (1992) and in Chari, Christiano and Kehoe (1994): the capital tax in a stochastic setting can act as a perfect substitute for state-contingent debt of the type discussed in Lucas and Stokey (1983).

As in the monetary model of the previous section, the Ramsey policy can be sustained as the outcome in a reputation equilibrium in which the behavior of the government is disciplined by the threat of reversion to the repeated one-shot Nash equilibrium. In the latter equilibrium households have no incentive to invest. They hoard their goods endowment and all spending is financed with contemporaneous labor taxes. If spending is the labor tax rate is zero, $\tau^N_{\ell 1} = 0$. If spending is high the labor tax $\tau^N_{\ell 2} > 0$ is set at the minimum level needed to raise the required revenue g. Any capital tax policy that violates (9) can be used, but no revenue is collected from it.

Notice that there are temptations to deviate in both states of the world. In the low spending state there is a one-time gain from setting both tax rates to zero, and in the high spending state there is a one-time gain from using a large capital levy. But for β sufficiently close to one the Ramsey government resists both temptations.

3. Mixed types ($\lambda > 0$)

The equilibrium described above is valid for an economy in which it known with certainty that the government in office is a Ramsey government. Suppose instead that the government's type is i.i.d., and let λ be the probability of the 'other' type. Let mx and Rx denote values under myopic and Ramsey governments respectively in this mixed environment. We will look at equilibria in which households still have an incentive to invest in the mixed economy, so $\theta^x = 1$. If λ is not too large and β is sufficiently close to one, such equilibria exist.

The behavior of the myopic government is straightforward. In the low spending state it sets both tax rates to zero, $\tau_{k1}^{mx} = \tau_{\ell 1}^{mx} = 0$; and in the high spending state it raises all of the required revenue from a capital tax, setting the labor tax to zero, $\tau_{k2}^{mx} = g/\omega$ and $\tau_{\ell 2}^{mx} = 0$. The household's problem is as in (6)-(7). Since the labor tax is the same in both states, it follows immediately that consumption is equal in both states, $c_1^{mx} = c_2^{mx} = c^{mx}$. Labor supplies in the two states are then determined by (8).

In a world with a positive (but small enough) probability of a myopic government, the Ramsey government must alter its strategy, since otherwise households will not be willing to invest. Conditional on the myopic type holding office, the capital tax is $\tau_{k2}^{mx} = g/\omega$ with probability $(1 - \pi)$ and zero otherwise. Hence a household faces an expected utility loss of

$$L \equiv u'(c^{mx}) \left\{ \left[(1 - \pi) \frac{g}{\omega} - r \right] \omega \right\} > 0$$

if it invests its entire endowment. The assumption in (5) implies that the term in braces is positive. The Ramsey type must offset this loss by offering an expected gain when it is in office.

In particular, the Ramsey type must raise the subsidy on capital in the low spending state and/or cut the capital tax in high spending state so that, averaging over both types of government, the household faces a nonnegative expected rate of return. Thus, in the mixed economy with probability λ of a myopic government, the rate of return constraint for the Ramsey government is

$$\sum_i \pi_i u'(c_i^R)\omega\left(r - \tau_{ki}^R\right) \geq \frac{\lambda}{1-\lambda}L. \tag{10}$$

For $\lambda = 0$ this inequality reduces to the one in (9), but for $\lambda > 0$ the right side is positive and increasing in λ.

The problem of the Ramsey government in the mixed economy is as before, with (10) in place of (9). As shown in Appendix B, for any fixed $\lambda > 0$ the solution $\{\theta^{Rx}, \left(c_i^{Rx}, \ell_i^{Rx}, \tau_{ki}^{Rx}, \tau_{\ell i}^{Rx}\right), \ \ i = 1,2\}$ retains many of the qualitative features of the solution for $\lambda = 0$. Properties (i) - (iii) are unchanged: consumption and the labor tax are the same across the two states, and labor supply is higher by g in the second state. The analog of property (iv) says that (10) holds with equality. Property (v) continues to hold if λ is not too large. In principle, however, the Ramsey type might subsidize capital in both states if λ is large enough.

Changes in the probability of a myopic administration affect the allocation under the Ramsey government as one would expect: consumption c^{Rx} is decreasing in λ; the labor tax τ_ℓ^{Rx} is increasing in λ; and both capital taxes τ_{ki}^{Rx} are decreasing in λ. That is, the subsidy on capital in the low spending state is larger, and the tax on capital in the high spending state is smaller. Expected utility, conditional on a Ramsey government being in office, is decreasing in λ.

As λ rises the Ramsey government must increase the distorting labor tax to subsidize capital more heavily when spending is low and to finance a greater share of expenditure when spending is high. These costs are endured because there is a substantial gain to maintaining the incentives to invest.

4. A Policy Rule

Alternatively, society could adopt a simple Policy Rule mandating a cap on the capital tax that is low enough to insure that households have an incentive to invest. In our simple model the optimal cap is $\overline{\tau}_k = r/(1-\pi)$. Both the Ramsey and myopic types use the same policy under the Rule. In the zero spending state tax rates are zero, $\tau^u_{k1} = \tau^u_{\ell 1} = 0$. In the high spending state the capital tax is set at the mandated maximum, $\tau^u_{k2} = \overline{\tau}_k$, and the labor tax $\tau^u_{\ell 2} > 0$ at the lowest rate consistent with budget balance. Expected utility under this Policy Rule is not as high as under the Ramsey policy, but the Rule is robust against the blunders of the myopic government.

5. An example

In this section we will look at a simple numerical example that illustrates an important point: the difference in expected utility under the reputation equilibrium compared with the Policy Rule is quite modest, even if the government is certain to be the Ramsey type. In addition, expected utility in the reputation equilibrium declines as the probability of the myopic type rises, and eventually the Policy Rule dominates. By contrast, the expected utility gain from using the Rule rather than enduring the one-shot Nash outcome is very substantial. This result reflects the fact that the Rule was deliberately constructed to exploit a large potential gain, ignoring small ones.

Utility is logarithmic, $u(c) = a \ln c$, and the parameter values are

$$a = 10, \quad w = 1, \quad \omega = 3, \quad r = 0.2, \quad g = 2, \quad \pi = 1/2.$$

The discount factor β is assumed to be sufficiently close to one so that the reputation equilibrium exists.

Figures 5a - 5d display the equilibrium outcomes as the probability of the myopic type increases from 0% to 70%. Obviously, nothing happens to the policies or outcomes under the myopic type or under the Policy Rule. What do change are the policies adopted by the Ramsey type and the weighted averages in the economy with mixed types.

Figure 5a displays the tax rates. Under the myopic type the average capital tax rate is 33% (an average of 67% and 0%), well above the 20% rate of return on capital. The labor tax is zero. Under the Ramsey type the average capital tax is 20% (an average of 51% and -11%) with $\lambda = 0$ and declines monotonically as λ rises. The labor tax is positive and increases with λ, offsetting the declining revenues from the capital tax. The reason for this pattern is clear: the Ramsey type adjusts its policy to maintain the incentive for households to invest. Under the Policy Rule the average capital tax is 20% (an average of 40% and 0%), and the average labor tax is a little over 5%.

Figure 5b displays revenue from the capital tax. Recall that government expenditure is 2 or 0. Under the myopic type revenue from the capital tax exactly covers spending: it is 2 or 0, depending on the state, and the labor tax is not used. Under the Ramsey type, if $\lambda = 0$ revenue from the capital tax is 1.55 or -0.35, depending on the state. As λ increases, both figure decline (the subsidy in the zero spending state gets larger). Under the Policy Rule revenue from the capital tax is $\overline{\tau}_k \omega = r\omega / (1 - \pi) = 1.2$ or 0, and the labor tax is used when spending is high.

Figure 5c displays consumption. Consumption is the same in both states under the myopic or Ramsey types, since each type sets the same labor tax in both states. Consumption falls rather sharply under the Ramsey government as λ rises. This change is a direct consequence of the rising labor tax. Under the Policy Rule consumption differs in the two states, since the labor tax varies.

Figure 5d displays expected utility under the myopic and Ramsey governments, as well as the weighted average, and under the Policy Rule. The Rule delivers higher expected utility if $\lambda > 40\%$.

