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Method of Moments

1. Introduction

In many empirical investigations of dynamic economic systems, statistical analysis of a fully specified stochastic process model of the time series evolution is too ambitious. Instead it is fruitful to focus on specific features of the time series without being compelled to provide a complete description. This leads to the investigation of partially specified dynamic models. For instance, the linkages between consumption and asset returns can be investigated without a full de-

scription of production and capital accumulation. The behavior of monetary policy can be explored without a complete specification of the macroeconomic economy. Models of inventory behavior can be estimated and tested without requiring a complete characterization of input and output prices. All of these economic models imply moment conditions of the form:

$$Ef(x_t, \beta_o) = 0 \qquad (1)$$

where f is known a priori, x_t is an observed time series vector, and β_o is an unknown parameter vector. These moment relations may fall short of providing a complete depiction of the dynamic economic system. Generalized Method of Moments (*GMM*) estimation, presented in Hansen (1982), aims to estimate the unknown parameter vector β_o and test these moment relations in a computationally tractable way. This entry first compares the form of a *GMM* estimator to closely related estimators from the statistics literature. It then reviews applications of these estimators to partially specified models of economic time series. Finally, it considers *GMM*-related moment-matching problems in fully specified models economic dynamics.

2. Minimum Chi-square Estimation

To help place *GMM* estimation in a statistical context, I explore a closely related minimum chi-square estimation method. Statisticians developed minimum chi-square estimators to handle restricted models of multinomial data and a variety of generalizations. Neyman (1949) and Burankin and Gurland (1951), among others, aimed to produce statistically efficient and computationally tractable alternatives to maximum likelihood estimators. In the restricted multinomial model, estimators are constructed by forming empirical frequencies and minimizing Pearson's chi-square criterion or some modification of it.

The method has direct extensions to any moment-matching problem. Suppose that $\{x_t\}$ is a vector process, which temporarily is treated as being *iid*. Use a function ψ with n coordinates to define target moments associated with the vector x_t. A model takes the form:

$$E[\psi(x_t)] = \phi(\beta_o)$$

where β_o is an unknown parameter. The moment-matching problem is to estimate β_o by making the empirical average of $\{\psi(x_t)\}$ close to its population counterpart $\phi(\beta_o)$:

$$\min_{\beta} \frac{1}{T} \sum_{t-1}^{T} [\psi(x_t) - \phi(\beta)]' \ V \sum_{t-1}^{T} [\psi(x_t) - \phi(\beta)] \qquad (2)$$

where V is the distance or weighting matrix. The distance matrix sometimes depends on the data and/or

the candidate parameter vector β. The use of V to denote a weighting matrix that may actually depend on parameters or data is an abuse of notation. This simple notation is used because it is the probability limit of the weighting matrix evaluated at the parameter estimator that dictates the first-order asymptotic properties.

The limiting distribution of the criterion in Eqn. (2) is chi-square distributed with n degrees of freedom if the parameter vector β_o is known, and if V is computed by forming the inverse of either the population or sample covariance matrix of $\psi(x)$. When β is unknown, estimates may be extracted by minimizing this chi-square criterion; hence the name. To preserve the chi-square property of the minimum (with an appropriate reduction in the degrees of freedom), we again form the inverse sample covariance matrix of $\psi(x)$, or form the inverse population covariance matrix for each value of β. The minimized chi-square property of the criterion may be exploited to build tests of over-identification and to construct confidence sets for parameter values. Results like these require extra regularity conditions, and this rigor is supplied in some of the cited papers.

While the aim of this research was to form computationally tractable alternatives to maximum likelihood estimation, critical to statistical efficiency is the construction of a function ψ of the data that is a sufficient statistic for the parameter vector (see Burankin and Gurland 1951). Berkson (1944) and Taylor (1953) generalize the minimum chi-square approach by taking a smooth one-to-one function h and building a quadratic form of $h[\frac{1}{T}\sum_t \phi(x_t)] - h[\psi(\beta)]$. Many distributions fail to have a finite number of sufficient statistics; but the minimum chi-square method continues to produce consistent, asymptotically normal estimators provided that identification can be established.

