The main ancient sources of early medieval logic were Boethius's translations of Aristotle's *Categories* and *De interpretatione* and Porphyrius's introduction to the *Categories* (*Isagoge*), Boethius's commentaries on these works and on Cicero's *Topica*, as well as a number of other logical treatises by Boethius, including *De syllogismis categoricis*, *De hypotheticis syllogismis*, *De topicis differentiis*, and *De divisione*.¹ Some historians, following medieval usage, call early medieval logic based on these sources the old logic, as distinct from the new logic the textual basis of which was extended by twelfth-century recovery of the rest of Aristotle's *Organon* (*Prior Analytics*, *Topics* and *On Sophistic Refutations*, translated by Boethius, and *Posterior Analytics*, translated by James of Venice in the second quarter of the twelfth century).² The elements of distinctly medieval logic and semantics had been introduced since the twelfth century in what was called


modern logic (logica moderna) and some historians call terminist logic because of its concentration on an analysis of the properties of terms, such as signification, supposition, appellation, and ampliation. This trend was also associated with the rise of some new genres in logic: the treatises on sophisms, which were influenced by Aristotle’s On sophistic refutations, insoluble propositions, syncategorematic terms, and the logic of ‘obligational’ disputations. The subjects of the old and new logic were combined with those of terminist logic in many early and mid-thirteenth century introductions and compendia, the most influential of which proved to be Peter of Spain’s Tractatus.

Ancient modal theories were extensively dealt with in Boethius’s two commentaries on De interpretatione, early medieval authors being acquainted with Aristotelian and other ancient modal paradigms through these works before the later Aristotelian reception. The modal themes which were brought into the discussion by the new logic were mainly the modal syllogistics and the conversion of modal propositions which formed part of it. Early medieval theories of modality were also influenced by Augustine’s ideas about divine power and freedom which deviated from the philosophical assumptions of ancient views of possibility. There were analogous discussions of ancient philosophical theories and their relationship to divine modalities in Arabic philosophy. Arabic modal theories influenced Latin discussions mainly through the translations of the works of Averroes.


4physik (Stuttgart and Bad Canstatt: Frommann-Holzbock, 2001), 125-45.
In Section (1) I shall deal with some influential medieval interpretations of modal concepts from Boethius to Aquinas. I do not mean that these were shared by everybody before Aquinas and by nobody after him. Since the eleventh century recovery of philosophical modal theories, Boethian formulations of central ancient conceptions had often been considered congenial, but were also qualified by theological considerations. These discussions were accompanied by some attempts to redefine modal concepts using the idea of alternativeness. While all these trends influenced mid-thirteenth century discussions, many thinkers were particularly interested in interpreting modal terms in the light of Aristotelian essentialism. This approach is also found in Robert Kilwardby’s commentary on Aristotle’s *Prior Analytics* (c. 1240) which became the standard thirteenth-century textbook for modal syllogistics. Many thirteenth-century paradigms lost their significance in early fourteenth-century discussions of modal theory, as will be shown in section (3). Before this, however, I shall discuss the details of earlier logical modal theories including modal syllogistics in section (2). Section (3) is about fourteenth-century developments and section (4) about medieval theories of applied modalities.

1 GENERAL SEMANTIC PARADIGMS FROM BOETHIUS TO THOMAS AQUINAS

1.1 Extensional Interpretation of Modality

In the introductory remarks of his commentary on Chapter 9 of Aristotle’s *Peri hermeneias*, Thomas Aquinas classifies various types of propositions on the basis of their ‘matter’:

In necessary matter all affirmative propositions are determinately true; this holds for propositions in the future tense as well as in the past and present tenses; and negative ones are false. In impossible matter the contrary is the case. In contingent matter, however, universal propositions are false and particular propositions are true. This is the case in future tense propositions as well as those in the past and present tenses. In indefinite ones, both are at once true in the future tense propositions as well as those in the past and present tenses. (*In Peri herm. I.13, 168 [5], trans. Oesterle, with changes*)

The matter of a proposition is associated with the habitude of a predicate to a subject and is explained as follows:

If the predicate is *per se* in the subject, it will be said to be a proposition in necessary or natural matter, for example ‘Man is an animal’

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and ‘Man is risible’. If the predicate is *per se* repugnant to the subject, as in a way excluding the notion of it, it is said to be a proposition in impossible or remote matter, for example ‘Man is an ass’. If the predicate is related to the subject in a way midway between these two, being neither *per se* repugnant to the subject nor *per se* in it, the proposition is said to be in possible or contingent matter. (In *Peri herm.* I.13, 166 [3], trans. Oesterle, with changes)

Aquinas employs the terms *enunciatio* and *propositio* as synonyms and takes them to mean statement making sentences. For reasons of simplicity, I shall use the term ‘proposition’ for these and related terms in medieval authors.7

The ancient theory of the matter of propositions was often associated with the rules of contraries, subcontraries and contradictories in the traditional square of opposition. While these rules defined how the members of various opposed pairs were related to truth and falsity, it was thought that they could be further specified by classifying propositions on the basis of their matter. An interesting feature in Aquinas’s account of the contingent matter is that universal affirmative and negative propositions are false and particular affirmative and negative propositions are true. Comparing this with what is said about propositions in other matters, modal differences can be characterized as corresponding to a descending order in the frequency of true cases: the predicate is not truly said of any subject in impossible matter, it is truly said of some subjects in contingent matter and of all subjects in necessary matter. Aquinas’s formulations are possibly influenced by Ammonius’s commentary on Aristotle’s *Peri hermeneias*, translated into Latin by William of Moerbeke in 1268, but the ancient theory of the matter of propositions was also known through the works of Boethius and found in many Latin authors before the translation of Ammonius’s work.8

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8Ammonius, *Commentaire sur le Peri hermeneias d’Aristote. Traduction de Guillaume de Moerbeke*, ed. G. Verbeke, Corpus Latinum Commentariorum in Aristotelis Graecorum (Louvain: Publications Universitaires, Paris: Béatrice-Nauwelaerts, 1961), 168.41-169.57; 175.69-176.95; 178.27-32; 179.38-43; 180.61-181.82; 199.23-201.63; the Greek text is edited by A. Busse in Commentaria in Aristotelis Graeca 4.5 (Berlin, 1897). Boethius was acquainted with the same doctrine which Ammonius explains, though he does not use the same terminology. Ammonius’s work was not known to Boethius; see R. Sorabji, ‘The Tree Deterministic Arguments Opposed by Ammonius’, in Ammonius, *On Aristotle: On Interpretation 9*, trans. D. Blank, with Boethius, *On Aristotle: On Interpretation 9*, first and second Commentaries, trans. N. Kretzmann, with Essays by R. Sorabji, N. Kretzmann and M. Mignucci (London: Duckworth 1998), 3-15. In dealing with the opposition between the pairs of universal and particular propositions, Boethius distinguishes between predications which are natural (necessary), impossible and neither natural nor impossible. While universal affirmative and negative contraries of the third case are both false, the corresponding particular propositions are both true. A universal affirmative proposition is true in the first case and a universal negative proposition in the second case (In *Periherm.* II, 177.18-178.8; see also 303.15-306.13; 325.8-15.) Garland the Computist explains the same classification, distinguishing between propositions with natural, remote and impossible matter (*Dialectica*, ed. L. M. de Rijk, Wijsgerige teksten en studies 3 (Assen: van Gorcum, 1959), 54.21-30; 82.25-30.) In a late eleventh century logical treatise attributed to William of Cham-
Some of those who discussed the matter of propositions did not maintain that both of the particular sub-contraries are true in contingent matter, apparently thinking that it was somewhat strange to exclude the possibility, say, that at a certain time no human being is sitting. Those who did not find this problematic could think that universal and particular propositions should basically be understood as making assertions without temporal restrictions (simpliciter) and not how things are at the moment of utterance (ut nunc). In this approach universal and particular propositions are regarded as statements about the generic features of the order of being, such as natural kinds and the necessary or contingent properties of their members. It is possible that a true predication in contingent matter is universal ut nunc though it is particular simpliciter.

The Thomist version of the doctrine of the matter of propositions is in agreement with what some historians call the statistical or frequency model for modality in ancient philosophy. This refers to the habit of associating the notion of necessity with omnitemporal actuality or actuality in all members of a species, contingency with actuality at some times or in some members of a species, and impossibility with the lack of actuality in these respects. Possibilities are dealt with from the point of view of their actuality in history without an idea of alternative domains. This way of thinking, which was known to medieval thinkers through Boethius, was later employed by many authors, although it never was the only modal paradigm.

Aristotle made use of modal terms in this manner, particularly in the contexts in which he discussed eternal beings, the natures of things, the types of events, or generic statements about such things. (See, e.g., Met. IX.8, 1050b6-34; IX.10, 1051b9-30; XII.6, 1071b18-22; XIV.2, 1088b14-25; De caelo 1.12, 282a4-25; Phys. III.4, 203b29-30; De generatione et corruptione II.11, 338a1-3). According to the temporal version of this paradigm as it is found in Boethius, what always is, is by necessity, and what never is, is impossible. Possibility is interpreted as expressing what is at least sometimes actual (In Periherm. I, 200.20-201.3, II,
Correspondingly, a property which belongs to all members of a group is a necessary property. If it is not actual at all in that group, it is impossible; and if it is exemplified at least in one member, it is possible (In Periherm. I, 120.24-121.16).

Aristotle sometimes treats affirmative sentences as token reflexive type sentences which express how things are at the moment of their utterance and consequently have changing or unchanging truth-values (e.g., Cat. 5, 4a23-b2). In Metaphysics IX.10 this is associated with the frequency paradigm as follows:

Regarding contingent things, then, the same opinion or the same proposition comes to be false and true, and it is possible at one time to have truth and at another time to be in error, but regarding things that cannot be otherwise opinions are not at one time true and at another false, the same opinions being always true or always false. (1051b13-17)

When contingency is equated with the change in things and truth-values, it follows that unchangingly true sentences are necessary and unchangingly false ones are impossible. This seems to be the background of some prima facie obscure formulations in Boethius:

For it is only those things that can both be and not be that not always are and not always are not. For if they always were, their status could not change, and so they would of necessity be; but if they always were not, it would be necessary that they not be. For of course just as the very nature of the things, events or states of affairs coming about is various, so also does the one or the other part of the contradiction have variable truth. And, indeed, it is always true or false – not, however, one definitely, in such a way that this one is determinately true, or that one – but in either of two ways. And so just as the very status of things, events or states of affairs is mutable, so also is the truth or falsity of the propositions dubitable. And indeed it comes about that as regards some things the one is more often but not always true and the other is more rarely true although it is not necessary that it be false. (In Periherm. I, 124.30-125.14; trans. Kretzmann)  

This text is part of Boethius's discussion of the truth of singular future contingent propositions. He does not argue that these as such have a changing truth-value, but assumes that as their objects belong to changeable things which make present

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11See also Thomas Aquinas, In duodecim libros Metaphysicorum Aristotelis expositio, ed. M.-R. Cathala and R.M. Spiazzi (Turin: Marietti, 1964), IX.11 (1900): ‘Regarding to those things which can be compounded and divided one and the same proposition is clearly sometimes true and sometimes false, for example ‘Socrates is sitting’ is true when he sits and false when he stands up. And the same holds true of an opinion. But regarding the things which cannot be otherwise, namely those which are always compounded or always divided, it is not possible that a same proposition or opinion is sometimes true and sometimes false, but that which is true is always true and which is false is always false.’

12For the translation, see note 8 above.
tense propositions about their prevalence sometimes true and sometimes false, it is illuminating to characterize future propositions about the objects of this kind as variably true or false. Since ‘Socrates is sitting’ is sometimes true and sometimes false, depending on when it is uttered, the proposition as such is not simply true or false. It is this indeterminacy which Boethius tries to express in calling temporally determinate future contingent propositions variably true or false.  

There is a similar shift from temporally definite cases to temporally indefinite ones in Boethius’s discussion of Aristotle’s remark that what is necessarily is when it is (De int. 19a23). Boethius draws a distinction between simple necessity and temporal or conditional necessity (In Periherm. I, 121.20-122.15; II, 241.1-242.15).

By temporally conditional necessity he means the immutability of a state of affairs as long as a condition is actual. One example of unconditional (simple) necessity is the motion of the sun, of which he says that, as distinct from temporal necessity, ‘it is not because the sun is moved now, but because it will always be moved, that there is necessity in its motion’ (II, 241.5-7). Conditional necessities are of two sorts: the condition may be the actuality of what is said to be necessary, e.g. Socrates is necessarily seated when he happens to be seated, or the actuality of the subject, e.g. fire is necessarily hot as long as it exists (II, 187.24-188.2; 243.1-6).

Boethius is particularly interested in the first group of conditional necessities. He thinks that this necessity expresses the necessity of the present and stresses that it does not imply any other necessity which has deterministic implications.

The Boethian view of the necessity of the present shows how it was understood in ancient modal thought which did not involve the idea of synchronic alternatives. Boethius typically maintains: ‘For since the same person cannot be sitting and not sitting at the same time, whoever is sitting cannot be not sitting at the time when he is sitting; therefore, it is necessary that he be sitting’ (II, 241.10-13). It is assumed here that

\[
(1) \quad \neg M(p_t \& \neg p_t)
\]

implies

\[
(2) \quad \neg(p_t \& M_t \neg p_t)
\]

which is equivalent to

\[
(3) \quad p_t \rightarrow L_t p_t.
\]

Boethius thought that the temporal necessity of the present is qualified by the possibility to be otherwise at another time. Referring to the consequent in (3), he states that ‘if we separate from the proposition . . . the temporal condition, then truth vanishes from the whole proposition’ (242.1-3). One might wonder how the alleged necessity of Socrates’s sitting at a certain moment of time is qualified

\[13\]See also In Periherm. I, 106.11-23; 123.16-24.

\[14\]In his similar account Ammonius writes: ‘That you are necessarily seated or walking, as long as one of these holds of you, is true, but not absolutely, for we are not always walking or sitting’ (153.24-6).
by what he does at other times. According to Boethius, this shows that sitting as such does not inhere in Socrates by necessity (II.242.22-243.2). Many twelfth and thirteenth century authors applied this analysis in discussing the sentence ‘A standing man can sit’, adding to the true divided (or de re) reading ‘at another time’ or something similar. Another Boethian strategy was to refer to antecedent contingency; I shall return to this in 1.3 below.

Even though there are many examples of the frequent use of modal terms in Western authors from Boethius to Thomas Aquinas, they did not define modal terms through the ideas of various degrees of generality or changing and unchanging truth. These aspects of modalities were regarded as concomitant rather than definitional. Modifying Boethius’s classification of ancient theories from this point of view, Thomas Aquinas writes that:

Some who distinguished them by results – for example Diodorus – said


16For an equation of modal and temporal notions (necessarily-always, possibly-sometimes), see Averroes’s commentary of Aristotle’s *De caelo* in *Aristotelis opera cum Averrois commentarius* (Venice 1562-74), vol. V, 84A-D, 87H-M, and Avicenna’s treatise *al-Masa'il*, quoted in T. Street, ‘FahraddSn Ar-R~zi’s Critique of Avicennan Logic’ in D. Perler and U. Rudolph (eds.) *Logik und Theologie. Das Organon im arabischen und lateinischen Mittelalter*, Studien und Texte zur Geistesgeschichte des Mittelalters 84 (Leiden: Brill, 2005), 104. Further typical examples are the principles that if something can be destroyed, it will be destroyed (Moses Maimonides, *The Guide of the Perplexed*, trans. with introduction and notes by Sh. Pines (Chicago: University of Chicago Press, 1974), II, 247, 249; Thomas Aquinas, *Summa theologae*, ed. P. Caramello (Turin: Marietti, 1948-50) I.2.3; In *Post. an. I.13, 117*; cf. Aristotle, *Metaphysics* XII.6), what is possible with respect to a species will be actualized (Moses Maimonides in a letter to Samuel ibn Tibbon, quoted in Ch.H. Manekin, ‘Problems of ‘Plenitude’ in Maimonides and Gersonides’ in R. Link-Saling, R. Long and Ch.H. Manekin (eds.), *A Straight Path: Studies in Medieval Philosophy and Culture. Essays in Honor of Arthur Hyman* (Washington: Catholic University of America Press, 1988), 187, and that universal propositions are false and particular propositions are true in contingent matter (De Rijk 1967, II.2, 115.5-12; 138.24-6; Albert the Great, *Commentarius in Perihermenias*, in *Opera omnia*, ed. A. Borgnet, vol. I (Paris: Vivès, 1890), 5, 422, and the texts mentioned in note 8 above). Gersonides also assumed that all genuine possibilities will be actualized; see the *Book of Correct Syllogism* I.1.14 in *The Logic of Gersonides. A Translation of Sefer ha-Heqqesh ha-Yashar* (The Book of the Correct Syllogism), with introduction, commentary, and analytical glossary by Ch.H. Manekin, The New Synthese Historical Library 40 (Dordrecht: Kluwer, 1992). One of distinctions between modalities per se and per accidens, which were employed in logical treatises, was based on the idea that the modal status of a temporally indefinite tensed proposition may be changeable. According to the early thirteenth-century *Logica ‘Ut dicit’,* true future tense singular propositions referring to non-existent future things, such as ‘The Antichrist will be existent’, are necessary *per accidens*, for they cannot have been false in the past nor false now, though they will be false in the future (de Rijk 1967, II.2, 390.21-2); ‘necessarily’ is said to mean the same as ‘in all times’ in this treatise (411.8-9). It was more usual to associate accidental necessities and impossibilities with past tense singular propositions; see note 111 below. Peter of Spain wrote that ‘Antichrist has not been’ will be accidentally impossible, though it is true now (*Tractatus IX.4*).
that the impossible is that which never will be, the necessary that which always will be, and the possible that which sometimes will be, sometimes not. The Stoics distinguished them according to exterior restraints. They said the necessity was that which could not be prevented from being true, the impossible that which is always prevented from being true, and the possible that which can be prevented or not. However, the distinctions in both of those cases seem to be inadequate. The first distinctions are a posteriori, since something is not necessary because it always will be, but will always be because it is necessary; this holds for the possible as well as the impossible. The second designation is taken from what is external and accidental, since something is not necessary because it does not have an impediment, but does not have an impediment because it is necessary. (In Peri herm. I.14, 183 [8], trans. Oesterle, with changes)

Aquinas criticized Diodorus for regarding the effects of necessity or possibility as their definitions and the Stoics for similarly considering something external as the basis of definitions. He then describes the true model based on the nature of the things:

Others distinguished these better by basing their distinctions on the nature of things. They said that the necessary is that which in its nature is determined only to being, the impossible that which is determined only to non-being, and the possible that which is not altogether determined to either. (Ibid.)

It may be noticed that when the characterizations of modalities are turned in the way suggested by Aquinas, one gets the assumption typical of the frequency model that what is possible is sometimes actual. Combining frequential and essentialist ideas was not unusual among Western thinkers before Aquinas, but this is also found in Moses Maimonides, Avicenna and Averroes. I shall return to early medieval qualifications of this model because of divine possibilities and some other considerations.

1.2 Possibility as a Potency

Another Aristotelian modal paradigm used by Boethius was that of possibility as potency. In Met. V.12 and IX.1 potency is said to be the principle of motion or change either as an activator (in Latin potentia activa) or as a receptor of a relevant influence (in Latin potentia passiva). One group of possibilities is defined as those which are based on potency. The types of potency-based possibility belonging to a species are recognized as possibilities because of their sometime actualization – no natural potency type can remain eternally frustrated (De caelo I.12). According to Boethius, some potencies are never unrealized, their nature being such that they are always actual and as such necessarily so. When potencies
are not actualized, their ends are said to exist potentially (In Periherm. II, 453.10-455.19). Necessarily actual potencies leave no room for the potencies of their contraries. There are no contrary potencies in these cases, Boethius says, because they would remain unrealized forever and the constitution of nature cannot include elements which are in vain (II, 236.11-18). It is implied here that all natural types of potency must show their reality through actualization. The potencies of non-necessary features of being do not exclude contrary potencies. They are not always and universally actualized, but as potency-types even these are taken to fulfil the criterion of genuineness mentioned (II, 237.1-5). This is in agreement with the 'statistical' model of modality.

