Toward A Term Structure of Macroeconomic Risk

Pricing Unexpected Growth Fluctuations

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[1] Based in part joint work with John Heaton, Nan Li, Jose Scheinkman and Tom Sargent

Introduction

- Gordon growth model:

$$\textit{value} = \frac{\textit{cash \ flow}}{\textit{discount \ rate} - \textit{growth \ rate}}$$

- Characterize the dynamics of valuation
- Deconstruct risk premia into interpretable components
  - Exposure of the alternative consumption profiles or cash flows to underlying shocks
  - Prices or imputed compensations for alternative exposures to shocks
  - Evolution of values across an array of investment horizons
- Use Markov formulations and martingale methods

Why long run?

- growth uncertainty has important consequences for welfare
- stochastic component growth can have a potent impact on asset values
- economics more revealing for modeling long-run phenomenon

Related literature

- Alvarez and Jermann, Econometrica
- Bansal and Lehmann, Macroeconomic Dynamics
- Bansal and Yaron, Journal of Finance
- Campbell and Vuolteenaho, American Economic Review
- Croce, Lettau and Ludvigson
- Hansen, Heaton and Li
- Hall, AER
- Lettau and Wachter, Journal of Finance
- Moskowitz and Vissing-Jorgensen
- Parker, American Economic Review
- Piazzesi and Schneider, Macroeconomic Annual
- Shaliastovich and Tauchen

Game plan

- Review basic asset pricing concepts that I will build upon;
- Develop novel decompositions of economic models with value implications;
- Apply methods and explore statistical and model sensitivity to some of the essential inputs;
- Speculate on productive future directions of research.

Components of asset values

1. One period returns: bundles of state-contingent payoffs in a single period or Arrow securities.
   - An economic model predicts prices for the components of single-period payoffs - assigns values to one period risk exposures
2. Intertemporal counterpart: price bundled claims across states and time periods; equity portfolios, term structure
   - An economic model predicts prices of intertemporal cash flows or hypothetical consumption processes - assigns values to risk exposures at alternative points in time

Asset valuation and stochastic discount factors

$$\pi = \sum_{t=0}^{\infty} E\left[S_t D_t | x_0\right]$$

where π is the date zero price of a "cash flow" or "dividend" process.

$\{S_t\}$ is a stochastic discount factor process. Encodes both discounting and adjustments for risk. Satisfies consistency constraints - Law of Iterated Values.

- Benchmark example - Rubinstein, Lucas and Breeden

$$S_t = \beta^t \left(\frac{C_t}{C_0}\right)^{-\rho} \quad \text{Intertemporal MRS}$$

- Dynamics of pricing are captured by the time series behavior of the stochastic discount factor.

Review of empirical findings

- Short run links between consumption and risk prices as implied by the Rubinstein-Breeden-Lucas model are too small.
- Variety of modifications in the underlying economic model have been proposed and investigated. Some changes are transient in nature and some are strongly persistent.
- Time variation in risk premia are arguably important.
- Intriguing low frequency links between consumption and asset prices have been found.

Alternative economic models

- Recursive utility Kreps-Porteus model
- Habit persistence, consumption externalities
- Model ambiguity and robustness
- Incomplete markets
- Ad hoc models of local risk prices

Use long run limits as a frame of reference

Intertemporal structure of risk premia reflect both the dynamics of cash flow exposure to risk and of price of that risk exposure.

To uncouple these effects and to characterize the structure of risk premia, I will explore valuation of conveniently constructed martingales.

- Directly construct these martingales from primitive shocks.
- Extract martingales from alternative cash flows and consumption processes.

Why new tools?

Assess formally the role of statistical and economic inputs in the valuation of macroeconomic risk.

- Support model comparisons: To which components of uncertainty are valuations (market or shadow prices) most sensitive?
- Allow for stochastic changes in macroeconomic volatility.
- Discriminate between transient and persistent implications: What hypothetical changes in preferences, technology and macroeconomic policy have the most potent impact on the long run? What changes are transient?

Constructing Markov paths

Let $\{x_t\}$ be a Markov process.

Path:

$$A_t = \sum_{j=1}^{t} \kappa(x_j, x_{j-1}) \qquad A_t^* = -\sum_{j=1}^{t} \kappa^*(x_j, x_{j-1})$$

$$\uparrow \qquad\qquad\qquad\qquad \uparrow$$

arithmetic growth **arithmetic decay**

$\kappa(x_t, x_{t-1})$ is a state dependent growth rate and $\kappa^*(x_t, x_{t-1})$ is a state dependent decay rate.