GMM estimators can have a structure very similar to the minimum chi-square estimators. Notice that the core ingredient to the moment-matching problem can be depicted as in Eqn. (1) with a separable function f:

$$f(x, \beta) = \psi(x) - \phi(\beta)$$

used in the chi-square criterion function. Target moments are one of many ways for economists to construct inputs into chi-square criteria, and it is important to relax this separability. Moreover, in GMM estimation, the emphasis on statistical efficiency is weakened in order to accommodate partially specified models. Finally, an explicit time series structure is added, when appropriate.

3. GMM Estimation

Our treatment of GMM estimation follows Hansen (1982), but it builds from Sargan's (1958, 1959) analyses of linear and nonlinear instrumental variables (see *Instrumental Variables in Statistics and Econ-*

ometrics). See Ogaki (1993) for a valuable discussion of the practical implementation of GMM estimation methods. GMM estimators are constructed in terms of a function f that satisfies Eqn. (1) where f has more coordinates, say n, than there are components to the parameter vector β_o. Another related estimation method is M-estimation. M-estimation is a generalization of maximum likelihood and least squares estimation. M-estimators are typically designed to be less sensitive to specific distributional assumptions (see *Robustness in Statistics*). These estimators may be depicted as solving a sample counterpart to Eqn. (1) with a function f that is nonseparable, but with the same number of moment conditions as parameter estimators.

In Sects. 4–6 we will give examples of the construction of the f function, including ones that are not separable in x and β and ones for which there have more coordinates than parameter vectors. A minimum chi-square type criterion is often employed in GMM estimation. For instance, it is common to define the GMM estimator as the solution to:

$$b_T = \arg\min_\beta Tg_T(\beta)' V g_T(\beta)$$

where

$$g_T(\beta) = \frac{1}{T}\sum_{t-1}^T f(x_t, \beta)$$

and V is a positive definite weighting matrix. This quadratic form has the chi-square property provided that V is an estimator of the inverse of an appropriately chosen covariance matrix, one that accounts for temporal dependence.

The sections that follow survey some applications of GMM estimators to economic time series. A feature of many of these examples is that the parameter β by itself may not admit a full depiction of the stochastic process that generates data. GMM estimators are constructed to achieve 'partial' identification of the stochastic evolution and to be robust to the remaining unmodeled components.

3.1 Time Series Central Limit Theory

Time series estimation problems must make appropriate adjustments for the serial correlation for the stochastic process $\{f(x_t, \beta_o)\}$. A key input into the large sample properties of GMM estimators is a central limit approximation:

$$\frac{1}{\sqrt{T}}\sum_{t-1}^T f(x_t, \beta_o) \Rightarrow \text{Normal}(0, \Sigma_o)$$

for an appropriately chosen covariance matrix Σ_o. An early example of such a result was supplied by Gordin (1969) who was used martingale approximations for

partial sums of stationary, ergodic processes. See Hall and Heyde (1980) for an extensive discussion of this and related results. The matrix Σ_o must include adjustments for temporal dependence:

$$\Sigma_o = \sum_{j=-\infty}^{+\infty} Ef(x_t, \beta_o) f(x_{t-j}, \beta_o)'$$

which is the long-run notion of a covariance matrix that emerges from spectral analysis of time series. In many *GMM* applications, martingale arguments show that the formula for Σ_o simplifies to include only a small number of nonzero terms. It is the adjustment to the covariance matrix that makes the time series implementation differ from the *iid* implementation (Hansen 1982).

