Until Death Do Us Part?
The Marriage Model with Divorce

Extended Abstract

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We construct an economic model to show that, despite its negative image in society, divorce can increase aggregate welfare. We start with the standard two-sided search and matching model, where agents on both sides of the market are heterogeneous in their quality and each agent must search for a partner from the other side of the market in order to obtain positive utility. Search is costly in that agents are impatient and prefer an immediate match to one that occurs later in life. Each agent's payoff is increasing in the partner's quality. Furthermore, the payoffs are supermodular: the increase in agent X's payoff that results from matching to a higher-quality partner is higher when agent X's own quality is higher. Utility is non-transferable: no payments can be made between agents. We add to this standard model that agents can continue to search while matched, divorce, and rematch if they find a better partner. Our main result is that the ability to separate results in a steady-state matching pattern that is more efficient than that observed in a model without divorce.

To see concretely how divorce improves matching and increases efficiency, first consider the world without divorce. Because matches are permanent and waiting is costly, even the highest-quality agents accept an entire range of partners, not just ones of the highest-quality. In the resulting equilibrium, agents of a given quality will therefore be matched to a set of partners of varying qualities. Consequently, there will be unequal matches. But if all parties can search while matched, such matches will not survive in equilibrium. Because the quality distribution and the total mass of agents is the same on both sides of the market, for each woman W_H of quality higher than x who is matched to a man of quality less than x, there must be a man M_H of quality higher than x who is matched to a woman of quality less than x. Eventually, our high-quality friends W_H and M_H will meet each other, and, since both prefer each other to their current partners, they will divorce and marry each other instead. Divorces and remarriages will continue until everyone is matched to their own type. At this point, no further divorces will occur: if a man of quality x (an x-man) is married to a y-woman, he would be willing to divorce her only for women of quality $y > x$, but all of these women are already happily married to spouses worthier than our x-man! Comparing the resulting matching pattern to the no-divorce equilibrium, we see that some people (the higher-quality ones previously married to lower-

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quality partners) are now better off, while others (the lower-quality ones who were previously lucky to have higher-quality spouses) are worse off. However, because of supermodularity in payoffs, the total utility gain of the higher-quality agents marrying up exceeds the utility loss of the lower-quality agents marrying down.

To understand both the roots of our model and the contributions that the model makes, it is necessary to take a short detour into the history of the economic study of matching. This literature began with a seminal paper by Gale and Shapley [1962], in which they demonstrated that certain marriage markets always allow "stable" matchings, in which no woman and man both prefer each other to their current spouses; this equilibrium concept is equivalent to the core in this situation (as noted by Shapley and Shubik [1972]).[1] One of the key insights in this class of matching models is due to Becker [1973]: when the payoff function is supermodular (i.e., there are complementarities in production), the only efficient allocation, and hence the unique core allocation, is the one where each agent matches only to her own type. We call this matching pattern *perfect positively assortative matching* (perfect PAM).

Models such as these typically examine the core, or perhaps competitive equilibria, of matching markets. To achieve these outcomes, the models tend to rely on a massive clearinghouse, which solicits preference reports from every agent on each side and constructs a matching according to some algorithm. In most real-life settings, however, no equivalent of such a clearinghouse exists. Instead, agents must engage in costly search over time.

This more complex (and arguably more realistic) setting of matching with search has been explored in a large number of papers in the 1990s, starting with the seminal paper of McNamara and Collins [1990] and culminating with the insightful papers by Shimer and Smith [2000, with transferable utility] and Smith [2006, with nontransferable utility]. Like our paper, these papers explore the steady state of the system, similar to the standard approach in evolutionary game theory, where the pivotal concept is that of an evolutionarily stable strategy (ESS), as introduced by Smith and Price [1973].[2]

When utility is nontransferable, papers in this literature commonly find the phenomenon of *block segregation* to which we alluded earlier: agents on each side of the market separate themselves into fixed bands or intervals of quality, such that an agent in a given band can be matched with any agent from the corresponding band on the other side, but to no agent from any other band. In particular, this implies that a small change in an agent's quality results either in no change or a discontinuous jump in the set of possible partners for that agent. This peculiar discontinuity in the otherwise continuous matching model was first discovered by McNamara and Collins [1990] and was subsequently revisited by authors such as Bloch and Ryder [2000]; Burdett and Coles [1997]; Chade [2001]; Eeckhout [1999]; Morgan [1998]. Smith [2006] was the first to use a general payoff function to show that block segregation arises with any payoff function that is multiplicatively separable in the two partners' payoffs, and to show *why* this phenomenon occurs.

Block segregation has always raised criticism because it simply does not seem to fit with our sense of

[1] Marriage is a standard interpretation of the matching model. It considerably simplifies the terminology, without loss of generality. An alternative (but equivalent) interpretation is a labor market, in which firms and workers seek to match with one another. Such markets have been studied by Crawford and Knoer [1981]; Demange and Gale [1985]; Kelso and Crawford [1982]; Shapley and Shubik [1972], often with the additional element of a monetary transfer between firms and workers (namely, a salary). Roth and Sotomayor [1990] constructed a thorough discussion of two-sided matching theory, with particular attention to the empirical case of the National Resident Matching Program (NRMP).

