SOME KEY CONCEPTS
- Long-run Economic Growth
- Growth Accounting
- Solow Growth Model
- Endogenous Growth Model

EXERCISE

Question 1 (Growth Accounting)
A researcher analyzes economic growth using the production function $Y = AF(K, L)$. Suppose the output of an economy has grown by 150% over the past 75 years. It is found that capital stock and labor force have grown by 100% and 50%, respectively. The output elasticity of capital and labor are $a_K = 0.3$ and $a_L = 0.7$, respectively.

1) Write down the growth accounting equation.
2) What are the contributions to economic growth from growth in capital, growth in labor force, and growth in productivity?

Question 2 (Solow Growth Model: Dynamics, Steady-state and Golden-rule Level of Consumption)
This question walks you through the dynamics of Solow growth model. Consider the usual constant return to scale production function at time $t$ given by $Y_t = F(K_t, L_t)$. Population save constant saving rate $s$ of the output. At per-worker level, gross investment must at least replace the worn out capital $(dk_t)$ and must expand the capital stock as population grows $(nk_t)$. The per-worker gross investment at time $t$ is therefore given by $(d + n)k_t$. Hence, the dynamics of per-worker capital stock can be expressed by the equation $k_{t+1} = k_t - (d + n)k_t + sy_t$

1) Rewrite all relevant variables – $Y_t$, $S_t$, $C_t$ and $K_t$ – as per-worker quantities.
2) The steady-state level of capital corresponds to the situation where saving equals to gross investment. Denote such point on the Solow diagram.
3) Denote the per-worker consumption at the steady-state level of per-worker capital.
4) Suppose the economy starts with the level of per-worker capital below the steady-state level, this leads to positive net investment and the economy accumulates capital. Explain in a diagram with saving and gross investment. Show the capital accumulation on a diagram with time and per-worker capital on horizontal and vertical axes, respectively.
5) The golden-rule level of consumption corresponds to a particular saving rate such that, at the steady-state, the steady-state level of capital leads to the highest level of consumption. Graphically explain the golden-rule level of consumption by considering three saving rates: below, above and the golden-rule saving rates.

Question 3 (Solow Growth Model: Numerical Example)
In the Solow model, suppose per-worker production function is $y = 10k^{0.5}$. Saving rate, population growth rate and depreciation rate are $s = 0.05$, $n = 0.02$ and $d = 0.03$, respectively. Calculate the steady-state equilibrium levels of per-worker capital stock, $k^{ss}$, per-worker output, $y^{ss}$, and per-worker consumption, $c^{ss}$.

Question 4 (Solow Growth Model: Comparative Static Analysis)
In each case, assuming that the economy is initially in its steady state. Distinguish the following scenario towards the diagram of the Solow growth model. What curve(s) are shifted? What happens to the level of per-worker capital stock at steady state? Plot the dynamic effects of per-worker capital stock over time.

1) A permanent increase in saving rate
2) A one-time increase in saving rate
3) A permanent decrease in population growth rate
4) An increase in productivity
5) An event which results in an abrupt decrease in capital stock

**Question 5 (Endogenous Growth Model)**
1) True/False: The production function in endogenous growth model exhibits diminishing marginal returns.
2) True/False: The endogenous growth model has a steady-state level of capital stock and output.
SOLUTION

Question 1
1) Growth accounting equation:
\[
\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + a_K \frac{\Delta K}{K} + a_L \frac{\Delta L}{L}
\]

2) Contribution to economic growth from growth in capital = \(0.3 \times 100\% = 30\%\)
Contribution to economic growth from growth in labor force = \(0.7 \times 50\% = 35\%\)
Since \(\frac{\Delta Y}{Y} = 150\%\), then the contribution to economic growth from growth in productivity is \(\frac{\Delta A}{A} = 150\% - 30\% - 35\% = 85\%\).

Question 2
1) Let \(k_t\) be the level of per-worker capital stock, then \(y_t = f(k_t), s_t = sy_t, c_t = (1 - s)y_t\). The evolution of per-worker capital stock is \(k_{t+1} = k_t - (n + d)k_t + sf(k_t)\).
2) - 5) See drawing sheet

Question 3
Solve for steady state per-worker capital stock \(s = (n + d)k\), so \(0.05 \times 10k^{0.5} = (0.03 + 0.02)k\). Hence, \(k^{ss} = 100, y^{ss} = 10 \times 100^{0.5} = 100,\) and \(c^{ss} = (1 - 0.05)\times 100 = 95\).

Question 4
1) - 5) See drawing sheet

Question 5
1) False
2) False
2), 3), 4)

\[ y = f(k) \]

\[ (n + d)k \]

\[ s \cdot y = s \cdot f(k) \]

Steady-state

\[ k_1, k_2, k_3, k_4, k^{ss} \]

\[ k^{ss}, K_4, K_3, K_2, K_1 \]

\[ 1, 2, 3, 4 \] time
Let $s_{GR}$ be the golden-rule saving rate. Let $s_1 < s_{GR} < s_3$. 

(graphical representation of economic curves and savings rates)
Question 4

1) Permanent $s^\uparrow$

2) One-time $s^\uparrow$

3) Permanent $n^\downarrow$