Adapting the minimum chi-square apparatus to this environment requires that we estimate the covariance matrix Σ_o. Since f is not typically separable in x and β, an estimator of Σ_o requires an estimator of β_o. This problem is familiar to statisticians and econometricians through construction of feasible generalized least squares estimators. One approach is to form an initial estimator of β_o with an arbitrary nonsingular weighting matrix V and to use the initial estimator of β_o to construct an estimator of Σ_o. Hansen (1982), Newey and West (1987) and many others provide consistency results for the estimators of Σ_o. Another approach is to iterate back and forth between parameter estimation and weighting matrix estimation until a fixed point is reached, if it exists. A third approach is to construct an estimator of $\Sigma(\beta)$ and to replace V in the chi-square criterion by an estimator of $\Sigma(\beta)^{-1}$ constructed for each β. Given the partial specification of the model, it is not possible to construct $\Sigma(\beta)$ without the use of the time series data. Long run covariance estimates, however, can be formed for the process $\{f(x_t, \beta)\}$ for each choice of β. Hansen et al. (1996) refer to this method as *GMM* estimation with a continuously updated weighting matrix. Hansen et al. (1996), Newey and Smith (2000), and Stock and Wright (2000) describe advantages to using continuous-updating, and Sargan (1958) shows that in some special circumstances this method reproduces a quasi-maximum likelihood estimator.

3.2 Efficiency

Since the parameter vector β that enters moment condition (1) may not fully characterize the data evolution, direct efficiency comparisons of *GMM* estimators to parametric maximum likelihood are either not possible, or not interesting. However, efficiency statements can be made for narrower classes of estimators.

For the study of *GMM* efficiency, instead of beginning with a distance formulation, index a family

of *GMM* estimators by the moment conditions used in estimation. Specifically study

$$a g_T(b_T) = 0$$

where a is a k by n selection matrix. The selection matrix isolates which (linear combination of) moment conditions will be used in estimation and indexes alternative *GMM* estimators. Estimators with the same selection matrix have the same asymptotic efficiency. Without further normalizations, multiple indices imply the same estimator. Premultiplication of the selection matrix a by a nonsingular matrix e results in the same system of nonlinear equations. In practice, the selection matrix can depend on data or even the parameter estimator provided that the selection matrix has a probability limit. As with weighting matrices for minimum chi-square criteria, we suppress the possible dependence of the selection matrix a on data or parameters for notational simplicity. The resulting *GMM* estimators are asymptotically equivalent (to possibly infeasible) estimators in which the selection matrix is replaced by its probability limit. As has been emphasized by Sargan (1958, 1959) in his studies of instrumental variables estimators, estimation accuracy can be studied conveniently as a choice a^* of an efficient selection matrix. The link between a weighting matrix V and selection matrix a is seen in the first-order conditions:

$$a = V \frac{1}{T} \sum_{t=1}^{T} \frac{\partial f(x_t, b_T)}{\partial \beta}$$

or their population counterpart:

$$a = Vd$$

where

$$d = E\left[\frac{\partial f(x_t, \beta_o)}{\partial \beta}\right]$$

Other distance measures including analogs to the ones studied by Berkson (1944) and Taylor (1953) can also be depicted as a selection matrix applied to the sample moment conditions $g_T(\beta)$. Moreover, *GMM* estimators that do not solve a single minimization problem may still be depicted conveniently in terms of selection matrices. For example, see Heckman (1976) and Hansen (1982) for a discussion of recursive estimation problems, which require solving two minimization problems in sequence.

Among the class of estimators indexed by a, the ones with the smallest asymptotic covariance matrix satisfy:

$$a = ed' \Sigma_o^{-1}$$

where e is any nonsingular matrix, and the best covariance matrix is $d'(\Sigma_o)^{-1}d$. This may be shown by imitating the proof of the famed Gauss-Markov Theorem, which establishes that the ordinary least squares estimator is the best, linear unbiased estimator.

3.3 Semiparametric Efficiency

Many *GMM* applications, including ones that we describe subsequently, imply an extensive (infinite) collection of moment conditions. To apply the previous analysis requires an *ad hoc* choice of focal relations to use in estimation. Hansen (1985) extends this indexation approach to time-series problems with an infinite number of moment conditions. The efficiency bound for an infinite family of *GMM* estimators can be related to the efficiency of other estimators using a semiparametric notion of efficiency. A semiparametric notion is appropriate because an infinite-dimensional nuisance parameter is needed to specify fully in the underlying model. It has been employed by Chamberlain (1987) and Hansen (1993) in their study of *GMM* estimators constructed from conditional moment restrictions.

