EXAMINING MACROECONOMIC MODELS WITH FINANCE CONSTRAINTS THROUGH THE LENS OF ASSET PRICING

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Examining macroeconomic models with financing constraints through the lens of asset pricing

Dynamic stochastic equilibrium models of macro economy are designed to match transient time series properties including impulse response functions. Gertler, Kiyotaki and others have extended this literature by introducing financing constraints that alter investment opportunities. Since these models aim to be structural, they have implications for asset pricing. To assess these implications, we consider asset pricing counterparts to impulse response functions. We quantify the exposures of alternative macroeconomic cash flows to shocks over alternative investment horizons and the corresponding prices or compensations that investors must receive because of their exposure to such shocks. We build on the continuous-time methods developed in Hansen and Scheinkman and our earlier work and construct discrete-time shock elasticities that measure the sensitivity of cash flows and their prices to economic shocks including economic shocks featured in the empirical macroeconomics literature.

Outline

- Construct shock price and shock exposure elasticities as inputs into valuation accounting.
- Go beyond log-linearization methods to accommodate stochastic volatility and other sources of nonlinearity in the dynamic evolution.
  - Previously - long-run cash flow predictability and time-variation in expected returns.
  - Our aim - dynamic risk or shock exposure elasticities for cash-flows and dynamic risk or shock price elasticities.

  References:
  - Pricing Growth Rate Risk, Hansen and Schienkman, *Finance and Stochastics*
  - Risk Price Dynamics, Borovička, Hansen, Hendricks and Scheinkman, *Journal of Financial Econometrics*
- Explore these elasticities in the context of a recent macroeconomic model with financial frictions. Ongoing work with Borovička.

Talk Outline

- Elasticities and valuation accounting
- Log-exponential parameterization
- Recursive utility revisited
- Financial market wedges

Setup

Suppose X is first-order Markov, and W is an iid sequence of multivariate, standard normally distributed random vectors.

- Conditional Gaussian model in logarithms:

$$Y_t = \sum_{s=0}^{t-1} \left[\beta\left(X_s\right) + \alpha\left(X_s\right) \cdot W_{s+1} \right].$$

- Levels $M_t = \exp(Y_t)$. Examples of M include a macroeconomic growth functional G such as consumption or capital and a stochastic discount factor functional S used to price assets.

Single-period asset pricing

Suppose that

$$\log G_1 = \beta_g(X_0) + \alpha_g(X_0) \cdot W_1$$
$$\log S_1 = \beta_s(X_0) + \alpha_s(X_0) \cdot W_1$$

$$R_1 = \frac{G_1}{E(S_1 G_1 | X_0)}$$

Logarithm of the expected return is:

$$\log E(G_1 | X_0 = x) - \log E(S_1 G_1 | X_0 = x) =$$
$$- \beta_s(x) - \alpha_g(x) \cdot \alpha_s(x) - \frac{|\alpha_s(x)|^2}{2}$$

Then $-\alpha_s$ is the risk price vector for exposure to the components of W_1.

Asset pricing puzzle: Modeled versions of $|\alpha_s|$ are too small.

Recursive utility gives one way to address using seemingly large values of γ.

Alternative approach that extends to other investment horizons

Compute elasticities.

- Consider a parameterized family of payoffs.

$$H_1(\mathsf{r}) = \mathsf{r}\alpha_h(X_0) \cdot W_1 - \frac{r^2}{2}|\alpha_h(X_0)|^2$$

where

$$E[|\alpha_h(X_0)|^2] = 1.$$

Then α_h gives an exposure direction and $H_1(\mathsf{r})$ has conditional expectation equal to one.

- Form $G_1 H_1(\mathsf{r})$ where

$$\log G_1 + \log H_1(\mathsf{r}) = [\alpha_g(X_0) + \mathsf{r}\alpha_h(X_0)] \cdot W_1 + \beta_g(X_0) - \frac{(\mathsf{r})^2}{2}|\alpha_h(X_0)|^2$$

Parameterized family of asset payoffs to be priced.