The theory of active and passive potencies was originally meant to explain how and why a singular change takes place. Possibilities as active and passive potencies are the dynamic aspects of an actual change. This background to the potency paradigm made it a cumbersome model for singular possibilities. While it allowed Aristotle and his medieval followers to speak about unrealized possibilities in the sense of partial possibilities, i.e., as the correlates of active or passive potencies, full singular possibilities were actualized when they could be actualized. Natural passive potencies could not be actualized without an active power and they were necessarily actualized when an active power activated them and there was no external hindrance (Met. IX.5, 1048a5-21).

According to Thomas Aquinas, the generic natural potentialities, divided into passive propensities (potentia passiva) and related activating principles (potentia activa), are necessary features of things, determined by the nature of the subjects in which they occur. These necessary structures cannot be violated, and therefore miraculous events, which take place against the common course of nature, must occur through a special supernatural causation. Corresponding to the supernatural active power, there is a passive potentia obedientiae by which creatures may receive exceptional influences from the divine cause.

Boethius believed that the passive and active conditions of a great number of events were embedded in the antecedent causes of these events, but in criticizing the Stoic causal determinism he also taught that according to an Aristotelian view there were indeterminate events based on free choice, chance and ad utrumlibet contingency which were not determined by preceding causes. As for the efficient causes, he distinguished between necessary causes which always produced their effects and lower causes which may be prevented because of an indeterminacy factor in the causal nexus of nature. Related frequential ideas became popular

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18 Summa theologiae I.82.1; In Metaph. V.6, 832-4; IX.1, 1782; for obedient powers, see De potentia, ed. P.M. Pession in Quaestiones disputatae, vol. II (Turin: Marietti 1965), 1.3, ad 1; 6.1, ad 18.
20 In Periherm. II.197.10-198.3; In Ciceronis Topica, Patrologia Latina 64, 1148, 1152, trans-
through Averroes’s works in the thirteenth century. Following the Averroistic view, Thomas Aquinas and Siger Brabant, his contemporary in Paris, defined necessary natural causes as causes which, when actual as cause, always bring about their effect. Contingent causes are divided into those which bring about the effect in most cases (\textit{ut in pluribus}) and are in a few cases prevented by chance and those which are not associated with a natural tendency to a definite result (\textit{ad utrumlibet}). A particular cause is here considered necessary or contingent depending on how causes of the same type usually behave. Similarly, the effect which is necessary with respect to its actual causes can be called contingent by referring to what happens in other similar cases. In asking how things can be contingent if they are eternally known by divine omniscience and included in the immutable providential plan, Aquinas sometimes states that things are contingent, if their proximate causes are not necessary, i.e., causes which always bring about their effect.

While the conceptions of power and potentiality were widely regarded as essential elements in understanding modality, Anselm of Canterbury attempted to base the whole of modal semantics on these notions. Putting forward a detailed analysis of direct (proper) and indirect (improper) modes of agency, he suggested that the

\begin{footnotesize}
\footnotetext{21}{For Averroes’s view of causation, see A. Maier \textit{Die Vorläufer Galileis im 14. Jahrhundert} (Rome: Edizioni di Storia e Letteratura, 1949), 219-50.}


\footnotetext{23}{\textit{Scriptum super libros Sententiarum} I.38.1.4-5. While repeating this view in his later works (\textit{Summa contra gentiles} I.85.4; III.72.2; \textit{Summa theologicae} I.14.13, ad 1), Aquinas qualified it stating that he did not mean that things could be considered necessary as such. He stressed that God as the first cause provided things which naturally necessary or contingent causes so that contingent events, particularly the acts of free will, were not reducible to a necessitating ultimate cause, though dependent on God’s irresistible will which transcended the natural orders of necessity and contingency (\textit{Quaestiones disputatae I: De veritate}, ed. R.S. Spiazzi (Turin: Marietti, 1964), 23.5; \textit{In Peri herm.} I.14, 197; see also H. Goris, \textit{Free Creatures of an Eternal God: Thomas Aquinas on God’s Infallible Foreknowledge and Irresistible Will} (Utrecht: Thomas Instituut, Leuven: Peeters, 1996), 293-302). Siger of Brabant refers to some Parisian masters who argued that associating the modality of an effect with that of its proximate cause was compatible with metaphysical determinism (\textit{De necessitate et contingentia causarum} 26.67-79). In order to avoid this, he stated that the first cause left room for individual coincidences the results of which were not known even by divine foreknowledge (\textit{De necessitate et contingentia causarum} 26.80-28.9; 31.78-32.00; 42.10-25). When accused of heresy, he became more cautious when speaking about indeterminate events in nature (\textit{Quaest. in Met.} (Vienna), 370.1-387.89; R. Hissette, \textit{Enquête sur les 219 articles condamnés à Paris le 7 Mars 1277}, Philosophes médiévaux 22 (Louvain: Publications Universitaires, Paris: Vander-Oyez, 1977), 42-3.)}
\end{footnotesize}
concepts of potency and impotency should be treated in the same way. According to Anselm, 'A is necessarily B' properly ascribes a constraint to A and 'A is possibly B' properly ascribes a capacity to A. Modal terms proper refer to properties of things, such as the power to bring something about, or constraints caused by other things. If modal terms are used in another way, they are used improperly. Since Anselm regards the notion of capacity as the basic modal notion, something is necessary in an improper sense with respect to causes which cannot prevent it, and something is similarly impossible with respect to causes which cannot bring it about. There are no constraining necessities proper in God, and divine impossibilities are similarly analysed by referring to external impossibilities. In trying to solve problems associated with the notions of freedom, sin and grace, Anselm also employed the distinction between full and partial potencies. The shortcomings of these analyses and some artificial constructions of the meaning of modal statements based on the distinction between proper and improper capacity show that the notion of potency is too narrow a basis for modal semantics.24

1.3 Temporal modalities

A third ancient modal paradigm in Boethius meant for discussing singular possibilities could be called the diachronic model of modality. In chapter 9 of Peri hermeneias, Aristotle tries to qualify the necessity of the present — a corollary of the lack of simultaneous alternatives in his theory — stating that not everything which is actual is necessary simpliciter, without qualification (19a23-4). If necessity without qualification means 'necessary even without the temporal qualification', as Boethius reads it in his first commentary, Aristotle's point would be that the temporal necessity of actual events does not imply that such events are necessary in themselves in the sense that they always take place in similar circumstances.25 As already mentioned, this is a problematic attempt to qualify the necessity of a definite singular event with the help of frequential considerations. Some medieval and modern interpreters have opted for another interpretation: Aristotle wanted to show that the necessity of an event at a certain time does not imply that it would have been antecedently necessary. There are some places in which Aristotle speaks about genuine singular possibilities with respect to definite future points of time which may be realized or remain unrealized (De int. 19a13-17, EN III.5, 1114a17-19, Met. VI.3).26 This idea of diachronic modalities was considered important in the later Peripatetic tradition. It was argued against Stoic determinism that there are genuine future alternatives which remain open

25See Hintikka 1973, 147-175; Boethius, In Periherm. 1.121.25-122.4.
until the moment of time to which they refer. Even the Stoics spoke about alternative prospective possibilities which are not yet fixed at the level of known causes, but they also regarded fate as an active potency which ultimately necessitates everything. In the Peripatetic theory of diachronic modalities it is assumed that there are transient individual alternative possibilities, but those which will not be realized disappear.\(^{27}\) When Boethius refers to chance, free choice, and possibility, which restrict the domain of causal necessity, his examples include temporalized modal notions which refer to diachronic prospective possibilities at a given moment of time.\(^{28}\) A temporally determinate prospective possibility may not be realized at the time to which it refers, in which case it ceases to be a possibility. Boethius did not develop a theory of simultaneous synchronic possibilities which remain intact even when diachronic possibilities have vanished, insisting that only what is actual at a certain time is possible at that time with respect to that time.

The model of diachronic modalities provided Boethius with a more satisfactory tool for qualifying the necessity of the present than the frequency interpretation did. Instead of arguing that when the condition of the conditionally necessary proposition is removed, the proposition itself is contingent, he could also remark that when the antecedent conditions of a temporally necessary state of affairs are considered, it may be realized that it was not necessary before it was actualized.\(^{29}\) (See also 2.3 below.)

1.4 Divine Modalities

While Boethius's works formed the main source of knowledge about philosophical modal theories for early medieval thinkers, Augustine's doctrine of creation made them aware of a theological discussion of possibility and necessity which was based on different ideas. Augustine argued that God simultaneously created the first things and the seminal reasons for later things out of nothing. The creation was based on an eternal free act of God's perfectly good will, and took place through his omnipotence. In Augustine's Trinitarian view, the Son is a perfect image of the Father and, as the Word, the seat of the ideas of finite beings which in a less perfect manner can imitate the highest being. The ideas refer to possible actualization in the domain of mutability. In this sense the possibilities have an ontological foundation in God's essence.\(^{30}\) This became the dominating conception of theological modal metaphysics until Duns Scotus departed from it.\(^{31}\)

\(^{28}\)See, e.g., *In Periherm.* I, 106.11-14; 120.9-16; II, 190.14-191.2; 197.20-198.3; 203.2-11; 207.18-25.
\(^{29}\)See, e.g., *In Periherm.* II, 245.4-246.19. See also Thomas Aquinas, *In Periherm.* I.15, 201; Albert the Great, *Comm. in Periherm.* I.5.6, 421; Siger of Brabant, *De necessitate et contingentia causarum* 32.4-18.
\(^{31}\)Knuuttila 1993, 135-6.
Augustine thought that the world was not necessary and that many possibilities remained unrealized. In book 12 of *The City of God*, he criticizes the ancient doctrines which claimed that the only permissible notion of infinity is that of potential infinity, arguing that an infinite series of numbers actually exists in God’s mind, and that God could create an infinite number of individuals and know each of them simultaneously. Augustine regards God’s omnipotence as an executive power with respect to his free choice, which is conceptually preceded by knowledge about alternative possibilities. God could have done other things, but did not want to. Augustine’s remarks on various unrealized possibilities remained sketchy, but the basic idea is pretty clear and was often repeated in the early medieval doctrine of God as acting by choice between alternatives. This involved an intuitive idea of modality based on synchronic alternativeness.

The discrepancy between the Catholic doctrine of God’s freedom and power and the philosophical modal conceptions was stressed in Peter Damian’s *On Divine Omnipotence* and formulated in a less ambiguous manner by Anselm of Canterbury. Like Peter Damian, Anselm also avoided speaking about the restrictions of divine omnipotence and preferred formulations according to which all necessities and impossibilities are subject to God’s will. He did not mean that all necessary truths were dependent on God’s will, for example those about God’s existence or properties or the principles of logic. His point was that the natural order was created. Events which take place in accordance with the common course of nature or against it are naturally necessary or impossible with respect to the laws of the created order, but God as the Lord of this order can bring about effects which are naturally impossible. Miraculous divine interventions do not strictly speaking violate natural patterns, since they are meant to be subordinate. These ideas were codified in the commonly employed distinction between possibilities *secundum inferiorem causam* as possibilities *secundum cursum naturae* and possibilities *secundum superiorum causam* as God’s possibilities. Some theologians

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34Plotinus also thought that the various levels of reality outside the One, the first principle, are caused and thus different from Aristotle’s conception of higher spheres. However, the power of being eternally proceeds from the One and does not leave any genuine possibility of being unrealized. See H. Buchner, *Plotins Möglichkeitlehre*, Epimeleia 16 (Munich and Salzburg: Anton Pustet, 1970); for the principle of plenitude in Proclus, see T. Kukkonen, ‘Proclus on Plenitude’, *Dionysius 18* (2000), 103-28.
35For Peter Damian, see *Lettre sur la Toute-Puissance Divine (De divina omnipotencia)*, introduction, critical text, translation and notes by A. Cantin, Sources Chrétienes 191 (Paris: Cerf, 1972), 603a-604b, 610d-615b, Knuuttila 2004, 115-18.
36See, for example, *Car Deus homo* 2.17 in F. Schmitt (ed.), *Opera omnia* (Edinburgh: Nelson, 1946-61), vol. II, 122.25-30; *Meditatio redemptionis humanae (Opera omnia* III, 86.60-2); *De conceptu virginali et de originali peccato (Opera omnia* II, 153.7-9, 154.4-15); for non-created modalities, see, for example, *Proslogion* 3-5 (*Opera omnia* I, 102-4); *De grammatico* 1-2 (*Opera omnia* I, 145-6).
37Peter Lombard, *Sententiae in IV libris distinctae* ed. PP. Collegium S. Bonaventurae
asked whether one should apply the notion of possibility without qualification to
divine possibilities and the notion of qualified possibility, such as natural possi-
Bility, to those spoken of in accordance with the lower causes, or vice versa. This is
a sign of an increasing awareness of the fact that the conception of omnipotence,
which included the idea of eternal alternatives, was very different from philosophi-
cal modalities, and there were some twelfth-century attempts to explicate the idea
of synchronic alternatives more systematically.

In addition to the doctrines of omnipotence and eternal choice between providi-
tional options, divine modalities were discussed in treating the compatibility of
divine omniscience and the contingency of events in the created world. In his
longer commentary on De interpretazione, Boethius says of God that ‘he knows
future events as happening contingently and not necessarily so that he does not
ignore the possibility that something else might take place.’ In his later work
Consolation of Philosophy, Boethius argues that God is atemporal and has timeless
knowledge of everything. God’s timelessness involves his having the whole history
present to him simultaneously. God’s knowledge is not foreknowledge, since it is
not temporally located, but the predictions of future contingents are true or false
from the point of view of God’s eternal knowledge of the things referred to. It
is necessary that if God knows that $p$, then $p$. This ‘conditional necessity’ does
not imply the ‘simple necessity’ of $p$. This approach was very influential and
was further developed in Aquinas’s theory of God who grasps all combinations of
things at particular times by one eternal vision. God has an immediate knowl-
edge of all things and their relative temporal order, though none of them is past
or future with respect to His cognition. The objects of divine omniscience are
necessary by supposition (i.e., with respect to God’s knowledge and providential
plan), but many of them are contingent with respect to their proximate causes.
God’s eternal and immutable knowledge includes the knowledge of things which are
future contingents in the temporal order, and he can supranaturally inform lower
intellects about these things – otherwise there could not be true prophetic predic-

(Grottaferrata: Collegium S. Bonaventurae ad Claras Aquas, 1971-81), II.18.5-6; Simon of Tour-
nai, Disputationes, ed. J. Warichez, Spicilegium Sacrum Lovaniense, Études et Documents 12
(Louvain: Spicilegium Sacrum Lovaniense,1932), 35.2 (104.32-105.29); Peter of Pottiers, Senten-
tiae, I.7 (65.444-66.452, 67.498-68.502); Alan of Lille, Regulae caelestis iuris, ed. N.M. Här
ing, Archives d’histoire doctrinale et littéraire du moyen âge 78 (1981), 164-5. This was the back-
ground of the scholastic doctrine of God’s absolute and ordained power. For similar discussions
in medieval Arabic thought, see Kukkonen 2000.

38 William of Auxerre, Summa aurea, ed. J. Ribaillier, Spicilegium Bonaventurianum 16-20
1.11.2 (206-9).

39 In Periherm. II. 226.9-12.

40 Philosophiae consolatio, ed. L. Bieler, Corpus Christianorum Series Latina 94 (Turnhout:
atemporal knowledge is meant to explain how future contingent events retain their indetermi-
nateness while being certainly known as atemporally present. See also J. Marenbon, Boethius

41 Scriptum super libros Sententiarum, 1.38.1.4-5; Summa contra gentiles, I.66-7; Questiones
tions. While prophetic statements are formally future contingent propositions, their being true is based on a revelation of atemporal knowledge. This is how Aquinas tries to combine the doctrine of divine omniscience and the lack of definite truth and falsity in future contingent propositions. (See also 1.6 below.)

An influential formulation of divine foreknowledge was put forward by Peter Abelard. William of Champeaux, one of Abelard's teachers, dealt with an argument against the compatibility between contingency and divine omniscience which was discussed in Augustine's *City of God* (V.9). In response to Cicero's *De fato* and *De divinatione*, Augustine refuted the claim that the possibility of events having happened otherwise implies the possibility of error in God. William stated that the antecedent of the argument is true but the consequent is false and therefore the consequence is not valid. Peter Abelard, in discussing the same example in his *Dialectica* and *Logica 'Ingrdientibus'*, applied a systematic division which he elsewhere drew between modal statements *de sensu* or in the compound sense and modal statements *de re* or in the divided sense. Abelard's analysis of Cicero's argument was often repeated in medieval theology, since it was presented in slightly modified form in Peter Lombard's *Sententiae*, which became the standard medieval introduction to theology. Abelard states that when the proposition ‘A thing can be otherwise than God knows it to be’ is taken to mean that it is possible that a thing is otherwise than God knows, which corresponds to what he elsewhere calls the *de sensu* reading, the antecedent is false. When the antecedent is taken to say that a thing may be otherwise than God knows it to be, which corresponds to what Abelard elsewhere calls *de re* possibility, the antecedent is true, but the consequent is false, since if things were otherwise, God would possess different knowledge of them. This shows that the consequence is not valid. Following Peter Abelard, Peter Lombard formulated the same view in stating that ‘Things cannot be other than as God foreknows them’ is true in the compound sense and false in the divided sense. The truth of the compound sense saves God's infallibility and the falsity of the divided sense expresses God's freedom and the

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42 *Summa theologiae* II-2.171.3.
metaphysical contingency of the future. It is assumed that when something is, it is possible that it is not at that very instant of time at which it is actual.\textsuperscript{46}

1.5 \textit{Modality as Alternativeness}

Augustine's doctrine of God's eternal choice involved an intuitive idea of modality as alternativeness which influenced early medieval theological discussions of divine omnipotence and omniscience. Some authors regarded this as a special theological matter which did not affect the use of traditional ideas in other disciplines, an attitude supported by the general Aristotle reception in the thirteenth century, but there were twelfth-century thinkers who realized the philosophical significance of this new modal conception.

According to Abelard, the possibilities and necessities belonging to the individuals of a species are determined by their shared nature. What nature demands or allows and what is repugnant to it is the same for all members of a species, and what is possible is seen in what has taken place in the representatives of the species.\textsuperscript{47} One could regard these essentialist possibilities as abstract Philonian possibilities, many of which never become or even can become actual in an individual.\textsuperscript{48} This is one aspect of Abelard's modal views, but there are others. He assumes that what is actual is temporally necessary at a certain point of time as no longer avoidable, but he also argues that unrealized alternatives are possible at the same time in the sense that they could have happened at that time. Some of the alternatives of a singular being are real counterfactual alternatives. These are unrealizable because of some previous changes in the conditions of the subject. There are also merely imaginable alternatives, such as Socrates' being a bishop, which never had a real basis in things.\textsuperscript{49} (See also 2.3 below.)

