# Career Uncertainty and Dynamic Incentives

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**Abstract**

Career concerns are known to provide incentives even when performance-based compensation is not feasible. Previous literature assumes ability equally valuable everywhere. When ability is career-specific, individuals can escape bad reputation by changing their careers. The possibility of changing the career makes collection of reputational rewards less likely and therefore dampens incentives. However, I show that the wage becomes more sensitive to the reputation since the market anticipates the workers with good career matches to exert more effort. This effect countervails the direct incentive-weakening effect of career uncertainty. In fact, it may be so strong that the expected marginal return on the reputation increases and the worker who is less certain about her career prospects puts in more effort as a result. I show that equilibrium effort is higher for workers facing moderate career uncertainty if their effort is sufficiently responsive to incentives. In general, both oversupply and undersupply of effort can occur in the equilibrium. One way to control the strength of reputational incentives is to manipulate the timing of information release: delaying the release of performance data weakens the incentives and can help avoiding excessive effort supply early in the career.

## 1 Introduction

Reputational considerations motivate individuals to supply effort even in the absence of explicit contractual links between pay and performance. The implicit link arises as the market learns about the worker's productivity through the history of performance and competitive forces adjust the wage as the worker's expected productivity evolves. Hence future wages respond to current performance even though the current wage does not. Such implicit incentives are especially

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strong in the beginning of the worker's career when performance information is most scarce and sensitivity of the market's perception to new incoming information is the highest. However, it is also in the early years on the labor market when the workers are the least committed to their careers. Poor performance may be a signal of a bad career match, and often workers interpret it so and respond by moving to other industries and starting new careers. Neal (1999) documents that more than half of young men move across sectors when they change their jobs. It has also been documented that the likelihood of career change decreases with the career tenure.[1] The natural explanation to these phenomena is that a significant component of the worker's ability is industry-specific and is revealed to the worker gradually over time.

The possibility to change careers effectively protects the worker from the risk of bad productivity realizations. This could potentially weaken incentives to invest in a career by building up a good reputation. Indeed, the previously acquired reputation becomes less informative of the worker's potential upon a career change. Therefore the worker who believes there is a significant chance of changing the career in future must be less concerned of establishing a good reputation for the current career and less compelled to put in effort for that purpose. However, it turns out that the effect of career uncertainty on the worker's incentives is not so straightforward. As I argue below, if effort is sufficiently responsive to incentives, moderate career uncertainty actually makes reputational incentives stronger because wages become more sensitive to the worker's ability.[2]

I analyze the decision problem of a worker who seeks to establish a reputation in a competitive industry where firms gradually learn about he worker's productivity though realizations. I depart from the setup of Holmstrom (1999) by assuming that productivity is industry-specific, and the worker has an option to leave the industry and collect a fixed outside option. In the context of career search the fixed outside option can be interpreted either as an alternative career with fixed productivity or, as more common in search and matching models, it can be viewed as value of further search that I do not model explicitly. The outside option can also be thought of as the value of leisure, which gives the model an important interpretation different from career seach: the worker decides whether to participate in the labor force or not. The assumption of constant outside option makes reputation valuable only to those workers who stay in the industry. The higher is the worker's posterior expectation of talent, the lower is the chance to leave the career. This generates the first key distinction from the original model of Holmstrom (1982): incentives

[1] See Mergenhagen (1991), Neal (1999).

[2] Career uncertainty is modelled in this paper through the worker's outside option. Workers whose expected gains in a current career are close to their outside options are more likely to change careers in future than those whose expected gains are well above the outside options. An alternative way to think about career uncertainty is through the variance of the quality of career match. I discuss some comparative static results with respect to this parameter too.

depend not only on the worker's tenure but also on the history of performance.[3] The history of good performance suggests that the career match is good and the probability of changing the career is low. This makes reputation in the current career more valuable and induces the worker to invest in reputation more by exerting more effort. Competitive firms anticipate the worker with higher perceived ability to work harder and offer a higher wage to compensate not only for higher ability but also for higher effort. Therefore, the wage increases in perceived ability at a slope greater than one due to variation in effort across workers of different talent. The increased wage sensitivity to perceived talent strengthens the worker's reputational incentives and countervails the direct incentive weakening career uncertainty effect. The overall strength of reputational incentives in fact can increase due to career uncertainty if effort is sufficiently sensitive to incentives.

Variation of effort over time across workers of different talent in career concerns models is clearly inefficient. In fact, inefficiencies can arise due to both oversupply and under supply of effort. One way to reduce variability in effort would be to complement reputational incentives with explicit performance pay, as in Gibbons and Murphy (1992). The alternative route that I explore in this paper is changing the time of information release. I show that the strength of reputational incentives depends on timing of information release, and that even in the Holmstrom's benchmark model it is not optimal to release performance data evenly over time.

In the context of career search, timing of information release plays another important role – it determines the speed of matching of workers to careers. If this consideration was the only one, competitive firms would always release performance data as soon as it becomes available, thus getting rid of the workers who are overpaid relative to their productivity, and compensating the highly productive workers fairly. This is no longer true when career concerns are taken into account. Delaying the performance data release may be welfare-improving since it helps to reduce the excessive strength of incentives early in the career, and also defers the moment when effort drops to suboptimal level. I provide analytical and computational results on how optimal timing varies across the workers with different outside options. In general, it is more costly for workers with higher outside option to stay in the industry and therefore they value early feedback more. However, since effort is non-monotonic in the outside option, neither is the optimal time of information release. A familiar example of a situation where most of these issues arise is the tenure decision for an assistant professor. There are typically two evaluations before the tenure decision is made, and effort is an important determinant of the candidate's success. The implication of the model is that universities with higher turnover of junior faculty

[3] Holmstrom (1999) assumes that effort is constant across types conditional on tenure solves for an equilibrium in which this is true. Although he does not argue that this equilibrium is unique, Martinez (2006) proves it in a model with finite horizon.

should evaluate sooner.

There are two strands in the literature this paper is related to. The literature on job matching with gradual learning about the match productivity stems from Jovanovic (1979). The focus of this literature is the job turnover and gradual sorting of workers through jobs. The key predictions of Jovanovic (1979) are that wages grow with tenure and frequency of separation of workers with firms decline with tenure. A more recent work by Neal (1999) distinguishes career search from job search within the career. Neal (1999) argues that a typical pattern of job search can be separated in two phases: first the worker sorts through careers and then, once a good career match is found, the search is focused on jobs within the career. He also finds empirical support for this result. The primary focus of the job-matching literature is learning about the worker's ability, and the incentives aspect is typically not explored. This paper focuses on the reputational incentives of a worker who sorts through careers, and so complements the job-matching literature.

The career concerns literature started by Fama (1980) and Holmstrom (1999) focuses exclusively on the incentives of a worker whose productivity is gradually revealed through the history of performance in a competitive labor market. The key results of Holmstrom (1999) are that the strength of reputational incentives declines with tenure, unless productivity evolves over time, and that the strength of incentives depends on tenure only, and not on the specific history of the worker's performance. Martinez (2006, 2007) analyzes models with career concerns where the payoffs that are not linear in the worker's ability: the slope can vary depending on a task assigned and there can be discrete jumps in wages asociated with promotions. While the obvious aspect that distinguishes this paper from the rest of career concern literature is career uncertainty, there is another key distinction that is important. In this paper I argue that the link between market's expectations of the worker's ability and effort is an important determinant of the strength of reputatoinal incentives, and that responsiveness of wages to the worker's perceived ability may vary depending on the equilibrium effort variation across workers of different talent. This is what drives the counterintuitive result about the increasing strength of incentives due to career uncertainty.

The rest of the paper is organized as follows. In Section 2, I set up the model and characterize the equilibrium behavior of the worker. In Section 3, I show that incentives can be stronger due to career uncertainty and present some computational comparative statics. In Section 4, I present some analytical and computational results on timing of information release. In Section 5, I summarize main findings and discuss promising extensions.