Elasticities

- Compute expected return:

$$\log E[G_1 H_1(\mathsf{r})|X_0 = x] - \log E[S_1 G_1 H_1(\mathsf{r})|X_0 = x]$$

- Differentiate:

$$\frac{d}{d\mathsf{r}} \log E[G_1 H_1(\mathsf{r})|X_0 = x]|_{\mathsf{r}=0} - \frac{d}{dr} \log E[S_1 G_1 H_1(\mathsf{r})|X_0 = x]|_{\mathsf{r}=0}$$

- Component elasticities:
  1. shock-exposure elasticity:

$$\varepsilon_g(x) = \frac{d}{d\mathsf{r}} \log E[G_1 H_1(\mathsf{r})|X_0 = x]|_{\mathsf{r}=0} = \alpha_g(x) \cdot \alpha_h(x)$$

  2. shock-value elasticity:

$$\varepsilon_v(x) = \frac{d}{dr} \log E[S_1 G_1 H_1(\mathsf{r})|X_0 = x]|_{\mathsf{r}=0} = \alpha_s(x) \cdot \alpha_h(x) + \alpha_g(x) \cdot \alpha_h(x)$$

  3. shock-price elasticity:

$$\varepsilon_p(x) = \varepsilon_g(x) - \varepsilon_v(x) = -\alpha_s(x) \cdot \alpha_h(x)$$

EXTENDING THE INVESTMENT HORIZON

- Construct payoff: $G_t H_1(\mathsf{r})$.
- Compute price: $E\left[S_t G_t H_1(\mathsf{r}) | X_0 = x\right]$
- Form elasticities:
  1. shock-exposure elasticity for horizon t:

$$\varepsilon_g(x, t-1) = \frac{d}{d\mathsf{r}} \frac{1}{t} \log E[G_t H_1(\mathsf{r}) | X_0 = x]|_{\mathsf{r}=0}$$

  2. shock-value elasticity for horizon t and (and shock date one):

$$\varepsilon_v(x, t-1) = \frac{d}{dr} \frac{1}{t} \log E[S_t G_t H_1(\mathsf{r}) | X_0 = x]|_{\mathsf{r}=0}$$

  3. shock-price elasticity for horizon t:

$$\varepsilon_p(x, t-1) = \varepsilon_g(x, t-1) - \varepsilon_v(x, t-1).$$

Representations

Let M be a multiplicative functional (either G or SG). Then

$$\varepsilon_m(x,t) = \alpha_h(x) \cdot \frac{E\left(M_t W_1 | X_0 = x\right)}{E\left(M_t | X_0 = x\right)}$$

Observations

- When M is log-linear, essentially recovers the impulse response function for $\log M$ in response to a shock $\alpha_h \cdot W_1$. Shock exposure elasticities reflect impulse response functions for $\log G$, shock-price elasticities reflect impulse response functions for $-\log S$
- With stochastic volatility and other sources of nonlinearity, the choice of G matters for computing the shock-price elasticities.
- The elasticities are inputs into valuation accounting.

Representation continued

Recall the change of measure based on the factorization:

$$M_t = \exp(\eta t)\hat{M}_t \frac{e(X_0)}{e(X_t)}$$

where $\hat{M}$ is multiplicative martingale. Then

$$\varepsilon_m(x, t-1) = \alpha_h(x) \cdot \frac{\hat{E}\left[\hat{e}(X_t)W_1 | X_0 = x\right]}{\hat{E}\left[\hat{e}(X_t) | X_0 = x\right]}.$$

where $\hat{e} = \frac{1}{e}$.
Observations:

- Under the change of measure, the expectation of W_1 is not zero and determines limiting value: $\alpha_h(x) \cdot \hat{E}\left[W_1 | X_0 = x\right]$.
- Reflects the state-dependent counterpart to the impulse response function for $\hat{e}$ using Markov diffusions.

Shifting the shock exposure date

Shock date is at τ and the impact date at $t+\tau$.

The resulting elasticity is:

$$\frac{\hat{E}\left[\hat{e}(X_{t+\tau})\varepsilon(X_{\tau-1},t)|X_0=x\right]}{\hat{E}\left[\hat{e}(X_{t+\tau})|X_0=x\right]}=\frac{\hat{E}\left[\hat{e}(X_{t+\tau})\alpha_h(X_{\tau-1})\cdot W_\tau|X_0=x\right]}{\hat{E}\left[\hat{e}(X_{t+\tau})|X_0=x\right]}$$

The shifted elasticity depends on $\hat{e}$ and on the alternative probability distribution.