Gilbert of Poitiers stresses the idea that natural regularities which are called natural necessities are not absolute, since they are chosen by God and can be overridden by divine power. This had become a widespread theological view in the twelfth century. Gilbert explained it in the light of the Augustinian view of God's acting by divine will, which chooses between alternative providential plans, and divine omnipotence as an executive power. It has been sometimes thought

\textsuperscript{46}\textit{Sententiae} I.38.2.
\textsuperscript{47}\textit{Dialectica} 193.36-194.3; 385.1-8.
\textsuperscript{48}Philo's definitions of modal concepts are described in Boethius's commentary on \textit{De interpretatione} (II, 234.10-21). One of his examples was that it is possible for a piece of wood at the bottom of the sea to be burnt, in virtue of the fitness of the subject. See also Alexander of Aphrodisias, \textit{In Aristotelis Analyticorum priorum librum I commentarium}, ed. M. Wallies, Commentaria in Aristotelem Graeca 2.1 (Berlin: Reimer, 1883), 184.6-12; John Philoponus, \textit{In Aristotelis Analytica Priora commentaria}, ed. M. Wallies, Commentaria in Aristotelem Graeca 13.2 (Berlin: Reimer, 1905), 169.19-20.
that Gilbert wanted to deny the necessity of the past in this context, but the
texts interpreted in this way can be read as meaning that temporal necessities are
based on God’s free choice and do not imply any lack of Divine power. There is an
interesting formulation of Plato’s ‘Platonitas’ in Gilbert. He says that this includes
all what Plato was, is and will be as well as what he could be but never is. Even
though Gilbert does not explain why there is a modal element in the individual
concept, it was probably needed in order to speak about Plato in alternative
possible histories or, as Abelard did, about Socrates as a bishop. Gilbert seems to
have been the first to formulate an individual concept in this way.50

A third context of the systematic interest in synchronic alternatives was the
new twelfth-century theory that declarative singular propositions should be pri-
marily treated as temporally definite and as having an unchanging truth-value.
This approach was developed by twelfth-century authors who were later called
nominales. One of their theses was that ‘What is once true is always true’.51 This
was applied in discussing of the question of how the beliefs of Abraham and others
who lived before the coming of Christ and believed various things about him were
the same as the beliefs of those who live after His coming – the previous beliefs
were formulated in future tense statements and the latter in present or past tense
statements. According to the nominales, one could regard as basic a temporally
definite propositional content which is expressed by various tensed expressions
depending on when they are uttered. While tensed statements about temporally
definite singular events have a changing truth-value, the corresponding non-tensed
propositions are unchangingly true or false.52

Peter of Poitiers, one of the twelfth-century authors taking this approach, argued
that propositions pertaining to contingent things have a truth-value on the basis
of God’s eternal choice. These truth-values would be otherwise, if the providential
design of the world were different in relevant respects. The contingent truth-
values of future contingent propositions do not prevent future events from being
indeterminate. They may begin to be temporally unavoidable, but they do not
begin to be metaphysically contingent nor do the propositions pertaining to them
begin to be true or false.53 This is in accordance with the immutability of God’s
knowledge. God does not know contingent things through tensed propositions,

50See Knuttila 1993, 75-82.
51See Y. Iwakuma and S. Ebbesen, ‘Logico-Theological Schools from the Second Half of the
12th Century: A List of Sources’, Vivarium 30 (1992), 196, n. 46; 199, n. 50a, b; 200, n. 51a;
201, n. 52d; 205, n. 62d; 206, n. 64a. For the history of the principle, see also J. Marenbon, ‘Vo-
calism, Nominalism and the Commentaries on the Categories from the Earlier Twelfth Century’,
(30), 1992, 73-4. For the history of twelfth-century nominales, see W. Courtenay, ‘Nominales
and Nominalism in the Twelfth Century’ in J. Jolivet, Z. Kaluza, A. de Libera (eds.), Lectionum
52Nuchelmans 1973, 177-89; for some later examples of distinguishing between temporally
definite and temporally indefinite declarative sentences, see H. Goris, ‘Tense Logic in 13th-
53Peter of Poitiers, Sententiae I, ed. P.S. Moore and M. Dulong, Publications in Medieval
Studies 7 (Notre Dame: The University of Notre Dame Press, 1961), I.7.133-43; I.12.164-82;
since their truth-value is changeable. If God’s knowledge is described by using tensed propositions, analogously to the articles of faith before and after the coming of Christ, one should read them so that they signify the same. This became a well-known position, since it was also employed in Peter Lombard’s *Sententiae*. 54

The formulations by Peter Abelard, Gilbert of Poitiers, Peter Lombard and Peter of Poitiers discussed above exemplify twelfth-century deviations from the Aristotelian thesis ‘What is necessarily is when it is.’ This was traditionally understood as implying the principle of the necessity of the present, which was not questioned in ancient modal theories. Since God’s knowledge about contingent things was regarded as unchangeable, the contingency of this knowledge also implied the denial of the Aristotelian equation of immutability with necessity, a denial regarded as an explicit doctrine of the *nominales*. 55

The idea of modal alternativeness was also discussed by Robert Grosseteste in early thirteenth century. Grosseteste taught that the opposites of actualized contingent things are no longer realizable possibilities, though they are possible in the sense that they could have been included in God’s eternal providential choice. Actual history is an explication of one of the divine alternatives with respect to which things are primarily called necessary, possible or impossible. Modalities at this basic level are called modalities ‘from eternity and without beginning’. Mathematical truths are necessary in this way. In addition to these ‘simple’ necessities, there are necessities and impossibilities which have a beginning and which are eternal contingencies in the sense that God could have chosen their opposites. 56

The contingency of the divine acts of knowledge and will is based on an atemporal causal priority between the power and its acts. 57

1.6 Future Contingents

Some Stoics took Aristotle to deny that future contingent propositions are true or false, as Boethius reports in his second commentary on Aristotle’s *De interpretatione* (208.1-4). Future contingent propositions were regarded as true or false in Stoic logic, the Stoics taking the universally valid principle of bivalence to imply the predetermination of all future events. (See Cicero, *De fato*, 20-1.) 58 Boethius regarded the Stoic view of future contingent propositions as well as the Stoic characterization of Aristotle’s position as false, his interpretation being based on

57Ibid. 178.24-6.
58On the basis of a note in Simplicius’s commentary on Aristotle’s *Categories* (407.6-13), it is argued that some Aristotelians also qualified the principle of bivalence; for this and other ancient deviations from the principle, see R. Sorabji, *Necessity, Cause, and Blame: Perspectives on Aristotle’s Theory* (Ithaca, NY: Cornell University Press, 1980), 92-3. For the Stoic views, see also Bobzien 1998, 59-86.
the distinction between definite and indefinite division of truth and falsity in the contradictory pairs of propositions. This terminology was also used in Ammonius’s commentary. Since this work was not known to Boethius, both authors apparently based their commentaries on earlier Greek discussions in which these qualifications were introduced. According to Boethius, Aristotle argues that if all pairs of contradictory propositions definitely divide truth and falsity and all propositions are definitely true or definitely false, then everything necessarily occurs as it does. The fact that contingent future things, events and states of affairs are not determined refutes the thesis that all affirmations or negations are definitely true or definitely false. The disjunctive pairs of contradictory future contingent propositions can be said to divide truth and falsity only in a very special sense; ‘the whole body of the contradiction does indeed separate into truth and falsity, but this truth and falsity is undifferentiated and alterable.’ Boethius interprets Aristotle’s final solution as follows:

For it is necessary as regards future and contingent contradictions that the whole contradiction have one part true and the other false. For example, if someone affirms that there is going to be a sea battle tomorrow ... and if someone denies it ... the whole contradiction will indeed have one part true, the other false; but there will not be one of them definitely true and the other definitely false. (In Periherm. I, 122.26-123.10, trans. Kretzmann)

The majority interpretation of contemporary commentators is that Ammonius and Boethius ascribe to Aristotle the view that the predictions of future contingent things and their denials differ from other contradictory pairs, because truth and falsity are not definitely distributed between these propositions, which are consequently neither definitely true nor definitely false. This is taken to mean that they are not true or false. In answering the Stoic criticism, Boethius might have thought that future contingent propositions have the disjunctive property of being true-or-false, which would mean something other than simply lacking a truth value.  

59 Sorabji 1998. Ammonius’s commentary was translated by William of Moerbeke in 1268 and was used by Thomas Aquinas. See note 8.
60 Boethius thinks that the term ‘definitely’ can be added to Aristotle’s argument as part of what Aristotle means (In Periherm. I, 108.18-26; 125.20; II, 204.8-25; see also I, 109.9-17; 110.28-112.4; 114.8-24; II, 208.7-23; 211.26-213.4).
61 In Periherm. II, 219.5-17.
63 This contradiction is said to be indefinitely both true and false (I, 124.28); according to the longer commentary, ‘one part of the contradiction is true and the other false only indefinitely’ (II, 246.12-13).
Another interpretation of Boethius and Ammonius holds that future contingents are not definitely true or false, because their truth-makers are not yet determined, but are true or false in an indeterminate way. No qualification of the principle of bivalence is involved. True statements are either determinately true or simply (indeterminately) true.\textsuperscript{65} While Ammonius and Boethius assumed that Aristotle denied the definite truth of predictions which they took to imply determinism, it is less clear how they understood the indefinite truth of these. Boethius’s formulations often suggest that future contingent propositions are true-or-false without being simply true or false, but perhaps he was not quite sure about this.\textsuperscript{66}

The past and the present are necessary in Boethius. Prospective contingent alternatives with respect to a future event remain open until the relevant causes are settled or the event takes place and the alternative options vanish. Correspondingly he seems to take the truth of future propositions to mean that things cannot be otherwise, for the antecedently assumed actuality of future truth-makers implies that alternative prospective possibilities refer to things which are rendered temporally impossible by the actualized alternatives.

In his early commentary on \textit{De interpretatione}, Abelard follows Boethius’s analysis of future contingent propositions. He understands Boethius’s comments in the way most contemporary commentators do and accepts the idea that future contingent propositions are merely true-or-false. Contradictory present tense propositions are determinately true and determinately false and also disjunctively determinately/necessarily true or false (\textit{etiam sub disjunctione}), whereas contradictory future contingent propositions are merely disjunctively true or false (\textit{tantum sub disjunctione}).\textsuperscript{67}

Abelard changed his view in the \textit{Dialectica} and his longer commentary on \textit{De interpretatione} in \textit{Logica ‘Ingredientibus’}. The historical order of these two works is not quite clear. Many scholars have argued that the \textit{Dialectica} is earlier\textsuperscript{68}, but it is also possible that the texts contain parts written at different times. While the main structure of Aristotle’s argument in \textit{De interpretatione} \textit{9} is understood in the same way in the \textit{Dialectica} and the early commentary on \textit{De interpretatione}, Abelard now argues that future contingent propositions are true or false, although not determinately or necessarily so, and takes this to be Aristotle’s view as well. The difference between future contingent propositions and other propositions has nothing to do with bivalence; it concerns the determinateness or indeterminateness of the truth of propositions and what is signified by propositions. Misguided opinions are based on the mistaken idea that the necessity or determinateness of


\textsuperscript{66}See also Sorabji 1998.


\textsuperscript{68}See Marenbon 1997, 40-8.
a disjunction implies that the disjuncts are necessary or determinate.\textsuperscript{69} This is close to what Mignucci regards as Boethius’s position.

The commentary on \textit{De interpretatione} 9 in \textit{Logica ‘Ingredientibus’} involves Abelard’s most detailed analysis of the structure of Aristotle’s argument. Chapter 9 is described as a dialectical discussion of the thesis that in all contradictories one part is necessarily true and the other false. The correct reading of this is that of all contradictory pairs necessarily one part is true and the other false. This does not imply that one is necessarily true and the other necessarily false, although one is true and the other false.\textsuperscript{70}

Abelard pays attention to some propositions about past and present states of affairs the truth and falsity of which depend on future, such as ‘Socrates is the name of a man going to eat tomorrow’ or ‘He has spoken falsely’, when this is said of a man who said yesterday that Socrates will eat tomorrow. The truth and falsity of propositions of this kind are not knowable without knowledge of future contingent states of affairs which are indeterminate and epistemically inaccessible to human beings. Hence the propositions have an indeterminate truth and falsity and do not differ from future contingent propositions in this respect. In discussing these examples, Abelard states that ‘Aristotle calls propositions determinately or indeterminately true with respect to the determinate or indeterminate states of affairs they propose’. This terminology is part of Abelard’s rational reconstruction of Aristotle’s discussion; being probably aware that Aristotle did not use these terms at all, Abelard suggests that Aristotle would consider propositions indeterminately true or false if their truth depended on indeterminate future contingent states of affairs. Propositions which propose the actual inherence of truth in a future contingent proposition are of the form: ‘Socrates will eat tomorrow’ is true. As far as determinateness is understood in accordance with what Abelard calls Aristotle’s view, this is no more determinate than ‘Socrates will eat tomorrow’. As for the truth as a present fact, Abelard repeats the remark from the \textit{Dialectica} that one could call this present state of affairs determinate and, correspondingly, the propositions determinately true or false. In this sense all true propositions are determinately true, but this has nothing to do with causal determination. Abelard also remarks that God knows whether any proposition is true and false, and all true propositions are determinately true with respect to this omniscience. The determinateness in the sense of divine knowability pertains to truth-values and truth-makers, but this supranatural knowability does not make things necessary.\textsuperscript{71} While the truth-values of these propositions are in principle knowable, human beings cannot know them without supranatural illumination. The present truth of a future contingent proposition implies that what is predicted will obtain but not that it is determinate, since future contingent states of affairs are inde-

\textsuperscript{69}Dialectica 211.28-32; 212.36-213.7; 221.15-24.
\textsuperscript{70}Logica ‘Ingredientibus’ 431.13-432.9; 445.22-446.29.
terminate. In discussing the necessity of the present, Abelard follows Boethius in stating that what is actual at a certain point of time is necessary in the sense that it can no longer be avoided, but he also argues that unrealized alternatives may be possible at the same time in the sense that they could have happened at that time. The actuality of a contingent state of affairs at a specified future time does not exclude the non-temporal possibility of simultaneous alternatives and the truth of a proposition about this state of affairs does not make it necessary. This seems to be the background of Abelard’s deviation from Boethius’s approach to the truth and falsity of future contingent propositions.

According to Albert the Great and Thomas Aquinas, Aristotle argues in *De interpretazione* 9 that future contingent propositions differ from other assertoric propositions in not being determinately true or determinately false. Their general view of the structure of Chapter 9 is similar to that of Boethius. Instead of the Boethian definite-indefinite distinction, Albert and Aquinas employed the terms ‘determinate’ and ‘indeterminate’, as most medieval commentators did. Assertoric propositions are related to truth or falsity in the same way as their correlates are related to being or non-being. When future things are indeterminate with respect to being and non-being, the contradictory propositions about them must also be indeterminate with respect to truth and falsity. A future contingent proposition and its denial form a disjunction which is necessarily true. The members of this disjunction are disjunctively true or false. The indeterminate truth or falsity of a member of the disjunction does not imply that it is true or that it is false. It is merely true-or-false. Because of the prospective indeterminateness of a sea-battle, the propositions pertaining to it ‘must be true or false under disjunction, being related to either, not to this or that determinately.’ According to Aquinas, truth is not altogether lacking in a pair of contradictory singular future contingent propositions. It not true to say that both of these are false, for if one of these is false, the other is true, and *vice versa*.

Following Boethius, Aquinas thinks that contingency pertains to the future. Past and present things are necessary. The absence of the idea of synchronic alternatives in Boethius’s approach makes a true proposition about a future contingent event determinately and necessarily true. Aquinas describes this assumption as follows:

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72 Logica ‘Ingredientibus’ 422. 33-40.
74 See note 49 above.
77 *In Peri herm.* I.13, 175.
If it is always true to say of the present that it is, or the future, that it will be, it is not possible that this not be or that it will not be. The reason for this consequence is evident, for these two are incompossible, that something truly be said to be, and that it not be, for this is included in the signification of the true that what is said to be is. If therefore that which is said concerning the present or the future is posited to be true, it is not possible that this not be in the present or future. (In Peri herm. I.13, 173 [10]; trans. Oesterle, with changes)\textsuperscript{78}

While the necessity of the disjunction of future contingent contradictory propositions does not imply that one of these is necessarily true, the truth of one part implies that it is necessarily true, because it excludes the truth of the other as impossible. In commenting on Aristotle's thesis that everything is necessarily when it is, though not simply necessary, Aquinas writes:

This necessity is founded on the principle that it is impossible at once to be and not be, for if something is, it is impossible that it not be at the same time; therefore it is necessary that it be at that time ... Similarly, if something is not, it is impossible that it be at the same time. This is not absolute necessity, but necessity by supposition. (I.15, 201 [2], trans. Oesterle, with changes)

Necessity by supposition is the impossibility of being otherwise when one is actual. This necessity of the present does not imply the antecedent necessity of what is actual. Similarly the determinate division between truth and falsity in a contradictory pair of present propositions does not imply that even beforehand one or the other is determinately true.\textsuperscript{79} Following Boethius, Aquinas assumes, as distinct from Abelard, that the disjunctive necessity of a contradictory pair of future contingent propositions implies that if one part were true, the other would be impossible, because the truth implies the actuality of the predicted state of affairs, and it is necessarily actual when it is actual, since antecedently possible alternatives are excluded.\textsuperscript{80}


\textsuperscript{79}In Periherm. I.15, 201.

\textsuperscript{80}See also 1.4 above. After Aquinas, many authors endorsed the interpretation that Aristotle did not regard future contingent propositions as true or false. John Buridan was one of the few who read Aristotle like Abelard. All assertoric statements are true or false though those about future contingents are not determinately true or false; Questiones longe super librum Perihermeneias, ed. R. van der Lecq, Artistarium 4 (Nijmogen: Ingenium Publishers, 1983), I.10. Since most theologians thought that divine omniscience presupposed bivalence, the discussion of future contingents was divided into historical constructions of Aristotle's view and the systematic discussions in theology which usually followed the Abelardian lines. Peter Auriol argued that since future contingent propositions lack a truth value, even God is aware of the future in a way
1.7 Essentialist Assumptions

In his treatise *On Hypothetical Syllogisms* Boethius speaks about two kinds of conditionals which express a necessary consequence between the antecedent and the consequent. The consequence is accidentally necessary when the antecedent and consequent are immutably true but have no internal link between them, for example 'If fire is hot, the heavens are spherical'. In a non-accidental consequence, which Boethius calls natural, there is a conceptual connection between the parts; for example, 'If something is human, it is an animal'. Abelard also teaches that a genuine conditional expresses a necessary consequence in which the antecedent of itself requires the consequent. These were taken to express immutable laws of nature derivable from the nature of things. A related distinction between *per se* and *per accidens* necessary propositions was employed in mid-thirteenth century discussions of modal conversion and modal syllogistic. Robert Kilwardby states that some necessary connections between terms are merely accidentally necessary in the sense that the things signified are inseparable. These necessities are not dealt with in modal syllogistic the necessity propositions of which express *per se* necessities explained in *Posterior Analytics* I.4. The first type is said to occur when the definition of the subject includes the predicate and the second type when the definition of the predicate includes the subject. Typical *per se* necessary propositions were those expressing the properties determined by the substantial form of a subject or, as in the second class, other features based on the genus-species structure. Terms themselves were necessary if they stood necessarily for what they signified, for example ‘horse’. Other terms were accidental, for example ‘white’ or ‘walking’. (See also 2.4 below.)

Necessary propositions which were not *per se* necessary were often exemplified by propositions about inseparable accidents. In the *Isagoge*, Porphyry defines the inseparable accident as something which cannot actually be removed from its subject though the subject can be conceived of without it (3.5-6). The idea of the degrees of necessity and impossibility was also developed in late ancient discussions of indirect proofs and impossible hypotheses. In order to defend Aristotle’s indirect proofs with impossible premises, Alexander of Aphrodisias argued that Aristotle had in mind impossibilities which were not nonsensical. Some late ancient authors were interested in impossible hypotheses as tools for conceptual

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which does not imply that future contingent statements are true or false. This was an exceptional view. See C. Schabel, *Theology at Paris, 1316-1345: Peter Auriol and the Problem of Divine Foreknowledge and Future Contingents* (Aldershot: Ashgate, 2001).

81Boethius, *De hypotheticis syllogismis*, 1.3.7; Abelard, *Dialectica*, 253.28-30; 279.12-14; 280,12-18; 283.37-284.17; see also Garland the Computist, *Dialectica*, 141.7-22. Some later twelfth-century masters regarded the principle that the antecedent is not true without the consequent as a sufficient condition for the truth of a conditional and accepted the so-called paradoxes of implication. See C.J. Martin, ‘Logic’ in J. E. Brower and K. Guilfoyl (eds.), *The Cambridge Companion to Abelard* (Cambridge: Cambridge University Press, 2004), 164-5, 179-81.

analysis. In the arguments which were called Eudemian procedures something impossible was assumed in order to see what followed. The impossibilities discussed in this way by Philoponus and Boethius show similarities with Porphyry’s characterization of inseparable accidents as something which cannot occur separately but can be separated in thought. This analysis, which was counterpossible rather than counterfactual, influenced the branch of medieval obligations logic which was called *positio impossibilis*. The rules of this logical theory were often applied to theological problems – impossible hypotheses were regarded as conceivable but doctrinally impossible.\textsuperscript{83}

In thirteenth-century philosophy, Aristotelian impossible hypotheses were also explained with the help of special abstract possibilities referring to states of affairs which could not exist. It was thought that the possibilities of a thing are determined by its genus, species, and matter. Something which is possible for a subject as a member of a genus may be impossible for it as a member of a species, and the same holds of its being a member of a species and a singular being individuated by matter. This was an essentialist counterfactual analysis without the idea of alternative possibilities.\textsuperscript{84} In explaining why Aristotle says that the elements can be continuous with each other and with the celestial bodies, Aquinas writes as follows:

But it must be said that the contingent and the impossible are taken in different ways when something is demonstrated of a genus and when something is demonstrated of a species. For when a species is treated, that must be taken as impossible which is repugnant to either the genus or the specific difference from which the nature of the species is constituted. But when a genus is treated, everything which is not repugnant to the nature of the genus is taken as contingent, even though it may be repugnant to the differentia which constitutes the species. For example, if I am speaking of animal, I can say that every animal being winged is contingent. But if I descend to the consideration of man, it would be impossible for this animal to be winged. Now Aristotle is speaking here about movers and mobile objects in general ... Therefore, he states as a contingency that all mobile objects are continuous with each other.