The figures for the one-shot Nash equilibrium are not displayed since they are—literally—off the charts. Households do not invest, so there is no interest income and all revenue must be raised from the labor tax. When $g = 0$, labor supply is 7 and consumption is 10. When $g = 2$, the labor tax is 40%, labor supply is 5, and consumption is 10. Expected utility is 14.5. This dismal outcome deters the Ramsey type from abandoning the reputation equilibrium for reasonable β values.

This simple model illustrates several points. The first is quantitative. A Policy Rule that is simple but well designed can capture much of the benefits available from commitment. Here the first-order effect comes from maintaining the incentive to invest, as can be seen by comparing expected utility under the Nash regime and under the Policy Rule. The simple Rule cannot capture the further gains available from implicit insurance, but these are much smaller. Indeed, they vanish altogether if λ is large enough.

In addition, the behavior of the Ramsey government in this simple model suggests that the political economy issues surrounding the reputational equilibrium cannot be neglected. Running for election on the Ramsey platform in this economy would be a difficult task indeed!

Finally, note that the damage a 'bad' government can inflict is much larger if capital is long lived. To keep the model here simple, capital was assumed to last for only one period. If capital is durable and expensive, 'bad' government behavior may have much worse consequences.

III. CONCLUSION

To conclude it is useful to touch on some issues that the two formal models do not address. We begin with issues related to the monetary model.

As noted above, and as many authors have emphasized, models like the one analyzed here have a vast multiplicity of equilibria. We compared monetary instruments by looking at the best equilibrium within a certain class. But why should we suppose that the best equilibrium is likely to arise? In addressing this very practical question, it is useful to keep in mind that many of the equilibria in these games have similar outcome paths. In particular, there are many equilibria in which the Bank plays the Ramsey strategy as long as its reputation is intact.

These equilibria differ in their description of how the Bank loses its reputation, resulting in a reversion to a 'bad' outcome, and in the precise description of the nature of the reversion. Here we assumed that one-shot Nash behavior prevailed during reversions and that the end of a reversion episode was linked to an observation of the signal, but neither feature is critical. For example, the most severe 'punishment' could be used instead of one-shot Nash. And even if one-shot Nash is used, the reversions could be of fixed length, of completely random length, allow returns to the 'good' state as a complicated function of current and past signals, etc. Indeed, the return probability could be interpreted as the (random) length of time required to reorganize the Central Bank or to install a new head of the Bank in office. As an empirical matter it would be very difficult to distinguish sharply among these equilibria. They differ only in their descriptions of reversion behavior, and reversions are (necessarily) rare.

More importantly, the model's description of behavior during a reversion episode seems better taken with several grains of salt. During 'good' times the Central Bank's behavior is stable and predictable. This is also roughly true in practise, and the model captures this behavior quite well. Reversions are not so precisely scripted, in reality or in the model. Choosing among reputation equilibria (on a theoretical level) and distinguishing among them (empirically) are equally difficult tasks. But they are also unimportant tasks, in the sense that the important aspect of behavior in the model is the robust feature shared by all the reputation equilibria, behavior during 'good' times.

Having argued that choosing among reputation equilibria is not terribly important, there remains the issue of how reputations are established. Formal models that permit reputation equilibria always have a large multiplicity of other equilibria as well. Indeed, simply repeating the 'bad' outcome is one possibility. As an empirical matter, countries with stable governments often manage to build and maintain reputations for 'good' behavior in areas where it matters most: the public debt is honored; capital taxes are stable and not too high; intellectual property rights are protected by patents, copyrights, and trademarks; and monetary policy is fairly stable. Reputations are central in explaining 'good' outcomes in settings like these, where the policy maker has substantial discretion, but on a theoretical level little is understood about how reputations are built, how credibility is established.[11]

Since theory provides little or no guidance here, it may be more fruitful to view this as an empirical issue. Perhaps this is the role of the Central Banker (an individual) as opposed to the Central Bank (an institution). A successful Central Banker is one who can steer the economy toward a 'good' equilibrium. Success requires that the Central Bank take the appropriate actions, but that is not enough. The Central Banker must convince the private sector that the Bank will behave that way. Indeed, he (or she) must persuade the private sector that there is a commonly held belief that the Bank will behave appropriately. Perhaps 'leadership' is the name we give to the elusive qualities that enable some individuals to succeed at this task.

Initially establishing a reputation for good behavior is a critical task for a Central Banker. Adopting a more observable instrument for conducting policy, pegging an exchange rate rather than using the money growth rate, may ease the Banker's task during the critical initial phase when he is attempting to establish a reputation for 'good' behavior. Establishing a Currency Board is another way to accomplish the same task, in the sense that it acts as an easy-to-monitor instrument for conducting monetary policy.[12]

Of course, in the long run monetary and fiscal policy are linked through the government's budget constraint. Good monetary policy is simply infeasible without a conservative (balanced budget) fiscal policy. A government that runs substantial deficits, with no prospect of surpluses to retire the accumulating debt, will eventually fail in its efforts to float new bonds issues. The problem is exacerbated if, as is typically the case, old debt must be rolled over as well. At some point the only feasible options are outright default, a large devaluation, or both. A government facing that situation typically find the seignorage revenue from a large devaluation too attractive to resist, and monetary policy becomes the fiscal policy of last resort.[13]

The model analyzed here focuses on an issue that is critical for some Central Banks, those for whom establishing and maintaining a reputation is a first priority. After that task has been largely accomplished, as it has been in the U.S., Japan and elsewhere, other issue take center stage: which targets should be used, which monetary aggregates should be given the greatest attention, etc. These are important issues, but only after a reputation for good conduct has been fairly well established. If a bridge is in danger of collapsing, there is little point in repairing potholes. Only after the structural problems have been addressed is it useful to think about the quality of the road surface.

The model of fiscal policy analyzed above illustrates one danger from allowing too much discretion. The myopic government in the model can be thought of as representing administrations subject to a variety of short-run political pressures, arising from many possible sources. The model highlights the fact that a well designed Policy Rule is one that pays attention to first-order effects (here, the incentive to invest), although it may neglect more subtle issues (here, the insurance available from a more subtle capital tax).

The model here has a representative household, but the same issue arises when there is heterogeneity. Differences among households create some divergence in views about fiscal policy, but if those differences are modest there may still be a fair amount of common ground. There may be a set of Policy Rules that are advantageous to all, even if there is disagreement about the *optimum optimorum*.

The essence of good government is to design institutions that permit solution of the repeated moral hazard problem. The goal of the models here has been to provide some insight into that problem, and how it affects decisions about policy regimes.

APPENDIX A

The set of simple two-state Markov equilibria for the money growth model is described in detail here. First the incentive compatibility (IC) constraints for the Central Bank are derived. Then the equilibria that minimize total expected discounted costs are characterized.

Suppose the economy is in the good state. Then households expect the Central Bank to permit money growth (net of real output growth) at the Ramsey rate, $\mu = \mu^g = 0$, so they set wages at $w^g = 0 + \alpha$. Households then observe the signal s and the actual inflation rate π. Households use a one-sided threshold to decide if the Bank has deviated. If the signal lies below the threshold, $s \leq S^g$, they assume that the Central Bank has behaved as anticipated, and the state next period is 'good.' Otherwise they assume the Central Bank has deviated, and the state next period is 'bad.' In equilibrium the Central Bank sets $\mu = \mu^g = 0$, so $s \leq S^g$ if and only if $\varepsilon \leq S^g$.

When the economy is in the bad state, households expect the Central Bank to set money growth at the Nash rate, $\mu = \mu^b = \mu^N$, so they set wages at $w^b = \mu^N + \alpha$. They then observe s and π. Household use a two-sided test in the 'bad' state, so the state next period is good if and only if the signal lies with the tolerance level set by the test, $s \in \left[\mu^N - \varepsilon^b, \mu^N + \varepsilon^b\right]$. Since the Central Bank sets money growth at the Nash rate μ^N, the signal s lies in the acceptable region if and only if $\varepsilon \in \left[-\varepsilon^b, \varepsilon^b\right]$.

In a two-state Markov equilibrium the expected discounted value of current and future losses from any period on depends only on the current state. Let c^g and c^b denote those expected values, and let

$$\Delta \equiv c^b - c^g,$$

denote the difference between the two.