Consumption or cash flows: $G_t = \exp(A_t)$. Model geometric, stochastic growth. Encodes risk exposure or exposure to shocks over multiple horizons.

Pricing : $S_t = \exp(A_t^*)$. Model stochastic discount factors. Encodes risk prices associated with shocks over multiple horizons.

Example

Partition:

$$x_t = \begin{bmatrix} y_t \\ z_t \end{bmatrix}$$

State evolution: $y_{t+1} - y_t = \Lambda y_t + \Theta z_t + \Gamma(z_t) w_{t+1}$

Signal evolution: $e_{t+1} = \Psi y_t + \Phi z_t + \Xi(z_t) w_{t+1}$

where $\{z_t\}$ is a finite state Markov chain with transition matrix P and $\{w_{t+1} : t = 0, ...\}$ is a sequence of multivariate standard normally distributed random vectors.

- $\{z_t\}$: regime shifts in means and volatility.
- $\{e_t\}$: observed by an econometrician.

Example: Markov paths

State evolution: $y_{t+1} - y_t = \Lambda y_t + \Theta z_t + \Gamma(z_t) w_{t+1}$.

Signal evolution: $e_{t+1} = \Phi z_t + \Psi y_t + \Xi(z_t) w_{t+1}$.

Markov path:

$$A_t = \sum_{\tau=1}^{t} u' e_\tau = \sum_{\tau=1}^{t} u' \Phi z_{\tau-1} + u' \Psi y_{\tau-1} + u' \Xi(z_{\tau-1}) w_\tau .$$

Exponentiate. $G_t = \exp(A_t)$ stochastic growth process and $S_t = \exp(A_t^*)$ for stochastic discount factor process.

Valuation

Build stochastic discount factor process S_t by exponentiating a Markov path: $S_t = \exp(A_t^*)$ and build a stochastic growth process G_t similarly. Then

$$S_t G_t = \exp(A_t + A_t^*)$$

Compute values recursively via:

$$f_1(x) = E\left[S_1 G_1 f(x_1) | x_0 = x\right] = \mathbb{V} f(x).$$

$$f_t(x) = E\left[S_t G_t f(x_t) | x_0 = x\right] = \mathbb{V}^t f(x)$$

where f_t is the value as a function of the Markov state of the cash flow or consumption payoff $D_t = G_t f(x_t)$.

Intertemporal risk-return relation

expected return $\frac{E[G_t|x_0]}{E[S_t G_t|x_0]}$

riskfree return $\frac{1}{E[S_t|x_0]}$

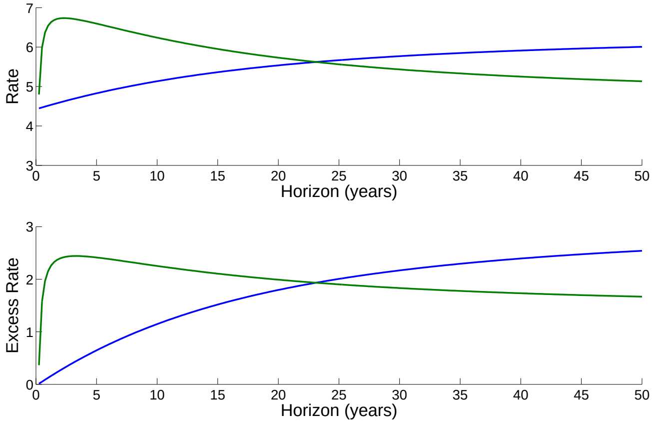
Risk premium:

$$\underbrace{\frac{1}{t}\log E\left[G_t|x_0\right]}_{\uparrow \text{ expected growth}} - \underbrace{\frac{1}{t}\log E\left[S_t G_t|x_0\right]}_{\uparrow \text{ price}} + \underbrace{\frac{1}{t}\log E\left[S_t|x_0\right]}_{\uparrow\ -\text{ riskfree rate}}$$

first two terms - logarithm of the expected rate of return
all three terms - logarithm expected excess return

To examine transient adjustment to cash flows, replace G_t by $D_t = D_0 G_t f(x_t)$ for positive f.