## 2 The Model

Consider a worker who starts a career in an industry with competitive labor market. There are three periods. At the beginning of each period the worker has the choice either to stay in the industry and receive a wage $w_t$, or to quit and collect a fixed monetary payoff $u$ per period. The decision to quit is irreversible: the worker who quits in period $t$ collects payoff $u$ in all subsequent periods.[4] The worker who continues the career must also decide in each period how much effort to put in. The choice of effort is not observed by the market. The disutility of effort, $C(a_t)$, is an increasing and convex function satisfying the following conditions: $C'(0) = 0$, $\lim_{a_t \to \infty} C'(a_t) = \infty$. Thus the worker's utility it period $t$ is

$$v_t = \begin{cases} w_t - C(a_t), & \text{if continues the career,} \\ u, & \text{if quits the career in period } t \text{ or before.} \end{cases}$$

The worker's industry-specific talent $\eta$ is symmetrically unknown but its distribution is known to be normal with mean $\eta_1$ and precision (inverse of the variance) $h_1$. The output of the worker is a sum of three components, $y_t = \eta + a_t + \varepsilon_t$, the worker's unknown ability $\eta$, her privately chosen effort $a_t$, and a noise component $\varepsilon_t$ whose distribution is normal with mean zero and precision $h_\varepsilon$: Output is symmetrically observed by the market and by the worker, but is non-contractible.

The industry is competitive and the worker receives the wage equal to her expected output, $w_t = \eta_t^* + a_t^*$. The "starred" variables, $\eta_t^*$ and $a_t^*$, correspond to the market's expectation in period $t$ of the worker's talent and effort respectively. It must be noted that the term "expectation" is not used here in the statistical sense. Since the worker's effort is not observed, inferring the worker's talent from the history of output observations is not a purely statistical exercise. It entails some guesswork about the worker's strategy, and therefore requires understanding of the worker's incentives.

If the worker's strategy is guessed correctly, the standard linear filter can be used to update the market's expectation of the worker's talent upon realization of the output:

$$\eta_t^* = \frac{h_1}{h_t}\eta_1^* + \frac{h_\varepsilon}{h_t} \sum_{\tau=1}^{t-1} (y_\tau - a_\tau^*), \quad t = 2, 3$$

where $\eta_1^* = \eta_1$, and $h_t$ is the precision of the posterior beliefs in period $t$, that evolves deterministically according to the rule $h_t = h_1 + (1 - t)h_\varepsilon$. The worker's own beliefs, $\eta_t$, evolve in the same manner but are based on the actual actions, $a_\tau$. To simplify notation, let $r_t$ denote the

[4] Irreversibility of the quitting decision is assumed only to simplify the analysis. It can be shown that the worker will never choose to go back even if allowed.

sensitivity of the posterior in period $t$ to the performance in the previous period, $r_t = h_\varepsilon / h_t$. In this notation the updating rule can be written as follows

$$\eta_t^* = (1 - r_t)\,\eta_t^* + r_t\left(y_{t-1} - a_{t-1}^*\right), \tag{1}$$

$$\eta_t = (1 - r_t)\,\eta_t + r_t\left(y_{t-1} - a_{t-1}\right), \quad t = 2, 3. \tag{2}$$

The last component of the worker's strategy is the quitting rule, that specifies the level of career-specific talent $\underline{\eta}_t(\eta_t^*)$ at which the worker is indifferent between staying in the industry and quitting the career, given the market's expectation $\eta_t^*$. It is straightforward to verify that workers with $\eta_t > \underline{\eta}_t(\eta_t^*)$ will prefer to stay in the industry, and workers with $\eta_t < \underline{\eta}_t(\eta_t^*)$ will choose to quit. The competitive equilibrium can now be defined as follows.

**Definition 1** *The competitive equilibrium is a sequence of wages* $\{w_t(\eta_t^*)\}_{t=1}^3$*,market's expectations,* $\left\{\eta_t^*(y^t), a_t^*(\eta_t^*)\right\}_{t=1}^3$*, the worker's expectations,*$\left\{\eta_t(y^t)\right\}_{t=1}^3$ *, and the worker's strategy* $\{\underline{\eta}_t(\eta_t^*), a_t(\eta_t, \eta_t^*)\}_{t=1}^3$ *that satisfy the following conditions:*

1. *The worker's strategy* $\{\underline{\eta}_t(\eta_t^*), a_t(\eta_t, \eta_t^*)\}_{t=1}^3$ *maximizes the worker's utility given* $w_t(\eta_t^*)$*,* $\left\{\eta_t(y^t)\right\}_{t=1}^3$ *and* $\left\{\eta_t^*(y^t), a_t^*(\eta_t^*)\right\}_{t=1}^3$*.*
2. *The wage equal's the worker's expected output,* $w_t(\eta_t^*) = \eta_t^* + a_t^*(\eta_t^*)$*.*
3. *The market expectation of the worker's effort coincides with the worker's strategy,* $a_t^*(\eta_t^*) = a_t(\eta_t, \eta_t^*)|_{\eta_t = \eta_t^*}$*.*
4. *The expectations are formed according to the Bayes rule, (1) and (2), with* $\eta_1^* = \eta_1$*.*

One technical remark about the above definition is that $a_t(\eta_t, \eta_t^*)$ must maximize the worker's utility **conditional** on staying in the industry **for all** $\eta_t$ and $\eta_t^*$, including the off-equilibrium case $\eta_t < \underline{\eta}_t(\eta_t^*)$, i.e., when a worker who was supposed to change careers in the beginning of the period decides to stay in the industry. This is necessary so that the market's expectations are well defined in condition 3. Also, the above definition requires that the worker does not randomize in equilibrium. The difficulty associated with mixed strategies is that they create information asymmetry: the worker knows exactly what effort level has been chosen in each period, while the market cannot infer it from the history of performance. This significantly complicates the analysis without apparent gain in terms of economic insights. Specific assumptions that guarantee that the worker's problem has a unique solution in each period will be discussed below.

One straightforward consequence of equilibrium conditions 3 and 4 is that the expectations of the worker's ability must be the same for the market and the worker.

**Claim 2** *In equilibrium* $\eta^*_t(y^t) = \eta_t(y^t)$ *for all* $t$.

The equilibrium behavior of the worker can be described using backward induction.

**Period Three:** The worker has no incentives to exert effort in the third period: the current wage is already fixed and future reputation does not matter because the third period is terminal. Zero effort implies that the wage in the third period is equal to the worker's expected talent, $w_3 = \eta^*_3$. Also, the decision of the worker about whether to quit or stay is straightforward in this period. The worker stays if $w_3 \geq u$ and quits otherwise. Let $V_3(\eta_3, \eta^*_3)$ denote the worker's value in the third period, so that

$$V_3(\eta_3, \eta^*_3) = \max\{\eta^*_3, u\}.$$

**Period Two:** In the second period the worker has two non-trivial choices: he must choose his effort and decide whether to continue with the career. The worker's Bellman equation is

$$V_2(\eta_2, \eta^*_2) = \max\left\{2u, \max_{a_2} w_2 - C(a_2) + E\left[V_3(\eta_3, \eta^*_3)\right]\right\}. \quad (3)$$

The first-order condition of this problem with respect to effort is

$$C'(a_2) = r_3\left[1 - F_{23}(u + \Delta\eta_3|\eta_2)\right], \quad (4)$$

where $F_{23}(.|\eta_2)$ is normal c.d.f. with mean $\eta_2$ and precision $h_2 h_3/h_\varepsilon$, and $\Delta\eta_3 = \eta^*_3 - \eta_3$.[5] Note that while both $\eta^*_3$ and $\eta_3$ are random variables from the period two perspective, $\Delta\eta_3$ is not. Indeed, (1) and (2) imply that $\Delta\eta_3 = r_3(\Delta a_1 + \Delta a_2)$, where $\Delta a_t = a_t - a^*_t$.

It is evident from the first-order condition that the workers with higher expected ability exert more effort in the second period, since $F_{23}(u|\eta_2)$ is decreasing in $\eta_2$. Intuitively, since ability is industry-specific, the workers with high $\eta_2$ are more likely to stay in the industry in the future and therefore are more likely to benefit from the effect of their effort on future reputation.