\textsuperscript{83}C.J. Martin, ‘Thinking the Impossible: Non-Reductive Arguments from Impossible Hypotheses in Boethius and Philoponus’, *Oxford Studies in Ancient Philosophy* 19 (1999), 279-302, id., ‘Impossible *positio* as the Foundation of Metaphysics or, Logic on the Scotist Plan’, in C. Marmo (ed.), *Vestigia, imaginis, verba. Semiotics and Logic in Medieval Theological Texts* (Turnhout: Brepols, 1997), 255-76; id., ‘Obligations and Liars’ in S. Read (ed.), *Sophisms in Medieval Logic and Grammar* (Dordrecht: Kluwer, 1993), 357-81. An example analysed in twelfth century Christological discussion was that a person is both man and donkey. This was regarded as naturally impossible and supernaturally possible since Jesus Christ was a human being and a divine being simultaneously. See S. Knuuttila, ‘*Positio impossibilis* in Medieval Discussions of the Trinity’ in Marmo (ed., 1997), 277-88.

This, however, is impossible if mobile objects are considered according to their determinate natures. (*In Phys. VII.2.896*)

Aquinas's formulations illustrate the nature of medieval essentialism, also showing some baroque aspects of the theory.

2 MODAL SEMANTICS AND MODAL LOGIC IN TWELFTH AND THIRTEENTH CENTURIES

2.1 Equipollences and Oppositions

In the Latin translation of Aristotle's *Peri hermeneias* by Marius Victorinus, Aristotle's two terms for possible, *dunaton* and *endechomenon*, were apparently rendered by *possibile* and *contingens*. Boethius states that these words may be taken as meaning the same, although they are not used in quite the same way: while the privation of possibility is expressed by *inpossibile*, the corresponding word *incontingens* is not used. Many twelfth- and thirteenth-century authors thought that the terms *possibile* and *contingens* are used as synonyms or, if not, the term *possibile* is used for what is not impossible (possibility proper) and *contingens* for what is neither impossible nor necessary (two-edged possibility). Using the term 'possible' for the basic modality which includes necessity and contingency became

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87*In Periherm. II*, 382.17-22; 384.6-7; 392.17-393.12.

88For the synonymous use, see Peter Abelard, *Dialectica* 193.31; *Super Periermenias XII-XIV*, 8, 37; William of Sherwood, *Introductio in logicam*, 44-5; trans. Kretzmann (1966), 47, 50; Peter of Spain, *Tractatus*, I.24. In an anonymous twelfth-century commentary on Aristotle’s *Prior Analytics* (Ms. Orléans BM 283) *possible* refers to what is not impossible and *contingens* to this or to what is neither impossible nor necessary; f. 181a. (For this work, see S. Ebbesen, ‘Analyzing Syllogisms or *Anonymus Aurelianensis III* — The (Presumably) Earliest Latin Commentary on the *Prior Analytics*, and Its Greek Model, *Cahiers de l’Institut du Moyen-Âge grec et latin, Université de Copenhagen* 37 (1981), 1-20.) In the *Dialectica Monacensis* the term *possible* is characterized as expressing something like a genus of two species of modality, namely the necessity and the contingent (De Rijk II-2, 481.9-13, 20-21). Lambert of Auxerre and Robert Kilwardby suggest that while the common term may be ‘contingent’ or ‘possible’, ‘contingent’ is used for two-edged possibilities (Lambert of Auxerre, *Logica*, 40.1-42.2; Robert Kilwardby, *In libros Priorum Analyticorum expositio* (Venice, 1516, reprinted Frankfurt am Main: Minerva 1968), Tra-b); see also Albert the Great, *Commentarius in librum I Priorum Analyticorum*, ed. A. Borghnet in *Opera omnia* I (Paris: Vivès, 1890), IV 4, 546. Roger Bacon and some others stated that *contingens* is used of what is true or actual but can be false or un-actual, while *possible* refers to what is false or un-actual but can be true or actual; Roger Bacon, *Summulae dialectics, I: De termino, II: De enuntiatione*, ed. A. de Libera, *Archives d’histoire doctrinale et litteraire du moyen âge* 61 (1986), II.1.6, 247, 260-1, 395-6, 408-10; de Rijk 1967, II.1, 467; II-2, 391.19-19.
more common in late medieval logic. The contingency which was separated from
necessary and impossible was often divided into natural (naturale or naturum) and
indefinite (infinitum) depending whether the contingency was founded on a natu-
ral tendency which was realized in most cases or was without such a tendency.99 In
Boethius’s frequential classification, the contingency was divided into what takes
place in most cases, in few cases, and indeterminately, all these being cases of
utrumlibet.90 While some medieval authors called non-necessary contingency ad
utrumlibet,91 it became more usual to equate ad utrumlibet with the indefinite
contingency.92

Modifying Boethius’s systematization of Aristotle’s remarks in De interprcta-
tione 12 and 13, the authors of this period often presented the equipollences be-
tween modal terms and opposed relations between modal propositions with the
help of the following diagram:

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non possibile est non esse     contrariae
non contingens est non esse    contradictoryae
impossible est non esse        subcontrariae
necesse est esse

possible est esse             non possibile est esse
contingens est esse           non contingens est non esse
impossible est non esse        non possibile est non esse
non possibile est non esse    non necesse est esse
necesse est esse
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The square could be taken to refer to modals de dicto or singular modals de re.93

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99The background of this division was Aristotle’s Prior Analytics I.13, 32b4-14; see In Pr. an.
(Orléans) 186b; Lambert of Auxerre, Logica, 42.5-25; Robert Kilwardby, In Pr. an. 7vb- 8rb;
Albert the Great, In Pr. an. I.12, 477a.
90In Periherm. II, 240.6-21; 248.11-26.
91In Pr.an. (Orléans) 181a; Dialectica Monacensis, 481.18-21; Lambert of Auxerre, Logica,
42.4-5.
92Robert Kilwardby, In Pr. an. 7vb, 8rb; Roger Bacon, Summulae dialectics III: De argumen-
tatione, ed. A. de Libera, Archives d’histoire doctrinale et littéraire du moyen âge 62 (1987),
III.1.2, 62. Both uses are found in Albert the Great, In Pr. an. I.12, 476a-477a; 1.14, 481b;
see also K. Jacobi, Die Modalbegriffe in den logischen Schriften des Wilhelm von Sherwood und
in der logischen Analyse (Cologne: Brill, 1980), 92-4; H. Lagerlund, Modal Syllogistics in the
Middle Ages, Studien und Texte zur Geistesgeschichte des Mittelalters 70 (Leiden: Brill, 2000),
23-5.
93de Rijk 1967, II-1, 469-70; II-2, 393.6-394.5; 431.19-26; 483.1-484.5; William of Sherwood,
Introductiones in logican, 45.11; Peter of Spain, Tractatus I.25, 16.12-13; Roger Bacon, Summu-
lae dialectics II.1.6, 354; Thomas Aquinas, De propositionibus modalibus, ed. H.-F. Dondaine,
Opera omnia 43 (Rome: San Tommaso, 1976), 422. Peter of Abelard explained the relations
without a diagram (Super Periermenias 22.4-25.5).
Peter Abelard tried to define the opposed relations between quantified *de re* modals as well. He thought that these were the same as those between singular modal propositions, which is completely wrong. This question was not much discussed before its satisfactory solution in the early fourteenth century. (See p. 554 below.)

In discussing the imperfect first figure assertoric-contingency syllogisms, Aristotle formulated the principle 'If when A is, B must be, then when A is possible, B must be possible'. On the basis of this, he describes syllogisms as follows: 'If someone were to put the premises as A and the conclusion as B, it would not only follow that B is necessary, if A is necessary, but also that B is possible, if A is possible' (*Prior Analytics* 1.15, 34a5-7, 22-4). These are the rules of inference

\[(4) \quad p \to q \vdash Lp \to Lq \]

and

\[(5) \quad p \to q \vdash Mp \to Mq. \]

In *Metaphysics* IX.9, 1047b14-20, Aristotle argues for (5) by applying the characterization of potentiality in *Metaphysics* IX.3, 1047a24-6, which is based on the definition in *Prior Analytics* 1.13, 32a18-19: 'I use the expressions 'to be possible' and 'what is possible' in application to something if it is not necessary but nothing impossible will result if it is put as being the case.' If this is applied to possibility proper, it could be formulated as

\[(6) \quad Mp & (p \to q) \vdash Mq. \]

Aristotle did not develop his general remarks on propositional modal logic, but these principles were dealt with in later ancient discussions of the logic of conditionals and related issues. Similar rules were often put forward in early medieval logical treatises.

### 2.2 Modalities *de dicto* and *de re*

In dealing with modal propositions in his later commentary on *De interpretatione*, Peter Abelard argues that modal terms in the proper grammatical sense of the word are adverbs which modify the inherence between a subject and a predicate. Terms like 'necessarily', 'well' or 'rapidly' are used in this manner; adverbs like 'possibly' or 'falsely' are analogously called modals, although they do not modify actual inferences. Nominal modal terms occur in propositions like 'Necesse est

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95 See, e.g., Peter Abelard, *Dialectica*, 202.6-8; 219.5; *Logica 'Ingredientibus'* 429.36-7; 430.16-17.
Socratem currere'. When this is understood in accordance with its grammatical construction, it is not a genuine modal proposition, the mode itself being said of the grammatical subject which is the accusative and infinitive phrase. The nominal mode can also be understood in an adverbial manner, however, in which case the proposition expresses a modalized predication. In Abelard’s view, these and some further ambiguities demanded a systematic investigation of the properties of various propositions including modal terms. In his approach, grammatical distinctions were transformed into a logical distinction between de sensu modes and de re modes. Abelard states that in the present treatise he uses the terms de sensu and de re in the same way as Aristotle uses the expressions ‘in a compound sense’ (per compositionem or coniunctionem) and ‘in a divided sense’ (per divisionem) in Sophistici elenchi 4, 166a23-30, and that the distinction, understood in this way, maintains an inference from an affirmative possibility proposition de sensu to a corresponding affirmative possibility proposition de re. When possibility propositions de sensu are interpreted in a compound sense, both de sensu readings and de re readings are in fact de re readings which express modalized predications about actual things. The difference between these is that a modal proposition de sensu in a compound sense asserts that a subject which is said to have a predicate possibly, necessarily, or impossibly has it together with all descriptions of the subject mentioned in the proposition. A modal proposition de re does not imply this combination. The compound de sensu reading of ‘An F is necessarily/possibly/impossibly G’ is:

(7) An x which is F is necessarily/possibly/impossibly G while remaining F

and the de re reading is

(8) An x which is F is necessarily/possibly/impossibly G.

In addition to this analysis, Abelard mentions that de sensu modalities can also be understood in an impersonal manner in which case they do not refer to capacities or incapacities embedded in actual beings but instead to what can or cannot be. A de sensu possibility proposition in this strict sense does not imply a de re possibility proposition with an existential import. Abelard’s examples are ‘Every substance is a spirit’ and ‘My son is living’ said by a person not having a son. These are not true in the Aristotelian sense, but possible in the sense of what is not repugnant to nature. The differences between Abelard’s strict de sensu — reading and (7) and (8) could be expressed in standard modern notation as follows:

(9) ⊢Ex(Fx & Gx)

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96 Super Periermenias 3.7-11.16.
(10) \( \text{Ex}(Fx & \Diamond (Fx & Gx)) \)

(11) \( \text{Ex}(Fx & \Diamond (Gx)) \).

As already mentioned, Abelard applied the distinction between compound and divided meanings, without using these terms, in his influential analysis of the compatibility between contingency and divine omniscience. After Abelard the sections about modal propositions in logical treatises often begin with remarks on adverbial and nominal modes. According to the *Dialectica Monacensis*, the modal adverb qualifies the composition between the subject and the predicate. Modal propositions with nominal modes can be taken to mean the same as corresponding adverbial modal propositions or can be taken to mean that what is expressed by the accusative and infinitive element is necessary, possible, or impossible. The adverbial modes are said to be dealt with in Aristotle's *Prior Analytics* and the non-adverbial nominal modes in his *De interpretatione*.99 The structure of adverbial modal propositions without negation is:

(12) quantity/subject/mode, copula/predicate.

The question of the quality (affirmative, negative) is associated with the problem that the sign of negation can be located in two different places:

(13) quantity/subject/negation, mode, copula/predicate

or

(14) quantity/subject/mode, negation, copula/predicate.

If modal propositions with a negation are read in accordance with (13), then the mode is denied; if they are read in accordance with (14), the modal adverb qualifies a negated predication. The quality and quantity of modal propositions are determined by those of the corresponding non-modal propositions. Thus (14) is a negative modal proposition and (13) an affirmative proposition with a negated mode. Propositions with non-adverbial nominal modes are always singular, their form being:

(15) subject/copula/mode.

In the *Dialectica Monacensis*, modal propositions with the structure of (12)–(14) are also called *de re* or divided modalities and those with the structure of (15) *de dicto* or compound modalities.100 This terminology was commonly employed in later medieval treatises.101 In the twelfth century some logicians began to apply the grammatical distinction between categorematic and syncategorematic terms to

100 de Rijk 1967, II-2, 570.22-9.
101 See Jacobi 1980, ch. 4.
modal terms. These were taken to have a categorematic meaning as de re modes and a syncategorematic meaning as de dicto modes.\textsuperscript{102}

The distinction between various interpretations of possibility propositions was extensively dealt with in twelfth- and thirteenth-century discussion of Aristotle’s examples of fallacies based on shortcomings in distinguishing between compound and divided meanings. The compound interpretation of ‘A standing man can sit’ was usually taken to be ‘It is possible that a man sits and stands at the same time’. Many authors formulated the corresponding divided interpretation as involving a reference to a later or earlier time.\textsuperscript{103} The reference to another time was based on the assumption that the necessity of the present prevents the acceptance of

\[(16) \; p \text{(now)} \& \Diamond \neg p \text{(now)}.
\]

The authors who regarded (16) as false did not operate with counterfactual alternatives. They thought that if an unactualized present possibility is assumed to be actualized, something impossible follows, as is seen in the rule of thirteenth-century obligations logic, according to which one should deny that the ‘now’ of a disputation is the present ‘now’, if it is assumed for the case of argument that a contingent proposition which is false now is true.\textsuperscript{104}

In discussing the proposition ‘That which is standing can sit’ the author of the \textit{Dialectica Monacensis} states that the de re possibility can be predicated of the subject abstracting from the subject’s actual qualifications; this may imply a counterfactual interpretation of the modal proposition. Another example of the idea of modal alternativeness in this and some other works is found in the theory that the word ‘can’ may amplify the subject term so that it stands for actual and merely possible beings.\textsuperscript{105}


\textsuperscript{103}In \textit{De obligationibus} attributed to Willam of Sherwood, the author writes: ‘When a false contingent proposition concerning the actual instant of time has been posited one must deny that it is the actual instant. This is proved as follows. Let A be the name of the actual instant; it is discrete name and not a common name. When it is false that you are in Rome, it is impossible that it is true then or in A. It can become true only through a motion or through a change, but it cannot become true through a motion in A, because there is no motion in an instant. And it cannot become true through a change, since if there were a change to truth in A, the truth would be in A, for whenever there is a change, there is the term of that change. So it is impossible that this false proposition becomes true in A. Therefore, if it is true, A is not actual.’ The text is edited in R. Green, \textit{The Logical Treatise ‘De obligationibus’: An Introduction with Critical Texts of William of Sherwood (?) and Walter Burley} (PhD diss., University of Louvain, 1963).

\textsuperscript{104}Rijk 1967, II-2, 570.18-571.9; 623.33-624.26. Cf. 1.5 above.
2.3 Modalities with Temporal Determinations

Avicenna and Abelard showed particular interest in alethic modalities with determinations. Like Boethius, Abelard divided propositions with a *dum*-phrase (‘so long as’) into two groups, depending on whether the determination was intrinsic or extrinsic, i.e., whether it is the same as the modalized predicate or not. Both of these can be understood in two ways. The word *dum* can be taken to indicate time and to form a compound predicate. In this ‘modal’ reading the intrinsic or extrinsic *dum* phrase is included as a restriction in what is predicated with a modal qualification. Or they may be considered as complex propositions which have as parts a modal proposition and a non-modal *dum* proposition and assert that something is necessary or possible or impossible and things are as the *dum* phrase says they are.¹⁰⁶ There were similar discussions in Avicenna. In the part on logic of his *Pointers and Reminders*, Avicenna distinguishes between absolute necessity propositions (like ‘God exists), which are always true and refer to eternal things, and various conditional necessity propositions. These are true while the non-eternal substances referred to by the subject terms exist (like ‘Man is necessarily a rational body’), some descriptions are attached the substances (like ‘All that is moving is necessarily changing’), what is predicated is actual or a definite or indefinite time is actual.¹⁰⁷ Propositions which are necessary on the condition that the subject is qualified by a description show similarities to Abelard’s *de re* compound modal propositions.

Apart from the distinction between two basic readings Abelard’s remarks on the propositions which contain a modal term and a *dum* phrase are not always easily understood. If the temporal *dum* phrase is included in the modal proposition, one may wonder how to understand the difference between simple modal propositions and modal propositions with temporal determinants. Abelard states that a determined possibility proposition implies a corresponding simple possibility proposition, but not *vice versa*, and a simple necessity proposition implies a determinate necessity proposition, but not *vice versa*.¹⁰⁸ These principles could be derived from the standard idea that simple possibilities are temporally indefinite and may be prevented from being actualized as temporally qualified, but Abelard also refers to a distinction between treating the determination as pertaining to a modal term and treating it as pertaining to the scope which is modalized.¹⁰⁹

¹⁰⁶ *Dialectica* 206.14-37; *Super Perihermenias* 36.22-37.23.
¹⁰⁸ *Super Perihermenias* 36.11-21.
'Socrates necessarily sits when he sits' is true in the first case but false in the second case. Perhaps he means that while the modal status of a proposition pertaining to a temporally definite event is basically independent of temporal changes, it can be changeable when the modal qualification is evaluated from a temporal point of view. Things which are not necessary now may be so to-morrow. This is in agreement with Abelard's view about unchangeable metaphysical necessities and possibilities and temporal modalities which are associated with real potencies.\textsuperscript{110}

After Abelard, an often discussed theory about time and modality was associated with the distinction between temporal modalities \textit{per se} and \textit{per accidens}. \textit{Per se} necessary propositions were said to be true whenever they were uttered and \textit{per accidens} necessary propositions, true past tense singular propositions referring to a definite event, were unchangeably true after having begun to be true. The same analysis was applied to impossible propositions as well.\textsuperscript{111}

\section*{2.4 Modal Conversion}

While the conversion of assertoric propositions was frequently discussed in early medieval logic, the modal conversions were not often dealt with before the \textit{Prior Analytics} began to be used in logic teaching in the thirteenth century. One earlier example is Peter Abelard's remark that modal propositions in the divided sense are only convertible into assertoric propositions, not into modal propositions; for example, ‘For every man it is possible to run’ converts into ‘Something for which it is possible to run is a man’.\textsuperscript{112} This was found problematic when it was realized that conversion of modals into modals played an important role in Aristotle's modal syllogistics which seemed to concentrate on modalities in the divided sense.\textsuperscript{113} According to Aristotle (\textit{An. pr.} I.3), necessity propositions are converted in the same way as the corresponding assertoric propositions: a universal affirmative predication (AaB) implies a converted particular predication (BiA), a particular affirmative predication (AiB) is equivalent with a converted predication (BiA), and a universal negative predication (AeB) is converted into universal negative predication (BeA). The first conversion was called accidental and the others simple. While these rules are not problematic with respect to modals in the compound sense, Aristotle employed them in proving modal syllogisms some of which seem to be acceptable only when the premises are modal propositions in


\textsuperscript{111}See de Rijk II-1, 371; II-2, 429.1-10; 481.22-482.14; Roger Bacon, \textit{Summulae dialectices} 2.1.6, 366-72; William of Sherwood, \textit{Introductiones in logicam} 41.8-16, trans. Kretzmann 1966, 41, and note 16 above.