Risk premia for aggregate cash flows



Corporate Earnings
Aggregate Dividends

Questions

- How much of the dynamics is coming from predictability in cash flows and how much from the dynamics of risk prices?

- When does changing the econometric specification have important consequences in valuation problems?

These questions require an explicit economic model to answer.

Multiplicative martingales

Construction:

$$M_t = \exp\left[\sum_{\tau=1}^{t} \kappa_m(x_j, x_{j-1})\right].$$

where

$$E\left(\exp\left[\kappa_m(x_t, x_{t-1})\right] | x_{t-1}\right) = 1.$$

Observations:

- Constructed from shocks, normal shocks and surprise movements discrete state
- Best forecast is current value
- Does not converge - shocks have permanent consequences

Three uses for multiplicative martingales

1. Change probability measures - construct a likelihood ratio.
2. Decompose cash flows and stochastic discount factors.
3. Represent benchmark cash flows for characterizing risk prices.

1. Martingales and changes in probability

Let ϕ_t be a random variable in the date t information set $\mathcal{F}_t$.

Construct the corresponding conditional expectation operator:

$$\tilde{E}\left(\phi_t | x_0\right) = E\left(M_t \phi_t | x_0\right).$$

Martingale imposes the Law of Iterated Expectations for the alternative probability measure. M_t is the likelihood ratio of an alternative model with respect to an initial model.

Impact:

- Changes the conditional distribution of w_{t+1} from a standard normal to a normal with a state dependent mean.
- Change the discrete-state distribution of z_{t+1} conditioned on z_t.

2. Multiplicative martingale decomposition

Follow Hansen-Scheinkman

Solve the following equation for any t:

$$E\left[\exp(A_t)f^*(x_t)|x_0 = x\right] = \exp(\eta t)f^*(x)$$

where $f^* > 0$. (f^* is the "dominant" eigenfunction. η is real.)

Construct

$$M_t = \exp(-\eta t)\exp(A_t)\frac{f^*(x_t)}{f^*(x_0)}.$$

M_t is a martingale.

Martingale decomposition continued

$$\begin{array}{cccccc} \exp(A_t) & = & \exp(\eta t) & M_t & \frac{g(x_t)}{g(x_0)} \\ & & \text{dominant} & \text{dominant} & \text{transient} \\ & & \text{eigenvalue} & \text{eigenfunction} & \text{component} \end{array}$$

where $g = 1/f^*$ and

- η is a growth (or discount rate);
- $\{M_t\}$ is a (multiplicative) martingale that implies a change of probability measure used for approximation.

Distinct from martingale decompositions of the Markov path $\{A_t\}$.

Valuing martingale cash flows

Recall:

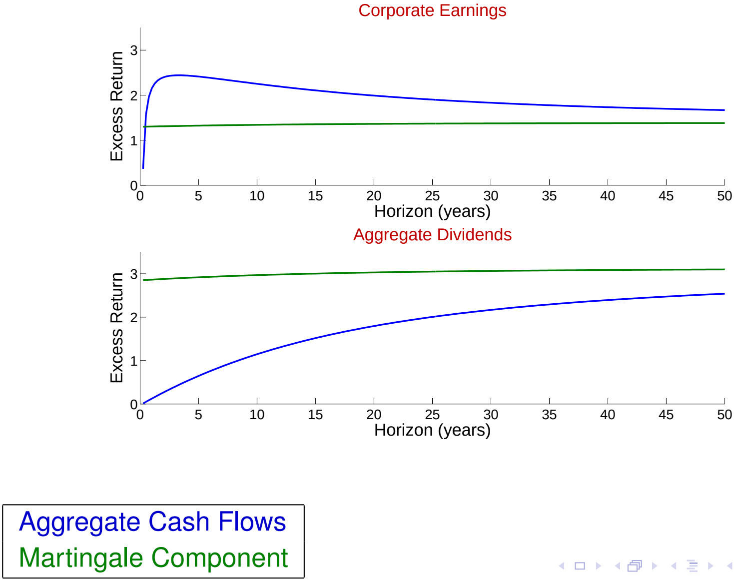
$$\frac{1}{t}\log E\left[G_t | x_0\right] \quad - \quad \frac{1}{t}\log E\left[S_t G_t | x_0\right] \quad + \quad \frac{1}{t}\log E\left[S_t | x_0\right]$$

$$\underset{\textit{expected growth}}{\uparrow} \qquad \underset{\textit{price}}{\uparrow} \qquad \underset{-\ \textit{riskfree rate}}{\uparrow}$$

Logarithm expected excess return for horizon t.