Applying the implicit function theorem to the first-order condition yields

$$\frac{\partial a_2(\eta_2, \eta^*_2)}{\partial \eta_2} = \frac{r_3 f_{23}(u + \Delta\eta_3|\eta_2)}{C''(a_2)}, \quad (5)$$

where $f_{23}(u|\eta_2)$ is normal p.d.f. with mean $\eta_2$ and precision $h_2 h_3/h_\varepsilon$. This result will be essential for understanding the worker's incentives in the first period. Equilibrium condition 3 implies that the market should expect better workers to exert more effort in the second period,

[5] See the appendix for details on the derivation of the first-order condition.

and this should be reflected in the wages offered to workers of different talent. It is useful to contrast this result with the findings of Holmstrom (1982). In the limiting case as $u \to -\infty$, the first-order condition (4) takes the form $C'(a_2) = r_3$. This implies that regardless of their talent all workers exert the same effort, and the only source of variation in the expected output among workers is their talent. This means, in particular, that $w_2'(\eta_2^*) = 1$ in the world without career uncertainty. When career uncertainty is introduced, the wage becomes more sensitive to the worker's talent, and this is summarized in the following claim.

**Claim 3** *In equilibrium,* $w_2'(\eta_2^*) = 1 + r_3 f_{23}(u|\eta_2^*)/C''(a_2) > 1$.

**Proof.** See Appendix. ■

Intuitively, the above claim means that due to career uncertainty marginal returns to reputation in the second period become higher. How significant this increase in the wage sensitivity is depends on how convex is the disutility of effort, as measured by the second derivative. If effort disutility is "not too convex", then variation in returns to effort leads to significant changes in the effort supplied. Therefore the wage, which is equal to the expected output, is sensitive to the changes in probability of staying in the industry.

From the perspective of the first period, higher wage sensitivity in the second period means that conditional on staying in the industry returns to effort are higher in period one. The overall strength of incentives also depends on how fast the probability of staying in the industry declines with $u$. To determine which effect is stronger one has to analyze the worker's problem in the first period. But before proceeding to the first period problem, it is useful to establish one more property of the period two wage. Let $\underline{\eta}_t^*$ denote the equilibrium level of expected talent of the worker who is indifferent between changing the career or staying in the industry in the beginning of period $t$. Then the following result must be true.

**Lemma 4** *In equilibrium,* $w_2(\underline{\eta}_2^*) < w_3(\underline{\eta}_3^*)$.

**Proof.** See Appendix. ■

The above lemma states that the worker is willing to tolerate a lower wage in period two than in period three. This is a version of a well-known result of Jovanovic (1979) on the job turnover: the worker becomes more demanding to a current wage as variance of future wages decreases and the chances of having a good wage draw in subsequent periods diminish. However, the above result is stronger: the reservation wage increases from period two to period three *despite the fact* that the worker is compensated for non-negative effort in period two, so the difference in reservation wages does not follow from $\underline{\eta}_t^*$ increasing over time. However, it is a straightforward corollary from this result that $\underline{\eta}_2^* < \underline{\eta}_3^* = u$, which will be useful in the subsequent analysis.

**Period One** The worker's Bellman equation in period one is

$$V_1(\eta_1) = \max\left\{3u, \max_{a_1} w_1 - C(a_1) + EV_2(\eta_2^*, \eta_2)\right\}.$$

After some manipulation[6], the first-order condition simplifies to

$$C'(a_1) = E\left(r_2 + r_2 r_3 f_{23}(u|\eta_2)/C''(a_2) + r_3\left[1 - F_{23}(u|\eta_2)\right] \middle| \eta_2 \geq \underline{\eta}_2^*\right)\left[1 - F_{12}(\underline{\eta}_2^*)\right], \qquad (6)$$

where $F_{12}()$ is a normal c.d.f. with mean $\eta_1$ and precision $h_1 h_2 / h_\varepsilon$. In the first period the choice of effort affects the worker's future payoffs through the following channels: (1) it increases perceived ability in period two, (2) it increases effort expected by the market in period two, and (3) it increases perceived ability in period three. These three channels correspond to the three components of the sum at the left-hand side of equation (6). In this limiting case as $u$ goes to $-\infty$ and career uncertainty disappears the first order condition simplifies to $C'(a_1) = r_2 + r_3$, the Holmstrom's benchmark. Only the first and the third components of the sum remain, and they both increase. It is only the disappearing second component that could make incentives weaker as career uncertainty vanishes. One parameter that determines the significance of this component and nothing else in the model is convexity of the disutility of effort function, $C''()$. If $C''()$ is small, then the effort varies a lot among workers of different ability, and the the wage is more sensitive to the worker's ability.

To determine from (6) whether it is possible that incentives become stronger as the outside option $u$ increases and career change becomes more likely, it is necessary to characterize the career changing rule, $\underline{\eta}_2^*$, as a function of $u$. This rule, $\underline{\eta}_2^*(u)$, must solve the following indifference condition

$$w(\underline{\eta}_2^*) - C(a_2(\underline{\eta}_2^*)) + E(\eta_3^* | \underline{\eta}_2^*, \eta_3^* \geq u) \Pr(\eta_3^* \geq u | \underline{\eta}_2^*) + u \Pr(\eta_3^* < u | \underline{\eta}_2^*) = 2u,$$

and there is no closed-form solution to this equation. However, there is a way to circumvent this problem which I describe in the following section.

# 3 Does Career Uncertainty Weaken Incentives?

While it is not feasible to evaluate the strength of incentives in the first period directly from the first-order condition, one can construct a lower bound in a way described below. The strategy will be to show that for some model parameters this lower bound will be greater than $r_2 + r_3$, the

[6] See appendix for details. To keep the expression simple the equilibrium conditions $\Delta\eta_2 = 0$ and $\Delta\eta_3 = 0$ are imposed.

solution for the case without the outside option. To keep derivation simple I will assume that the disutility of effort is a quadratic function, $C(a) = a^2/2c$ so that $C'(a) = a/c$, $C''(a) = 1/c$.[7] In this case the first-order condition (6) takes the following form:

$$a_1 = cE\left(r_2 + cr_2r_3f_{23}(u|\eta_2) + r_3\left[1 - F_{23}(u|\eta_2)\right] | \, \eta_2 \geq \underline{\eta}_2^*\right)\left[1 - F_{12}(\underline{\eta}_2^*)\right]. \tag{7}$$

**Proposition 5** *The equilibrium effort level $a_1$ can be bounded from below as follows:*

$$a_1 \geq c\left(r_2 + cr_2r_3f_{13}(u) + r_3\left[1 - F_{13}(u)\right]\right)\left[1 - F_{12}(u)\right], \tag{8}$$

*where $f_{13}(.)$ and $F_{13}(.)$ are normal p.d.f and c.d.f. respectively with mean $\eta_1$ and precision $h_{13} = h_1h_3/\left(2h_\varepsilon\right)$.*

**Proof.** In the Appendix. ■

The above proposition significantly simplifies the task of evaluating the strength of incentives in period one. In particular, if $u = \eta_1$, then (8) simplifies to $a_1 \geq c\left(2r_2 + 2cr_2r_3\sqrt{h_{13}/2\pi} + r_3\right)/4$, and it can be verified that the right-hand side exceeds $c(r_2 + r_3)$, the Holmstrom's benchmark, if $5h_1 + 7h_\varepsilon < c\sqrt{h_\varepsilon h_1\left(h_1 + 2h_\varepsilon\right)/\pi}$. Intuitively, this condition requires that $c$ must be sufficiently high, i.e., the effort must be sufficiently responsive to incentives. It is the variation in expected effort that increases wage sensitivity to perceived talent, and the higher is wage sensitivity, the stronger are incentives. Figure 1 depicts a comparative statics results: the right-hand side of (8) as a function of $u$. If $h_{13}$ is high enough, the equilibrium choice of effort is non-monotone in the outside option.

[7] The main purpose of this assumption is to simplify expositon. The results can be generalized for the case of an arbitrary convex function with second derivative bounded from above.