Fix $S^g \geq 0$, and suppose that the economy is in the good state. The Ramsey rate of money growth $\mu^g = 0$ is incentive compatible for the Central Bank if and only if any other growth rate $\mu \neq 0$ leads to a (weakly) greater sum of current and future expected losses. If money grows at the rate μ, then the expected loss in the current period is $\Lambda(0, \mu)$. The state next period, and hence the payoffs from next period on, depend only on the signal s that is observed today. If money grows at the rate at μ and the signal s is observed, the error term was $\varepsilon = s - \mu$. Therefore, if S^g is the threshold for the accept region, the expected cost from next period on is

$$\begin{aligned} &c^g, \quad \text{if } \varepsilon \leq S^g - \mu, \\ &c^b, \quad \text{if } \varepsilon > S^g - \mu. \end{aligned}$$

Hence the Ramsey growth rate $\mu = 0$ is incentive compatible for the Central Bank in the good state if and only if

$$\begin{aligned} &\Lambda(0, \mu) + \beta \left\{ F\left(S^g - \mu\right) c^g + \left[1 - F(S^g - \mu)\right] c^b \right\} \\ &\quad \geq \Lambda(0, 0) + \beta \left\{ F\left(S^g - 0\right) c^g + \left[1 - F(S^g - 0)\right] c^b \right\}, \quad \text{all } \mu. \end{aligned}$$

Rearranging terms and using the definition of Δ, we can write this constraint as

$$\begin{aligned} \beta\Delta\left[F(S^g) - F(S^g - \mu)\right] &\geq \Lambda(0, 0) - \Lambda(0, \mu) \\ &= b\alpha\mu - \mu^2, \quad \text{all } \mu. \end{aligned} \tag{11}$$

The expression on the left side of (11) is zero at $\mu = 0$. Under Assumption 1 it is continuous and increasing in μ, convex for $\mu < S^g$, and concave for $\mu > S^g$. The expression on the right side is also zero at $\mu = 0$, and from (1) we see that it is increasing for $\mu < b\alpha$, decreasing for $\mu > b\alpha$, and everywhere concave. Hence (11) holds near $\mu = 0$ if and only

$$\beta\Delta f(S^g) \geq b\alpha. \tag{12}$$

Condition (12) is the basic IC constraint that equilibria must satisfy. The interpretation is straightforward: $b\alpha$ is the marginal gain from increasing the expected inflation rate in the current period, $f(S^g)$ is the marginal increase in the probability of reversion to the bad state from that change, and $\beta\Delta$ is the discounted expected loss if a reversion occurs.

Similarly, the Nash rate of money growth, $\mu^b = \mu^N$, is incentive compatible for the Central Bank in the bad state if and only if any other growth rate $\mu \neq \mu^N$ leads to (weakly) greater expected losses. Hence the required condition is

$$\beta\Delta\left\{\left[F\left(\varepsilon^b\right) - F\left(-\varepsilon^b\right)\right] - \left[F\left(\left(\mu^N - \mu\right) + \varepsilon^b\right) - F\left(\left(\mu^N - \mu\right) - \varepsilon^b\right)\right]\right\}$$
$$\geq \Lambda(\mu^N, \mu^N) - \Lambda(\mu^N, \mu), \quad \text{all } \mu.$$

Under Assumption 1 the term on the left is strictly positive for all $\mu \neq \mu^N$, and it follows immediately from the definition of μ^N that the term on the right is negative for all $\mu \neq \mu^N$. Hence any value $\varepsilon^b \geq 0$ satisfies the incentive constraint for the bad state.

For a an equilibrium characterized by $\left(S^g, \varepsilon^b\right)$, the probability of a reversion to the bad state (along the equilibrium outcome path) is $p = 1 - F(S^g)$, and the probability of a return to the good state is $q = \left[F(\varepsilon^b) - F(-\varepsilon^b)\right]$. Under Assumption 1 the relationship between the thresholds $\left(S^g, \varepsilon^b\right)$ and the probabilities (p, q) is invertible, so we can formulate the problem in terms the latter. Thus, the next step is to solve for Δ in terms of the probabilities (p, q).

Suppose (p,q) is an equilibrium pair. In equilibrium, the money growth in the good state is $\mu^g = 0$, and the probability of an accidental reversion is p. Hence the expected discounted value of current and future losses satisfies the recursive relation

$$c^g = \Lambda(0,0) + \beta\left[c^g + p\Delta\right]. \tag{13}$$

Similarly, the money growth rate in the bad state is $\mu^b = \mu^N$, and $1-q$ is probability of remaining in the bad state. Hence c^b satisfies the recursive relation

$$c^b = \Lambda(\mu^N, \mu^N) + \beta\left[c^g + (1-q)\,\Delta\right].$$

The difference between the two is

$$\begin{aligned} \Delta(p,q) &= \frac{\delta}{1-\beta\,(1-p-q)} \\ &= \delta\psi(p,q), \end{aligned} \tag{14}$$

where $\delta = \Lambda(\mu^N, \mu^N) - \Lambda(0,0)$.

Since $f(S^g) = \gamma(p)$, it then follows from (12) that the Ramsey rate of money growth is incentive compatible in the good state for any $p, q \in [0,1]$ satisfying

$$\gamma(p)\beta\delta\psi(p,q) \geq b\alpha. \tag{15}$$

Since f is symmetric, so is γ. Hence if (15) holds for $\hat{p} \geq 1/2$, it also holds for $p = 1-\hat{p} \leq 1/2$, and we can limit attention to p values in the upper half of the distribution, $p \in (0, 1/2]$.

Hence if the density $f(\varepsilon)$ satisfies Assumption 1, there exists a simple two-state Markov equilibrium for any pair $p \in (0,1/2]$ and $q \in [0,1]$ satisfying (15). Since $\psi(p,q)$ is decreasing in q, for fixed p, if (15) holds anywhere, it holds for $q = 0$, establishing parts (a) and (b) of the Proposition.

Substituting from (14) into (13), we find that expected cost per period in the (p,q) equilibrium is

$$(1-\beta)\,c^g(p,q) = \Lambda(0,0) + \beta p\delta\psi(p,q).$$

Hence expected costs are minimized if and only if (p, q) solves (3), establishing part (c) of the Proposition.

APPENDIX B

Ramsey behavior for the tax model is characterized here. The cases $\lambda = 0$ and $\lambda > 0$ are similar and can be treated together.

It is convenient first to reformulate the problem as one of choosing the allocation $\{(c_i, \ell_i)\,,\ i = 1, 2\}$ and the capital tax rates $\{\tau_{ki},\ i = 1, 2\}$. To this end, note that the condition for the consumer's maximum is

$$(1 - \tau_{\ell i})\, w u'(c_i) - 1 = 0, \quad i = 1, 2. \tag{16}$$

Multiply the budget constraint (7) by $u'(c_i)$ and substitute from (16) to get

$$u'(c_i)\left[c_i - (1 + r - \tau_{ki})\,\omega\right] - \ell_i = 0, \quad i = 1, 2. \tag{17}$$

The constraints for the Ramsey government's problem are (8), (9) or (10), and (17). Let $\pi_i\mu_i$, ϕ, and $\pi_i\lambda_i$, be the multipliers for the three constraints. Then the conditions for a maximum are

$$\begin{aligned} 0 &= u_i' + \lambda_i \left[u_i'' \cdot \left[c_i - (1 + r - \tau_{ki})\,\omega\right] + u_i'\right] - \mu_i - \phi u_i'' \omega \left[\tau_{ki} - r\right], \\ 0 &= -1 - \lambda_i + \mu_i w, \\ 0 &= (\lambda_i - \phi)\, u_i' \omega, \qquad\qquad i = 1, 2. \end{aligned}$$

The last two lines imply

$$\lambda_i = \phi, \quad \text{and} \quad \mu_i = \frac{1 + \phi}{w}, \quad i = 1, 2.$$

Then substituting into the first line gives

$$(1 + \phi)\left(u_i' - \frac{1}{w}\right) + \phi\,(c_i - \omega)\, u_i'' = 0, \quad i = 1, 2,$$

which suggests a solution with the same private consumption level in the two state, $c_1 = c_2 = c^R$. The remaining task is to find values for $\left(c^R, \left\{\ell_i^R, \tau_{ki}^R,\ i = 1,2\right\}\right)$ that satisfy the constraints, (8), (9) or (10), and (17).