Replace G_t by its martingale component. Make the first contribution zero for all horizons.

Risk premia for martingale components



Recursive utility, distorted beliefs and martingales

$$V_t = (C_t)^{1-\beta}[\mathrm{R}(V_{t+1}|\mathcal{F}_t)]^{\beta}$$

where C_t is consumption, V_t is the continuation value and

$$\mathrm{R}(V_{t+1}|\mathcal{F}_t) = \left(E\left[(V_{t+1})^{1-\gamma}|\mathcal{F}_t\right]\right)^{\frac{1}{1-\gamma}}$$

$$S_t = \beta^t M_t^* \left(\frac{C_t}{C_0}\right)^{-1}$$

where $\{M_t^*\}$ is a martingale.

Recursive utility, distorted beliefs and martingales

$$S_t = \beta^t M_t^* \left(\frac{C_t}{C_0}\right)^{-1}$$

where $\{M_t^*\}$ is a martingale.

Observations:

- M_t^* constructed from continuation values and depends on investor perceptions of the future and risk aversion;
- alternatively M_t^* captures investor concern about model misspecification through distorting the probability measure;

Dual role of the long run

- Long-run growth prospects affects valuation of durable assets

- Beliefs about the long run affect short-run pricing

Empirical example

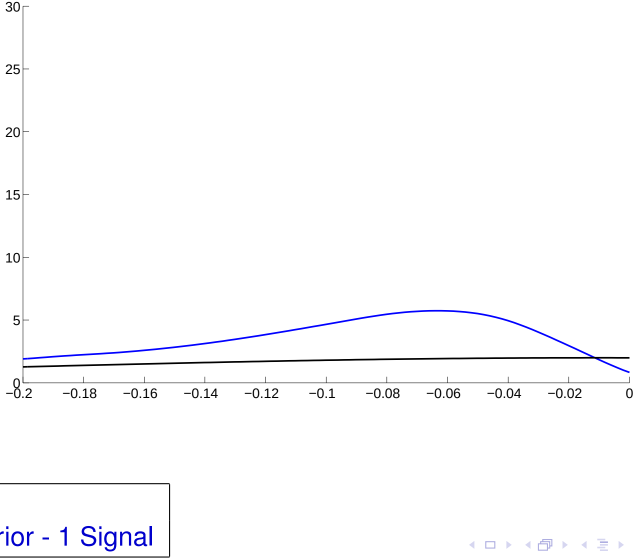
$$y_{t+1} - y_t = \lambda y_t + \begin{bmatrix} \nu & 0 & 0 \end{bmatrix} w_{t+1}$$

$$\begin{bmatrix} c_{t+1} - c_t \\ h_{t+1} - c_{t+1} \end{bmatrix} = \phi + \begin{bmatrix} 1 \\ \psi \end{bmatrix} y_t + \begin{bmatrix} 0 & \xi_{12} \cdot z_t & 0 \\ 0 & \xi_{22} \cdot z_t & \xi_{23} \end{bmatrix} w_{t+1}$$

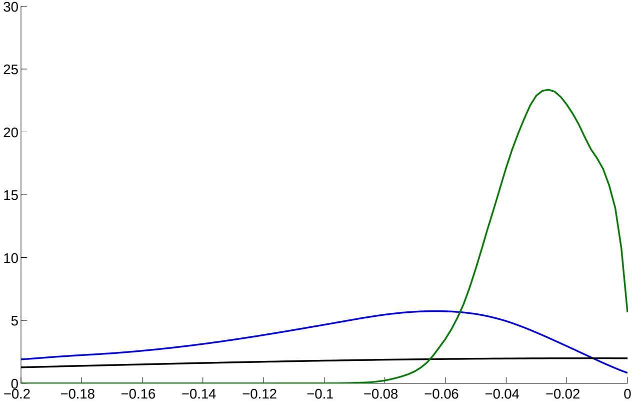
Observations:

- Three shocks: consumption growth, consumption and corporate earnings.
- Two very persistent consumption volatility states.
- Corporate earnings h_t relative to consumption c_t is an additional signal.
- Sometimes include aggregate dividend growth rate with a fourth shock and additional volatility states.