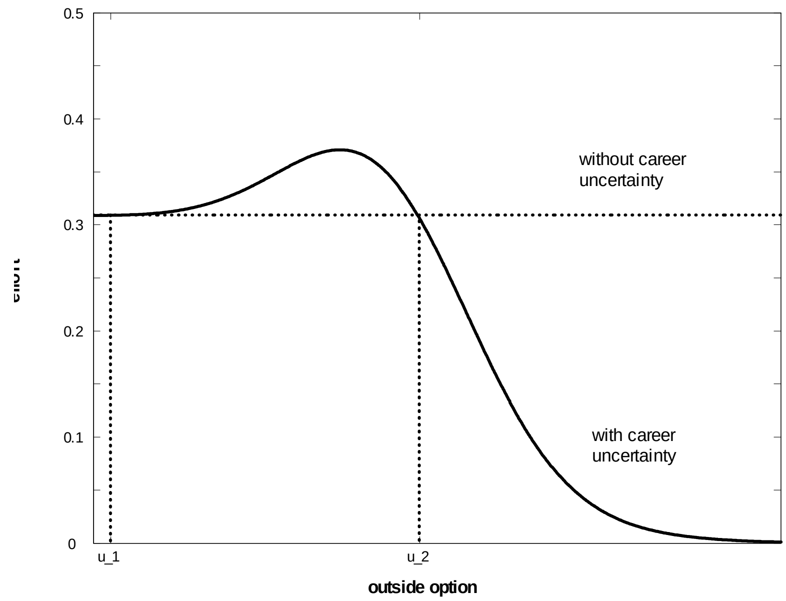



Figure 1: The choice of effort as a function of the outside option, a lower bound for $h_1 = 4$, $h_\varepsilon = 1$

As $u \to -\infty$ and the career uncertainty disappears, the equilibrium effort converges to the benchmark value given by $C'(a_1) = r_2 + r_3$, that corresponds a three-period version of the original model analyzed by Holmstrom (1999). In the interval $(u_1, u_2)$ the equilibrium effort exceeds the benchmark value. In this region career uncertainty strengthens the incentives to build up reputation. In terms of model parameters, this is more likely to occur when $h_1$ and $h_\varepsilon$ are high, i.e., when there is not too much talent heterogeneity in the population of workers and learning is relatively fast. As $u$ increases, the the probability to stay in the industry in the next period goes to zero, and so does the equilibrium effort.

### 3.1 More Comparative Statics

Below I present some computational comparative statics results. Figure 2 depicts how the equilibrium effort in period 1 changes depending on the precision of the signal. The more informative is the signal, the more sensitive is the next period wage to current performance. Therefore, the effort curve shifts up as $h_\varepsilon$ increases.

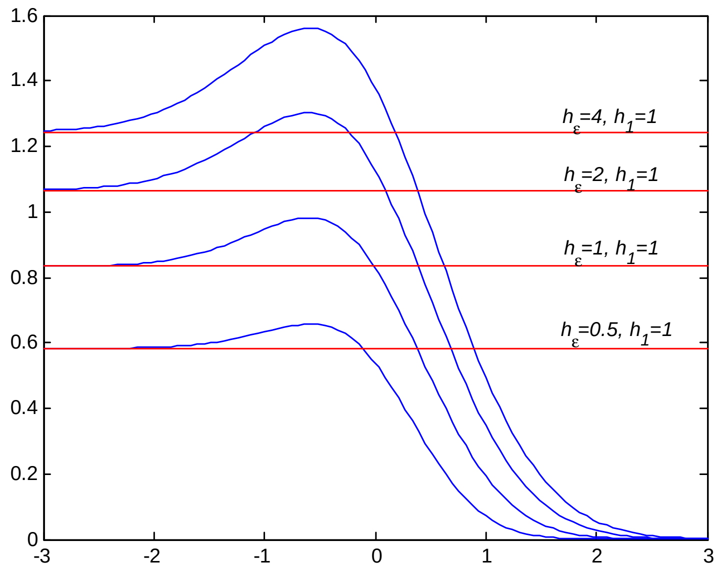



Figure 2: Effort increases as the quality of signal improves.

The variable on the vertical axis of figure is the marginal cost of effort, which in case of quadratic disutility of effort function is just rescaled effort. The efficient level of effort corresponds to $C'(a_1) = 1$, and it is evident from the above figure that the equilibrium effort may exceed this level if $h_\varepsilon$ is high enough. Excessive effort in the early years of a career is typical in career concerns models and is usually treated as inadvertent evil. In Section 4 I study how by manipulating the timing of information release firms can vary the strength of incentives and improve efficiency.

Figure 3 shows how the equilibrium effort changes depending on the precision of the prior beliefs about the worker's ability, $h_1$. The more dispersed is the prior, the less it matters, and the wider rage of workers is affected by the career uncertainty. While only upper and lower bounds on effort can be derived analytically (the dashed curves), the actual curve (solid line) can be obtained computationally.

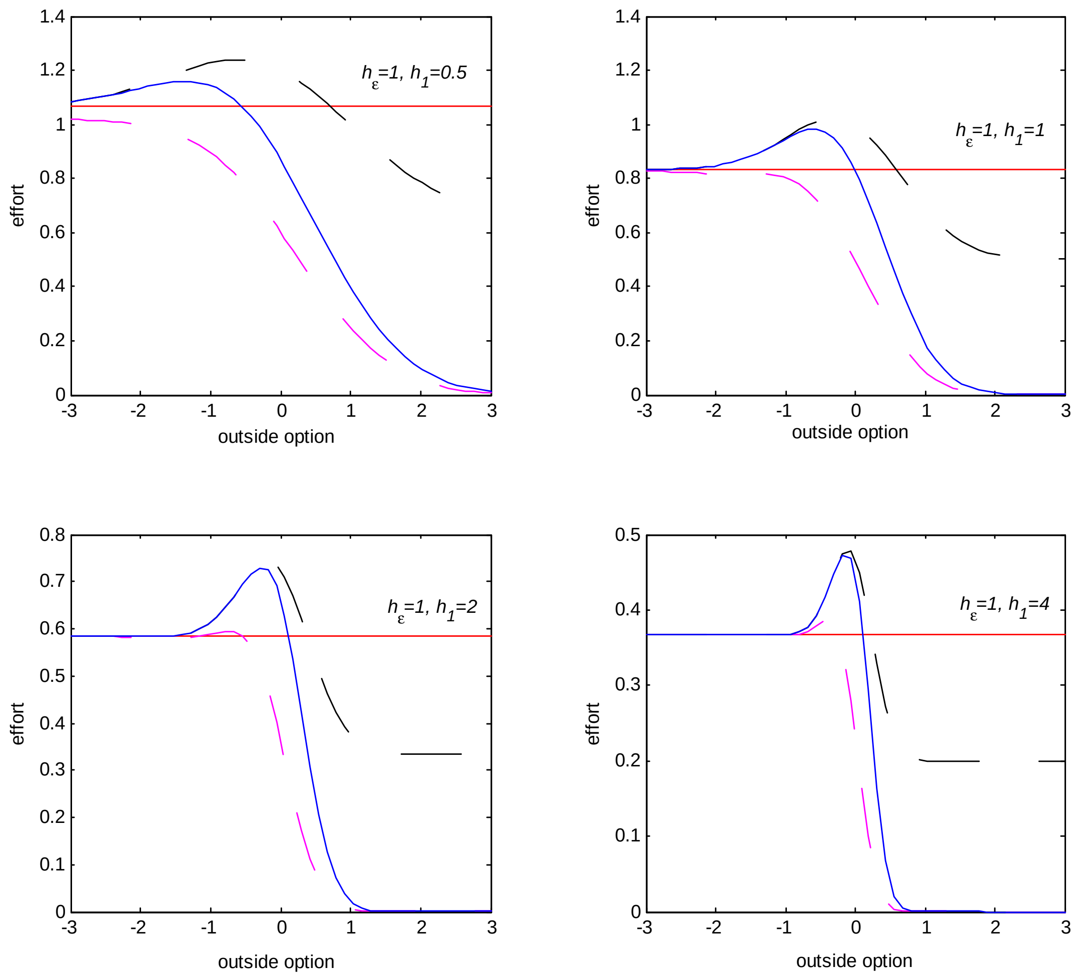
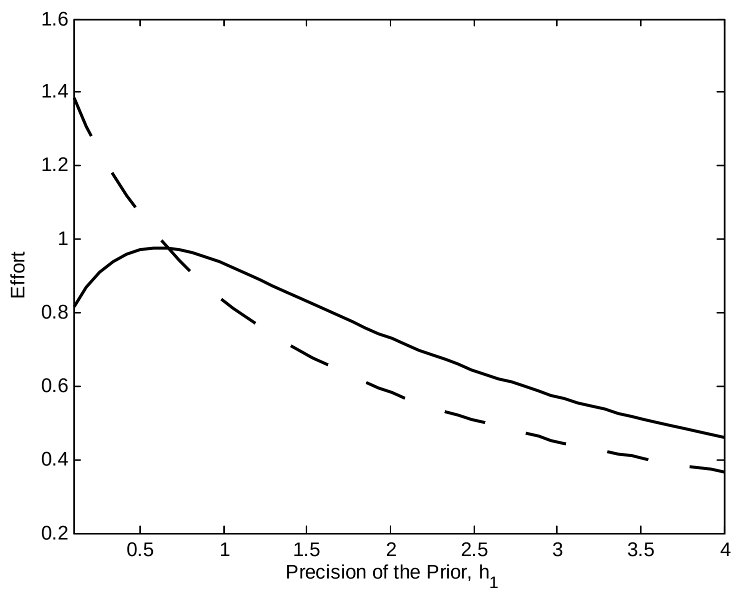



Figure 3: Equilibrium effort (blue solid), lower and upper bounds (magenta and black dashed lines respectively).

