



MCMCpack: Markov Chain Monte Carlo in R

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Abstract

We introduce **MCMCpack**, an R package that contains functions to perform Bayesian inference using posterior simulation for a number of statistical models. In addition to code that can be used to fit commonly used models, **MCMCpack** also contains some useful utility functions, including some additional density functions and pseudo-random number generators for statistical distributions, a general purpose Metropolis sampling algorithm, and tools for visualization.

Keywords: Bayesian inference, Markov chain Monte Carlo, R.

1. Introduction

The Bayesian paradigm for statistical inference is appealing to researchers on both theoretical (Jeffreys 1998; Bernardo and Smith 1994) and practical (Gelman, Carlin, Stern, and Rubin 2003) grounds. The interest in Bayesian methods in the social sciences is growing, and a number of researchers are using these approaches in substantive applications dealing with: deliberative bodies (Clinton, Jackman, and Rivers 2004; Martin and Quinn 2002), economic performance (Western 1998), income dynamics (Geweke and Keane 2000), legislative redistricting (Gelman and King 1990), mass voting (King, Rosen, and Tanner 1999), party competition (Quinn, Martin, and Whitford 1999; Quinn and Martin 2002), social networks (Hoff, Raftery, and Handcock 2002; Hoff and Ward 2004), and historical changes (Park 2010). Despite this interest, the social scientific community has yet to take full advantage of Bayesian techniques when approaching research problems. This is due primarily to two distinct problems. The first problem, which has been largely solved, was the inability to compute the high dimensional integrals necessary to characterize the posterior distribution for most models. To a large extent, this has been solved by the advent of Markov chain Monte Carlo (MCMC) methods (Tanner and Wong 1987; Gelfand and Smith 1990; Besag, Green, Higdon,

and Mengersen 1995) and the dramatic increases in computing power over the past twenty years. For a comprehensive treatment of MCMC methods, see Robert and Casella (2004).

MCMC methods are widely considered the most important development in statistical computing in recent history. MCMC has allowed statisticians to fit essentially *any* probability model—including those not even considered a few years ago. Unfortunately, statisticians have been about the *only* people who have been willing, and able, to write the computer code necessary to use MCMC methods to fit probability models.

Which brings us to the second problem; namely, the lack of flexible, yet easy-to-use software for social scientists unwilling or unable to devote substantial time and energy writing custom software to fit models via MCMC. Since reasonably efficient MCMC algorithms exist to sample from the posterior distribution for most classes of models, developing software to meet the needs of social scientists is feasible.

MCMCpack (Martin, Quinn, and Park 2011) is an R (R Development Core Team 2011b) package that contains functions to perform Bayesian inference. It provides a computational environment that puts Bayesian tools (particularly MCMC methods) into the hands of social science researchers so that they (like statisticians) can fit innovative models of their choosing. Just as the advent of general purpose statistical software (like SPSS and SAS) on mainframe and then personal computers led to the widespread adoption of statistical approaches in the social sciences, providing easy-to-use general purpose software to perform Bayesian inference will bring Bayesian methods into mainstream social science.

MCMCpack currently contains the eighteen statistical models: linear regression models (linear regression with Gaussian errors, a singular value decomposition regression, and regression for a censored dependent variable), discrete choice models (logistic regression, multinomial logistic regression, ordinal probit regression, and probit regression), measurement models (a one-dimensional IRT model, a k -dimensional IRT model, a k -dimensional ordinal factor model, a k -dimensional linear factor model, a k -dimensional mixed factor model, and a k -dimensional robust IRT model), a model for count data (a Poisson regression model), models for ecological inference (a hierarchical ecological inference model and a dynamic ecological inference model), and time-series models for change-point problems (a binary change-point model, a probit change-point model, an ordinal probit change-point model, and a Poisson change-point model). Many of these models, especially the measurement models, are otherwise intractable unless one uses MCMC.

