KEY CONCEPTS
Chapter 15 Open Economy

The budget constraint for the home country is

\[ PY_t + R_{t-1}B_{t-1}^f = P(C_t + I_t + G_t) + (B_t^f - B_{t-1}^f) + (H_t - H_{t-1}) \]

- Current account balance = \( B_t^f - B_{t-1}^f \)
- Capital account balance = \( B_t^f - B_{t-1}^f \), which is also equal to current account balance when \( H_t = H_{t-1} \)
- Outflow of capital: \( B_t^f - B_{t-1}^f > 0 \)
- Inflow of capital: \( B_t^f - B_{t-1}^f < 0 \)
- Trade Balance: \( PY_t - P(C_t + I_t + G_t) = \) Net Exports
- National Saving: \( S_t = PY_t + R_{t-1}B_{t-1}^f - P(C_t + G_t) \)
- Net foreign investment = \( PY_t - P(C_t + I_t + G_t) + R_{t-1}B_{t-1}^f \)
- Trade Balance + Net factor income from abroad = \( S_t - PI_t = \) National Saving net of Domestic Investment
- Surplus (Deficit) on current account: \( CAB > 0 (< 0) \)

Terms of Trade

\( P \) denotes the price of goods produced in home country. \( \bar{P} \) denotes the index price of a basket of world goods. Suppose the home country exports all the produced good and domestic consumption and investment rely on imported goods.

\[ \text{Terms of trade} = \frac{P}{\bar{P}} \]

The current balance now becomes:

\[ CAB = PY_t + R_{t-1}B_{t-1}^f - \bar{P}(C_t + I_t + G_t) \]

EXERCISES

1. Current Account Balance for a Small Open Economy
   The Current Account Balance is initially positive. Assume that a temporary positive shock to production affects a small country in an open economy. Assume that the shock increases the MPK. What are the effects on CAB and interest rate? What are the effects if the same shock hit a large country in an open economy?

2. Current Account Balance for Large Open Economy
   Consider a two-country model. Assume that the Current Account Balance is initially positive for one country. Assume that a permanent positive shock to production affects the country which initially had a positive current account balance. Assume that the shift increases the MPK. Graphically indicate the effect on the commodity market clearing. Be clear to indicate what happens to the real interest rate and the CA balance. Explain the intuition behind the graphs.
3. **(HW 15.9) A change in the Terms of Trade.**
   We considered in the text a change in the terms of trade that reflected a disturbance from the rest of the world. Suppose instead that a negative supply shock at home leads to an increase in the relative price of the home country’s tradable goods. What are the effects from this type of disturbance on the home country’s
   a. Wealth and consumption of various good?
   b. Current account balance?

4. **(HW 15.7) A Two-Country Model of the Current-Account Balance**
   Suppose that the world consists of two countries, A and B, which are roughly of the same size. Consider a temporary supply shock that affects the production function in country A but not in country B. What are the effects on the real interest rate and the current-account balances in the two countries?

5. **PPP and Interest Rate Parity**
   Derive the formulas for Purchasing Power Parity and Interest Rate Parity in absolute form. Derive the IRP in relative form.

6. **PPP and Interest Rate Parity**
   Derive the formulas for Purchasing Power Parity and Interest Rate Parity in relative form. Suppose that the return on domestic bonds held by foreigners in country i are subsidized at the rate $s$ and that returns on domestic bonds held by residents of country j are taxed at the rate $\tau$. Write down the interest parity condition (between countries i and j) as viewed by an investor in country k.

7. **Exchange Rate and Interest Rate Parity**
   At time period 0, suppose that the nominal rate of return on one-period bond is 14% in Germany, and 4% in the U.S. Further, suppose that at time period 0, the exchange rate is 2 German Marks / U.S. Dollar. All else equal, from what you know about purchasing power parity and interest rate parity, what would be reasonable expectations for you to have regarding the inflation rate in Germany and the U.S.? What should be the exchange rate of German Marks / U.S. Dollar at time period 1?
Small open economy

\[ Y^s + RBf \]

\[ Y^d = C_d + I_d + G \]

Large open economy

CAB decreases

\[ Y^s + RBf \]

\[ Y^d \]

CAB decreases, \( r^w \uparrow \)

Large open economy with

\[ \Rightarrow \] the increase in investment from abroad as well due to permanent increase in MPK, a better opportunity to invest.

CAB decreases, \( r^w \uparrow \)
SOLUTION

3. (HW 15.9) A change in the Terms of Trade
   a. If the demand for coffee is inelastic (elastic), so that total revenues rise (fall) with the increases in the price of coffee, a harvest failure raises (lowers) Brazil’s wealth. The change in the wealth is larger for a permanent change. If the wealth effect is positive, private consumption will rise, more for the permanent change and less for the temporary change.
   b. The current account will run a surplus for a temporary change. In the case of a permanent change, the current account may show a deficit as foreigners increase their investment in Brazil.