The more precise is the prior, the lower is the value of additional information, so the incentives become weaker on average as $h_1$ increases. However, the spike around $u = 0$ becomes more pronounced in relative terms. For all computational comparative statics $\eta_1 = 0$, and this is where incentives seem to be the strongest. By Claim 3 the wage is most sensitive to the worker's reputation when $f_{23}(u|\eta_2^*)$ is maximized, and this occurs at $u = \eta_2^*$. Since in equilibrium the expectation of $\eta_2^*$ is $\eta_1$, it is natural that incentives must be strong around $u = \eta_1$. In fact, the peak of $a_1$ is slightly to the left of zero because there is also a negative effect on incentives associated with decreasing probability to stay in the industry.

The comparative statics with respect to the precision of the prior beliefs about the worker's ability is especially important because $h_1$ can be thought of as an alternative measure of career uncertainty. The more precise is the prior, the more can be learned about career prospects just from comparing the prior expectation of the ability $\eta_1$ to the outside option. It is also natural to expect the incentives to become weaker as $h_1$ increases. Intuitively, the more precise in the prior belief about the worker's ability, the less sensitive is the posterior expectation, and hence the future wage, to the worker's performance. Below figure demonstrates that incentives indeed become weaker monotonically as $h_1$ increases in the world without career uncertainty. However, when the outside option is introduced, the strength of incentives does not monotonically decrease in $h_1$. The non-monotonicity arises because $h_1$ affects the wage sensitivity to the worker's ability. According to Claim 3, $w_2'(\eta_2^*) = 1 + r_3 f_{23}(u|\eta_2^*)/C''(a_2)$, where precision of $f_{23}$ is $h_2 h_3/h_\varepsilon$, an increasing function of $h_1$. When $h_1$ is close to zero, the incentive-strengthening effect due to increasing $w_2'(\eta_2^*)$ dominates, and the equilibrium effort increases as quality of the prior information improves.




Equilibrium effort as a function of $h_1$, precision of the prior. The solid line corresponds to the optimal effort under career uncertainty, and the dashed line corresponds to the Holmstrom's benchmark, the world with a single career. Parameter values: $u = 0$, $\eta_1 = 0$.

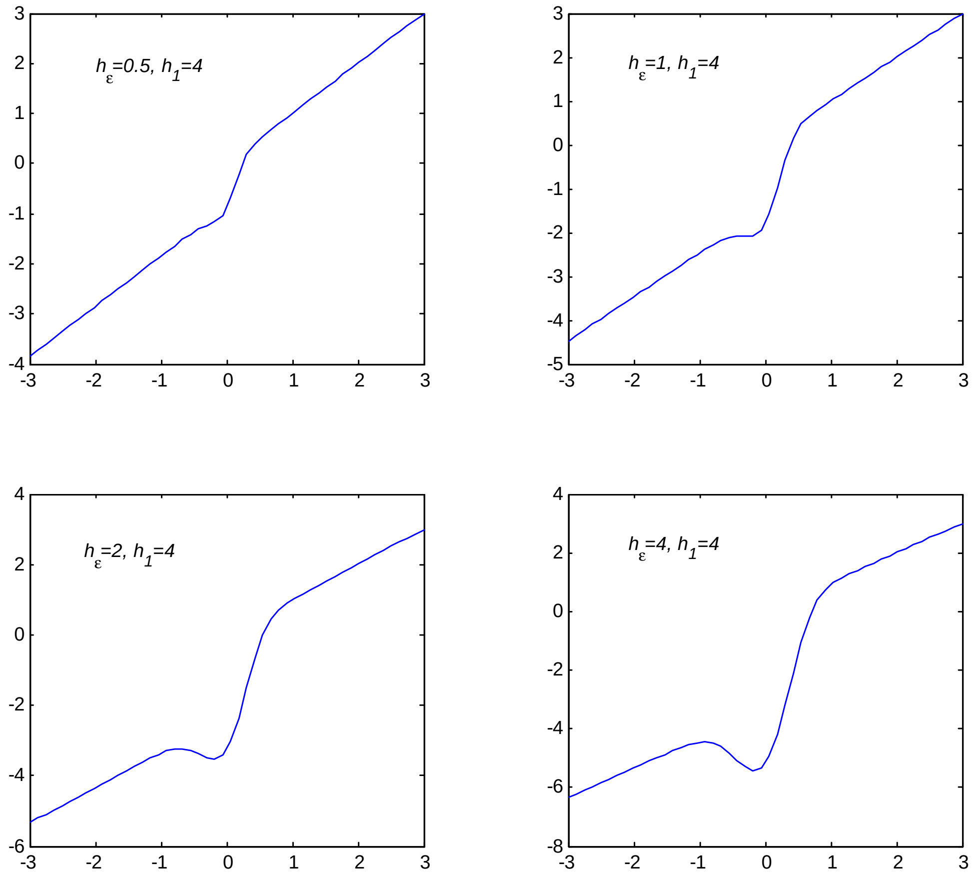



Figure 4: The worker's expected output in the first period as a function of expected ability, $\eta_1$.

These graphs depict the worker's expected output in period one as a function of expected ability. When the signal is sufficiently informative ($h_\varepsilon$ is high), the expected output is non-monotonic in ability. The decrease in expected effort is so sharp that it dominates the increase in productivity due to higher ability so that workers who are inherently more productive produce less. This local "overqualification" effect arises due to the decreasing expected marginal return on reputation that works through the next period wage.

## 4 Timing of evaluation

Typically the strength of reputational incentives declines over time as information accumulates and the worker's future perceived ability becomes less and less sensitive to his current performance. In fact, it is possible that for some time in the beginning of the career the worker puts in too much effort. In this section I show that it is possible to avoid the periods of excessive effort by manipulating the timing of information release. I assume that time is continuous $t \in [0, 3]$ and the worker's output follows a Brownian motion with drift $\eta + a(t)$, variance $1/h_\varepsilon$, so that the cumulative output $y(t)$ is normally distributed with mean $\eta t + \int_0^t a(\tau)d\tau$ and variance $t/h_\varepsilon$. Further I assume that the worker's performance is not observed continuously, but, just as in the previous section, there are two points in time when information is released, $t_1$ and $t_2$. It is important to emphasize that there is no informational asymmetry regarding the worker's output at any point. Neither the firm nor the worker has access to the performance data before it is released, and once it is released it is symmetrically observed by all market participants. In the previous section these points of observation were exogenously fixed to $t_1 = 1, t_2 = 2$. In this section the firms will be allowed to vary $t_1$ at their choice. As it will be evident from the analysis, the earlier the worker is evaluated the first time, the stronger are the incentives to work hard prior to the evaluation, and the weaker are incentives after the evaluation. Therefore, by manipulating the timing of the first evaluation the firm can vary the aggregate surplus in the relationship. And since in the competitive environment all surplus is captured by the worker, the market outcome will result in maximal welfare of the worker.

The timing of evaluation also has important turnover implications. For a fixed outside option the worker's with lower ability prior generally prefer to be evaluated sooner. These workers work at a lower starting wage are more likely to be sorted out of the industry in the future. They prefer the sorting to occur sooner and therefore benefit from earlier evaluation. However there is also a sorting-related cost associated with early evaluation. The sooner the first evaluation occurs the less precise is the signal released at that point. So even in the absence of the effort-smoothing considerations the decision when to release the performance information is non-trivial.