Use the market clearing condition (8) to write labor supply in the two states as

$$\ell_i^R = \frac{1}{w}\left[c^R - (1+r)\,\omega + g_i\right], \quad i = 1, 2. \tag{18}$$

Then use this fact and the budget constraint (17) to obtain

$$\tau_{ki}^R \omega u' = (1 - wu')\,\ell_i^R + g_i u', \quad i = 1, 2. \tag{19}$$

The rate of return constraint (9) or (10) holds with equality.

Since consumption is the same in the two states, the rate of return constraint is

$$u'(c^{Rx})\omega\left(r - \Sigma_i \tau_{ki}^{Rx}\right) \geq \frac{\lambda}{1-\lambda} L.$$

Notice that since consumption is different under the two types of government, expected returns must be weighted by marginal utilities as well as probabilities.

Hence

$$\omega u' \Sigma_i \pi_i \tau_{ki}^R = \omega u' r - \frac{\lambda}{1-\lambda} L. \tag{20}$$

Taking the probability-weighted sum of the two equations in (19), substituting from (20), and using the fact that $\ell_2 = \ell_1 + g/w$, we obtain

$$r\omega u' - \Lambda = (1 - wu')\,\ell_1^R + \pi_2 \frac{g}{w}.$$

Using (18) we find that c^R satisfies

$$r\omega u' - \Lambda = \left(\frac{1}{w} - u'\right)\left[c^R - (1+r)\,\omega\right] + \pi_2 \frac{g}{w}.$$

The labor supplies ℓ_1^R, ℓ_2^R can then be determined from (18), the capital tax rates from (19), and the labor tax—which is the same in both states—from (16).

REFERENCES

[1] Atkeson, A. and P.J. Kehoe (2001) The advantage of transparent instruments of monetary policy. Federal Reserve Bank of Minneapolis Staff Report 297.

[2] Barro, R.J. and D.B. Gordon (1983) Rules, discretion and reputation in a model of monetary policy. *Journal of Monetary Economics* 12, 101-121.

[3] Canzoneri, M.B. (1985) Monetary policy games and the role of private information. *American Economic Review 75, 1056-1070.*

[4] Chari, V.V., L.J. Christiano, and P.J. Kehoe (1994) Optimal fiscal policy in a real business cycle model. *Journal of Political Economy* 102, 617-652.

[5] Chari, V.V. and P.J. Kehoe (1990) Sustainable plans. *Journal of Political Economy* 98, 783-802.

[6] Chari, V.V., P.J. Kehoe, and E.C. Prescott (1989) Time consistency and policy. In R.J. Barro, ed., *Modern Business Cycle Theory*, Harvard University Press.

[7] Clarida, R., J. Gali, and Mark Gertler (1999) The science of monetary policy: a new Keynesian perspective. *Journal of Economic Literature 37, 1661-1707.*

[8] Cole, H. and L. Ohanian. 2001. New Deal policies and the persistence of the Great Depression. Federal Reserve Bank of Minneapolis.

[9] Faust, J. and L.E.O. and Svensson (2001) Transparency and credibility: monetary policy with unobserved goals. *International Economic Review* 42.

[10] Fischer, S. (1980) Dynamics inconsistency, cooperation and the benevolent dissembling government. *Journal of Economic Dynamics and Control* 2, 91-108.

[11] Friedman, M. (1948). A monetary and fiscal framework for economic stability. *American Economic Review* 38, 245-264. Reprinted in M. Friedman, *Essays in Positive Economics*, University of Chicago Press, 1953.

[12] Goodfriend, M. (1986) Monetary mystique: Secrecy and central banking. *Journal of Monetary Economics* 17, 63-92.

[13] Green, E.J., and R.H. Porter (1984) Noncooperative collusion under imperfect price information. *Econometrica* 52, 87-100.

[14] Ireland, P.N. (1997) Sustainable monetary policies. *Journal of Economic Dynamics and Control* 22, 87-108.

[15] ———. (1999) Does the time-consistency problem explain the behavior of inflation in the United States? *Journal of Monetary Economics* 44, 279-291.

[16] Kydland, F.E. and E.C. Prescott (1977) Rules rather than discretion: the inconsistency of optimal plans. *Journal of Political Economy* 85, 473-491.

[17] Lucas, R.E. and N.L. Stokey (1983) Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics 12, 55-93.*

[18] McCallum, B.T. (1999) Issues in the design of monetary policy rules. In J.B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*, Elsevier.

[19] Persson, T. and G. Tabellini (1994) Introduction. In *Monetary and Fiscal Policy, Volume 1: Credibility*. M.I.T. Press.

[20] Phelan, C. (2001) Public trust and government betrayal. Federal Reserve Bank of Minneapolis working paper.

[21] Poole, W. (1970) Optimal choice of monetary instruments in a simple stochastic macro model. *Quarterly Journal of Economics* 84, 197-216.

[22] Rogoff, K. (1989) Reputation, coordination, and monetary policy. In R.J. Barro (ed.), *Modern Business Cycle Theory* 39, 83-96.

[23] __________. (1987) Reputational constraints on monetary policy. *Carnegie-Rochester Series on Public Policy* 26, 141-182.

[24] __________. (1985) The optimal degree of commitment to a monetary target. *Quarterly Journal of Economics* 100, 1169-1190.

[25] Stokey, N.L. (1991) Credible public policy. *Journal of Economic Dynamics and Control* 15, 627-656.

[26] Zarazaga, C.E.J.M. (1995) Recurrent hyperinflations in a dynamic game with imperfect monitoring in the appropriation of seignorage. Working paper, Federal Reserve Bank of Dallas.

[27] Zhu, Xiaodong. (1992) Optimal fiscal policy in a stochastic growth model. *Journal of Economic Theory* 58, 250-189.

FOOTNOTES

*I am grateful to Fernando Alvarez, V.V. Chari, Patrick Kehoe, Robert Lucas, Roger Myerson, and Christopher Phelan for useful discussions, and to the Research Division of the Federal Reserve Bank of Minneapolis for hosting me during a visit where much of this work was done.

[1]If the government is not benevolent, if it has objectives different from those of the private sector, the same incentive to mislead can arise. But with conflicting objectives that is unsurprising. What is astonishing about Kydland and Prescott's examples is that all parties seem to share the same objectives. This appearance is somewhat illusory, in the sense that the payoffs of the private sector agents are symmetric, not

identical: the private sector is not a 'team.' See Chari, Kehoe and Prescott (1989) for a more detailed discussion.

[2]Chari and Kehoe (1990) and Stokey (1991) offer two slightly different general frameworks.

[3]See Rogoff (1987, 1989) for an excellent survey of the early literature, Persson and Tabellini (1994) for a discussion of many applied questions, and McCallum (1999) for a discussion of issues related to monetary policy.

[4]There are many examples of policy that was arguably well intentioned but surely misguided. Cole and Ohanian (2001) argue that the National Labor Relations Act was important in prolonging the Great Depression by keeping wage rates too high. The inflationary episodes experienced in many countries during the 1970's may offer a another. See Ireland (1999) and Clarida, Gali and Gertler (1999) for a further discussion.

Phelan (2001) offers an interesting model in which a government that is greedy, but also intelligent and patient, may for long episodes behave like one that is benevolent.

[5]Observability is different from what most authors call transparency. In discussions of monetary policy the latter is typically used to refer to the clarity with which the private sector can observe the Central Bank's *objectives*. The term observability will be used here to refer to the clarity with which the private sector can observe the Bank's *actions*.

[6]Of course, many other issues affect this choice as well. For example, in an early contribution Poole (1970) focusses on the sources of shocks.

[7]To incorporate serial correlation in the shocks, define μ and e to include expected changes in velocity and foreign inflation, respectively. Then v and ζ, interpreted respectively as innovations in velocity and exchange rate depreciation, are serially uncorrelated and have means of zero.

