ELASTICITY ON THE LINEAR DEMAND CURVE

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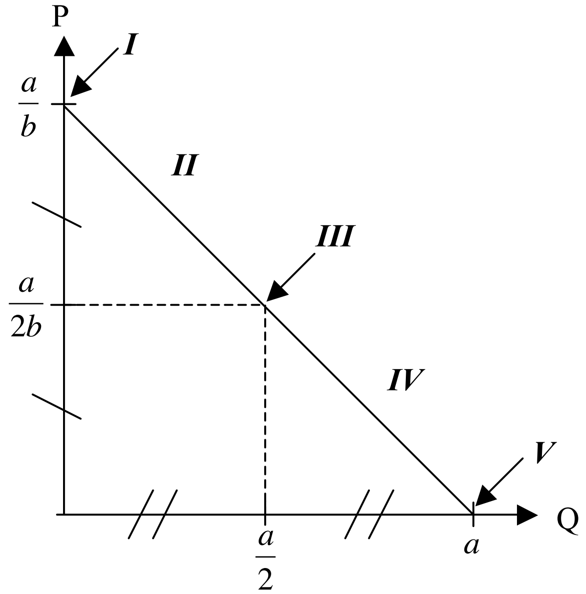

Consider a linear demand function denoted by

$$Q^d = a - bP$$

We find the endpoint co-ordinates on each axis:

at $P = 0 \Rightarrow Q^d = a$ … horizontal intercept

at $Q^d = 0 \Rightarrow P = \frac{a}{b}$ … vertical intercept

Also, find the midpoint co-ordinate of P and Q^d:

at $P = \frac{a}{2b} \Rightarrow Q^d = \frac{a}{2}$ … midpoint

Recall the own-price elasticity of demand:

$$\eta_{Q^d,P} = \frac{\%\Delta Q^d}{\%\Delta P} = \frac{dQ^d}{dP}\frac{P}{Q^d}$$

We need to find $\frac{dQ^d}{dP}$ from the demand function: $\frac{dQ^d}{dP} = -b$. Hence, we can simply find the own-price elasticity of demand by substituting P and Q^d into: $\eta_{Q^d,P} = -b\frac{P}{Q^d}$.

Observation I *– At the vertical-intercept endpoint:* $\eta_{Q^d,P} = -\infty$ *(perfectly elastic)*

Proof Substitute $Q^d = 0$ and $P = \frac{a}{b}$; therefore, $\eta_{Q^d,P} = -b\frac{\left(a/b\right)}{0} = -\frac{a}{0} = -\infty$.

Observation II *– At the quantity demanded lower than midpoint:* $\eta_{Q^d,P} < -1$ *(elastic)*

Proof If $Q^d < \frac{a}{2} \Rightarrow \frac{1}{Q^d} > \frac{2}{a}$ and $P > \frac{a}{2b}$; hence, $P\left(\frac{1}{Q^d}\right) > \frac{2}{a}\left(\frac{a}{2b}\right)$ $\therefore$ $\eta_{Q^d,P} = -b\left(\frac{P}{Q^d}\right) < -b\left(\frac{1}{b}\right) < -1$.

Observation III *– At the midpoint:* $\eta_{Q^d,P} = -1$ *(unitary elastic)*

Proof Substitute $Q^d = \frac{a}{2}$ and $P = \frac{a}{2b}$; therefore, $\eta_{Q^d,P} = -b\frac{\left(a/2b\right)}{a/2} = -\frac{a/2}{a/2} = -1$.

Observation IV *– At the quantity demanded higher than midpoint:* $-1 < \eta_{Q^d,P} < 0$ *(inelastic)*

Proof If $Q^d > \frac{a}{2} \Rightarrow \frac{1}{Q^d} < \frac{2}{a}$ and $P < \frac{a}{2b}$; hence, $P\left(\frac{1}{Q^d}\right) < \frac{2}{a}\left(\frac{a}{2b}\right)$ $\therefore$ $\eta_{Q^d,P} = -b\left(\frac{P}{Q^d}\right) > -b\left(\frac{1}{b}\right) > -1$.

Observation V *– At the horizontal-intercept endpoint:* $\eta_{Q^d,P} = 0$ *(perfectly inelastic)*

Proof Substitute $Q^d = a$ and $P = 0$; therefore, $\eta_{Q^d,P} = -b\frac{0}{a} = 0$.