To make the last step in the transition from continuous time to discrete time with periods of unequal length it remains to show that the worker will never find optimal to vary effort unless there is new information released.

**Lemma 6** *Let $a(t)$ be an arbitrary measurable policy function of the worker which specifies effort as a function of time for $t \in [t_a, t_b, ]$. Then there exist a constant-effort policy $\bar{a}$ such that*

$$\int_{t_a}^{t_b} a(t) - C(a(t))dt \leq (t_b - t_a)\left(\bar{a} - C(\bar{a})\right).$$

**Proof.** Consider $\bar{a} = \int_{t_a}^{t_b} a(t)dt/\left(t_b - t_a\right)$. Since $f(a) = a - C(a)$ is a concave function, the above result is a direct consequence of Jensen's inequality. ■

The setup modifications are as follows. Let $t_1$ be the time of the first evaluation (so that $t_1 = 1$ corresponds to the original setup). Then precision of the first signal is $t_1 h_\varepsilon$, and precision of the second signal is $(2 - t_1)h_\varepsilon$. The posterior after the first signal will be

$$\eta_2(t_1) = \frac{\eta_1 h_1 + \left[y_1(t_1)/t_1 - a_1(\delta)\right] h_\varepsilon t_1}{h_1 + t_1 h_\varepsilon} = \frac{\eta_1 h_1 + \left(y_1(t_1) - a_1(t_1)t_1\right) h_\varepsilon}{h_1 + t_1 h_\varepsilon},$$

and the posterior after the second signal will be

$$\eta_3(t_1) = \frac{\eta_1 h_1 + \left(y_1(t_1) - t_1 a_1(t_1) + y_2(t_1) - (2 - t_1)a_2(t_1)\right) h_\varepsilon}{h_1 + 2h_\varepsilon}$$

Let $r_2(t_1) = h_\varepsilon/(h_1 + h_\varepsilon t_1)$, and $r_3(t_1) = h_\varepsilon/(h_1 + 2h_\varepsilon) = r_3$ be the corresponding posterior sensitivities, the analogs of $r_2$ and $r_3$ in the original setup.

**Claim 7** *As the length of the first period increases, the effort in the first period decreases and the effort in the second period remains unchanged.*

**Proof.** In the Appendix. ■

While finding the optimal timing of evaluation analytically is not feasible, the diagram below depicts the optimal delta as a function of outside option that is computed numerically.

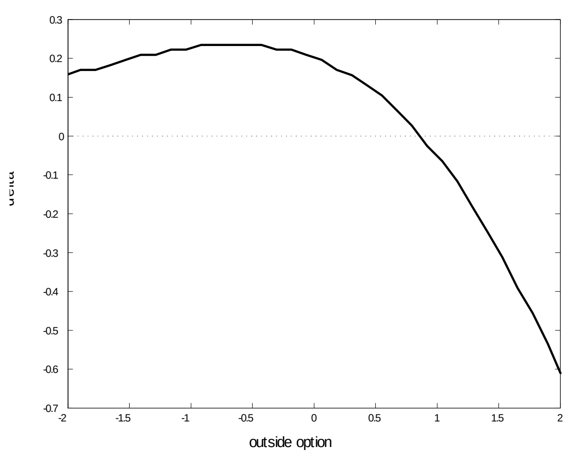



Figure 5. The optimal timing of the first evaluation as a function of the outside option. Parameter values: $h_1 = 1$, $h_\varepsilon = 0.5$, $C" = 0.15$

The workers with high outside option prefer early feedback, while those whose outside option is lower prefer their effort to be smoothed over time.

## 5 Conclusion

This paper has studied the reputational incentives of a worker facing career uncertainty. It has been shown that although the opportunity to "erase" bad histories by changing careers insures the worker against the low ability realizations, in does not necessarily weaken the incentives to build up reputation. While the probability of collecting reputational returns decreases due to career uncertainty, the reputation becomes more valuable conditional on keeping the career. The latter effect arises because the market expects workers with higher ability to exert more effort, since they are less likely to quit in the future and therefore are more likely to benefit from good reputation. This effect is not present unless there is a possibility to escape bad reputation, and it can be so significant that overall strength of incentives increases. Its strength is primarily determined by sensitivity of effort to rewards, as measured by the second derivative of the disutility of effort. If effort varies a lot depending on career prospects, then the competitive wage is more sensitive to reputation, and incentives to build up reputation are stronger. Also, higher precision of a signal and lower precision of prior expectations make incentives stronger early in the career. In fact, incentives can be excessively strong in the beginning of the career and this can cause efficiency loss. By changing timing of information release the employers can manipulate the strength of incentives and avoid this problem.

There are several labor market phenomena to which the results of this model are directly related. Firms are typically reluctant to hire "overqualified" workers. In the context of this model, this can be interpreted as comparing two workers of equal expected ability, but one of them having more skills that do not affect productivity in the position he or she seek but are productive elsewhere. This worker has a higher outside option and a higher chance to quit in the future. The direct effect of greater career uncertainty for overqualified workers is that their expected returns on reputation are lower and therefore they exert less effort. However, as I show in the model, under certain conditions the wage is more sensitive to reputation for workers with better outside options, and this effect countervails the direct incentive-weakening effect of career uncertainty so that "moderately overqualified" workers can be also more motivated.

The role of the outside option is in the model is to induce the worker to abandon the career after bad histories. Alternatively, one could argue that there are exogenous forces that prevent employment at low wages, e.g. minimal wage regulations or unions. The prediction of this model is that an increase in the minimum wage would have an adverse effect on all workers, since future separation becomes more likely, and therefore reputational incentives weaken. Importantly, the minimum wage regulation affects not only those whose expected productivity is close to the threshold, but also those who are at the beginning of their careers and are facing greater uncertainty about their labor market prospects. Unemployment benefits have a similar effect too since they make the search for a new career cheaper.

Finally, the role the outside option plays in determining the strength of reputational incentives increases the returns to specialization when workers choose the composition of skills at the pre-market human capital investment stage, as in Kovrijnykh and Kovrijnykh (2005). When career concerns are taken into consideration, workers have an incentive to bias[8] their skill composition and acquire more skills specific to a single industry. This not only increases the worker's productivity in the first-choice occupation but also acts as a commitment device by decreasing the outside option. The uncertainty about career choice therefore decreases, and the reputational concerns become stronger.

# Appendix

**Derivation of the first-order conditions for the second period.**

Holding the beliefs of the firm fixed, period two effort affects the worker's payoff in period three through two channels. It increases the expected output of period two, and which enters into the firm's period three estimate of the worker's talent. Higher perceived talent means (a) higher wage in period three, and (b) a lower probability of separation. The firm's period three estimate of the worker's talent is

$$\begin{aligned} \eta_3^* &= (1-r_3)\eta_2^* + r_3(y_2 - a_2^*) \\ &= (1-r_3)\eta_2^* + r_3(\eta + a_2 + \varepsilon - a_2^*). \end{aligned}$$

In equilibrium it is normally distributed with mean $\eta_2^*$ and precision $h_{23}$ given by

$$h_{23} = \frac{h_2 h_3}{h_\varepsilon}.$$

Let $f_{23}(.|\mu)$ denote the normal p.d.f. with mean $\mu$ and precision $h_{23}$, and let $\Delta a_t$ denote the worker's deviation from the effort expected by the market, $\Delta a_t = a_t - a_t^*$ Assume that $\Delta a_1 = 0$, so that the worker did not deviate in period one. This implies that $\eta_2^* = \eta_2$, and $\eta_3^* = \eta_3 + r_3\Delta a_2$. The worker is indifferent between staying in the industry or changing the career if $\eta_3^* = u$, or,$\eta_3 = u - r_3\Delta a_2$. In this notation the worker's problem can be written as

$$\max_{a_2} w_2 - C(a_2) + \int_{u-r_3\Delta a_2}^{\infty} (\eta_3 + r_3\Delta a_2)\, f_{23}\,(\eta_3|\eta_2)\, d\eta_3 + uF_{23}\,(u - r_3\Delta a_2|\eta_2) + C(0).$$

[8]The bias is relative to the optimal choice of skills when effort is inelastically supplied, i.e., in absence of career concerns.