With serial correlation we must also ask whether it remains reasonable to assume that e is perfectly observable. If the foreign rate of inflation is serially correlated, then e is observable only if $\mathrm{E}\left[\pi^f\right]$, the Central Bank's forecast of the foreign inflation rate, is observable. If the Central Bank announces its estimate $\mathrm{E}\left[\pi^f\right]$ each period, and if the private sector can verify this forecast independently, then the model goes through without change. As Goodfriend (1986) notes, a Central Bank's main forecasting advantage derives from its earlier access to data. But presumably the domestic Central Bank has little advantage in acquiring the data relevant for the foreign inflation rate. Hence the assumption that the private sector can verify the Bank's announcement seems reasonable.

[8]Our second model of money growth is similar in spirit to Canzoneri's (1985) model. There the Central Bank was assumed to have private information about a velocity shock; here it has private information about real output growth.

[9]See Ireland (1997) for a more detailed justification of a similar payoff function.

[10]If there are equilibria with $p > 1/2$, then there are also equilibria with $p \leq 1/2$, and the latter have lower costs. Hence we focus on them.

[11]See Faust and Svensson (2001) for an interesting exception.

[12]Rogoff (1985) suggests an intriguing solution: simply appointing a Central Banker who places more weight on price stability.

[13]See Zarazaga (1995) for a very interesting model in which episodic bouts of high inflation occur when decentralized fiscal policy is combined with centralized monetary policy.