\textsuperscript{112}\textit{Super Perihermenias} 11.17-12.20.

\textsuperscript{113}For this view of modal syllogistics, see \textit{Summe Metenses} in de Rijk 1967, II-1, 468; \textit{Dialectica Monacensis}, de Rijk 1967, II-2, 480.10-16; Lambert of Auxerre, \textit{Logica} 30.17-23.
the divided sense. One of the problems of his modal syllogistics is that reading these conversion rules in the divided sense leads to obvious difficulties: the actuality of subjects changes into necessity and possibility into actuality. Contrary to what one might expect, the relationship between compound and divided modalities was not dealt with in mid-thirteenth-century discussions of Aristotle's rules, these being associated with various philosophical ideas of the nature of necessity and contingency. Many authors tried to redefine syllogistic necessity propositions in a way which would match the conversion rules. This took place in treating some often repeated counter-examples, such as

(17) Everything literate is necessarily a human being

(18) Everything healthy (or awake) is necessarily an animal

or

(19) Everything white is necessarily a body.

It was considered an obvious fact that no animal is necessarily healthy or that ‘Some animal is healthy’ is not a necessary truth. Some authors argued that in examples like (17)–(19) the denominative subject terms refer to things which are qualified by non-essential forms and that these terms should be treated in the same way when they are predicates. 114 ‘An animal is necessarily healthy’ is taken to mean that an animal is necessarily that which is healthy. Accordingly, (17) could be read: ‘Every literate being is necessarily a member of the class of human beings’, but even then the conversion ‘Some human beings are necessarily members of the class of literate beings’ is false, if this is not understood, as Lambert of Auxerre suggests, to mean that some human beings are necessarily those who are literate, that is, human beings.

Robert Kilwardby, after mentioning this pretty artificial idea, moves to a ‘more probable’ interpretation which is based on the view that convertible necessity premises in modal syllogistic are necessity propositions per se and not per accidens, like (17)–(19), which are not convertible. In affirmative necessity propositions per se, the subject is per se connected to the predicate. In negative necessity propositions per se, the subject is apparently per se incompatible with the predicate. In accidental necessity propositions, the connection is not based on a per se inherence or repugnance. 115 In explaining the notion of per se later in his work,


115 In Pr. an. 7ra-b; cf. 45rb for essential negative propositions. Kilwardby's concludes in 7rb: 'Therefore, when Aristotle teaches that necessity propositions are convertible, he means that only per se necessity propositions are convertible'. Aristotle did not say this; Kilwardby followed the medieval commentator habit of maintaining that what commentators found reasonable was what Aristotle said or meant. Kilwardby seems to assume that syllogistic necessity propositions should be taken in the divided sense or at least not in the composite sense (27va). For per se necessity in Kilwardby, see also Lagerlund 2000, 25-42; Thom 2003, 93-6; P. Thom, Logic and Ontology in the Syllogistic of Robert Kilwardby, Studien und Texte zur Geistesgeschichte des Mittelalters 92 (Leiden: Brill, 2007), 19-28.
Kilwardby refers to Aristotle's discussion of different types of essential predication in *Posterior Analytics* I.4. He does not give a detailed explanation of why the *per se* inferences or incompatibilities make propositions convertible, but he does associate convertible propositions with terms signifying their subjects invariably and in this sense necessarily. This excluded the conversion of *per se* necessary predications involving accidental terms, such as 'All walking is necessarily moving.'

There were similar discussions among Arabic philosophers. Avicenna did not accept the conversion of affirmative necessary propositions, although he considered the conversion of universal negative necessity propositions as valid. Averroes restricted the syllogistic necessity premises to necessity propositions *per se*. In these propositions the terms, such as 'animal', are essential, invariably signifying necessary constituents of things. Accidental terms, such as 'walking' or 'white', signify accidental things. The background to this distinction is *Post. An. I.4*. The conversion of these propositions is regarded as unproblematic. The idea that the terms of syllogistic necessity premises are themselves essential also occurs in Kilwardby, although he does not use quite the same terminology.

Averroes, Robert Kilwardby, Albert the Great, and their followers regarded Aristotle's modal syllogistics as a correct theory of modalities, the explication of which often demanded metaphysical considerations. Restricting the modal conversion of necessity propositions to those involving essential terms is an example of this attempt to discern the unity and coherence of Aristotle's theory. The same orientation dominated the discussion of the conversion of contingent propositions.

According to Aristotle, a two-edged negative contingency proposition (neither necessary nor impossible) implies an affirmative contingency proposition of the same quantity, and all contingency propositions are converted by the conversion of terms into particular contingency propositions (*An. pr.* I.3, 25a37-b19; 13,

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116 In *Pr. an.* 8va; 22ra; 25rb; Roger Bacon, (*Summulae*, III.1.2, 59) refers to Aristotle's first and second mode of *per se* predication (*An. post.* I.4, 73a34-b5). Cf. Albert the Great, *In Pr. an.* IV.10,559a.

117 Avicenna's discussion of the conversion differs from other medieval approaches because of his special view that the subject terms and predicate terms of assertoric propositions stand for all possible applications. The instantiations of the subject term of a universal affirmative proposition are asserted to have the predicate at least once (*Pointers and Reminders* 2.5, trans. in Street 2002, 134). Other classes of proposition are treated in an analogous way. Avicenna took this to mean that universal negative assertoric propositions are not convertible (3.3; Street 2002, 143). The terms of modal propositions are amplified in the same way as the terms of assertoric propositions. Universal negative necessity propositions are simply convertible and other necessity propositions are not convertible. Universal negative possibility propositions do not convert. Affirmative possibility proper propositions convert like assertoric propositions (3.5; Street 2002, 144-5; Thom 2003, 65-6, 69).

118 Averroes's detailed discussion of the structure of modal propositions is included in his separate essay on modal syllogistics, translated into Latin in the *Quaesita octo in librum Priorum Analyticorum Aristotelis in Aristotelis Opera cum Averrois Commentariorum I.2b* (Venice 1562-1574); see IV.3, 83-4. In his commentary on the *Prior Analytics* (*Media expositio in libros Priorum Resolutiorum in Aristotelis Opera cum Averrois Commentariorum* (Venice, 1562-74), I.1) Averroes simply says that necessary predications express essential connections; see 45va. See also Thom 2003, 81-5.
32a29-b1). This was taken for granted in mid-thirteenth-century logical treatises. It was noted that while the converted propositions of indefinite contingency propositions are of the same type of contingency, the conversion of natural contingency propositions or of those of the minor part contingency, whether with respect to quality or terms, result in different modal propositions.\textsuperscript{119} There were extensive discussions of the kinds of contingency in the commentaries by Robert Kilwardby and Albert the Great and in other treatises by their contemporaries based on various philosophical ideas of contingency.\textsuperscript{120} Following Aristotle's remark in \textit{An. pr.} I.13, 32b23-32, according to which 'A contingently belongs to B' may mean either 'to that to which B belongs' or 'to that to which B contingently belongs', Kilwardby argues that the major premise in uniform first figure contingency syllogisms is read in the second way, having the form 'Everything/something that is contingently B is contingently A'.\textsuperscript{121} Contingency premises are amplified, if syllogistic relations do not demand restrictions.\textsuperscript{122} In explaining the difference between necessity propositions and contingency propositions in this respect, Kilwardby teaches that syllogistic terms are substantial or accidental. Substantial terms necessarily stand for the things they signify, while accidental terms contingently stand for them. Since the terms in \textit{per se} necessity propositions are substantial, 'Every A is necessarily B' and 'Whatever is necessarily A is necessarily B' mean the same.\textsuperscript{123}

Averroes did not have much to add to what Aristotle says about the convertibility of contingent propositions, except that the subject terms of syllogistic contingency propositions are always meant to be read with the prefix 'Everything/something which is or is contingently'.\textsuperscript{124} As distinct from the mid-
thirteenth century Latin commentators, Averroes taught that dealing with the various types of non-necessary contingent propositions is not relevant in logic.\textsuperscript{125} Aristotle’s *Prior Analytics* continued to be part of university logic teaching in the thirteenth century. Kilwardby’s commentary was used in this context as is shown by the fact that many of the questions dealt with by him were considered in later thirteenth-century treatises on the *Prior Analytics*. These have not yet been edited. The next edited work is Richard Campsall’s *Questions on Prior Analytics*, written in Oxford c. 1305.\textsuperscript{126} Campsall often refers to various views which show that he was familiar with modal ideas different from those in Kilwardby. Fourteen of twenty questions are about modal propositions and two are about past and future tense propositions. Campsall almost always begins his discussions of modal questions by referring to the distinction between compound and divided modal propositions, apparently thinking that one should deal with the logic of both readings, as he does to some extent. This became a standard procedure after Campsall. Analysing the fine structure of singular modal propositions and reducing the role of metaphysical principles were further new features of Campsall’s work in comparison to the mid-thirteenth century logic of modalities. He wanted to show that Aristotle’s theory was consistent, although it was associated with various problems. In trying to resolve these, Campsall was obliged to formulate some rules which were not used by Aristotle.

Campsall treats the conversions of necessity and contingency propositions in the compound sense as the conversions of their assertoric contents (*dicta*). These conversions are based on the rules that when assertoric propositions are qualified by the notions of necessity or contingency (neither necessary nor impossible), the converted propositions are qualified by necessity or contingency, with the exception that the conversion of universal affirmative contingency propositions does not always result in the same kind of contingency.\textsuperscript{127} Divided necessity propositions are also converted in the same way as assertoric propositions, and the same holds true of affirmative contingency propositions.\textsuperscript{128} Negative contingency propositions are regarded as equivalent to affirmative contingency propositions of the same quantity.\textsuperscript{129} Campsall regards these as Aristotelian modal principles and makes use of some additional rules in order to defend their questionable validity.

Campsall describes the notion of necessity as follows:

An affirmative necessity proposition in the divided sense is true only

\textsuperscript{125} *Media expositio in libros Priorum Resolutoriorum* 36rb.
\textsuperscript{126} *Quaestiones super librum Priorum Analyticorum* in *The Works of Richard Campsall* I, ed. E.A. Synan (Toronto: Pontifical Institute of Mediaeval Studies, 1968); there are a lot of mistakes in this edition. For unedited works, see Ch.H. Lohr, ‘Medieval Latin Aristotle Commentaries’, *Traditio* 23-30 (1967-74).
\textsuperscript{127} 5.39 (111); 6.21 (121); 7.32 (135). When ‘contingens’ and ‘possibile’ are not treated as synonyms, ‘contingens’ refers to what is neither necessary nor impossible and ‘possibile’ to what is not impossible (as in 7.32). Campsall does not discuss the distinctions between the kinds of contingency on which earlier authors spent much time.
\textsuperscript{128} 5.40 (111); 6.22-4 (121-2); 7.33 (135-6).
\textsuperscript{129} 16.21 (250).
if something contained under the predicate term is unchangeable with respect to something contained under the subject term, as long as this exists. This holds when the terms are common. When the terms are singular, it is required that what is signified by one is not changeable with respect to what is signified by the other, as long as this exists.\footnote{6.25 (122-3).}

Unchangeable features signified by common terms may be transcendental properties, such as 'something', substantial forms, properties based on the species nature of things, such as 'mortal', 'living', or other inseparable attributes.\footnote{5.43-5 (112-3); 9.18 (157).} Divided affirmative necessity propositions with common or singular terms include the assumption that the properties designated by the terms belong invariably to some same actual individuals, as long as these exist.\footnote{For the actuality condition, see 5.40 (111); divided negative propositions express that the things under the terms are necessarily separated (5.38 (110)).}

Campsall believes that this reading of divided necessity propositions explains why they are regulated by the Aristotelian rules of conversion. The strict restriction to actual things is not separately explained since Campsall probably thought that Aristotelian necessity premises should be treated in this way and that only actual things can have necessary properties. Much attention is paid to the example 'A pale Socrates is necessarily Socrates'. This is said to be false for the reason that what the accidental subject term signifies is not an invariable characterization of Socrates.\footnote{6.25-31 (122-5).} Campsall does not deal with the traditional universal or particular counter-examples, thinking that they can be analysed into the conjunctions or disjunctions of singular propositions with terms standing for accidental combinations.

The equation of necessity with unchanging relations pushes Campsall’s theory toward the temporal frequency approaches of necessity. It is also characterized by some versions of the principles that the present is necessary and that possibility implies actuality. Let us take a look at his proofs of the conversion of divided necessity propositions. According to Campsall, 'Every/some $B$ is necessarily $A$' is converted into 'some $A$ is necessarily $B$', because the negation of the converted proposition 'Every $A$ is possibly not $B$' is not compatible with the proposition to be converted. This is allegedly shown by the fact that were the negation true, a particular being, say $c$, which is $B$ and necessarily $A$ would be possibly not $B$. This is said to be impossible, apparently because $c$ is necessarily $B$ qua being unchangingly $A$.\footnote{6.22 (121-2).} Campsall's proof of the convertibility of universal negative divided necessity proposition is similar — this time he states that 'this consequence is necessary: $c$ can be one of those which are now contained under $B$; therefore it

\footnote{According to Lagerlund, Campsall assumes that if $c$ is possibly not $B$ now, it is not $B$ now (2001, 68). Campsall explicitly denies this principle later (19.21 (297)) and the argument can be understood as indicated above. Campsall maintains, however, that if $c$ is not $B$ now, it is necessarily not $B$ now (5.50 (114)). If all negative propositions about actual things are necessary, it is clear that there are no simultaneous de re alternatives. In this sense actuality implies necessity. It seems that Campsall was not fully aware of the problems embedded in his formulations.}
is one of those which are now contained under \( B' \).\(^{135}\) Possibility implies actuality in the sense that an affirmative possibility proposition about actual connections between things implies the corresponding affirmative proposition.

Campsall states that Aristotle’s rules for the conversion of divided contingency propositions can be applied to predications the subject terms of which stand for things which are under them or which are contingently under them. As far as the predicate term stands for actual things, the distinction pertains to contingent attributes of things which are necessarily (\textit{per se}) or contingently (\textit{per accidens}) under the subject term.\(^{136}\) Contingency means in this context that things signified by the terms are not invariable with respect to each other. Applying the standard supposition analysis, Campsall states that a universal affirmative contingency proposition in the divided sense means that \( A_1 \) is contingently \( B_1 \) or \( B_2 \) and so on and \( A_2 \) is contingently \( B_1 \) or \( B_2 \) and so on. A particular proposition is a disjunction of these. ‘This \( A \) is contingently this \( B' \)’ is said to express a conjunction of two possibility proper propositions one of which is affirmative and the other is negative, ‘This \( A \) is possibly this \( B \) and this same \( A \) is possibly not this same \( B \)’. The converted form can be neither necessary nor impossible, because the conversions of these would be incompatible with the original contingency proposition.\(^{137}\) If the terms stand for actual things, the negative possibility proposition denies that the attribute which possibly and therefore actually belongs to the subject does so necessarily, i.e., \( A \) and \( B \) as the attributes of this \( c \) are not invariable with respect to each other.

\section*{2.5 Modal Syllogistics}

Avicenna wrote a brief summary of Aristotle’s modal syllogistics, but his own theory was different, concentrating on necessities and proper possibilities understood in the same way as divided modal propositions in Latin discussions and being based on amplified terms and conversion rules mentioned above. He also sketched some ideas about modal syllogistics for propositions with temporal determinants. Insofar as Avicenna treated the terms of assertoric propositions as modally amplified and equated the truth-conditions of purely assertoric propositions and possibility propositions, syllogisms with assertoric and necessity premises are similar to syllogisms with possibility and necessity premises and syllogisms with assertoric and possibility premises or with assertoric premises to uniform possibility syllogisms.\(^{138}\)

While Averroes’s commentary on the \textit{Prior Analytics} follows the main lines of Aristotle’s text, his separate treatise on modality involves new systematic ideas, mainly the theory of accidental and \textit{per se} necessary essential terms. According to Averroes, necessary syllogistic premises are \textit{per se} necessary propositions with

\(^{135}\) 5.40 (111).
\(^{136}\) 7.33 (135), 17.56 (270).
\(^{137}\) 7.34-6 (135-7). See also Lagerlund 2000, 69-72.
\(^{138}\) See note 117 above; Street 2002; Thom 2003, 67-80. For later Arabic criticism of Avicenna’s temporal frequency approach to modality and modal syllogistic, see also Street 2005.
per se necessary essential terms. Since modal premises are of the divided type, assertoric premises in Aristotelian mixed necessary-assertoric-syllogisms must have a predicate term which in fact is necessary. Universal and particular propositions of this kind are called assertoric per se and necessary per accidens, since the predicate always belongs to the subject when the subject is actual. Propositions with a necessary subject term and a non-necessary predicate term are said to be temporally assertoric, since the predicate belongs to the subject merely at some times. The first premise in mixed assertoric-necessary syllogisms must be of this kind (or a proposition of necessary matter). The conclusion in the first case is necessary per accidens and in the second case temporally assertoric. This is a speculative explanation of Aristotle's asymmetric treatment of mixed necessary-assertoric syllogisms and mixed assertoric-necessary syllogisms. Analogous essentialist ideas were developed in thirteenth-century Latin discussions. According to Averroes, all syllogistic contingency propositions are amplified with respect to potential subjects. Assertoric premises in mixed contingency-assertoric syllogisms are based on universal predications which hold for all times for all subjects or at least for most subjects.

The first known Latin commentary on Prior Analytics (Ms. Orléans BM 283) involves fairly detailed explications of Aristotle's modal moods and discussions of many problems dealt with in ancient commentaries. A concise summary of Aristotle's modal syllogistic is also provided in the late twelfth-century Dialectica Monacensis, and the elements of modal syllogistics seem to have been discussed in logic courses in the first part of the thirteenth century. In the 1240's, Robert Kilwardby's commentary became an authoritative work on Aristotle's Prior Analytics. The remarks on modal syllogistics in Albert the Great's commentary were largely derived from Kilwardby.