4. Linear Models

Researchers in econometrics and statistics have long struggled with the idea of how to identify an unknown coefficient vector β_o in a linear model of the form:

$$\beta_o \cdot y_t = u_t$$

where y_t is a k-dimensional vector of variables observed by an econometrician. Least squares solves this problem by calling one of the variables, y_{1t}, the dependent variable and requiring the remaining variables, y_{2t}, to be orthogonal to the disturbance term:

$$E(u_t y_{2t}) = 0$$

Alternatively, as suggested by Karl Pearson and others, when there is no natural choice of a left-hand side variable, we may identify β_o as the first principal component, the linear combination of y_t with maximal variance subject to the constraint $|\beta| = 1$.

A third identification scheme exploits the time series structure and has an explicit economic motivation. It is a time-series analog to the instrumental variables and two-stage least squares estimators familiar to economists. Suppose that a linear combination of y_t cannot be predicted given data sufficiently far back into the past. That is,

$$E(\beta_o \cdot y_t | \mathscr{F}_{t-m}) = 0 \qquad (3)$$

where $\mathscr{F}_t$ is a conditioning information set that contains at least current and past values of y_t. This

conditional moment restriction can be used to identify the parameter vector β_o, up to scale. In this setup there may be no single variable to be designated as endogenous with the remainder being exogenous or even predetermined. Neither least squares nor principal components are appropriate for identifying β_o.

The model or the precise context of the application dictates the choice of lag m. For instance, restriction (3) for a specified value of m follows from martingale pricing relations for multiperiod securities, from Euler equations from the investment problems faced by decision-makers, or from the preference horizons of policy-makers.

To apply *GMM* to this problem, use Eqn. (3) to deduce the matrix equation:

$$E(z_{t-m} y'_t) \beta_o = 0$$

where z_t is an n-dimensional vector of variables in the conditioning information set $\mathscr{F}_t$ and $n \geqslant k - 1$. By taking unconditional expectations we see that β_o is in the null space of the n by k matrix $E(z_{t-m} y'_t)$. The model is over-identified if $n \geqslant k$. In this case the matrix $E(z_{t-m} y'_t)$ must be of reduced rank and in this sense is special. This moment condition may be depicted as in Eqn. (1) with:

$$f(x_t, \beta) = z_{t-m} y'_t \beta_o$$

where x_t is a vector containing the entries of z_{t-m} and y_t. The *GMM* test of over-identification, based on say a minimum chi-square objective, aims to detect this reduced rank. The *GMM* estimator of β_o seeks to exploit this reduced rank by approximating the direction of the null space.

For the *GMM* applications in this and Sect. 5, there is a related but independent statistics literature. Godambe and Heyde (1987) and others study efficiency criteria of martingale estimation equations for a fully specified probability model. By contrast, our models are partially specified and have estimation equations, e.g. $\sum_{t=1}^{T} [z_{t-m} y'_t] \beta_o$, that is a martingale only when $m = 1$. When the estimation equation is not a martingale, Hansen (1982) and Hansen (1985) construct an alternative martingale approximation to analyze statistical efficiency.

Given that the null space of $E z_{t-m} y'_t$ is not degenerate, it might have more dimensions than one. While the moment conditions are satisfied (the null space of $E(z_{t-m} y'_t)$ is nondegenerate), the parameter vector itself is under-identified. Recent literature on weak instruments is aimed at blurring the notion of being underidentified. See Stock and Wright (2000) for an analysis of weak instruments in the context of *GMM* estimation.

As posed here the vector β_o is not identified but is at best identified up to scale. This problem is analogous to that of principal component analysis. At best we might hope to identify a one-dimensional subspace. Perhaps a sensible estimation method should therefore

locate a subspace rather than a parameter vector. Normalization should be inessential to the identification beyond mechanically selecting the parameter vector within this subspace. See Hansen et al. (1996) for a discussion of normalization invariant *GMM* estimators. This argument is harder to defend than it might seem at first blush. Prior information about the normalized parameter vector of interest may eliminate interest in an estimation method that is invariant to normalization.

There are some immediate extensions of the previous analysis. The underlying economic model may impose further, possibly nonlinear, restrictions on the linear subspace. Alternatively, the economic model might imply multiple conditional moment relations. Both situations are straightforward to handle. In the case of multiple equations, prior restrictions are needed to distinguish one equation from another, but this identification problem is well studied in the econometrics literature.