Figure 1a: Money growth regime

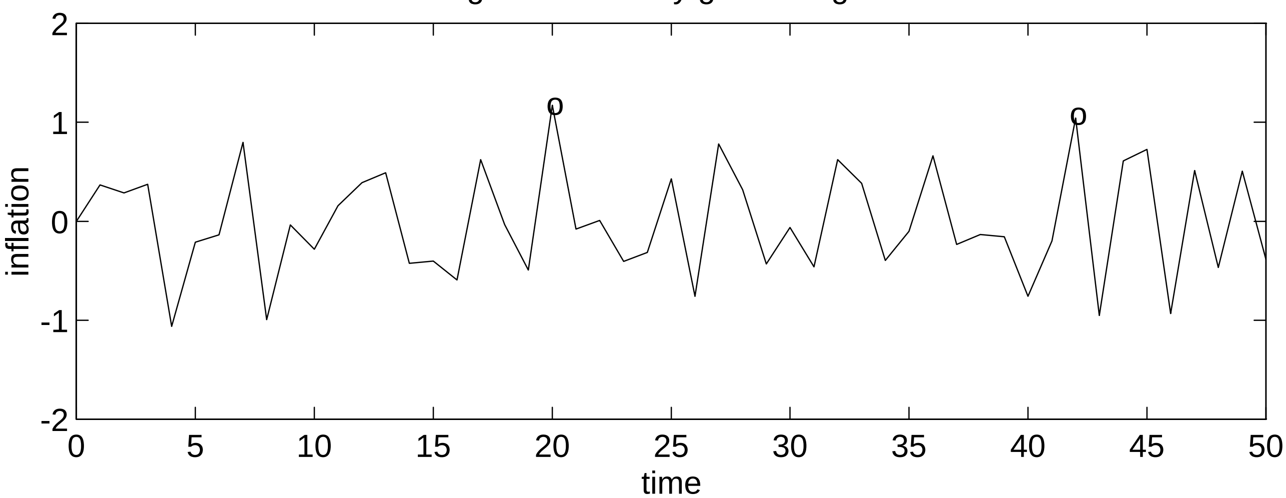


Figure 1b: Exchange rate regime

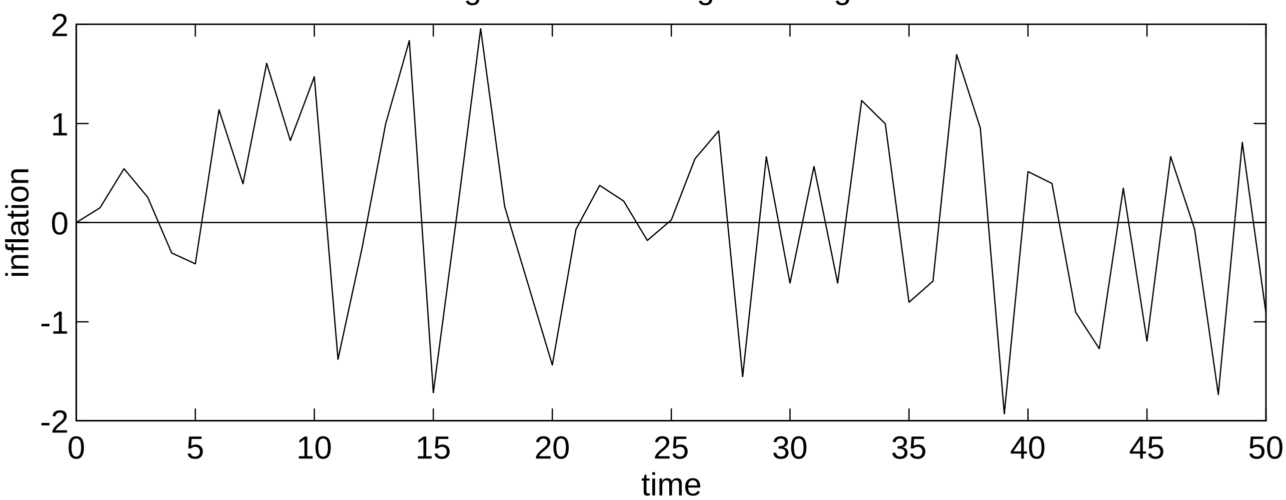


Figure 2: Equilibrium reversion probabilities

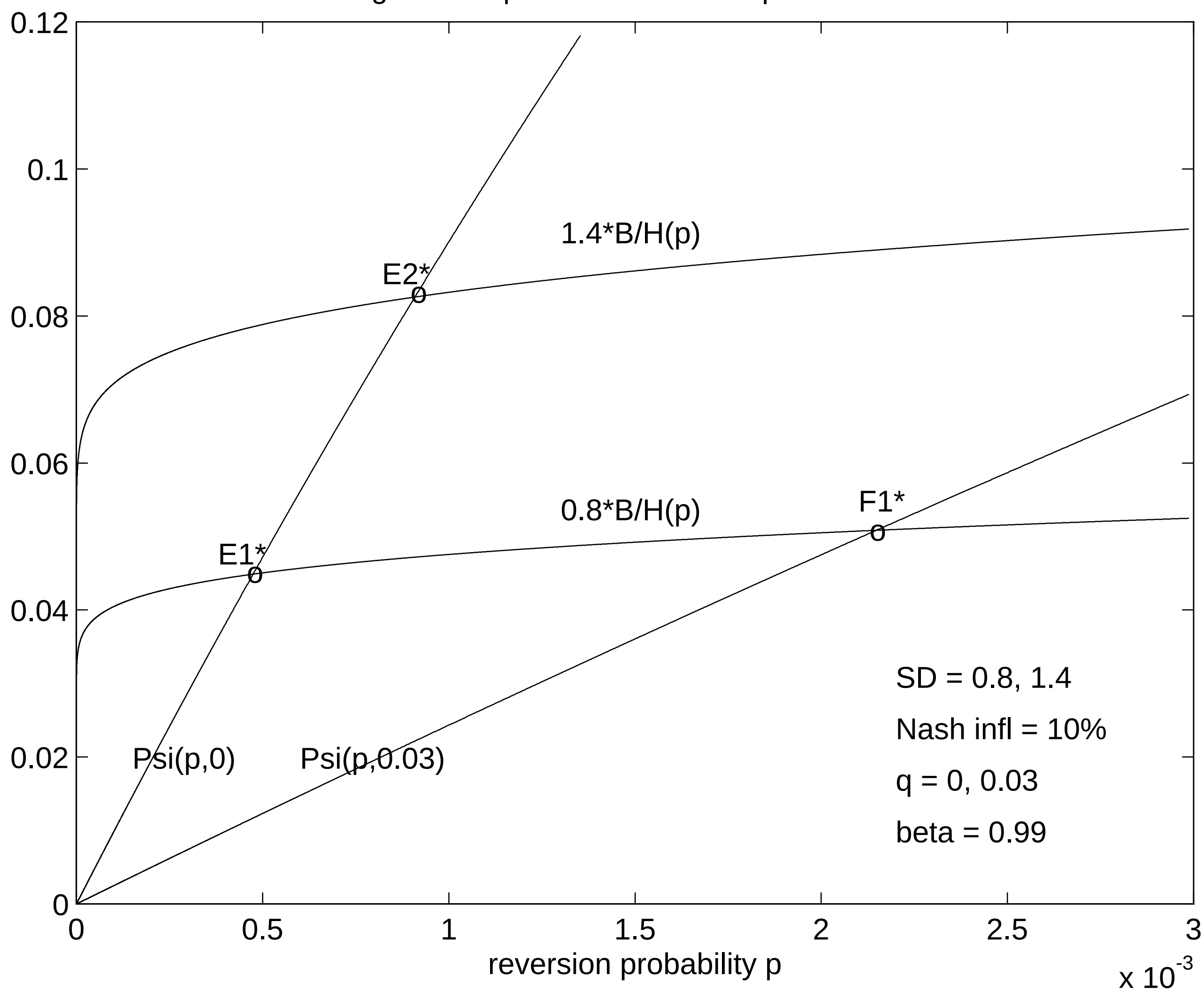
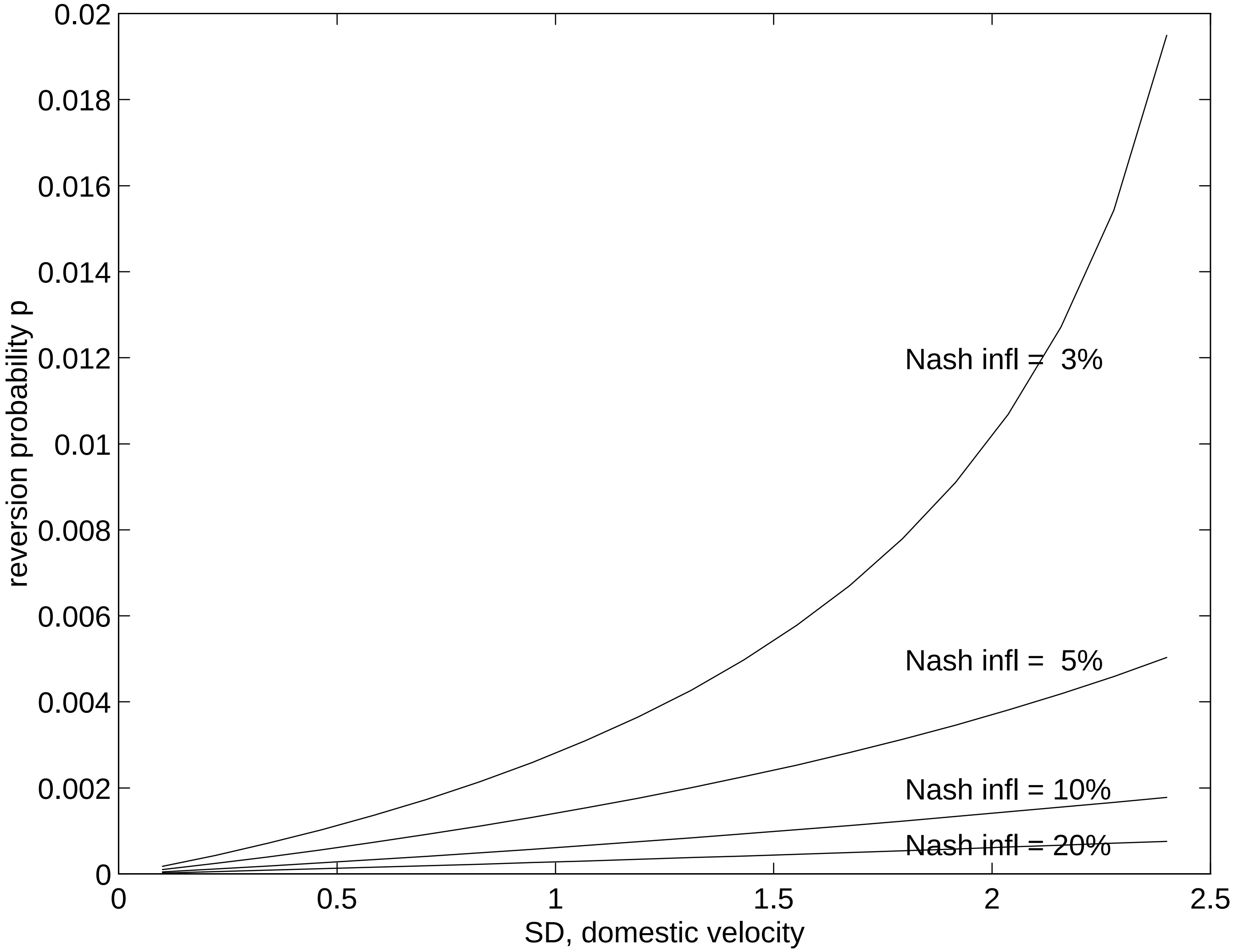