4. (HW 15.7) A Two-Country Model of the Current-Account Balance
   Presuming that the nations have not completely diversified their holdings of assets, residents in country A want to borrow. Thus, these people borrow from the citizens in country B. A’s citizens are able to borrow, but because the nations are the same size, the increased demand for loans drives up the real interest rate. Nation A runs a current account deficit (and capital account surplus) and Nation B runs a current account surplus (and capital account deficit).
\[
\begin{align*}
\dot{E}^i \left( \frac{1}{p^i} \right) &= \dot{E}^j \left( \frac{1}{p^j} \right) \\
&= \frac{\Delta E^i}{\Delta E^j} = \frac{\Delta p^i}{\Delta p^j} \\
\text{So:} \quad \frac{E^i}{p^i} = \frac{E^j}{p^j} \iff \frac{E^i}{E^j} = \frac{p^i}{p^j} \quad \text{PPP absolute form.}
\end{align*}
\]

Rewrite: \[
\frac{E^i + \Delta E^i}{E^j + \Delta E^j} = \frac{p^i + \Delta p^i}{p^j + \Delta p^j}
\]

Cross multiply:
\[
E^i p^j + p^j \Delta E^i + E^i \Delta p^j + \Delta E^i \Delta p^j = E^i p^i + \Delta E^i \cdot p^i + E^i \Delta p^j + \Delta E^i \Delta p^j
\]

since \[
\frac{E^i}{E^j} = \frac{p^i}{p^j} \iff E^i \cdot p^j = E^i \cdot p^i \quad \text{and} \quad \frac{\Delta E^i \cdot \Delta p^j}{E^i \cdot p^i} \to 0
\]

So:
\[
\frac{p^i \cdot \Delta E^i}{E^i} + E^i \cdot \Delta p^j + \Delta E^i \cdot \Delta p^j
\]

Divide through by \[
E^i \cdot p^j = E^i \cdot p^i
\]

\[
\frac{\Delta E^i}{E^i} + \frac{\Delta p^j}{p^j} = \frac{\Delta E^i}{E^i} \cdot \frac{p^i}{E^i} + \frac{\Delta p^j}{p^j} + \frac{\Delta E^i \cdot \Delta p^j}{E^i \cdot p^j}
\]

So:
\[
\frac{\Delta E^i}{E^i} + \frac{\Delta p^j}{p^j} = \frac{\Delta E^j}{E^j} + \frac{\Delta p^i}{p^i}
\]

\[
\Rightarrow \quad \frac{\Delta E^i}{E^i} - \frac{\Delta E^j}{E^j} = \frac{\Delta p^i}{p^i} - \frac{\Delta p^j}{p^j} = \pi^i - \pi^j \quad \text{PPP in relative form.}
\]

\[
\begin{align*}
\text{Invest in } j &\quad \Rightarrow \quad \text{Invest in } i \\
\frac{E^i (1 + R^i)}{E^i_{t+1}} &= \frac{E^j (1 + R^j)}{E^j_{t+1}} \\
\text{So:} \quad \frac{E^i (1 + R^i)}{E^i_{t+1}} &= \frac{E^j (1 + R^j)}{E^j_{t+1}} \quad \text{IRP in absolute form}
\end{align*}
\]

Rewrite:
\[
\frac{(1 + R^i)}{E^i_{t+1}/E^i_t} = \frac{(1 + R^j)}{E^j_{t+1}/E^j_t}
\]

and use that \[
\frac{E^i_{t+1}/E^i_t}{E^j_{t+1}/E^j_t} = 1 + \frac{\Delta E^i}{E^i}
\]

So:
\[
\frac{(1 + R^i)}{1 + \frac{\Delta E^i}{E^i}} = \frac{(1 + R^j)}{1 + \frac{\Delta E^j}{E^j}}
\]

Cross multiply:
\[
1 + \frac{\Delta E^i}{E^i} + R^i + R^i \cdot \frac{\Delta E^j}{E^j} = 1 + R^j + \frac{\Delta E^i}{E^i} + R^i \cdot \frac{\Delta E^j}{E^j}
\]

So:
\[
R^i - R^j = \frac{\Delta E^i}{E^i} - \frac{\Delta E^j}{E^j} = \pi^i - \pi^j
\]

PPP in relative form.
\begin{align*}
\text{Q6} & \quad (R = \text{USA}) \\
\text{Invest in i} & \quad \text{Invest in j} \\
\frac{E_t^i (1 + R^i(1+s))}{E_t^i} & \quad \frac{E_t^j (1 + R^j)}{E_t^j} \\
\text{IRP in absolute form:} & \quad \frac{1 + R^i(1+s)}{E_t^i E_t^j} = \frac{1 + R^j}{E_t^j E_t^j} \\
\text{Cross-multiply:} & \quad 1 + R^i(1+s) + \frac{\Delta E^j}{E^j} + R^j(1+s) \frac{\Delta E^i}{E^i} \\
& = 1 + R^j + \frac{\Delta E^i}{E^i} + R^j \frac{\Delta E^i}{E^i} \\
\Rightarrow & \quad \frac{R^i(1+s) - R^j}{E^i E^j} = \frac{\Delta E^i - \Delta E^j}{E^i E^j} = \pi^i - \pi^j \\
\text{PPP in relative form.} & \\
\text{IRP in relative form.} & \\
\end{align*}

\begin{align*}
\text{Q7} & \quad \text{Since } R^G - R^US = 14\% - 4\% = 10\% \\
& \text{By PPP in relative form} \\
& R^G - R^US = \Pi^G - \Pi^US = 10\% \\
& \Rightarrow \text{Inflation in Germany is 10\% higher than in USA.} \\
& \text{Also, since } R^G - R^US = \frac{\Delta E^G}{E^G} - \frac{\Delta E^US}{E^US} = 10\% \\
& \text{then exchange rate should be} \\
\text{2.2 German Marks / U.S. Dollar.}
\end{align*}