Some limits:

- For large t and a fixed τ the limiting elasticity is:

  $$\hat{E}\left[\alpha_h\left(X_{\tau-1}\right)\cdot W_\tau|X_0=x\right].$$

- For $t=0$ and large τ the limiting elasticity is:

  $$\frac{\hat{E}\left[\hat{e}(X_0)\alpha_h\left(X_0\right)\cdot W_1\right]}{\hat{E}\left[\hat{e}(X_0)\right]}.$$

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Exponential-quadratic framework

Triangular state vector system:

$$
\begin{aligned}
X_{1,t+1} &= \Delta_{10} + \Delta_{11} X_{1,t} + \Lambda_{10} W_{t+1} \\
X_{2,t+1} &= \Delta_{20} + \Delta_{21} X_{1,t} + \Delta_{22} X_{2,t} + \Delta_{23} \left(X_{1,t} \otimes X_{1,t} \right) \\
&\quad + \Lambda_{20} W_{t+1} + \Lambda_{21} \left(X_{1,t} \otimes W_{t+1} \right) + \Lambda_{23} \left(W_{t+1} \otimes W_{t+1} \right)
\end{aligned}
$$

- Stable dynamics if Δ_{11} and Δ_{22} have stable eigenvalues.
- Structure allows for stochastic volatility through $X_{1,t} \otimes W_{t+1}$

Additive functionals

$$
\begin{aligned}
Y_{t+1} - Y_t &= \Gamma_0 + \Gamma_1 X_t + \Gamma_2 \left(X_{1,t} \otimes X_{1,t} \right) \\
&\quad + \Theta_0 W_{t+1} + \Theta_1 \left(X_{1,t} \otimes W_{t+1} \right) + \Theta_2 \left(W_{t+1} \otimes W_{t+1} \right)
\end{aligned}
$$

- Use to model stochastic discount factor and growth functionals.

Component calculations

The framework allows for quasi-analytical formulas for conditional expectations of multiplicative functionals and for elasticities.

- Start with

$$\log f(x) = \phi + \Phi x + \frac{1}{2}(x_1)'\Psi(x_1)$$

- Then

$$\begin{aligned}
\log \quad & E\left[\left(\frac{M_{t+1}}{M_t}\right) f(X_{t+1}) | X_t = x\right] \\
&= \log E\left[\exp\left(Y_{t+1} - Y_t\right) f(X_{t+1}) \mid X_t = x\right] \\
&= \phi^* + \Phi^* x + \frac{1}{2}(x_1)'\Psi^*(x_1) \\
&= \log f^*(x)
\end{aligned}$$

Functional form for positive eigenfunctions and conditional expectations that accommodates growth and discounting and stochastic volatility.

Approximation

- Much of the existing macroeconomic literature uses "perturbation" methods.
  - log-linearization/ first-order approach
  - used to fit responses to macroeconomic *quantities* to small shocks
  - not suitable for stochastic volatility

Our approach

- log approximation of our one-period valuation operators second-order in x_1.
  - include both zeroth order and first-order terms in shock exposure
  - include zeroth and first-order terms in x_2 and zeroth, first and second-order terms in x_1
- impose stochastic stability - follow Schmitt-Grohe and Uribe, Kim, Kim, Schaumburg and Sims, and Lombardo.

Convenient, but ...

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Recursive utility

- Continuation values

$$V_t = \left[(\zeta C_t)^{1-\rho} + \exp(-\delta) \left[\mathcal{R}_t(V_{t+1}) \right]^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

  where

$$\mathcal{R}_t \left(V_{t+1} \right) = \left(E \left[(V_{t+1})^{1-\gamma} | \mathcal{F}_t \right] \right)^{\frac{1}{1-\gamma}}$$

- Intertemporal marginal rate of substitution

$$\frac{S_{t+1}}{S_t} = \exp(-\delta) \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left[\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} \right]^{\rho-\gamma}.$$

  Depends on continuation values, which gives a channel for sentiments to matter.

  Used to represent asset prices.

A LOGNORMAL EXAMPLE WITH $\rho = 1$

Use a specification from Hansen-Heaton-Li (JPE).

- State dynamics:

$$X_{t+1} = \Delta_0 + \Delta_1 X_t + \Lambda W_{t+1}$$

- Consumption dynamics:

$$Y_{t+1} - Y_t = \Gamma_0 + \Gamma_1 X_t + \Theta W_{t+1}$$

  where $\log C_t = Y_t$.