Figure 3a: Optimal reversion probabilities



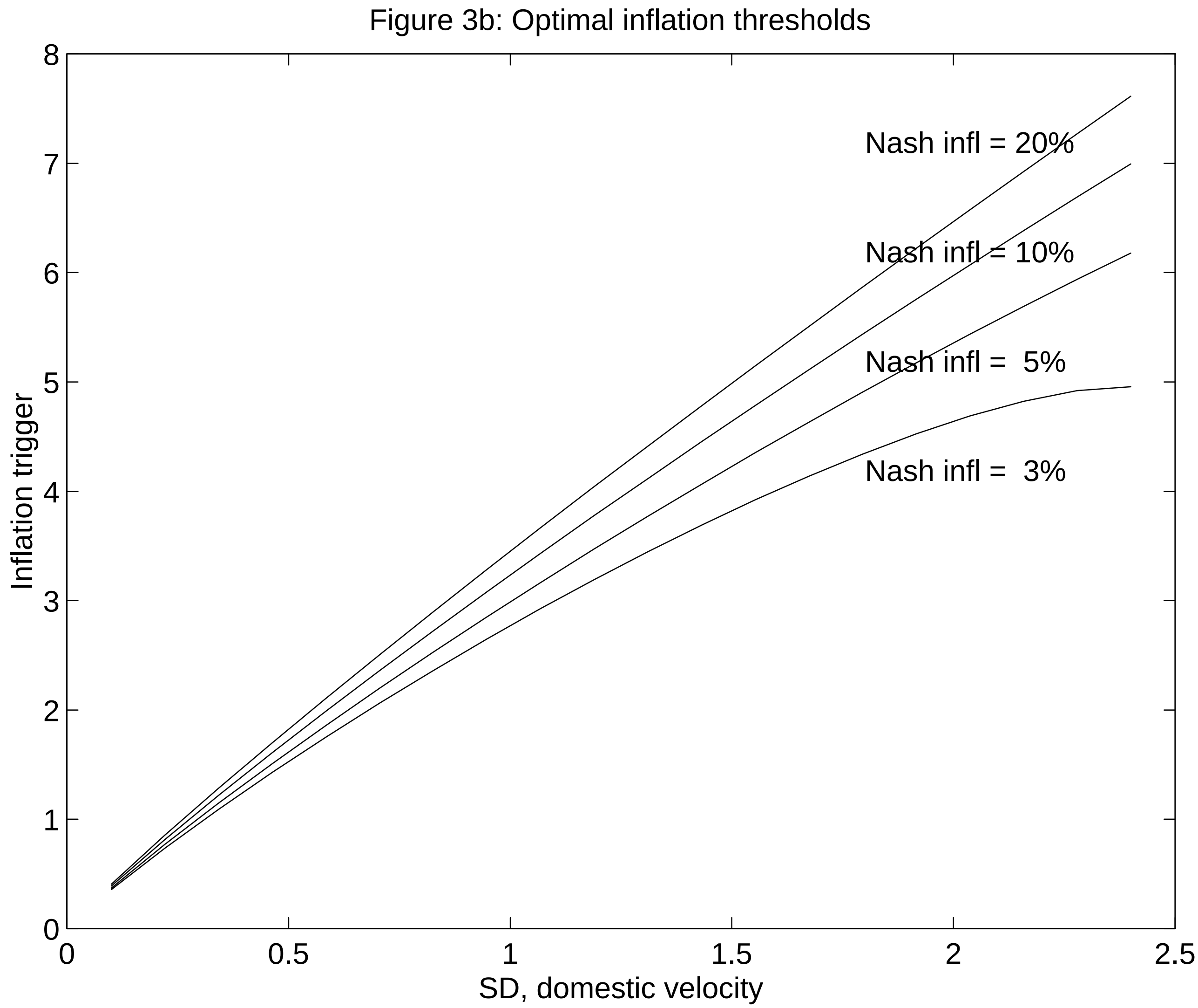


Figure 3b: Optimal inflation thresholds
Nash infl = 20%
Nash infl = 10%
Nash infl = 5%
Nash infl = 3%
Inflation trigger
SD, domestic velocity
0
1
2
3
4
5
6
7
8
0
0.5
1
1.5
2
2.5