The package also contains the density functions and pseudo-random number generators for the Dirichlet, inverse Gamma, inverse Wishart, noncentral Hypergeometric, and Wishart distributions. These functions are particularly useful for visualizing prior distributions. Finally, **MCMCpack** contains a number of utility functions for creating graphs, reading and writing data to external files, creating `mcmc` objects, and manipulating variance-covariance matrices. The `coda` package is currently used for posterior visualization and summarization (Plummer, Best, Cowles, and Vines 2006). **MCMCpack** is available from the Comprehensive R Archive Network at <http://CRAN.R-project.org/package=MCMCpack>.

The remainder of this paper is organized as follows. In Section 2 we discuss the package environment and features of **MCMCpack**. The following sections will show how model fitting functions in **MCMCpack** are implemented in a Gaussian linear model, and a Poisson change-point model. We conclude with a discussion of possible **MCMCpack** future developments.

2. The MCMCpack environment

We have chosen to make the R system for statistical computation and graphics the home environment for our software. R has a number of features that make it ideal for our purposes. It is open-source, free software that is distributed under the GNU General Public License. It is an extremely powerful programming environment that is designed for statistical data analysis and data visualization. R is very similar to the S language (Becker, Chambers, and Wilks 1988), and provides an easy-to-use interface to compiled C, C++ or Fortran code, as well as excellent facilities for debugging, profiling, and documenting native R programs. R is already the general purpose tool of choice for most applied statisticians and is well documented and supported (Venables and Ripley 2000, 2002; R Development Core Team 2011a,c). Evidence of the move toward R among social scientists can be found in the growing number of texts designed for social science graduate students that explicitly advocate R usage (Fox 2002; Gill 2002), and the decision of the Inter-University Consortium for Political and Social Research to require students to use R as a means of integrating its most advanced courses in their summer training program (Anderson, Fox, Frankin, and Gill 2003). Over the last five years R has become the *lingua franca* of applied statisticians working in the social sciences.

2.1. Design philosophy

In building the **MCMCpack** package we have attempted to adhere to a design philosophy that emphasizes: (a) widespread availability; (b) model-specific, computationally efficient MCMC algorithms; (c) compiled C++ code to maximize computational speed; (d) an easy-to-use, standardized model interface that is very similar to the standard R model fitting functions; and (e) compatibility with existing R packages (such as **coda**) for convergence assessment, posterior summarization, and data visualization.

From a purely practical perspective, the most important design goal has been the implementation of MCMC algorithms that are model-specific. The major advantage of such an approach is that the sampling algorithms, being hand-crafted to particular classes of models, can be made dramatically more efficient than black box approaches, such as those found in the **WinBUGS** software, while remaining robust to poorly conditioned or unusual data. In addition to using reasonably computationally efficient sampling algorithms, the **MCMCpack** model fitting functions are also designed to be fast implementations of particular algorithms. To this end, nearly all of the actual MCMC sampling takes place in C++ code that is called from within R.

The model fitting functions in **MCMCpack** have been written to be as similar as possible to the corresponding R functions for classical estimation of the models in question. This largely eliminates the need to learn a specialized model syntax for anyone who is even a novice user of R. For instance, to fit a linear regression model in R via ordinary least squares one could use the following syntax:

```
reg.out <- lm(y ~ x1 + x2 + x3, data = mydata)
```

Bayesian fitting of the same model with an improper uniform prior on the coefficient vector and the default settings for the parameters governing the MCMC algorithm can be accomplished with **MCMCpack** using the following syntax:

```
Bayes.reg.out <- MCMCregress(y ~ x1 + x2 + x3, data = mydata)
```

It is our experience that such similarities greatly decrease the amount of time it takes to become a competent user of the **MCMCpack** package.