According to Kilwardby, the premises and the conclusion in uniform necessity syllogisms should be necessary per se, as is clear from his interpretation of the conversion rules of necessary propositions. In mixed first-figure syllogisms with a major necessity premise and a minor assertoric premise, the non-modalized premise should be simpliciter assertoric rather than merely ut nunc (as-of-now). A simply assertoric premise of this mixture includes a predication which is per se

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139 Quaesita octo in librum Priorum Analyticorum, IV.3, 84.
140 See also the discussion in Thom 2003, 81-5.
141 Gersonides tries to develop further Averroes's remarks in the modal part of his Book of the Correct Syllogism; see Manekin, 1992, 309.
142 See note 124 above.
143 Media expositio in libros Priorum Resolutoriorum, 45va-b.
144 See, e.g., Braakhuis 1979, 243-4; de Rijk 1967, II-1, 468; Peter of Spain, Synogogorum corporum, 304; Compendium examinatorum Parisiense, ed. C. Lafleur and J. Carrier (Québec: Faculté de Philosophie, Université Laval, 1992), 176-94.
145 While Kilwardby does not analyse the structure of necessity premises in discussing the uniform necessity syllogism (15ra-16ra), he explains the nature of per se necessity premises elsewhere (16va; 21ra; 22ra; 25rb). In treating the exposition proofs of Baroco and Bocardo, Kilwardby states that either particular premises are changed into universal premises in expository syllogisms or expository syllogisms are formed by taking one singular example and having two singular premises (16ra).
in the first or second way (cf. Aristotle, An. *post.* I.4) and consequently necessary *per se* and not merely *per accidens*. Similarly Kilwardby states that in mixed first-figure syllogisms with contingent and assertoric premises the assertoric premise must be simply assertoric, but now not necessary. It is simply assertoric if the predicate belongs to the subject essentially, invariably or by natural contingency.146 Kilwardby states that even though the conclusions in mixed first-figure necessary-assertoric moods and mixed assertoric-necessary moods with simply assertoric premises are necessary, they are syllogistically derived as necessary only in the first case. The modal character of the predicate in the conclusion of perfect first-figure syllogisms follows that of the first premise which involves the whole syllogism in accordance with the *dici de omni et nullo*.147

In explaining the varieties in reading assertoric premises in mixed syllogisms, Kilwardby states that a first-figure major necessity premises 'appropriates' to itself a minor which is unrestrictedly assertoric, i.e., necessary *per se*. No such appropriation occurs in first-figure mixed assertoric-necessity syllogisms, the first premise being either necessary or true as-of-now. In the second figure, while the universal negative necessity premise appropriates to itself an unrestricted assertoric premise, a particular necessity premise cannot appropriate a universal to itself, nor an affirmative a negative. There are similar considerations about third-figure moods.148

In uniform contingency syllogisms, the subject term of the major premise is read 'Everything/something that is contingently A' but this ampliation does not concern mixed contingency syllogisms.149 This major contingency premise is always false when the subject is an accidental term and the predicate a substantial term. If the same applies to contingent minor premises, it seems that a major premise can be true with a minor only when the middle term is accidental, taking for granted that both terms cannot be substantial in a true contingency proposition. Kilwardby explains, however, that if the subject term of the minor premise is not amplified, it may be true even when the predicate is a substantial term.150

Kilwardby's considerations show similarities to Averroes's classification of modal propositions with essential and accidental terms. Both authors assume that propositions of the same form can have different interpretations depending on how they are related to other propositions in a syllogism. These interpretation rules are based on various metaphysical assumptions. Kilwardby's *ad hoc* remarks on contingency premises read in the sense of natural contingency in certain moods show the same approach.151

146Kilwardby, *In In Pr. an.* 16va; 25rb; see also Albert the Great, *In In Pr. an.* IV.10, 558b-559b.
147Kilwardby, op. cit. 16vb-17ra; 21ra-b; Albert the Great, op. cit. IV.4, 526a-b. For the application of the *dici de omni et nullo* in this context, see also Lagerlund 2001, 41-2.
14816vb; 17va-b; 18vb; 24vb. See also Thom 2007, 148, 160-1, 165-6, 172-4.
149Kilwardby, op. cit. 19vb, 21ra-b; Albert the Great, op. cit. IV.2, 540b-541a.
150Kilwardby, op. cit. 22ra; Albert the Great, op. cit. IV.4, 545a-b.
151In a mixed first-figure necessity-contingency mood the first negative premise 'appropriates'
As distinct from his forerunners, Campsall comments separately on Aristotelian modal syllogisms with premises in the compound and divided senses. Uniform necessity syllogisms are valid on both readings. Syllogisms in the compound sense are reduced to the principle

(20) If the premises are necessary, the consequent is necessary.

Syllogisms in the divided sense are valid because they are regulated by the same principles as the assertoric syllogisms, i.e., *dici de omni et nullo*.\(^{152}\) Mixed necessity-assertoric moods are valid in the compound sense on the basis of (20), when the assertoric premise \((de \ inesse \ proposition)\) is of the *de inesse simpliciter* type, i.e., necessarily true. These consequences are not syllogistic, however, and the same holds of mixed assertoric-necessity moods with a necessary conclusion. Mixed necessity-assertoric syllogisms in the divided sense are valid when the assertoric premise is *de inesse simpliciter* or *ut nunc primo modo*, i.e., a proposition which express a necessary relation between actual things. Propositions *de inesse secundo modo* express accidental relations and assertoric minor premises of this kind do not make valid syllogisms. Campsall describes the validity of the perfect necessity-assertoric syllogisms by referring, like Kilwardby, to the principle that the whole syllogism is implicitly included in the first premise. The middle term is said to be sufficiently the same in both premises only when the minor premise is in fact necessary.\(^{153}\)

If syllogistic premises are contingent in the compound sense, it does not follow that the consequent is contingent in the compound sense. Aristotelian uniform contingency syllogisms are valid for divided modals when the middle term is amplified with respect to contingency.\(^{154}\) Aristotelian first-figure mixed contingency-assertoric syllogisms are said to be valid for divided modals provided that the contingency premise is not amplified and the assertoric premise is assertoric *simpliciter* or *ut nunc primo modo*.\(^{155}\) Because affirmative and negative contingency propositions with terms standing for actual things imply affirmative assertoric propositions and the contingent premises in mixed necessary and contingent syllogisms are of this type, the conclusions are assertoric. Aristotle seemingly denied this, but in fact he wanted to say that the assertoric conclusion was evident in negative moods and less apparent in affirmative moods.\(^{156}\) The conclusions are

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\(^{152}\)Campsall 11.58-9 (194); see also Lagerlund 2000, 74.

\(^{153}\)12.33-5 (205). Campsall states that mixed assertoric-necessity Baroco and mixed necessity-assertoric Bocardo with necessary conclusions are syllogistically valid. When Aristote did not accept these, he did not mean that they were not acceptable — it was only because the indirect proof involved a possibility proper syllogism which was presented later in the *Prior Analytics*. The corresponding mixed necessity-assertoric Baroco and mixed assertoric-necessity Bocardo are valid consequences, but not syllogistic (14.16-19 (221-2)).

\(^{154}\)17.48-9, 17.70 (268-9, 274).

\(^{155}\)18.36-8 (283-4).

\(^{156}\)20.14-17 (304). Campsall does not deal with Aristotelian mixed contingency-necessity moods, apparently regarding them immediately obvious when the contingency premises are not ampli-
assertoric also in mixed assertoric-contingency syllogisms, for the predicate of the
contingency premise must stand for actual things and consequently be equivalent
to an assertoric premise, but this time Campsall says that the assertoric conclu-
sions do not follow syllogistically.\textsuperscript{157}

As far as the middle terms of contingent syllogisms stand for actual things, as is
the case with mixed moods with contingent minor premises, there is no difference
in the logic of affirmative premises and contingency premises. A negative assertoric
premise is said to imply the corresponding necessity premise. Why is this step asso-
ciated with negative propositions and not with affirmative propositions? Campsall
seems to think that negative premises express actual incompatibilities which are
analogous to essentialist \textit{de re} necessities. The possible denial of ‘Socrates is pale’
does not refer to the actual time at which Socrates is pale; Campsall says that as
long as Socrates exists, being pale is variable with respect to him and ‘sometimes
when he is ‘Socrates is a pale Socrates’ is true and sometimes it is false’.\textsuperscript{158} Even
though a \textit{de re} possibility of not being does not imply non-being now, it seems to
imply non-being later.

It has been argued that the asymmetry between affirmative and negative \textit{de re}
possibilities is based on the insight that if a contingency proposition with actual
terms implies two possibility propositions, one affirmative and one negative, and
the affirmative implies actuality, the negative cannot imply actuality. The no-
tion of contingency would otherwise be self-contradictory. Campsall was allegedly
familiar with the Scotist denial of the necessity of the present and applied it to af-
firmative \textit{ut nunc} propositions, although he continued to think that true negative
\textit{ut nunc} propositions were necessarily true.\textsuperscript{159} However, if Campsall made use of
the Scotist idea of simultaneous alternatives, why did he think that what can be
actual now is actual now and that actual things are contingent only if they change
later? Scotus’s idea was that contingently actual things can be otherwise at the
very moment of their actuality. Campsall cannot say this, because all proposi-
tions expressing present alternatives are necessarily false.\textsuperscript{160} While there are no
alternative possibilities with respect to the present, actual things are not simply
necessary if they can change later. This is one of the traditional ways to qualify
the necessity of the present.

\begin{footnotesize}
\textsuperscript{157}18.71-2 (291-2). Campsall states that when the first premise of a first figure mood is affir-
mative \textit{simpliciter} or \textit{ut nunc primo modo}, the syllogistic conclusions are contingent and not
merely non-impossible, as in negative moods. Aristotle says that all conclusions are possible,
and Campsall states that this follows when the major premise may also be \textit{ut nunc secundo
modo} (18.54-7 (287-9)). This is meant to explain Aristotle’s formulations. Campsall’s earlier
remark that Aristotle did not operate with assertoric premises of this kind in mixed syllogisms
apparently pertains to minor assertoric premises (12.32, (205)).
\textsuperscript{158}12.31 (204), 19.21 (297).
\textsuperscript{159}Lagerlund 2000, 87-90. For Scotus’s view, see 3.1.
\textsuperscript{160}See note 134 above.
\end{footnotesize}
3 LATER MEDIAEVAL DEVELOPMENTS

3.1 Modalities in Philosophy and Theology

It has been assumed that the increasing interest in modal syllogistic and modal logic in general in early fourteenth century was associated with certain philosophical and theological developments which added to the interest in modal theories. These often involved the idea of simultaneous alternatives which did not play a significant role in mid-thirteenth-century essentialist theory.

Following the twelfth-century model, Henry of Ghent (d. 1293) applied obligations logic to Trinitarian doctrine by assuming doctrinally impossible positions in order to see what followed from them and what did not.161 This was an influential idea. Several early fourteenth-century authors found an obligational analysis as a useful tool for analysing conceptual connections between theological concepts. The most popular version of this logic was called *positio*. It deals with how an increasing set of true and false propositions might remain coherent in a disputation in which an opponent puts forward a contingent and false initial proposition and a respondent accepts this and accepts or denies other propositions in a logically consistent way. Irrelevant propositions are granted, denied or doubted according to the best knowledge of the respondent. Relevant propositions should be treated in a consistent way. These either follow from the initial position and/or what has been granted and/or the opposites of what has been correctly denied or are incompatible with them. Thirteenth-century *positio* rules denied that a now false but possible position could refer to the time of an actual obligations discussion. This was in agreement with the traditional doctrine of the necessity of the present.162 As part of his new interpretation of modal concepts, John Duns Scotus dropped this rule, a revision which made it possible to understand obligational answers as partial descriptions of how things could be instead of regarding them as a internally consistent set of propositions without a sensible interpretation.163

Scotus’s revision of obligations rules was in agreement with his modal meta-

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162 See note 104 above.

physics. In arguing for the contingency of the created order Scotus writes:

I do not call something contingent because it is not always or necessarily the case, but because the opposite of it could be actual at the very moment when it occurs.¹⁶⁴

This is a denial of the traditional thesis of the necessity of the present and the temporal frequency characterization of contingency. In the Scotist definition, the meaning of the notion of contingency is spelt out by considering simultaneous alternatives. What is actual is contingent only if, instead of being actual, it could be not actual. This conception of simultaneous contingent alternatives is part of an argument that the first cause does not act necessarily. According to Scotus, the eternal creative act of divine will is free only if it is a choice between alternatives and could be other than it is in a real sense.¹⁶⁵

That God acts by choice between alternatives had been a common theological view since Augustine, but Scotus formulated its conceptual foundation in a new way, which had consequences for how modal terms in general were understood. In the Augustinian tradition, it was thought that metaphysical possibilities are ultimately based on the divine essence and represent the ways in which it could be imitated by created things. Scotus was the first to deviate from this metaphysical tradition in which possibilities are founded on divine being. According to Scotus, when God as an omniscient being knows all possibilities, he does not know them by turning first to his essence. Possibilities can be known in themselves.¹⁶⁶ In fact they would be what they are even if there were no God. Scotus states that if it is assumed that, per impossibile, neither God nor the world exists and that the proposition 'The world is possible' then existed, this proposition would be true. The actual world is possible as it is, and this possibility and the possibilities of unrealized things are primary metaphysical facts which are not dependent on anything else.¹⁶⁷

Scotus calls the propositional formulations of pure possibilities logical possibilities (possibile logicum). These express things and states of affairs to which it is not repugnant to be, which means that their descriptions do not involve a contradiction. Possibilities as such have no kind of existence of their own, but are real in the sense that they form the precondition for everything that is or can be.¹⁶⁸

A great deal of Scotus's discussion of metaphysical themes concentrates on the

¹⁶⁴ Ordinatio I.2.1.1-2, 86 (Opera omnia, vol. 2); De primo principio, ed. and translated by W. Kluxen (Darmstadt: Wissenschaftliche Buchgesellschaft, 1974), IV.4.


¹⁶⁷ Ord. I.7.1, 27 (Opera omnia, vol. 4); Lect.1.7, 32 (Opera omnia, vol. 16); Lect. I.39.1-5, 49; Quaestiones super libros Metaphysicorum Aristotelis libri VI-IX, ed. R. Andrews et al. (St. Bonaventure: The Franciscan Institute, St. Bonaventure University, 1997), 9.1-2, 18.

¹⁶⁸ Knuuttila 1996, 137-41.
modal explication of being and the disjunctive transcendental notions of necessity and contingency.  

God’s omniscience involves all possibilities and as objects of God’s knowledge they receive an intelligible or objective being. Some of these are included in God’s providential plan of creation and will receive an actual being. The description of a possible world at a certain moment consists of compossible possibilities. Though possibilities necessarily are what they are, the actualizations of non-necessary possibilities are not necessary but contingent. All things which are contingently actual at a certain moment could be not actual at that very moment. Since all finite beings are contingently actual, the contingent alternative possibilities are possible with respect to the same time, though they are not compossible with what is actual. According to Scotus, impossibilities are incompossibilities between possible ingredients, such as Socrates’s sitting at a certain time and Socrates’s not sitting at that same time. God could have chosen a world in which the first happens by Socrates’s free will or a world in which the second happens by Socrates’s free will. Since these possibilities are real possibilities, though not compossible, they are Socrates’s possibilities in alternative histories.

Scotus’s new modal metaphysics influenced early fourteenth-century philosophy and theology in many ways. From the point of view of logic and its applications to natural philosophy, the most significant ideas were the distinction between logical and real modalities and the association of logical modalities with alternative imaginative domains. Thirteenth-century essentialist assumptions were largely dropped from modal logic based on new modal semantics.

3.2 Early Fourteenth-Century Modal Logic

I shall sketch the main lines of early fourteenth-century modal logic by concentrating on three works. The first is William Ockham’s *Summa logicae* (OSL). In respect to the variety of the combinations of types of premises in modal syllogisms, this includes the most extensive fourteenth-century discussion of the topic. The

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second is John Buridan’s *Treatise on Consequences* (BC). According to the editors, Ockham’s *Summa logicae* was written in 1323 and Buridan’s *Tractatus* in 1335. The third is an anonymous *Questions on Prior Analytics* (PS). Its anonymous author is known as Pseudo-Scotus since it was included in the seventeenth-century edition of Duns Scotus’s works. I refer to this work as *PS*. Buridan discussed modal logic and modal syllogistic also in his *Summulae de Dialectica* and in *Quaestiones on Prior Analytics*. Some modal questions are dealt with in his *Questions on Peri hermeneias*. \(^{171}\) I refer mainly to the *Treatise on consequences* because it contains a carefully considered summary of his modal theory. It seems that the treatise by Pseudo-Scotus is later than the works of Ockham and Buridan, many of the questions being similar to those in Buridan’s *Questions on Prior Analytics*. \(^{172}\)

Buridan presents modal logic as part of his general theory of consequences, which includes sections on the equipollences between various modes in combination with negations and on the relations of the standard square of opposition between them, the nature and conversions of compound and divided (*de dicto* and *de re*) modal propositions, the mutual relationship between these two types of modal propositions and their relations to assertoric propositions, further modal consequences, and modal syllogisms. \(^{173}\) The same themes occur in all works mentioned. Fourteenth-century modal logic strove for generality, which had effects on the attitudes towards Aristotle’s modal syllogistics. For one thing, it was thought


\(^{173}\) In the first half of the fourteenth century, many logicians wrote detailed studies on consequences, either in comprehensive works on logic, such as Walter Burley’s *De puritate artis logicae* or William Ockham’s *Summa logicae* or in independent treatises, such as Burley’s *De consequentiis* and Buridan’s *Tractatus de consequentiis*. For recent works on late medieval discussions of consequences, see P. King, ‘Consequences as Inference: Medieval Proof Theory’ in M. Yrjönsuuri (ed., 2001), 117-45; I. Boh, ‘Consequence and Rules of Consequences in the Post-Ockham Period’, in M. Yrjönsuuri (ed., 2001), 147-81; C. Dutilh Novaes, ‘Buridan’s consequentia: Consequence and Inference within a Token-based Semantics’, *History and Philosophy of Logic* 26 (2005), 277-97. While Buridan and Pseudo-Scotus based their discussions on modal syllogistic on the theory of modal consequences, Ockham did not deal with modal syllogistic in the section on consequences of the *Summa logicae*. 
that the notion of possibility proper \((Mp = \neg Lp)\) must be added to modal syllogistics as the basic notion. Furthermore, it was considered imperative to distinguish between modal premises in the compound \((de\ dicto)\) and the divided \((de\ re)\) senses and to divide \(de\ re\) modals into two groups depending on whether the subject terms refer to actual things or possible things. Aristotle's modal syllogistics was regarded as a fragmentary theory in which the distinctions between different types of fine structure were not explicated.

The truth conditions of categorical propositions in the traditional square of opposition were given in terms of supposition theory as follows: provided that no semantic paradoxes are involved in the propositions, a universal affirmative proposition is true iff the predicate term stands for everything for which the subject term actually stands. A particular affirmative proposition is true iff the predicate term stands for some of those for which the subject term actually stands. Negative propositions did not have existential import, and negatives with empty subject-terms were considered true. Aristotelian assertoric conversion rules, viz., the simple conversions between universal negative propositions and affirmative particular propositions and the so-called accidental conversion from affirmative universal proposition to particular affirmative proposition are valid in this interpretation.\(^{174}\) These rules were in agreement with the identity theory of predication which was also employed by Campsall.\(^{175}\)

As far as these authors treated propositions as tensed, the above remarks were taken to pertain to propositions the copulas of which were understood as present tense verbs. Since the twelfth century, there had been various attempts to define the truth conditions of past and future tense propositions. This discussion continues in early fourteenth-century logic. According to Buridan, the supposition of the subject term of the past tense proposition ‘\(A\) was \(B\)’ is amplified so that it stands for past and present \(A\)s while the predicate term stands for past \(B\)s; ‘\(A\) was \(B\)’ is read as ‘What was or is \(A\) was \(B\)’. Similarly, in ‘\(A\) will be \(B\)’ the supposition of the subject term is amplified so that it stands for present and future \(A\)s. It is read as ‘What is or will be \(A\) will be \(B\)’. The restrictive phrase ‘which is’ \((quod\ est)\) prevented ampliation when it was added to the subject term.\(^{176}\) Some early thirteenth-century logicians applied the theory of ampliation to possibility propositions so that their subject terms associated with the verb ‘can’ were taken to stand for actual and merely possible beings, which is how the subjects in divided possibility propositions without the restriction \((quod\ est)\) were understood by Ockham, Buridan and Pseudo-Scotus.\(^{177}\) Following the traditional doctrine of the matter of propositions, fourteenth-century logicians stated that assertoric

\(^{174}\) OSL II.2-4, 249-66; II.21, 318-21; BC I.5, 25-6; I.8, 44-5; PS I.12-15, 290-96.

\(^{175}\) See Lagerlund 2000, 86.

\(^{176}\) See BC I.6, 26-30; I.8.14, 45-7; Summulae de Dialectica IV.6.2, 299-300; cf. OSL II.7, 269-72; II.22, 321-5; PS I.17, 297-9. In question 11 of his treatise on the Prior Analytics, Campsall discussed syllogisms with tensed premises, and this syllogistic tense logic was developed further in Ockham's Summa logicae III.1.17-19, 406-11.