The first-order condition for this problem is

$$-C'(a_2) + r_3 u f_{23}\left(u - r_3 \Delta a_2 | \eta_2\right) + \int_{u - r_3 \Delta a_2}^{\infty} r_3 f_{23}\left(\eta_3 | \eta_2\right) d\eta_3 - r_3 u f_{23}\left(u - r_3 \Delta a_2 | \eta_2\right) = 0,$$

which simplifies to

$$-C'(a_2) + r_3\left[1 - F_{23}\left(u - r_3 \Delta a_2 | \eta_2\right)\right] = 0.$$

This condition corresponds to the maximum if the second order condition is satisfied,

$$-C''(a_2) + r_3^2 f_{23}\left(u - r_3 \Delta a_2 | \eta_2\right) \leq 0.$$

Since $f_{23}(.)$ is bounded from above by $\sqrt{h_{23}/2\pi}$, the following condition on the model parameters must be satisfied

$$-C''(a_2) + r_3^2 \sqrt{h_{23}/2\pi} \leq 0.$$

**Lemma 4**

*In equilibrium,* $w_2(\underline{\eta}_2^*) < w_3(\underline{\eta}_3^*)$.

**Proof.** The indifference condition of the worker who decides whether to stay in the industry is

$$w(\underline{\eta}_2^*) - C(a_2(\underline{\eta}_2^*)) + E(\eta_3^* | \underline{\eta}_2^*, \eta_3^* \geq u) \Pr(\eta_3^* \geq u | \underline{\eta}_2^*) + u \Pr(\eta_3^* < u | \underline{\eta}_2^*) = 2u.$$

Since $a_2$ on the left-hand side of the indifference condition is chosen optimally, it must also be true that the value of the worker who exerts zero effort must be weakly less than $2u$ :

$$w(\underline{\eta}_2^*) - C(0) + E(\tilde{\eta}_3^* | \underline{\eta}_2^*, \tilde{\eta}_3^* \geq u) \Pr(\tilde{\eta}_3^* \geq u | \underline{\eta}_2^*) + u \Pr(\tilde{\eta}_3^* < u | \underline{\eta}_2^*) \leq 2u,$$

where $\tilde{\eta}_3^*$ is the expectation of the market given $a_2 = 0$, i.e., $\tilde{\eta}_3^* = \eta_3^* - r_3 a_2(\underline{\eta}_2)$. Since $C(0) = 0$, and $E(\tilde{\eta}_3^* | \underline{\eta}_2, \tilde{\eta}_3^* \geq u) > u$, the result follows:

$$w_2(\underline{\eta}_2^*) < u = w_3(\underline{\eta}_3^*).$$

■

**Derivation of the first-order conditions for the first period**

$$\max_{a_1} w_1 - C(a_1) + EV_2(\eta_2^*, \eta_2), \tag{9}$$

where $V_2$ is the continuation value of staying in the industry after period one, is defined as follows.

$$\begin{aligned} V_2 &= \max_{a_2} \left\{ w_2 - C(a_2) + E\left[V_3(\eta_3, \eta_3^*)\right], 2u \right\} \\ &= E_1\left[w_2(\eta_2^*) - C(a_2(\eta_2, \eta_2^*)) + E_2\left[V_3(\eta_3, \eta_3^*)|\eta_2\right]|\eta_2 \geq \underline{\eta}_2\right] \Pr\left(\eta_2 \geq \underline{\eta}_2\right) + 2u \Pr\left(\eta_2 \geq \underline{\eta}_2\right). \end{aligned}$$

The first-order condition therefore is

$$-C'(a_1) + r_2 \frac{\partial EV_2(\eta_2^*, \eta_2)}{\partial \eta_2^*} = 0.$$

By the envelope theorem,

$$\begin{aligned} \frac{\partial EV_2(\eta_2^*, \eta_2)}{\partial \eta_2^*} &= E_1\left[\frac{dw_2(\eta_2^*)}{d\eta_2^*} + \frac{\partial E_2 V_3(\eta_3, \eta_3^*)}{\partial \eta_3^*} \frac{d\eta_3^*}{d\eta_2^*}\right| \eta_2 \geq \underline{\eta}_2\right] \Pr\left(\eta_2 \geq \underline{\eta}_2\right) \\ &= E_1\left[1 + \frac{r_3 f_{23}(u|\eta_2^*)}{C''(a_2)} + \frac{r_3}{r_2}\left[1 - F_{23}\left(\underline{\eta}_3|\eta_2^*\right)\right]\right| \eta_2 \geq \underline{\eta}_2\right] \left(1 - F_{12}\left(\underline{\eta}_2\right)\right) \\ &= \int_{\underline{\eta}_2}^{\infty} \left[1 + \frac{r_3 f_{23}(u|\eta_2^*)}{C''(a_2)} + \frac{r_3}{r_2}\left[1 - F_{23}(u|\eta_2)\right]\right] dF_{12}(\eta_2). \end{aligned}$$

Given this result he first-order condition can be written as

$$-C'(a_1) + \int_{\underline{\eta}_2}^{\infty} \left[r_2 + r_2 r_3 \frac{f_{23}(u|\eta_2^*)}{C''(a_2)} + r_3\left[1 - F_{23}(u|\eta_2)\right]\right] dF_{12}(\eta_2|\eta_1) = 0.$$

To make sure this condition corresponds to a maximum one has to verify that the second-order condition holds.

$$-C''(a_1) + r_2^2 \frac{\partial^2 EV_2(\eta_2^*, \eta_2)}{\partial \eta_2^{*2}} < 0$$

Or, writing down $\partial^2 EV_2(\eta_2^*, \eta_2)/\partial \eta_2^{*2}$ explicitly,

$$\begin{aligned} &-C''(a_1) - \left[r_2 + r_2 r_3 \frac{f_{23}\left(u|\underline{\eta}_2\right)}{C''(a_2)} + r_3\left[1 - F_{23}(u|\underline{\eta}_2)\right]\right] f_{12}\left(\underline{\eta}_2\right) \frac{d\underline{\eta}_2}{da_1} \\ &+ \int_{\underline{\eta}_2}^{\infty} \left[r_2 r_3 \frac{2h_{23}(u - \eta_2^*) f_{23}(u|\eta_2^*)}{C''(a_2)} + r_3 f_{23}(u|\eta_2) \frac{d\underline{\eta}_3}{da_1}\right] dF_{12}(\eta_2|\eta_1) < 0. \end{aligned}$$

Substituting $d\underline{\eta}_2/da_1 = r_2$, $d\underline{\eta}_3/da_1 = r_3$, and using integration by parts, the above expression simplifies to

$$-C''(a_1) + \int_{\underline{\eta}_2}^{\infty} \left[r_2 + r_2 r_3 \frac{f_{23}(u|\eta_2)}{C''(a_2)} + r_3\left[1 - F_{23}(u|\eta_2)\right]\right] 2h_{12}(\eta_2 - \eta_1) f_{12}(\eta_2|\eta_1) d\eta_2 < 0.$$

This can be further simplified (at the cost of excessive restriction on parameters) as follows

$$
\begin{aligned}
& -C''(a_1) + \int_{\underline{\eta}_2}^{\infty} \left[ r_2 + r_2 r_3 \frac{f_{23}(u|\eta_2)}{C''(a_2)} + r_3 \left[1 - F_{23}(u|\eta_2)\right] \right] 2h_{12}(\eta_2 - \eta_1) f_{12}(\eta_2|\eta_1) d\eta_2 \\
< \quad & -C''(a_1) + \int_{\eta_1}^{\infty} \left[ r_2 + r_2 r_3 \frac{f_{23}(u|\eta_2)}{C''(a_2)} + r_3 \left[1 - F_{23}(u|\eta_2)\right] \right] 2h_{12}(\eta_2 - \eta_1) f_{12}(\eta_2|\eta_1) d\eta_2 \\
< \quad & -C''(a_1) + \int_{0}^{\infty} \left[ r_2 + r_2 r_3 \frac{f_{23}(u|\eta_2)}{C''(a_2)} + r_3 \left[1 - F_{23}(u|\eta_2)\right] \right] 2h_{12}\eta_2 f_{12}(\eta_2|0) d\eta_2 \\
< \quad & -C''(a_1) + 2h_{12}\left[r_2 + r_2/r_3 + r_3\right] \int_0^{\infty} \eta_2 f_{12}(\eta_2|0) d\eta_2 \\
< \quad & -C''(a_1) + \left[r_2 + r_2/r_3 + r_3\right] \sqrt{h_{12}/2\pi} < 0.
\end{aligned}
$$

The first step restricts the range of integration to the region where the integrand is positive. The second step is just a change of variables. The third step makes use of the second-order condion of the next period, $-C''(a_2) + r_3^2 f_{23}(u - r_3 \Delta a_2 | \eta_2) \leq 0$, and also replaces $r_3 \left[1 - F_{23}(u|\eta_2)\right]$ by $r_3$, so that the integrand increases. Finally, at the last step the integral is evaluated. Again, as in the second-order condition for period two, the above condition puts a lower bound on the second derivative of the disutility of effort.