4.1 Applications

A variety of economic applications produce conditional moment restrictions of the form in Eqn. (3). For instance, Hansen and Hodrick (1980) studied the relation between forward exchange rates and future spot rates where m is the contract horizon. Conditional moment restrictions also appear extensively in the study of a linear-quadratic model of production smoothing and inventories (see West 1995 for a survey of this empirical literature). The first-order or Euler conditions for an optimizing firm may be written as Eqn. (3) with $m = 2$. Hall (1988) and Hansen and Singleton (1996) also implement linear conditional-moment restrictions in the lognormal models of consumption and asset returns. These moment conditions are again derived from first-order conditions, but in this case for a utility-maximizing consumer/investor with access to security markets. Serial correlation due to time aggregation may cause m to be two instead of one. Finally, the linear conditional moment restrictions occur in the literature on monetary policy response functions when the monetary authority is forward-looking. Clarida et al. (2000) estimate such models in which m is dictated by preference horizon of the Federal Reserve Bank in targeting nominal interest rates.

4.2 Efficiency

The choice of z_{t-m} in applications is typically *ad hoc*. The one-dimensional conditional moment restriction of Eqn. (3) actually gives rise to an infinite number of unconditional moment restrictions through the choice of the vector z_{t-m} in the conditioning information set $\mathscr{F}_{t-m}$. Any notion of efficiency based on a preliminary choice z_{t-m} runs the risk of missing some potentially important information about the unknown parameter vector β_o. It is arguably more interesting to examine asymptotic efficiency across an infinite-dimensional family of *GMM* estimators indexed by feasible choices of z_{t-m}. This is the approach adopted by Hansen (1985), Hansen et al. (1988) and West (2000). The efficient *GMM* estimator that emerges from this analysis is infeasible because it depends on details of the time series evolution. West and Wilcox (1996) and Hansen and Singleton (1996) construct feasible counterpart estimators based on approximating the time series evolution needed to construct the efficient z_{t-m}. In particular, West and Wilcox (1996) show important improvements in the efficiency and finite sample performance of the resulting estimators.

Efficient *GMM* estimators based on an infinite number of moment conditions often place an extra burden on the model-builder by requiring that a full dynamic model be specified. The cost of misspecification, however, is not so severe. While a mistaken approximation of the dynamics may cause an efficiency loss, by design it will not undermine the statistical consistency of the *GMM* estimator.

5. Models of Financial Markets

Models of well-functioning financial markets often take the form:

$$E\left(d_t z_t \mid \mathscr{F}_{t-m}\right) = q_{t-m} \qquad (4)$$

where z_t is a vector of asset payoffs at date t, q_{t-m} is a vector of the market prices of those assets at date t, and $\mathscr{F}_t$ is an information set available to economic investors at date t. By assumption, the price vector q_t is included in the information set $\mathscr{F}_t$. The random variable d_t is referred to as a 'stochastic discount factor' between dates $t-m$ and date t. This discount factor varies with states of the world that are realized at date t and encodes risk adjustments in the security market prices. These risk adjustments are present because some states of the world are discounted more than others, which is then reflected in the prices. The existence of such a depiction of asset prices is well known since the work of Harrison and Kreps (1979), and its conditional moment form used here and elsewhere in the empirical asset pricing literature is justified in Hansen and Richard (1987). In asset pricing formula (4), m is the length of the financial contract, the number of time periods between purchase date and payoff date.

An economic model of financial markets is conveniently posed as a specification of the stochastic discount factor d_t. It is frequently modeled parametrically:

$$d_t = g(y_t, \beta_o) \qquad (5)$$

where the function *g* is given *a priori* but the parameter β_o is unknown, and a target of estimation. Models of investor preferences may be written in this manner as can observable factor models in which d_t is a function (often linear) or vector y_t of 'factors' observed by an econometrician. See Cochrane (2001) for examples and a discussion of how this approach is connected to other empirical methods in financial economics.