\(^{177}\) OSL I.72, 216; II.25, 331-2; BC I.6, 27; PS I.3, 277; I.26, 311. See also Lagerlund 2000, 108-12, 138-40, 171-6.
propositions are necessary, possible, or impossible depending on whether they can or cannot be true or false. Only those propositions were counted as modal, however, which included modal terms connected to the copula (divided modals) or connected by the copula to propositions or to dicta (compound modals).\footnote{BC II.1, 56; PS I.25, 309; William Ockham, _Expositio in librum Perihermenias Aristotelis_, ed. A. Gambatese and S. Brown, Opera philosophica II (St. Bonaventure: St. Bonaventure University, 1978), 450-66.}

The conversions of compound necessity and possibility propositions with respect to the dictum were not considered problematic. As in Campsall, these were said to hold by the rules that if the antecedent of a valid consequence is possible then the consequent is possible, and if the antecedent is of necessity then the consequent is of necessity or, as with simple conversions, if one of the convertibles is possible (necessary) then the other is possible (necessary). Of compound contingency propositions only those with simply convertible dicta are converted; the rule that if the antecedent is contingent then the consequent is contingent is not valid.\footnote{OSL II.24-5, 27, 327-8, 330-31, 334; BC II.7.12-14, 72-4; PS I.25, 310; I.30, 319.} The whole logic of compound modal propositions was in fact based on the Aristotelian principles for propositional modal logic which, as noted above, were widely known in early medieval logic as well. When modal syllogisms were regarded as syllogisms with respect to the dicta, connecting the mode ‘necessary’ with the premises and conclusions of valid assertoric syllogisms yielded valid modal syllogisms. This was based on (4) and the rule that if the conjuncts are necessary, the conjunction is necessary. Uniform syllogisms consisting of compound contingency or possibility modals were not considered valid, because the compossibility of two possible premises was not assured. Ockham and Pseudo-Scotus remark that because _de dicto_ necessities are compossible with any _de dicto_ possibilities or contingencies, mixed compound necessity and possibility or contingency syllogisms with possible or contingent conclusions are valid. Buridan did not mention this.\footnote{PS I.27, 313; I.33, 323; OSL III-1.20, 412-13; III-1.23, 419; III-1.44, 474; III-1.47, 479; BC IV.1.1, 113.}

The main object of the fourteenth-century modal logic was the theory of divided modals. Some treatises include discussions in which the logical relations between various divided modal propositions were codified in the same way as the relations between the types of assertoric propositions in the square of opposition. Buridan taught that there were two types of copula, the affirmative ‘is’ and the negative ‘is-not’, and that modality was part of the copula in divided modal propositions. Combining the equivalences between quantifying words with negations with equivalent modalities, Buridan arranged divided modals into eight groups of nine equivalent formulae. In the _Summulae_, these groups are presented in a diagram showing the relations of contradiction, contrariety, sub-contrariety, and sub-alternation between them.\footnote{For this octagon of opposition, see _Summulae de Dialectica_ I.8.4-7 and the diagram in _Questiones longe super librum Perihermenias_ II.9, 87; see also Hughes 1989, 109-10; E. Karger, ‘Buridan’s Theory of the Logical Relations between General Modal Formulae’ in H.A.G. Braakhuis and C.H. Kneepkens (eds.), _Aristotle’s Peri hermeneias in the Latin Middle Ages_, Artistorium supplementa 10 (Groningen - Haren: Ingenium Publishers, 2003), 429-44.}
According to Buridan and Pseudo-Scotus, the subject terms of all divided modals are amplified to stand for actual and possible beings which fall under those terms. The phrase ‘what is’ (quod est) attached to subject terms restrict them to standing for actual beings only.\(^{182}\) Divided necessity modals with restricted subject terms are not converted simply or accidentally, and the same holds true of the conversions of divided necessity propositions with non-restricted subject terms, with the exception that universal negative propositions are convertible simply.\(^{183}\) Unlike Buridan and Pseudo-Scotus, Ockham did not accept any conversions of terms of divided necessity propositions.\(^{184}\) In fact he did not treat divided necessity propositions with unrestricted subject terms at all in his modal logic.\(^{185}\)

As to unrestricted divided possibility propositions, Ockham, Buridan and Pseudo-Scotus state that affirmative modals are converted in the same way as assertoric propositions, those with restricted subject terms not being convertible.\(^{186}\) According to Buridan and Pseudo-Scotus, an unrestricted divided proposition de contingenti ad utrumlibet can be converted into one of the opposite quality, but no conversions of the terms are valid.\(^{187}\) They treated these propositions as amplified with respect to possibility. Ockham states that if the subject terms are amplified with respect to contingency, unrestricted universal contingency propositions convert into particular contingency propositions.\(^{188}\)

In discussing the logical properties of the unrestricted divided modal propositions, Buridan and Pseudo-Scotus made some comments on the question whether such propositions should be treated as categorical propositions with a disjunctive subject (what is or can be ...), as they did, or whether they should be taken as complex propositions.\(^{189}\) Pseudo-Scotus claimed that they could be read as conjunctions or disjunctions as follows: ‘Every A is-possibly(-not) B’ is a conjunction of ‘Everything which is A is-possibly(-not) B’ and ‘Everything which can be A, etc.’ and ‘Some A is-possibly(-not) B’ is the disjunction: ‘Something which is A is-possibly(-not) B’ or ‘Something which can be A, etc.’\(^{190}\)

It has been suggested that one could supply a Kripke-style possible worlds semantics for Buridan’s modal system as an axiomatic basis for it.\(^{191}\) I think

\(^{182}\) BC II.4, 58; II.6, 61, 63; IV.1, 111; Summulae de Dialectica IV.6.2, 299; PS I.26, 312-13.

\(^{183}\) BC II.6.6, 67; PS I.26, 312-13.

\(^{184}\) OSL II.24, 329-30; III-1.21, 416.

\(^{185}\) See also Lagerlund 2000, 112-15.

\(^{186}\) OSL II.25, 331-2; III-1.24, 423-4; BC II.6.5, 66-7; PS I.26, 312.

\(^{187}\) BC II.6.7, 68; PS I.30, 320.

\(^{188}\) OSL II.27, 338; III-1.27-8, 430-33.

\(^{189}\) BC II.4, 58-60; PS I.26, 310-11.

\(^{190}\) Pseudo-Scotus’s remarks on the conversion of these readings are sketchy and problematic. It is not clear why he thinks that possibility propositions would be convertible in the same was as assertoric propositions, the simple conversion of universal negative included. Necessity propositions are said to be non-convertible except when they are about the necessary characteristics of a necessary being. The author does not mention that a conversion from universal negative to particular negative necessity propositions would be acceptable in this approach. See also Lagerlund 2000, 172-6.

\(^{191}\) Hughes 1989.
that the general ideas of Buridan and Pseudo-Scotus can be described with the help of some models similar to those on which the possible worlds semantics is based, although many basic theoretical questions of modern formal semantics were beyond the purview of medieval logicians. Constructing the details of the intuitive model which the authors possibly had in mind is a cumbersome task. Ockham and Buridan state that the truth of ‘A white thing can be black’ demands the truth of ‘This can be black’ and that ‘This can be black’ and ‘This is black’ is possible mean the same.\textsuperscript{192} It is reasonable to suppose that the possible truth of ‘This is black’ means that it is true in at least one of the possible states of affairs in which the possible being referred to by ‘this’ occurs and that the necessary truth of ‘This is black’ means that it is true in all possible states of affairs in which the thing referred to by ‘this’ occurs. Buridan and Pseudo-Scotus assume that the subject terms of quantified divided modal propositions stand for possible beings. The truth of these propositions demand the truth of all or some relevant singular propositions of the type just mentioned. Compound possibility propositions maintain that assuming their dicta as true does not imply any contradictions. Buridan typically did not show any understanding of the idea of abstract unrealizable possibilities in Averroes and Aquinas – he took it for granted that if a counterfactual state of affairs is possible, it can be coherently imagined as actual.\textsuperscript{193}

One might wonder why, instead of regarding the reading with actual or possible subjects as basic for all modals, Ockham discussed the necessity propositions with restricted subject terms. This makes his modal syllogistics less systematic than those of Buridan and Pseudo-Scotus who argued that if divided possibility propositions are amplified, the equipollences between various modals requires that necessity propositions are amplified.\textsuperscript{194} It seems that the differences are related to the fact that the authors did not make use of the same notion of necessity while discussing divided modals. If the distinction between divided necessity propositions with restricted and unrestricted subject terms is not nugatory, one should admit that possible beings may have necessary properties without occurring in every possible state of affairs. This condition is fulfilled by a relative de re necessity as Buridan and Pseudo-Scotus understood it. That which can be $A$ is of necessity $B$ in this sense iff it could not be actual without being $B$. This notion of necessity is relative to possible actuality and is weaker than that of simple necessity by which a thing is necessarily something without qualification, i.e., always and in all thinkable states of affairs, and it is stronger than that of temporal necessity by which an actual thing is invariably something in the actual world.\textsuperscript{195} Some of Ockham’s examples also refer to necessity which is relative to possible actuality, but his syllogistic theory of divided necessity propositions is based on the view

\textsuperscript{192}Osl II.10, 276-9; III-1.32, 448; III-3.10, 632-4; BC II.7.16, 75-6. While compound and divided readings do not differ at this level, they are separated in dealing with standard universal and particular propositions.

\textsuperscript{193}For Averroes and Aquinas, see note 84 and Knuuttila 2001, 234.

\textsuperscript{194}BC II.6.2, 63; PS I.26, 312.

\textsuperscript{195}For these types of necessity, see Buridan, \textit{In An. pr.} I.25; BC V.1, 112; PS I.26, 311.
that *de re* necessities pertain to actual beings.\textsuperscript{196}

Buridan states that the relations between unrestricted divided modal propositions and the corresponding non-modal propositions are governed by the following principles. There are no valid consequences from divided necessity and possibility propositions to corresponding assertoric propositions, except that a universal negative assertoric proposition follows from an unrestricted universal negative divided necessity proposition. There are no valid consequences from assertoric propositions to corresponding divided necessity propositions. Particular divided possibility propositions follow from universal and particular affirmative assertoric propositions by valid consequences. Buridan adds that a possibility proposition follows from any necessity proposition.\textsuperscript{197} Ockham’s position with respect to these relationships is different for reasons already mentioned, compound and divided necessity propositions implying corresponding assertoric propositions.\textsuperscript{198} As for the relationships between compound and divided modal propositions, Buridan states that particular divided possibility propositions follow from compound universal and particular affirmative possibility propositions, and a universal negative compound necessity proposition follows from an unrestricted universal negative divided necessity proposition.\textsuperscript{199} According to Ockham, compound and divided readings of all singular modal propositions are equivalent, provided that the subject terms are demonstrative pronouns or proper names. Divided necessity propositions which are universal, particular, or have as their subject a common term with a demonstrative pronoun do not imply corresponding compound modals, and the same holds converso with the exception that affirmative compound propositions having as a subject of their *dicta* a common term with a demonstrative pronoun imply a corresponding divided necessity proposition. Ockham’s rules for universal and particular possibility propositions are the same as those of Buridan.\textsuperscript{200}

In discussing divided modal syllogistic in *Tractatus de consequentiis*, Buridan

\textsuperscript{196}See, e.g., *OSL* III-1.32, 448.119-122.
\textsuperscript{197}BC II.6, 64-6. Lagerlund (2001, 150) remarks that Buridan does not want to exclude the consequence from negative assertoric to divided negative possibility propositions which is not mentioned.
\textsuperscript{198}OSL III-3.11, 637-8.
\textsuperscript{199}BC II.7, 76-8. In denying that a universal affirmative compound necessity proposition implies a divided necessity proposition, Buridan states that while Aristotle says that ‘Every horse is an animal’ is necessary, no horse is necessarily an animal, for any horse can fail to exist and consequently fail to be an animal (77). This is a strange example, for it is said earlier that divided affirmative necessity propositions do not imply assertoric propositions, which is in agreement with the relative conception of necessity mentioned above. See also IV.1, 112 where Buridan says that ‘A horse is an animal’ is not necessarily true by a simple necessity, for it is possible that there will be no horses, although horses are necessarily animals as long as they exist. Perhaps the first example was about simple necessity which presupposed necessary existence. Some problems could be avoided by regarding compound necessity propositions as law-like statements with an amplified supposition. In this case ‘Every horse is an animal’ is necessarily true as well as the divided statement that every horse is necessarily an animal, while ‘Everyone walking is moving’ is necessarily true and the divided statement that all walking are necessarily moving is false. Cf. *In An. pr.* 1.25-6. See also the discussion in the introduction to the translation of *Summulae de Dialectica* by G. Klima, xlv-xlvii.
\textsuperscript{200}OSL III-3.10, 632-4.
first presents the valid moods with necessity or possibility premises. All four first-figure moods corresponding to direct assertoric ones are valid when they are modalized as follows: \( MMM, LLL, LML, MLM \). When assertoric premises are as-of-now statements, \( LAL \) and \( MAM \) with particular conclusions are valid as well as the universal negative mood in \( ALA \). All \( ALA \) moods are valid if the subject of necessity premise is restricted to stand for actual things. If assertoric propositions are assertoric *simpliciter*, equivalent to compound necessity propositions, all four first-figure moods are valid in \( LAL, ALL, AMM \). In outlining the syllogistics for contingency-propositions, Buridan states the validity of all first-figure moods in \( CMC, CCC, MCM, LCL, CLC \) and in \( CAC \) for particular moods. When the restrictive phrase ‘what is’ is prefixed to the subject of a divided modal \((U', M', C')\), all first figure moods are valid in \( L'U', L'AL', M'L'M', M'AM', C'AC' \).

Valid second and third figure moods are reduced to perfect first figure moods or proved by an expository syllogism or, on few occasions, by *reductio ad impossibile*. First figure divided modal moods without assertoric premises can be delineated as follows: if that which is or can be \( P \) is necessarily, possibly or contingently (not) \( Q \) and if that which is or can be \( S \) is necessarily, possibly or contingently \( P \), then that which is or can be \( S \) is necessarily, possibly or contingently (not) \( Q \). It does not matter by which modality the minor term is connected to the middle term—the modality of predication in the conclusion is always that of the first premise. The same principle holds true of the cases where the major premise is modalized and the minor premise is assertoric, but the conclusions are then always particular. Buridan regarded these moods as perfect and clear through the * dici de omni et nullo*.

Buridan made some historical comments on Aristotle’s theory claiming, for instance, that Aristotle meant by ‘simply assertoric propositions’ assertoric propositions which are necessary, that he considered propositions like ‘A horse is an animal’ as simply necessary because of his view of the eternity and unchangeability of natural species, and that he understood mixed modal syllogisms as divided modal syllogisms. However, he did not make any attempt to reconstruct Aristotle’s modal syllogistic as a uniform system; in the light of his modal principles, such a construction was impossible. Some Aristotelian moods are valid only as mixed compound and divided modal syllogisms, some are valid in this way and as divided modal syllogisms with non-ampliated subject terms, and some are valid

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201 BC IV.2.4, 115.
202 BC IV.2.10 (118-9), 15-16, 124-5.
203 BC IV.2.11, 120; IV.2.19, 127-8.
204 BC IV.2.21-4, 130-1.
205 BC IV.2.7, 117; IV.2.10, 119; *Summulae* V.7.2-4. Buridan says that if the major necessity premise is restricted and the minor is not, the conclusion is not restricted. He should have added that the conclusions are particular. (The same holds of \( ALM \) moods which are mentioned in *Summulae* V.7.3, 355.) If the major necessity premise is not restricted and the minor is, universal conclusions are said to be restricted. Lagerlund (2000, 239) does not pay attention to this qualification.
206 BC IV.2.4, 115; IV.2.10, 119; IV.2.16, 124-5.
207 BC IV.1, 111-112; *In Pr. an.* I. 30.
only with amplified subject terms. Buridan thought that Aristotle's modal syllogistic was a partial theory of valid modal deductions where the different types of fine structures made use of in dealing with modal premises were not distinguished.\textsuperscript{208} Some moods which are accepted in the \textit{Treatise on consequences} are not mentioned in the \textit{Quaestions on the Prior analytics} or in the \textit{Summulae}, and some valid moods which are discussed in the \textit{Summulae} are omitted in the \textit{Treatise on consequences}.\textsuperscript{209} Buridan's treatment of modal moods in all works is restricted to those the unmodalized counterparts of which are valid direct moods of three syllogistic figures. He does not treat valid indirect moods or moods with singular terms.

Ockham's divided modal syllogistic also concentrates on the moods which have valid assertoric counterparts, but his discussion is more throughout than that of Buridan. Ockham considers the indirect moods as well, and he treats the combinations in which one of the premise is a compound modal and the other a divided modal. He also mentions some moods with restricted premises which are valid in Buridan's system as well although Buridan does not mention them: e.g., all first figure moods $MM'M'$, $CC'C'$ and the third figure moods $MM'M$ and $M'MM$. As Ockham considered the subject terms of divided necessity propositions as restricted, he did not accept any second figure moods with divided necessity propositions as premises nor any moods with mixed divided modal premises and necessity conclusions.\textsuperscript{210} Pseudo-Scotus's discussion of divided modal moods is of the same type as that of Buridan, but it is less detailed.\textsuperscript{211}

\section*{4 APPLIED MODAL LOGIC}

Some twelfth-century thinkers considered that the logic of pure modal terms could be used as a model for studying the logical behaviour of related concepts. It was asked whether the basic rules for modal sentences \textit{de dicto}, viz. (4) and (5), held for other concepts showing \textit{prima facie} similarities with the notions of necessity and possibility. Many logicians in the fourteenth century were interested in this kind of applied modal logic. The extensive lists of terms which were found interesting in this regard included \textit{verum, falsum, per se, scitum, dubium, opinatum, creditum, apparens, notum, volitum, dilectum, obligatum, licitum, and illicitum}.\textsuperscript{212}

\textsuperscript{208}See also \textit{OSL} III-1, 31, 443.
\textsuperscript{209}See also Lagerlund 2000, 149-61; Thom 2003, 169-91.
\textsuperscript{210}\textit{OSL} III-1, 21 (416-7), 23 (420), 26 (428), 44 (476).
\textsuperscript{211}For divided modal syllogistics in Ockham and Pseudo-Scotus, see Lagerlund 2000, 196-29, 171-83; Ockham's modal syllogistic is also discussed in Thom 2003, 141-67. Nicholas Drukken's view of necessity moods was influenced by Ockham's theory; most later medieval authors followed Buridan. See Lagerlund 2000, 134-5, 184-227.
\textsuperscript{212}\textit{PS} I.25, 310; I.36, 328-9.
4.1 Epistemic Logic

Knowledge and belief were widely considered as partially analogous to necessity and possibility in late medieval times. Although the inference rules of modal logic de dicto were usually not accepted as rules for knowledge and belief, there was a lively discussion of some other important questions pertaining to epistemic logic. The standard Aristotelian reasons for not accepting (4) and (5) with respect to knowledge or belief is stated by John Buridan as follows:

From compound sentences de scito, de opinato, de dubitato, and other similar modals, there is no valid syllogism. It does not follow: ‘That every B is A is known by Socrates, and that every C is B is known by Socrates, therefore that every C is A is known by Socrates’, because although the two premises are known by Socrates, he nevertheless may perhaps fail to order them in a syllogism, or fail to see that the third follows from them.

Most medieval authors did not operate with the conception of logical omniscience that is included in some modern theories, treating the logic of epistemic notions from the point of view of factual attitudes. However, Peter of Poitiers mentions that according to some twelfth-century thinkers, knowing the antecedent of a sound consequence implies knowing the consequent. Another example is Ralph Strode’s (d. 1387) influential treatise on consequences. According to rule 13, if the antecedent of a sound consequence is known, the consequent is also known. The logicians, who commented on Strode’s rules, usually considered the reference to a person essential in epistemic contexts, adding the condition that the consequence must not only be valid, but must be known to be valid as well. Strode also stressed the same later in his work.