[2] A full dynamic analysis of the system is unfortunately exceedingly complex. For an idea of what happens out of steady state, see the biology paper by Alpern and Reyniers [2005] and the ongoing project by Smith [2002].

reality; Smith [2006, page 1134] writes that "there are no documented cases of block segregation." In addition, a discontinuous equilibrium in an entirely continuous model is in and of itself suspicious. It should also be noted that when payoffs are supermodular, block segregation is clearly inefficient. Both the implausibility and the inefficiency of the phenomenon prompt us to look for conditions that would eliminate this outcome.

Smith [2006] provides one set of conditions under which block segregation disappears and a more efficient matching pattern emerges. Because the set of potential partners for each agent in a model with costly search is a non-singleton set, the concept of perfect assortativeness is of no direct use as a measure of the degree of sorting in such a model. To extend the idea of "like matches to like" to this setting, Shimer and Smith [2000] introduce the concept of setwise positively assortative matching (setwise PAM): in setwise PAM, higher agents match to higher sets of agents. Smith [2006] shows that a sufficient condition for matching to be strictly assortative setwise is that the payoff function be strictly log-supermodular (note that strict supermodularity is not sufficient). When payoffs are multiplicatively separable (the borderline case between log-supermodularity and log-submodularity), block segregation obtains (which is only weakly positively assortative setwise).

Note, however, that the requirement on payoffs for strict setwise PAM is quite strong: the fact that strict supermodularity is not sufficient is quite troubling, and it is not entirely clear how to justify strict log-supermodularity conceptually or how to distinguish between log-supermodularity and simple supermodularity empirically. In addition, the sorting resulting from strict setwise PAM can be very weak and thus still highly inefficient. This prompts us to look for a different set of conditions that could improve efficiency in the search model. Noting that the inefficiency arises from the need of agents to weigh the benefits of marrying someone immediately against the opportunity cost of giving up the search for someone better in the future, it is natural to suspect that the situation could be improved if agreeing to match with someone did not entail giving up search for better matches in the future. This points toward the possibility of continued on-the-match search and divorce as a vehicle for improvement.

Our model builds on the seminal work of Smith [2006]. There are two sides of the market, men and women, and each agent has a given level of quality.[3] Every man prefers higher quality women and every woman prefers higher quality men. Agents meet in continuous time according to a Poisson process generated by mutual search with quadratic search technology. Each pair of agents who have met decide whether to marry. A married agent receives a positive flow of payoffs, which is a function of the agent's and his or her partner's qualities (increasing in the quality of the partner). Single agents receive no payoffs and must continue searching. Agents are impatient, which causes waiting for another match to be costly. All of these features have been used in existing models. Where we diverge from these models is that we allow agents to continue searching for better partners while married. Whereas in the standard model married agents exited the market immediately, in our model they remain on the market, and either one or both sides can continue searching. If a married agent finds a better match, he or she can divorce the current partner and remarry with the newly met partner instead. Divorce itself is costless.

Like Smith [2006] and most of the literature discussed above, we operate in a world of nontransferable utility (NTU), where wages are not available to equilibrate matches, and therefore payoffs are not transferable or quasi-linear in a transferable resource (see Shimer and Smith [2000] for the current state of the art in the

[3]There are no distinguishable sides in Smith [2006]; however, his model can easily be adapted to introduce such a distinction. Furthermore, the two sided-model can be reduced to a model with no sides under symmetry.

transferable utility framework). There are two main reasons we concentrate on the NTU case. First, even though some intramatch transfers (such as making compromises within a marriage) do exist, the extent of these transfers in many contexts is limited in practice. Second, a key objective for our paper is to examine the robustness of block segregation to the possibility of on-the-match search, and block segregation is exclusively an NTU phenomenon.[4]

As mentioned earlier, we find that block segregation does in fact disappear when we introduce on-the-match search and divorce. In particular, if at least one side of the market has the ability to search while matched and to break an inferior match in favor of a better one, then the "banding" of block segregation is no longer a steady-state equilibrium, and *strict* setwise PAM obtains with *weakly* log-supermodular payoffs (and thus also with multiplicatively separable payoffs). Furthermore, when both sides are allowed to divorce, rather weak conditions on payoffs and search technology guarantee the existence of an equilibrium in which matching converges to *perfect* (not just setwise) PAM.

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[4]Chiappori and Weiss [2000] is the closest paper that conducts an equilibrium analysis of matching and divorce. That paper, however, focuses on the transferable utility case, i.e. examining the division of the surplus between partners. Because we operate in an NTU world, such intramatch welfare comparisons are not relevant. The NTU and TU matching models operate in parallel literatures; we proceed exclusively in the NTU world. An important parallel between our work and that of Chiappori and Weiss, however, is that both their study and ours find that divorce can be welfare-improving.

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