**Proposition 5** *The equilibrium effort level $a_1$ can be bounded from below as follows:*

$$
a_1 \geq c\left(r_2 + c r_2 r_3 f_{13}(u) + r_3\left[1 - F_{13}(u)\right]\right)\left[1 - F_{12}(u)\right], \tag{10}
$$

*where $f_{13}(.)$ and $F_{13}(.)$ are normal p.d.f and c.d.f. respectively with mean $\eta_1$ and precision $h_{13} = h_1 h_3 / (2h_\varepsilon)$.*

**Proof.** Since $a_2(\eta_2)$ increases monotonically in $\eta_2$, the participation constraint defines a lower bound on perceived talent, $\underline{\eta}_2$, at which the worker is willing to continue with his current career. It follows from Lemma 4 that $\underline{\eta}_2 < u$, so the participation constraint is satisfied in particular if $\eta_2 \geq u$. With this in mind, it is straightforward to verify that (7) implies that

$$
a_1 \geq cE\left(h_3 + c r_2 r_3 f_{23}(u) + r_3\left[1 - F_{23}(u)\right]\right) \Pr\left[\eta_2^* \geq u\right]. \tag{11}
$$

Let $F_{12}$ denote the c.d.f. of $\eta_2^*$, which is normal with mean $\eta_1^*$ and precision $h_{12} = h_1 h_2 / h_\varepsilon$. Then (11) can be rewritten as

$$
a_1 \geq c\left[r_2 + c r_2 r_3 \int_{-\infty}^{\infty} f_{23}(u) dF_{12}(\eta_2^*) + r_3 \int_{-\infty}^{\infty} \left[1 - F_{23}(u)\right] dF_{12}(\eta_2^*)\right]\left[1 - F_{12}(u)\right]. \tag{12}
$$

■

Below I demonstrate how expression (12) can be simplified to obtain (8).

$$\begin{aligned}
& \int_{-\infty}^{\infty} f_{23}(\eta_2^*) dF_{12}(\eta_2^*) \\
= & \int_{-\infty}^{\infty} \sqrt{\frac{h_{12}h_{23}}{2\pi}} \exp\left\{-h_{12}\left(\eta_2^* - \eta_1^*\right)^2 - h_{23}\left(u - \eta_2^*\right)^2\right\} d\eta_2^* \\
= & \int_{-\infty}^{\infty} \sqrt{\frac{h_{12}h_{23}}{2\pi}} \exp\left\{-\left(h_{12}+h_{23}\right)\left(\eta_2^* - \frac{h_{12}\eta_1^* + h_{23}u}{h_{12}+h_{23}}\right)^2 - \frac{h_{12}h_{23}}{h_{12}+h_{23}}\left(u-\eta_1^*\right)^2\right\} d\eta_2^* \\
= & \sqrt{\frac{h_{12}h_{23}}{2\pi\left(h_{12}+h_{23}\right)}} \exp\left\{-\frac{h_{12}h_{23}}{h_{12}+h_{23}}\left(u-\eta_1^*\right)^2\right\} \\
& \times \int_{-\infty}^{\infty} \sqrt{\frac{h_{12}+h_{23}}{2\pi}} \exp\left\{-\left(h_{12}+h_{23}\right)\left(\eta_2^* - \frac{h_{12}\eta_1^* + h_{23}u}{h_{12}+h_{23}}\right)^2\right\} d\eta_2^* \\
= & \sqrt{\frac{h_{12}h_{23}}{2\pi\left(h_{12}+h_{23}\right)}} \exp\left\{-\frac{h_{12}h_{23}}{h_{12}+h_{23}}\left(u-\eta_1^*\right)^2\right\} = f_{13}(u),
\end{aligned}$$

where $f_{13}(.)$ is a normal p.d.f. with mean $\eta_1^*$ and precision $h_{13} = h_1 h_3 / \left(2h_\varepsilon\right)$. The simplified expression for $h_{13}$ can be obtained as follows:

$$h_{13} = \frac{h_{12}h_{23}}{h_{12}+h_{23}} = \frac{h_1 h_2^2 h_3}{h_\varepsilon^2\left(h_1h_2/h_\varepsilon + h_2h_3/h_\varepsilon\right)} = \frac{h_1h_2h_3}{h_\varepsilon\left(h_1+h_3\right)} = \frac{h_1h_2h_3}{h_\varepsilon\left(2h_2\right)} = \frac{h_1h_3}{2h_\varepsilon}.$$

Similarly,

$$\begin{aligned}
\int_{-\infty}^{\infty} \left[1 - F_{23}(u)\right] dF_{12}(\eta_2^*) &= 1 - \int_{-\infty}^{\infty} F_{23}(u) dF_{12}(\eta_2^*) \\
&= 1 - \int_{-\infty}^{\infty}\int_{-u}^{\infty} f_{23}(\eta_3^*) f_{12}(\eta_2^*) d\eta_3^* d\eta_2^* \\
&= 1 - \int_{-u}^{\infty} f_{13}(\eta_3^*) d\eta_3^* = 1 - F_{13}(u),
\end{aligned}$$

where $F_{13}(.)$ is a normal c.d.f. with mean $\eta_1^*$ and precision $h_{13} = h_1h_3/\left(2h_\varepsilon\right)$.

**Claim 7** *As the length of the first period increases, the effort in the first period decreases and the effort in the second period remains unchanged.*

**Proof.** The second period problem of the worker is

$$\max_{a_2} \left\{\left(w_2 - C(a_2)\right)\left(2 - t_1\right) + E\left[V_3(\eta_3, \eta_3^*)\right], 2u - \delta\right\}, \tag{13}$$

and the corresponding first-order condition is

$$(2 - t_1)C'(a_2) = (2 - t_1) r_3 \left[1 - F_{23}(u|\eta_2)\right].$$

Dividing both sides of the above first-order condition by $(2-t_1)$ yields (4), and this proves the first part of the claim: incentives in the second period must remain unchanged as $t_1$ changes. The first period problem of the worker is

$$\max_{a_1} (w_1 - C(a_1))\, t_1 + E\left[V_2^{\delta}(\eta_2^*, \eta_2)|\eta_1\right],$$

where $V_2^{\delta}(\eta_2^*, \eta_2)$ is given by (13). The corresponding first-order condition is

$$t_1 C'(a_1) = E\left(\left(r_2(t_1) + \frac{r_3 r_2\left(t_1\right) f_{23}(u|\eta_2)}{C''(a_2)}\right)(2-t_1)t_1 + r_3 t_1 \left[1 - F_{23}(u|\eta_2)\right]\middle|\, \eta_2 \geq \underline{\eta}_2\right)\left[1 - F_{12}(\underline{\eta}_2)\right]. \tag{14}$$

A straightforward application of the implicit function theorem completes the proof,

$$\begin{aligned}\frac{\partial a_1}{\partial t_1} &= \left(r_2'(\delta)t_1 - r_2(\delta)\right) E\left(1 + r_3 f_{23}(u|\eta_2)/C''(a_2)\middle|\, \eta_2 \geq \underline{\eta}_2\right)\left[1 - F_{12}(\underline{\eta}_2)\right]/C''(a_1) \\ &- \frac{\partial \underline{\eta}_2}{\partial t_1}\left[\left[r_2(t_1) + r_3 r_2\left(t_1\right) f_{23}(u|\underline{\eta}_2)\Big/ C''\left(a_2\left(\underline{\eta}_2\right)\right)\right](2-t_1)t_1 + r_3 t_1\left[1 - F_{23}(u|\underline{\eta}_2)\right]\right]/C''(a_1) < 0.\end{aligned}$$

■

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