- Impulse response/shock exposure elasticity for shock $\alpha_h \cdot W_1$ and $G = C$:

$$\varepsilon_g(0) = \Theta\alpha_h \quad \varepsilon_g(t) \quad = \left[\Theta + \Gamma_1 \sum_{j=0}^{t-1} (\Delta_1)^j \Lambda\right] \alpha_h$$

  No state dependence.

Recursive utility example continued

- Shock-exposure elasticity for shock $\alpha_h \cdot W_1$ and $G = C$:

$$\varepsilon_c(0) = \Theta\alpha_h \quad \varepsilon_g(t) \quad = \left[\Theta + \Gamma_1 \sum_{j=0}^{t-1} (\Delta_1)^j \Lambda\right] \alpha_h$$

- Shock-price elasticity for shock $\alpha_h \cdot W_1$

$$\varepsilon_p(j) = \varepsilon_g(j) + (\gamma - 1) \left[\Theta + \Gamma_1 \left(I - \beta\Delta_1\right)^{-1} \Lambda\right] \alpha_h$$

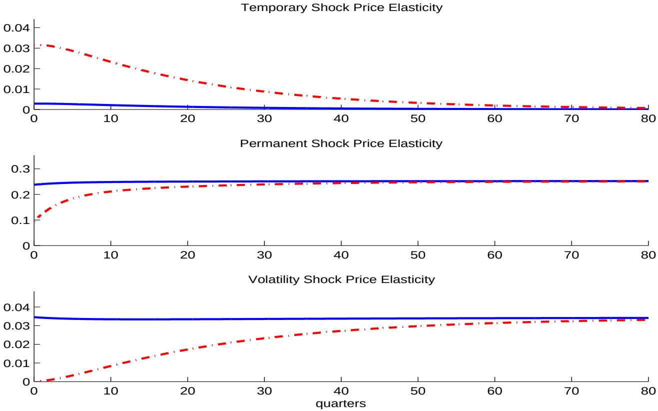
When γ is large the price elasticity is dominated by

$$(\gamma - 1) \left(\Theta + \Gamma_1 \left[I - \exp(-\delta)\Delta_1\right]^{-1} \Lambda\right) \alpha_h,$$

which does not depend on the investment horizon. When $\exp(-\delta) \approx 1$, this term coincides with the limiting consumption exposure elasticity scaled by $\gamma - 1$.

Recursive utility adds a forward-looking component to valuation.

Shock-price trajectories for power and recursive utility



Revisited from lecture one. Stochastic volatility is incorporated. Volatility state set at its unconditional mean.

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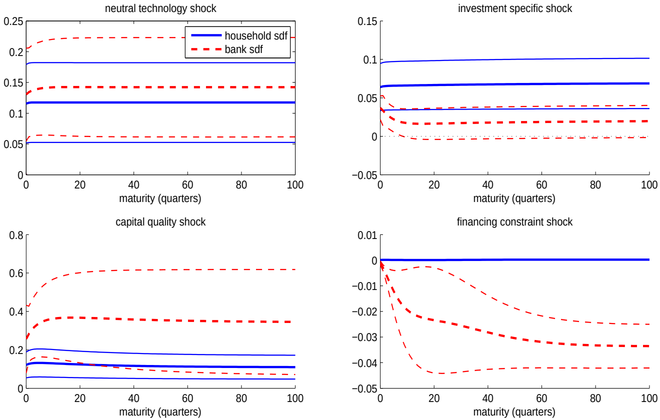
Model ingredients

Gertler-Kiotaki with recursive utility.

- Communication friction - firms that produce output return proceeds to consumers at random dates in the future.
- Wedge between internal and external financing of new capital - external financing recognizes that producers could confiscate a fraction of the capital after respecting commitments to internal financiers. Represent this wedge with two stochastic discount factors.
- No financial distortions for the production of new investment goods.
- Four shocks - neutral technology shock, investment specific shock, capital quality shock, and a financing constraint shock that represents the potency of the threat to confiscate.
- Stochastic volatility.

Shock-price elasticities



"Banks"are the internal financiers. Shock-price trajectories are depicted at the median state volatility state and at the upper and lower quartiles.