AR parameter estimator: statistical uncertainty

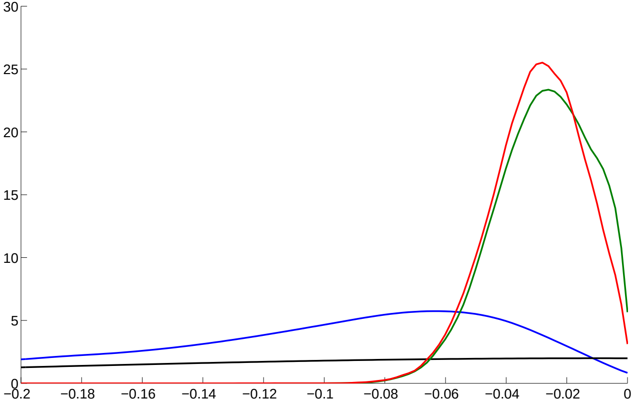


AR parameter estimator: statistical uncertainty



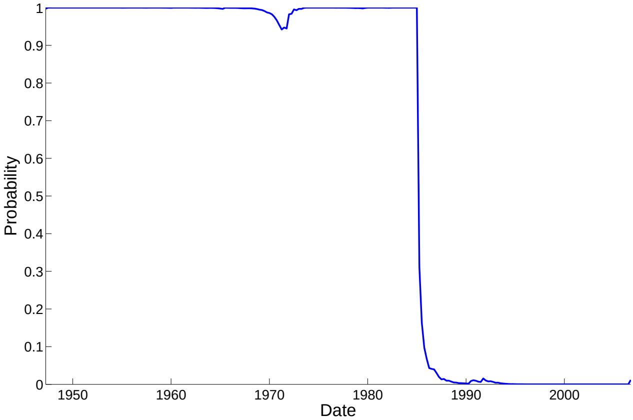
Prior	Posterior - 2 Signals
Posterior - 1 Signal	

AR parameter estimator: statistical uncertainty



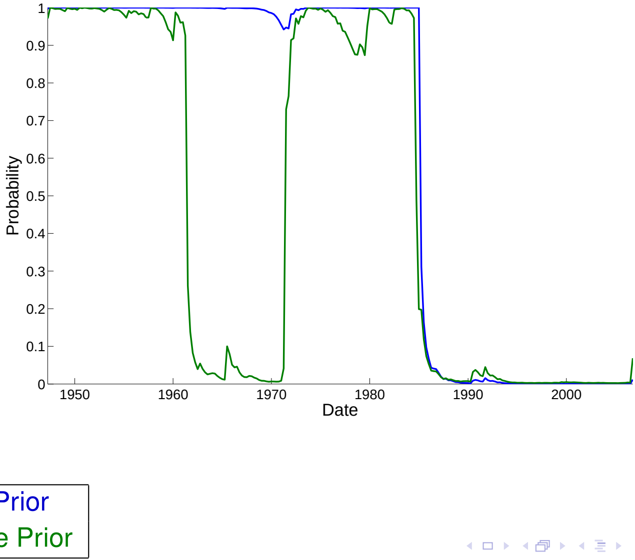
Prior	Posterior - 2 Signals
Posterior - 1 Signal	Posterior - 3 Signals