To apply *GMM* estimation, inference, and testing to this problem we do two things. First we replace Eqn. (4) by its unconditional counterpart:

$$E(d_t z_t - q_{t-m}) = 0$$

which can be justified by applying the Law of Iterated Expectations. Second we substitute Eqn. (5) into this unconditional moment implication:

$$E[g(y_t, \beta_o)z_t - q_{t-m}] = 0$$

This may be depicted as (1) by writing:

$$f(x_t, \beta) = g(y_t, \beta)z_t - q_{t-m}$$

where x_t contains the entries of the factors y_t, the asset payoffs z_t, and the corresponding prices q_{t-m}. As is evident from Hansen and Singleton (1982) and Kocherlakota (1996), *GMM*-based statistical tests of this model give rise to one characterization of what is commonly termed as the 'equity-premium puzzle.' This puzzle is a formal statement of how the observed risk–return trade-off from financial market data is anomalous when viewed through the guises of many standard dynamic equilibrium models from the macroeconomics and finance literatures.

Studying unconditional rather than conditional moment relations may entail a loss of valuable information for estimation and testing. As emphasized by Hansen and Richard (1987), however, information in the set $\mathscr{F}_{t-m}$ may be used by an econometrician to form synthetic payoffs and prices. This information is available at the initiation date of the financial contract. While use of conditioning information to construct synthetic portfolios reduces the distinction between conditional and unconditional moment restrictions, it introduces additional challenges for estimation and inference that are being confronted in ongoing econometric research.

6. *From Densities to Diffusions*

A century ago Karl Pearson proposed a family of density functions and a moments-based approach to estimating a parameterized family of densities. The densities within this family have logarithmic derivatives that are the ratio of a first to a second-order polynomial. More recently, Wong (1964) provided scalar diffusion models with stationary distributions in the Pearson family. Diffusion models are commonly used in economic dynamics and finance.

A scalar diffusion is a solution to a stochastic differential equation:

$$dx_t = \mu(x_t)\,dt + \sigma(x_t)\,dB_t$$

where μ is the local mean or drift for the diffusion, σ^2 is the local variance or diffusion coefficient, and B_t is standard Brownian motion. I now revisit Pearson's estimation problem and method, but in the context of a data generated by a diffusion.

The stationary density *q* of a diffusion satisfies the integral equation:

$$\int_r^{\bar{r}} \left(\mu\phi + \frac{\sigma^2}{2}\frac{d\phi}{dx} \right) q\,dx = 0 \qquad (6)$$

for a rich class of ϕ's, referred to as test functions. This gives an extensive family of moment conditions for any candidate (μ, σ^2). In particular, moment conditions of the form (1) may be built by parameterizing μ and σ^2 and by using a vector of test functions ϕ. Pearson's moment recursions are of this form with test functions that are low-order polynomials.

Pearson's method of estimation was criticized by R. A. Fischer because it failed to be efficient for many members of the Pearson family of densities. Pearson and Fischer both presumed that the data generation is *iid*. To attain asymptotic efficiency with *iid* data, linear combinations of $f(x, \beta_o)$ should reproduce the score vector for an implied likelihood function. Low-order polynomials fail to accomplish this for many members of the Pearson family of densities, hence the loss in statistical efficiency.

When the data are generated by a diffusion, the analysis of efficiency is altered in a fundamental way. Parameterize the time series model via $(\mu_\beta, \sigma_\beta^2)$. There is no need restrict μ and σ^2 to be low-order polynomials. Let Φ be a vector of test functions with at least as many component functions as parameters to estimate. Associated with each Φ is a *GMM* estimator by forming:

$$f(x, \beta) = \mu_\beta(x)\Phi(x) + \frac{1}{2}\sigma_\beta^2(x)\frac{d\Phi}{dx}(x)$$

The moment conditions (1) follow from Eqn. (6).