In the prologue to his Commentary on the Sentences, William Ockham states that some acts of the intellect are apprehensive and others are judicative. An apprehensive act can be directed to anything which may occur as the object of a cognitive power. It is an act of entertaining the content of a possible judgment. A judicative act is an act by which the intellect assents to or dissents from the complex objects of apprehension. We assent to what we regard as true and we dissent from what we regard as false. If the assent or dissent which is part of

\[213\] For medieval epistemic logic, see I. Boh, Epistemic Logic in the Later Middle Ages (London and New York: Routledge, 1993).
\[214\] Tractatus de consequentiis IV.1.3, 114.
\[215\] See, for example, J. Hintikka, Knowledge and Belief (Ithaca: Cornell University Press, 1962).
\[216\] Sententiae I.9 (82.139-83.154).
\[218\] For these comments and related discussions, see Boh 1993, 89-125; Boh, ‘Four Phases of Medieval Epistemic Logic’, Theoria 66 (2000), 137-40; Boh 2001, 161-70.
knowledge in Ockham is considered an act of judicative belief, Ockham's concept of knowledge could be regarded as implying the principle

(21) \( K_a p \rightarrow B_a p. \)

Robert Holcot explicitly treated assent as a belief, pointing out that the most common sense of believing (credere) is to assent to what is stated by a proposition. 'A proposition is believed when it is assented to, and in this way we believe what we know as well as what we formally opine.'

Ockham remarks that the term 'knowledge' is sometimes applied to firm cognition of a truth, but he considered this use less proper than those which included a reference to evidence as some kind of justification. Instead of the definition

\[ K_a p = B_a p \& p \]

he prefers

\[ K_a p = B_a p \& p \& J_a p. \]

\((J_a p \text{ stands for 'the person } a \text{ is justified in believing that } p')\) As shown by Ivan Boh, some later authors were reluctant to accept the first use of knowledge. Their reasons were similar to those which have been presented in the contemporary discussion of the Gettier problem. Some authors were interested in the question of the role of the will in assenting to a proposition. It was usually thought that evident assent was caused naturally and that there were types of non-evident assent which were freely caused, such as the religious assent of faith or the opinative assent based on probabilistic arguments. Robert Holcot argued that beliefs were not freely chosen, but this view was considered over-stated.

The theory of free assent was sometimes associated with the question of making moral decisions in uncertain cases, which anticipated the later controversy over...
probabilism and probabiliorism. Thomas Aquinas, Boethius of Dacia, and some other Parisian masters explained Aristotelian dialectical probabilities by stating that what is probable in the sense that most experts accept it is probably true because it is not probable that the majority of well informed experts would be mistaken in the same way. These and other similar examples show that, contrary to what has been sometimes maintained, an intuitive conception of objective frequency probability, different from epistemic probability, was developed in the Middle Ages.

The question of the relationship between epistemic propositions *de ditto* and *de re* belonged to standard fourteenth-century topics of epistemology. It was thought that knowledge statements *de ditto* did not imply knowledge statements *de re* or *vice versa*. Buridan says, however, that when a person knows that some A is B, then of something which is A he or she knows that it is B. The reason for denying this could be that Socrates does not know which A is B. Buridan would agree that in this sense the *de re* reading does not follow from the *de ditto* reading, but there is another kind of *de re* reading (or which might be called so) which does follow from the *de ditto* reading. This is a kind of intermediate reading between pure *de dicto* and *de re* readings. The idea can be formulated as follows. According to Buridan, statements of the type

\[(22) \quad K_s(Ex)(Fx)\]

imply that there are individuals having property F, although S does not necessarily know which they are. In principle they are identifiable, however, and if we suppose that one of them is z, we can write:

\[(23) \quad K_s(Ex)(Fx) \rightarrow (Ex)((x = z) \land K_s(Fx)).\]

From the *de dicto* statement ‘Socrates, who is sitting in a cellar, knows that a star is above’ it does not follow the *de re* reading understood as ‘There is a star which Socrates knows as the star which is above’, but the following *de re* reading does follow from it: ‘There is a star of which Socrates knows that it is above, although Socrates does not know which star it is.’

Buridan was not the first to employ this reading. In his *De obligationibus*, Walter Burley discussed the following case:

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226 See, for example, the extensive discussion of the question in the second chapter of William of Heytesbury’s *Rules for Solving Sophismata* (Venice 1494), translated in Kretzmann and Stump 1988, 436-72.

You know that Marcus is running. But Marcus is Tully. Therefore, you know that Tully is running. And furthermore, ‘Therefore, you know who is called Tully.’ But it was posited that you don’t know this. Solution. This is multiplex: ‘You know that Tullius is running.’ It may signify that you know the dictum: ‘That Tullius is running’, and this is false and the consequence is not valid. Or it may signify that of that person who is Tully you know that he is running, and this can be true, although you don’t know who is signified by this term ‘Tully’.228

The same analysis can be found in many fourteenth century writers.229

Let us turn to medieval deontic logic. The following equivalences analogous to those between modal concepts were used in the fourteenth-century discussions of the norms:

\begin{align*}
\text{(24)} & \quad -O \rightarrow p \leftrightarrow Pp \\
\text{(25)} & \quad -P \rightarrow p \leftrightarrow Op \\
\text{(26)} & \quad Op \leftrightarrow P \rightarrow p \\
\text{(27)} & \quad -Pp \leftrightarrow O \rightarrow p \\
\text{(28)} & \quad Op \leftrightarrow F \rightarrow p \\
\text{(29)} & \quad Fp \leftrightarrow O \rightarrow p.
\end{align*}

$O$ stands here for obligation ($obligatum$), $P$ for permission ($licitum$), and $F$ for prohibition ($illicitum$). One can find some of these equivalences used earlier and there were some twelfth-century writers who anticipated the later habit of treating deontic concepts as a species of modal concepts. According to one definition of modal terms sometimes used by Peter Abelard, necessity is identified with what nature demands, possibility with what nature allows, and impossibility with what nature forbids.230 At the beginning of his Ethics, Abelard asks whether it is possible that that the antecedent is permitted or obligatory while the consequent is forbidden.231 Later he discusses some problematic cases associated with this question. These remarks form the first known discussion of deontic consequences. Abelard also formulated the question of whether willing the antecedent of a good consequence which is known to be such implies willing the consequent. His view is that if will is understood in the sense of consent and not merely in the sense of wish and if the consequence expresses a relationship between a goal and a.

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228The text is edited in Green 1963, 39.29-40.7.
230See, e.g., Dialectica 385.1-8.
means, consenting to the antecedent (goal) implies consenting to the consequent (a means).\textsuperscript{232}

Some twelfth-century authors realized that even though willing an end effectively implies that a necessary means to it is also willed, a rule (W), analogical to (5) cannot be applied without restriction to efficient will in the form that if Socrates wills that \( p \) and he knows that there is a valid consequence from \( p \) to \( q \), he also wills that \( q \). The principle holds about the end-means relationship, but the necessary means are not the only consequents of what is willed. Peter of Poitiers formulated a counter-example as follows: if \( S \) repents of a sin, \( S \) is guilty of a sin, and \( S \) wills to repent of a sin, but \( S \) does not will to be guilty of a sin.\textsuperscript{233} Stephen Langton's counter-example was of the same type: if a man visits his sick father, the father is sick. This man wills to visit his sick father, but it does not follow that he wills the father to be sick.\textsuperscript{234}

When it was realized that one cannot apply (W) to effective will without qualification, medieval thinkers took an interest in finding cases in which a rational agent does not will the consequent of what he or she wills. The twelfth-century examples just mentioned specify the case in which to will something in certain circumstances implies that those circumstances prevail, although this is something which the agent does not will. This exception to (W) was thought philosophically interesting and was also used in later medieval deontic logic. Contrary to what Abelard thought, it was realized that (4) and (5) cannot be applied without qualification to the notions of obligation and permission. The reasons were similar to those which demanded the qualification of (W). Another relevant but more problematic idea was that a separately unwanted side-effect of what is willed is willed more indirectly than a separately unwanted means to an end. This line of thought forms the background to Thomas Aquinas's distinctions between directly and indirectly willed things.\textsuperscript{235}

One can follow the discussion of the question of how (W) should be qualified with respect to will by tracing the comments on the example of a man who 'wills to be in the mud and have 100 marks' (\textit{in luto esse cum 100 marchis}). This example was considered ambiguous, and it was possible to interpret it (i) as a case of willing reluctantly to become dirty as a means of receiving 100 marks, (ii) as a case of willing to become dirty as a side-effect or (iii) willing something in a situation in which one would become dirty in any case. The third alternative was particularly relevant to later discussions of conditional obligations.

The most extensive fourteenth-century discussion of the logical properties of deontic concepts is to be found in the first question of Roger Roseth's \textit{Lecture on...}

\begin{footnotesize}
\begin{enumerate}
\item[p.232] \textit{Ethics} 8.21-10.2; 14.14-19; 16.11-18.
\item[p.235] See, e.g., \textit{Summa theologiae} II.2.64.7-8; \textit{De malo} 2.1. See also Knuuttila 2006, 205-18.
\end{enumerate}
\end{footnotesize}
the Sentences, which he finished c. 1335. The opening question of this work is devoted to the problem of whether somebody can be obliged to do something against his or her conscience. In the second article, Roseth first formulates five general principles or rules which define the rationality of a system of norms, and the rest of the article consists of a discussion of various objections to the rules of rationality. In rules 1 and 2, Roseth defines formal rationality: describing behaviour in accordance with the norms must not yield contradictions, i.e., a person should not violate a norm by fulfilling a norm in the same system of norms. New obligations can be added to a system as rational norms without further changes only if they refer to omissions and commissions which one is permitted to will. Roseth pays particular attention to this point because he is interested in the fact that there are omissions or commissions which are permitted, although it is not permitted to will them; as they cannot be legitimately willed, they cannot be prescribed as such in a rational body of norms. Referring to the dictates of right reason, Roseth adds to the first rule that what is prescribed must be in the power of the agent in such a way that fulfilling the obligation does not result in a great disaster. Rules 3 and 4 are related to the Catholic doctrine of the salvation of the soul. Rule 5 refers to the doctrine of God's absolute power, through which an act which cannot be legitimately willed can be changed into an act which can be legitimately willed.

After these general rules, Roseth discusses various possible objections and develops some philosophical and logical ideas pertaining to norms. The second objection runs as follows. It is permitted to will to behave in a way which is not meritorious though the order that one should will to behave in a non-meritorious way is not rational. It is not rational because it could be violated only in a meritorious way. The first part is proved as follows. If you are sleeping, you do not behave in a meritorious way. Furthermore, you know that this is a good consequence: since you are permitted to will the antecedent, you are permitted to will the consequent.

The main point of Roseth's answer to the second objection is that it is mistakenly assumed that if one is permitted to will the antecedent of a good consequence, one is also permitted to will the consequent:

I accept the following consequence: you are sleeping; therefore you do not benefit from an elicited act. But I deny this consequence: this consequence is good and you know that it is good; therefore if you are permitted to will the antecedent, you are permitted to will the consequent. This is because there are consequences which are good.

236This work was copied several times during the second half of the fourteenth century and is preserved to us in eight manuscripts, only three containing the complete work. The first article of the book aroused interest as an independent study of natural philosophy and was circulated in several copies. It was printed, together with the second article of the first question, as the first question of Determinationes magistri Roberti Holcot at Lyon in 1518. The printed version is abridged and unreliable. The translations below are based on a transcription of manuscripts and quoted from S. Knuuttila and O. Hallamaa, ‘Roger Roseth and Medieval Deontic Logic’, Logique & Analyse 149 (1995), 75-87.

237These rules are translated in Knuuttila and Hallamaa 1995, 79-83.
and which I know to be good the antecedent of which I am permitted to will without being permitted to will the consequent. For example, this consequence is good and known to be good: I repent of my sin; therefore I am in sin. I am permitted to will the antecedent but I am not permitted to will the consequent, because I am permitted to repent of my sins, but I am not permitted to will to be in sin. Therefore the consequence is not valid.

Examples of this kind show that there are obligations which can be rationally fulfilled only in cases in which some norms have already been violated. Rules (4) and (5) could be applied to the notions of obligations and permissions only if there were no conditional norms by which one's conduct is regulated after having violated certain norms. Roseth realized that conditional norms of this kind, called contrary-to-duty imperatives in contemporary literature, prevent one from accepting the rules of inference of modal logic in deontic logic without qualification.

These are not the only problematic cases according to Roseth. There is a second class of conditional obligations and permissions regulating conduct in those situations in which the moral agent has not violated any rules but which he or she cannot will without violating some rule. This group of obligations and permissions could serve as Roseth's answer to the so-called paradox of the Good Samaritan. Roseth's example is that one may accept that one's father will be killed because of a serious transgression. A simpler case would have been a variant of Stephen Langton's example of conditional will; one should will to visit one's sick father, but one is not permitted to will that there be such a situation.

Roseth then discusses the question of how the conditional obligation should be formulated. He thinks that a reasonable prima facie formulation is

\[(30) \quad p \rightarrow Oq\]

where \(p\) may stand for something forbidden or permitted such that it is not permitted to will it. Roseth thought, however, that (30) was not a sufficient form. The first addition is formulated in his answer to the following question. If the conditional obligation is formulated so that it is obligatory and allowable to will to repent if one is guilty of a sin, one could ask whether Socrates, who wills to repent without having committed any sin, should repent or not. If he should, he is doing what he ought to do, but then he should not repent. If he should not, he violates a rule and he ought to see that he repents. So if he should not repent, he should repent and \textit{vice versa}.

According to Roseth, the intention of the rule-giver is that Socrates ought to will to repent of his sins only when he is guilty of sin and that the prescribed act of repentance is different from the act of willing to repent when one has not sinned before repenting. Thus Socrates, while willing to repent in this way, violates the intention of the rule-giver, although he seems to fulfil his obligation. In order to avoid difficulties of this kind, one apparently should add to formula (30) the qualification that fulfilling the condition of a conditional obligation does not fulfil the obligation:
(31) \((p \rightarrow Oq) \& - L(p \rightarrow q)\).

This form is similar to that which von Wright suggests in order to solve difficulties in defining conditional obligation.\(^{238}\) Rule (31) had certain antecedents in medieval logic. Conditional norms were also discussed in treatises *De obligationibus* which dealt with the logic of disputation and which as such had nothing particular to do with morality or ethics. One example of the interest in normative notions in obligations treatises is the discussion of disputation where the expression ‘must be granted’ is included in the initial *positum*. It was realized that a *positum* of the type ‘That you are in Rome must be granted’ can be read prescriptively as a norm or descriptively as a proposition expressing the existence of an obligation. The initial proposition including ‘must be granted’ is read descriptively in a *positio* disputation, but it is read normatively in the species of disputation called *petitio*. In his *De obligationibus*, Walter Burley discusses some problems of formulating conditional norms in connection with the disputation called *positio dependens*, *positio cadens*, and *positio renascens*. What they have in common is that a proposition begins or ceases to be the *positum* when a condition is fulfilled. The condition is a disputational act. Burley’s remarks on how the condition should be formulated show similarities with Roseth’s treatment of conditional obligations.\(^{239}\)

In order to show that (31) is not sufficient, Roseth deals with a variant of a well-known medieval sophism. Let us assume that those and only those who say something true will cross a bridge and that Socrates says: ‘I shall not cross the bridge.’ It is then asked whether this is true or false. According to Roseth, one should not accept the assumed case. Fulfilling the condition of a conditional obligation must not make it impossible that the obligation is fulfilled. When this restriction is added to (31), the resulting form is as follows:

(32) \((p \rightarrow Oq) \& - (p \rightarrow q) \& M(p \& q)\).

Roseth presents other similar problems associated with the questions of semantic paradoxes in this context.\(^{240}\)

**CONCLUDING REMARKS**

While early medieval discussions of modal matters were influenced by ancient modal paradigms such as temporal frequency interpretation of modal notions, the conception of possibility as a power, the ideas of diachronic modalities and essentialist modal assumptions, there were also new insights. Following Peter Abelard,
Peter Lombard wrote in his influential *Sentences* that 'Things cannot be other than as God foreknows them' is true in the compound sense and false in the divided sense. The truth of the compound sense saves God's infallibility, while the falsity of the divided sense expresses God's freedom and the metaphysical contingency of the future. It is assumed that when something is, it is possible that it is not at that very instant of time at which it is actual. These theological formulations exemplify twelfth-century deviations from the Aristotelian thesis 'What is necessarily is when it is'. This was traditionally understood as implying the principle of the necessity of the present, which was not questioned in ancient modal theories. Since temporally definite propositions about contingent things were regarded as unchangeably true in God's knowledge, the contingency of these propositions also implied the denial of the Aristotelian equation of immutability with necessity. The new modal idea could be characterized as the model of simultaneous alternatives. Its theological foundation was Augustine's idea of God, who as the creator and providence of the world, acts by free choice between alternatives.

In accordance with the new modal conception, possibilities as the objects of divine power were regarded as much more numerous than possibilities associated with natural powers. While exemplification in the actual world was often regarded as a criterion of the genuineness of the types of natural possibility, it was not relevant for divine possibilities. In twelfth-century theology, natural possibilities *secundum inferiorem causam* were said to be possibilities *secundum cursum naturae* and possibilities *secundum superiorem causam* meant divine possibilities. There were similar discussions in medieval Arabic philosophy.

In twelfth-century logical treatises, which were influenced by Boethius's works, one can find ideas not discussed in ancient sources in Abelard's analysis of the types of modal propositions, future contingents and the logic of conditionals and late twelfth-century and early thirteenth-century works on the logic of terms, time and modality, some of these being associated with the idea of modality as referential plurality. The increasing reception of Aristotle's philosophy in the thirteenth century gave support to traditional modal paradigms, as is seen in the discussions of modal conversion and Robert Kilwardby's very influential commentary on Aristotle's *Prior Analytics*, in which modal syllogistic is treated as an essentialist theory of the structures of being.

While twelfth-century modal innovations were used to some extent in thirteenth-century theology, they were not extensively discussed in philosophical contexts. Things became different when John Duns Scotus combined the various elements of the conception of modality as alternativeness into a detailed theory. Scotus describes the simultaneous alternatives as the domains with respect to which God chooses the actual world, but the structure of the theory is taken to pertain to logical modalities which are what they are independently of whether anything exists. A possible state of affairs is characterized as expressing something to which to be is not repugnant. What is impossible is a combination of elements which are incompossible.

Even Aristotle said that when a possibility is assumed to be actualized, noth-
ing impossible follows, but he had in mind actualization in the history of the actual world. The denial of an omnitemporally actual state of affairs is said to be impossible. In the thirteenth century, some thinkers qualified Aristotle’s characterization of possibility as something which can be assumed as actual without contradiction by explaining that possibilities referring to the same moment of time are not necessarily compossible in the sense that they could be actual at the same time. In Scotus’s theory, actualizability as a criterion of possibility does not refer to the already existing actual world, nor is it explained by referring to prospective options of which the unrealized will disappear, as in the theories of diachronic modalities. Temporally definite possibilities are primarily treated as referring to alternatives, as is shown by the definition of a contingent state of affairs: ‘I do not call something contingent because it is not always or necessarily the case, but because its opposite could be actual at the very moment when it occurs’.

In the fourteenth century, many authors followed Scotus in explicating the meaning of modal terms by referring to simultaneous alternatives as the basic model. This is also reflected in the reformulation of modal logic with the systematic distinction between modalities de dicto and de re and the division of de re modalities into those with actual subject terms and those with non-actual subject terms. The basic notion was that of logical possibility which was distinguished from the more restricted notion of natural possibility. John Buridan’s theory of modal consequences is considered as one of the greatest achievements of medieval logic. Aristotle’s modal syllogistics was regarded as a fragmentary theory in which the different types of the fine structure of modal statements were not distinguished. Contrary to thirteenth-century approaches, Ockham, Buridan and their followers did not try to reconstruct Aristotle’s theory as such into a uniform system. The new modal paradigms which show some similarities to the background ideas of possible worlds semantics also influenced fourteenth century logic of discourse (obligations logic) and the treatments of applied modal logic, the logic of knowledge and belief and the logic of norms.

BIBLIOGRAPHY

Primary Literature


ture University, 1997).


[Peter Damian, 1972] Peter Damian. *Lettre sur la Toute-Puissance Divine (De divina omnipotentia)*, introduction, critical text, translation and notes by A. Cantin, Sources Chrétien


[Peter of Ireland, 1996] Peter of Ireland. *Expositio et quaestiones in librum Aristotelis Perier-


Secondary Literature