Volatility regime probabilities



Tight Prior

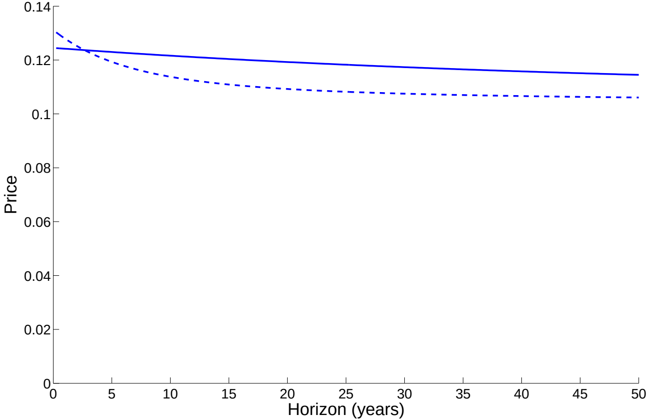
Volatility regime probabilities



Constructing risk prices

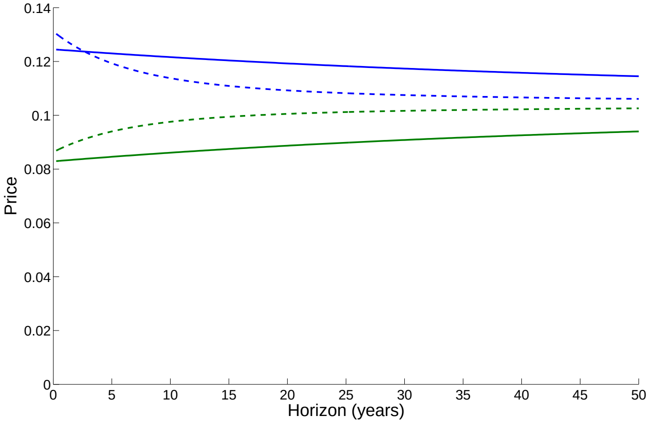
- Construct alternative martingale cash flows. Use martingale decomposition to motivate this construction. Strip away transient cash flow dynamics.
- Differentiate with respect to the risk exposure. Preserves martingale structure.

Consumption shock prices



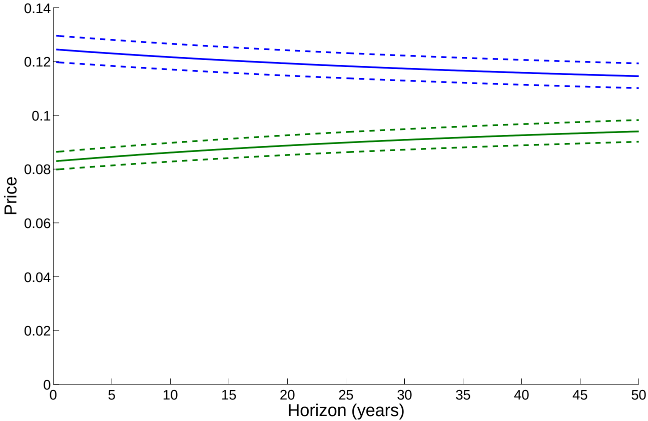
High Volatility Regime
- - Alt. Transition Prob.

Consumption shock prices



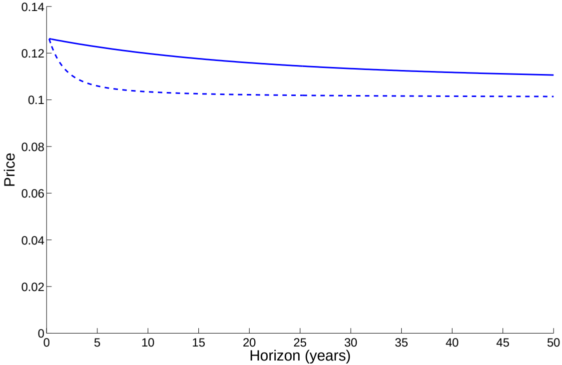
High Volatility Regime	Low Volatility Regime
- - Alt. Transition Prob.	- - Alt. Transition Prob.

Consumption shock prices: statistical uncertainty



High Volatility Regime	Low Volatility Regime
- - 25th,75th Quantiles	- - 25th,75th Quantiles