Conley et al. (1997) calculate the efficiency bounds for this estimation problem using the method described in Hansen (1985). They perform this calculation under the simplifying fiction that a continuous-data record is available and show that an efficient choice of Φ is:

$$\Phi_\beta(x) = \frac{\partial}{\partial\beta}\left[\frac{2\mu_\beta(x) + d\sigma_\beta^2(x)/dx}{\sigma_\beta^2(x)}\right] \qquad (7)$$

The choice of test function turns out to be the 'derivative' of the score vector with respect to the Markov state *x*. This efficient choice of Φ depends on

the true parameter vector β_0 and hence is infeasible to implement. Conley et al. (1997) show that the same efficiency can be attained by a feasible estimator in which Φ is allowed to depend on β as in Eqn. (7). The score function derivative is easier to compute than the score function itself because the constant of integration for implied density does not have to be evaluated for each β. While tractable, this efficient test function solution to the time series problem will typically not lead one to use low-order polynomials as test functions even for Pearson's parameterizations.

Pearson used moment recursions to construct tractable density estimates without numerical integration. Computational concerns have subsided, and the diffusion model restricts more than just the stationary density. Transition densities can also be inferred numerically for a given (μ, σ^2) pair. To motivate fitting only the stationary density, a model-builder must suppose potential model misspecification in the transition dynamics. One example of this form of misspecification is a subordinated diffusion model used to model financial time series in which calendar time and a more relevant information-based notion of economic time are distinct.

Integral equation (6) is 'localized' by using smooth test functions that concentrate their mass in the vicinity of given points in the state space. Banon (1978) exploited this insight to produce a nonparametric drift estimator using a locally constant parameterization of μ. Conley et al. (1997) justify a local linear version of the *GMM* test function estimator described in the previous subsection.

7. Related Approaches

Since around 1990 statisticians have explored empirical likelihood and other related methods. These methods are aimed at fitting empirical data distributions subject to *a priori* restrictions, including moment restrictions that depend on an unknown parameter. In particular, Qin and Lawless (1994) have shown how to use empirical likelihood methods to estimate parameters from moment restrictions like those given in (1) for *iid* data. While Qin and Lawless (1994) use the statistics literature on estimation equations for motivation, they were apparently unaware of closely-related econometrics literature on *GMM* estimation. Baggerly (1998) describes a generalization of the method of empirical likelihood based on the Cressie-Read divergence criterion. Imbens et al. (1998) and Bonnal and Renault (2001) use this generalization to unify the results of Qin and Lawless (1994), Imbens (1997), Kitamura and Stutzer (1997), and others and *GMM* estimation with a continuously-updated weighting matrix. (See Newey and Smith (2000) for a related discussion.) Within the *iid* framework, Bonnal and Renault (2001) show that the continuously updated *GMM* estimator is a counterpart to empirical likelihood except that it uses

Pearson's χ^2 criterion. The relative entropy or information-based estimation of Imbens (1997) and Kitamura and Stutzer (1997) is of the same type but based on an information criterion of fit for the empirical distribution.

Kitamura and Stutzer (1997) and others use clever blocking approaches for weakly dependent data to adapt these methods to time series estimation problems. Many *GMM* applications imply 'conditional' moment restrictions where the conditioning information is lagged m time periods. Extensions of these empirical distribution methods to accommodate time series 'conditional' moment implications is an important area of research.

8. Moment-matching Reconsidered

Statistical methods for partially specified models allow researchers to focus an empirical investigation and to understand sources of empirical anomalies. Nevertheless, the construction of fully specified models is required to address many questions of interest to economists such as the effect of hypothetical interventions or policy changes.

Models of economic dynamical systems remain highly stylized, however; and they are not rich enough empirically to confront a full array of empirical inputs. Producing interesting comparative dynamic results for even highly stylized dynamic systems is often difficult, if not impossible, without limiting substantially the range of parameter values that are considered. Since analytical solutions are typically not feasible, computational methods are required. As in other disciplines, this has led researchers to seek ways to calibrate models based on at least some empirical inputs. See Hansen and Heckman (1996) for a discussion of this literature. The analysis of dynamical economic systems brings to the forefront both computational and conceptual problems.

The computational problems are reminiscent to those confronted by the inventors of minimum chi-square methods. Two clever and valuable estimation methods closely related to moment matching have recently emerged from the econometrics literature. One method is called 'indirect inference' and has been advanced by Smith (1993) and Gourieroux et al. (1993). The idea is to fit a conveniently chosen, but misspecified approximating model that is easy to estimate. The empirical estimates from the approximating model are used as targets for the estimation of the dynamic economic model. These targets are 'matched' using a minimum chi-square criteria.