Figure 4: Instrument choice

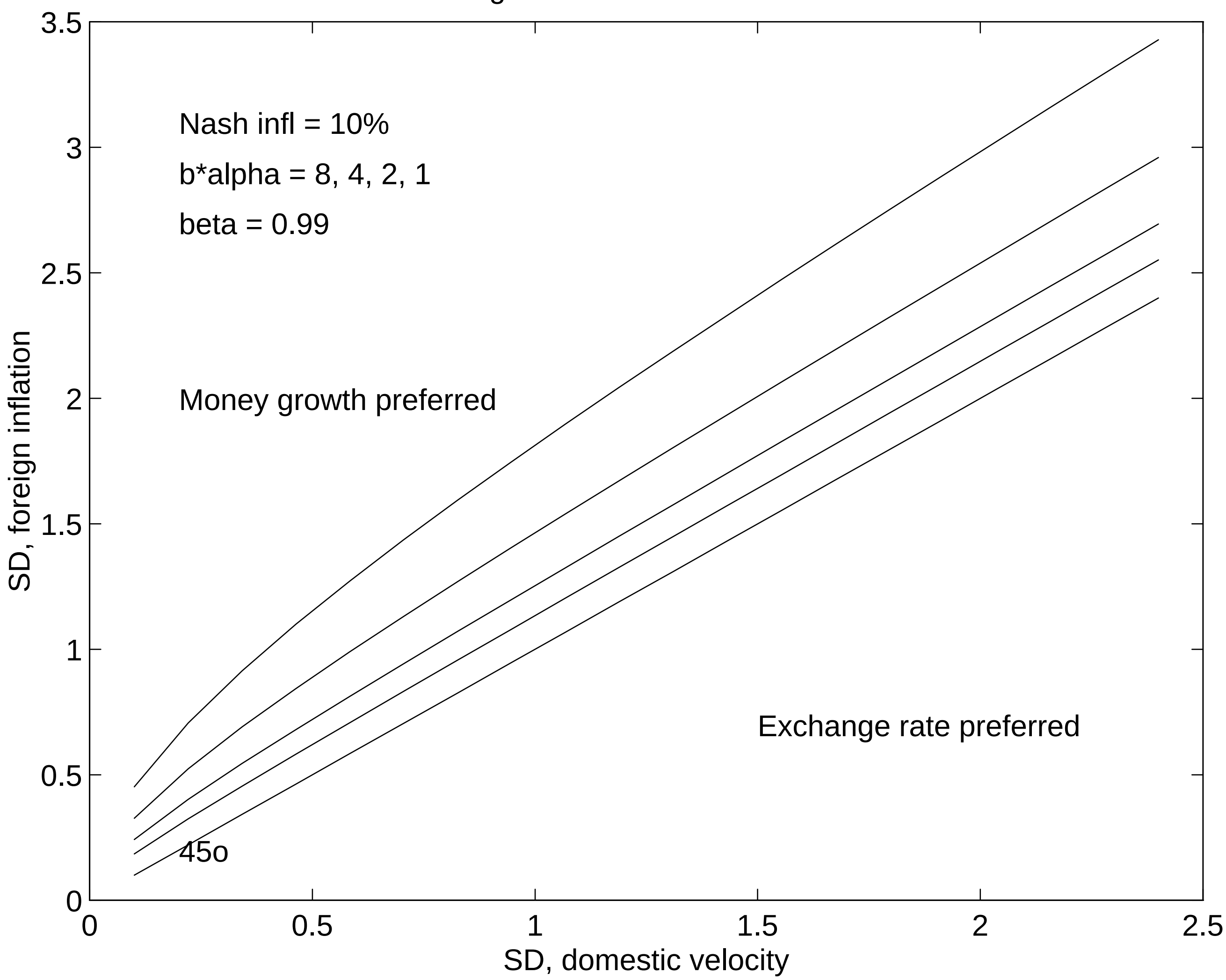


Figure 5a: Tax rates

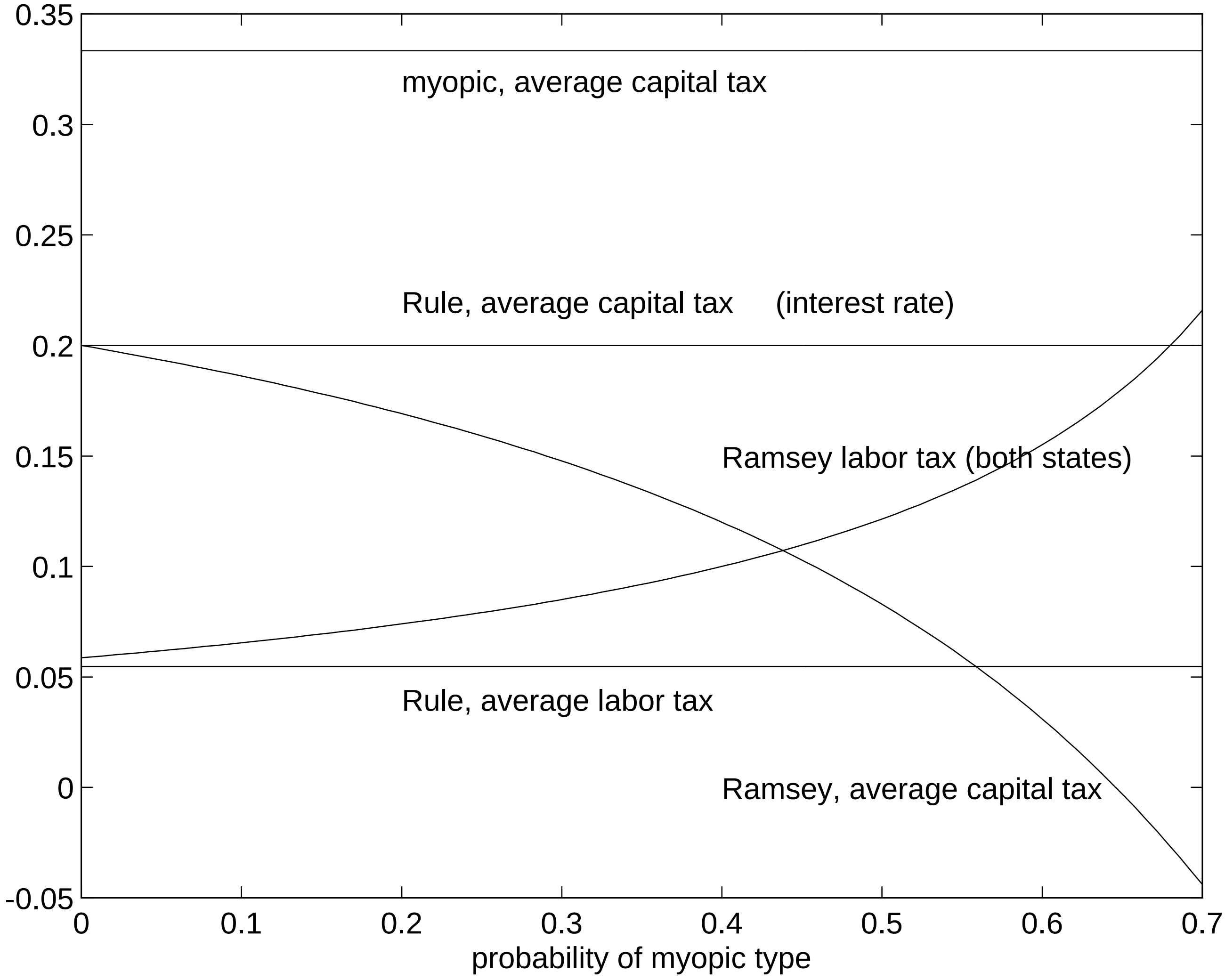


Figure 5b: Capital tax revenue

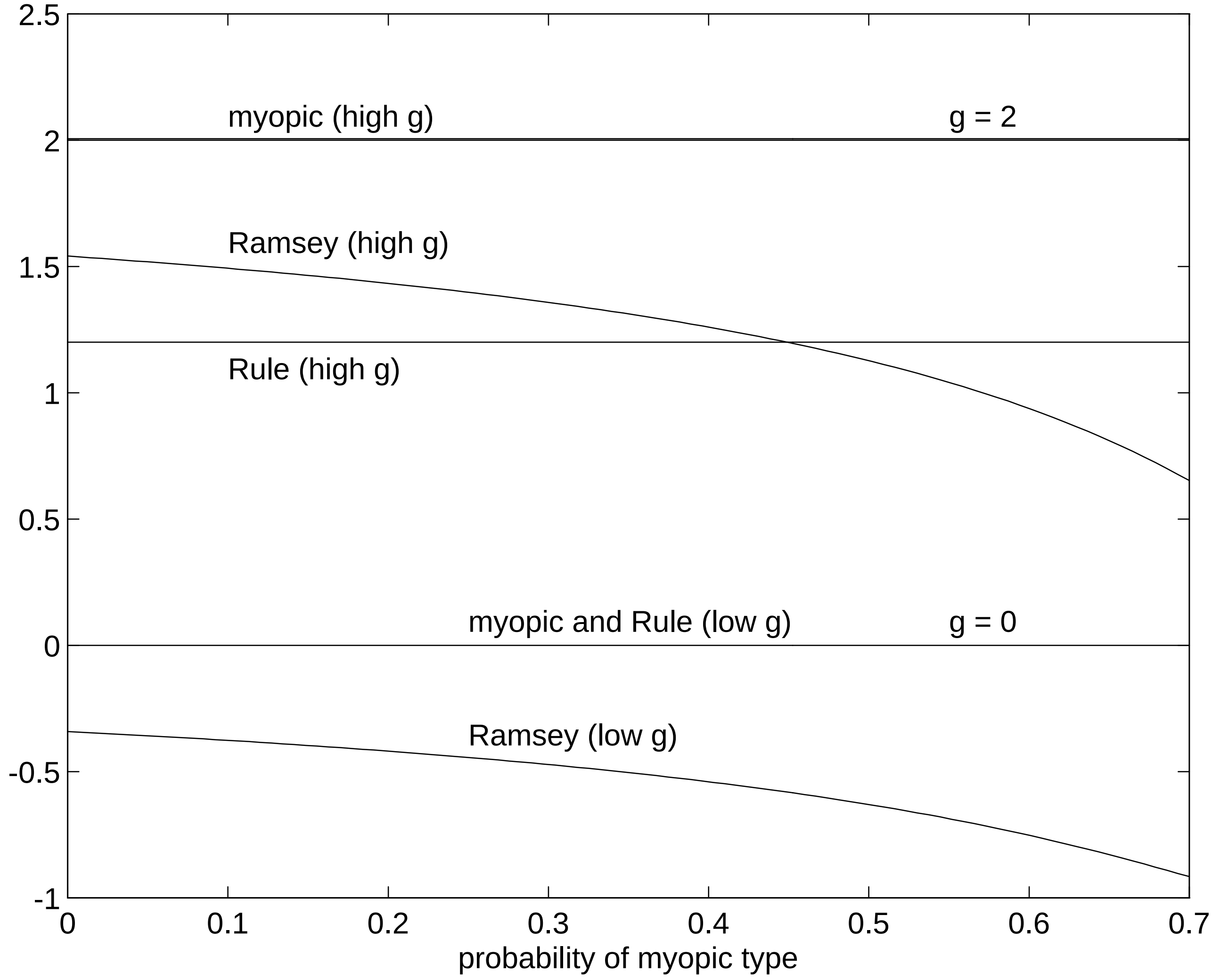


Figure 5c: Consumption

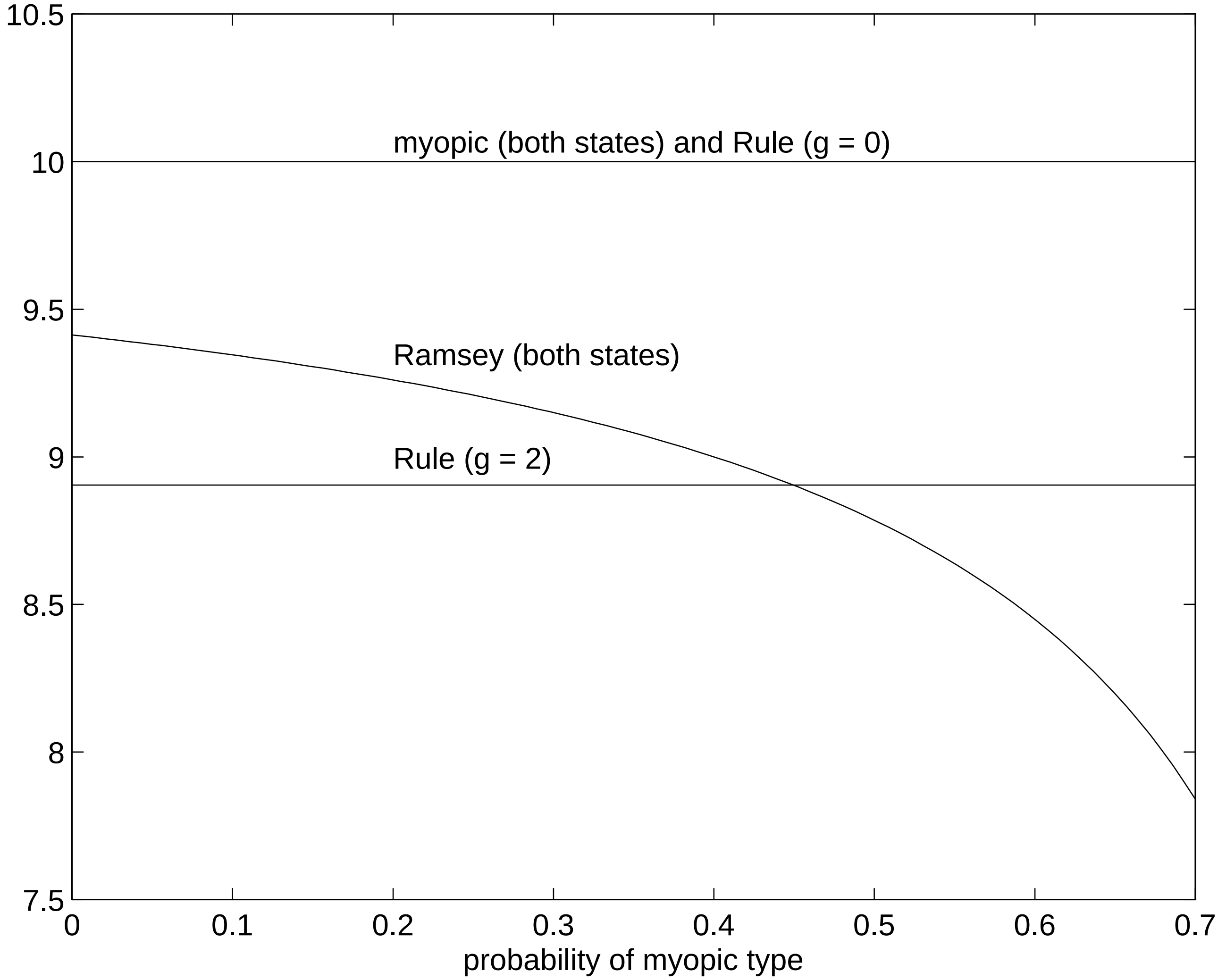


Figure 5d: Expected utility