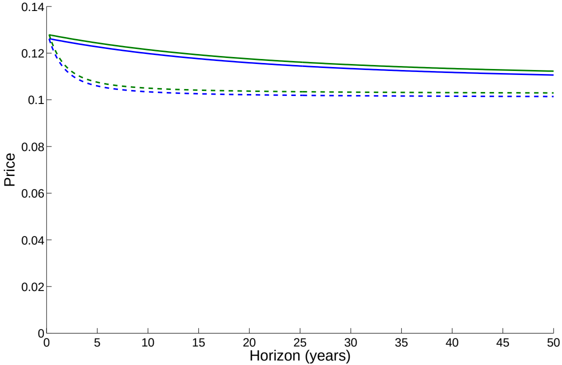
Volatility state prices



High Volatility Regime

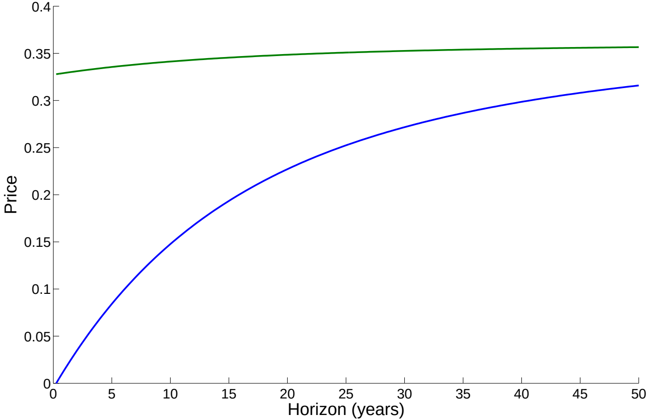
- - Alternative Transition Probability

Volatility state prices



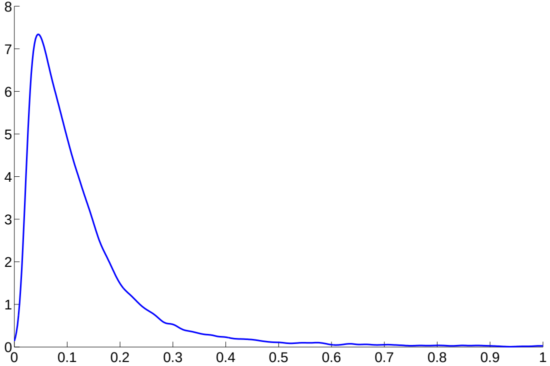
High Volatility Regime	Low Volatility Regime
- - Alternative Transition Probability	- - Alternative Transition Probability

Consumption growth shock prices



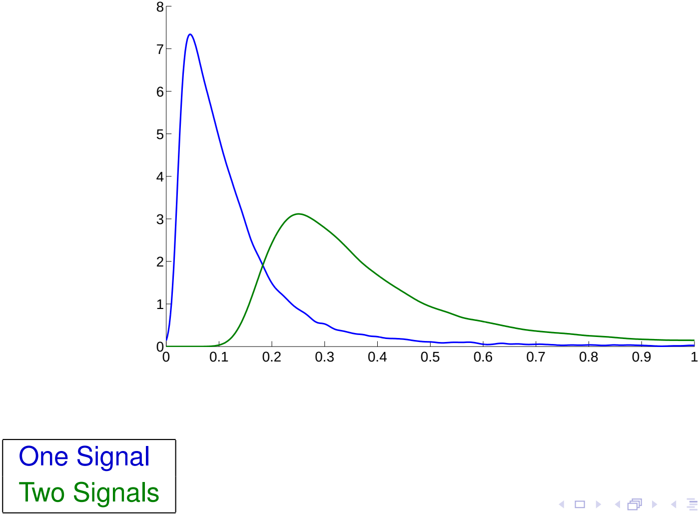
Recursive Utility Model
Expected Utility Model

Consumption growth shock prices: statistical uncertainty

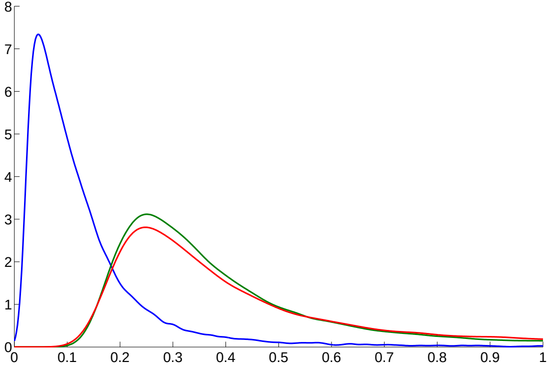


One Signal

Consumption growth shock prices: statistical uncertainty



Consumption growth shock prices: statistical uncertainty



One Signal	Three Signals
Two Signals	

Discussion

- Weak sample evidence (along some dimensions) does not remove interest in the calculations
  - Identify where economic inputs can have a big impact.
  - Suggest that ambiguity or fragility in the beliefs of investors who share some the same statistical challenges as econometricians will have important consequences on equilibrium valuation.
- Corresponding set of challenges for modeling and measurement of risk exposure.
- My focus has been on the low frequency component to macroeconomic volatility in contrast to a now extensive literature on high frequency financial volatility. Intriguing challenges in connecting these literatures.

Conclusions

I have presented tools for explicating the value implications of dynamic economic models. My specific aim is to understand better the role of statistical and economic inputs and the sensitivity to those inputs.