This method is sometimes difficult to implement because the implied approximating models associated with the dynamic economic model of interest may be hard to compute. Gallant and Tauchen (1996) circumvent this difficulty by using the score function for the approximating model (evaluated at the empirical estimates) as targets of estimation. The computational

burden is reduced to evaluating an empirical score expectation as a function of the underlying parameter of interest. This allows researchers to use an expanded set of approximating statistical models.

In both of these methods there are two models in play, an approximating statistical model and an underlying economic model. These methods address some of the computational hurdles in constructing parameter estimators through their use of convenient approximating statistical models. On the other hand, it is harder to defend their use when underlying dynamic economic model is itself misspecified. Gallant and Tauchen (1996), for instance, use statistical efficiency as their guide in choosing approximating statistical models. Better approximating models results in more accurate estimators of the parameters of interest. This justification, however, neglects the role of misspecification in the underlying dynamic economic model. For 'indirect inference' it is often not evident how to construct approximating models that leave the estimates immune to the stylized nature of the underlying economic model. Thus it remains an important challenge for econometricians to devise methods for infusing empirical credibility into 'highly stylized' models of dynamical economic systems. Dismissing this problem through advocating only the analysis of more complicated 'empirically realistic' models will likely leave econometrics and statistics on the periphery of important applied research. Numerical characterizations of highly stylized models will continue with or without the aid of statistics and econometrics.

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Methodological Individualism in Sociology

Methodological individualism in sociology refers to the explanatory and modeling strategies in which human individuals (with their motivations) and human actions (with their causes or reasons) are given a prominent role in explanations and models. Social phenomena are viewed as the aggregate results of individual actions. Explanation thus proceeds from the parts to the whole: individual action has an explanatory primacy in relation to social facts, society's properties, and observed macroregularities. Among the problems associated with this methodology, the following three are of special interest: (a) how should individual characteristics be selected and connected? (b) does the relevance of individualism as a methodology depend on the way models and theories are used?, and (c) how should individualistic social science take individual cognition into account?

1. Individualism and the Form of Explanations

1.1 Giving Primacy to Individuals

While the constitution of sociology as an autonomous discipline has involved the recognition of a separate layer of 'social facts,' methodological individualism, which presupposes the existence of social facts as an *explanandum* for social science, is not alien to the sociological tradition, as exemplified by the work of Weber, Pareto, and others. A general feature of individualistic explanations is that individual motivations, preferences, reasons, propensities, or individual characteristics generally speaking figure explicitly in the proposed models and explanations, together with the description of relevant technological and natural circumstances.

Methodological individualism has gained influence in the twentieth century through the work of such authors as Mises, Popper, and Hayek, but its emergence is traceable to the debates in nineteenth century Germany and Austria about the nature of explanation in history, the status of economic theory, and the respective scientific roles of nomological explanation and particular understanding. It is associated classically with the requirement of a real understanding of the motivations of the social actors themselves (as illustrated by Simmel 1900, Weber 1922).

This methodology can be contrasted with several types of nonindividualistic methods (Boyer 1992). It is violated by those theories which rely on the operation of impersonal forces (such as nature, mind, history, progress, or destiny), and by 'holistic' explanations in which the properties of collective entities (such as nations, classes, social groups, or society as a whole) or unconscious forces are given an independent explanatory role.

Methodological individualism is compatible with ontological holism about collective entities, in the following sense: individualistic social scientists may recognize the existence of social entities (such as 'cultures' or 'traditions') which are irreducible to individual component parts. But they postulate that only individuals have goals and interests, and that these have explanatory value with respect to human conduct. They reject or reinterpret the notion of collective belief. Finally, they recognize that the social set-up can be transformed through the action of individuals. This makes methodological individualism hardly compatible with the more deterministic versions of historical materialism, although it is congenial to some Marxian themes, in particular Marx's criticism of the belief in the historical effectiveness of abstract notions of man, society, and consciousness.

1.2 Individual Motivation

Some individual characteristics should figure in the *explanans* in an explicit manner. But which are the

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