In addition, the **MCMCpack** model fitting functions are designed to be as similar to each other as possible. As a result, once a user becomes familiar with the basics of using one model fitting function she can quickly move on to confidently use other model fitting functions. For instance, to fit a probit model in which the β vector is assumed to have a multivariate normal prior with mean m and precision P one would use the **MCMCpack** syntax:

```
Bayes.probit.out <- MCMCprobit(y ~ x1 + x2 + x3, b0 = m, B0 = P,
  data = mydata)
```

To fit a Poisson regression model in which the β vector is assumed to have a multivariate normal prior with mean m and precision P one would use the **MCMCpack** syntax:

```
Bayes.poisson.out <- MCMCpoisson(y ~ x1 + x2 + x3, b0 = m, B0 = P,
  data = mydata)
```

Using R as the home environment for **MCMCpack** allows us to make use of a wide range of software for MCMC convergence assessment, posterior summarization, and data visualization (such as the **coda** package of [Plummer *et al.* 2006](#)) that is already part of the R system. Output objects from all of the **MCMCpack** model fitting functions are formatted as **coda** **mcmc** objects, sometimes with additional attributes to allow for other types of analyses. This provides for a very easy-to-use interface between the two packages. For instance, to generate trace plots and density plots of the output in `Bayes.poisson.out` using the `plot` method, one would enter

```
plot(Bayes.poisson.out)
```

at the R command prompt. Similarly, to compute posterior means, standard deviations, quantiles, etc. using the `summary` method one would enter

```
summary(Bayes.poisson.out)
```

at the R command prompt. All other **coda** functions can be used in conjunction with **MCMCpack** with similar ease.

2.2. Comparison with existing software

Perhaps the most widely used piece of software for applied Bayesian inference is the **WinBUGS** package, distributed by the MRC Biostatistics Unit at Cambridge ([Spiegelhalter, Thomas, Best, and Lunn 2003](#)). **WinBUGS** is an important contribution, and is ideally suited for small models (both in the number of parameters and in the amount of data). **WinBUGS** is based on a set of general computational algorithms that can be used to estimate models specified using a model definition syntax. **WinBUGS** oftentimes does not exploit model-specific information for estimation. All models are estimated using general purpose black-box algorithms. For the most part, sampling is done parameter-by-parameter. There are two other implementations of the **BUGS** language: **JAGS** ([Plummer 2003, 2005](#)) and **OpenBUGS** ([Thomas, O'Hara, Ligges, and Sturtz 2006](#)). Furthermore, there are various R interfaces including **R2WinBUGS** ([Sturtz,](#)

Ligges, and Gelman 2005), **rjags** (Plummer 2009), or **BRugs** (Ligges, Thomas, Spiegelhalter, Best, Lunn, Rice, and Sturtz 2009).

While the **BUGS** language is useful for fitting many types of models, it has its limitations. The generic estimation engine used in **WinBUGS** is quite flexible, but because it does not exploit model-specific information, it is often inefficient, and sometimes ineffective. It is well known that **WinBUGS** has trouble with algorithms that require sampling from truncated distributions, such as the Albert and Chib (1993) algorithm for the binary probit model, and algorithms for fitting ordinal data models (Cowles 1996; Johnson and Albert 1999). The parameter-by-parameter approach is computationally inefficient, both in terms of computation time and clock time (Liu, Wong, and Kong 1994). While we think **MCMCpack** has definite advantages over **WinBUGS** for many users, we emphasize that we view **BUGS** and **MCMCpack** as complementary tools for the applied researcher. In particular, the greater flexibility of the **BUGS** language is perfect for users who need to build and fit custom probability models.

Other general purpose software packages with designs similar to that of **MCMCpack** exist. One is the **BACC** package (for Bayesian analysis, computation, and communication) discussed in Geweke (1999). The development of **BACC** was supported by the National Science Foundation, and the newest release was made available on 2003-06-03. **BACC** is available as a computational engine for MATLAB, R, S-PLUS, GAUSS, and the UNIX or Windows console. **BACC** shares a design component with **MCMCpack**, as each algorithm is coded in compiled code for each individual model. Another general purpose software package is the recently released **bayesm** package by Rossi and McCulloch (2006). This package is geared to analyses in marketing and applied econometrics. Both **BACC** and **bayesm** use an interface where users provide appropriate scalars, vectors, and matrices for data and priors, and where posterior density samples are returned as matrices, requiring the user to perform additional computation to summarize results, assess model fit, and the like. **MCMCpack**, on the other hand, has a more R-like interface. We also view these packages as complementary to **MCMCpack**, as they provide estimation engines for useful models.

In addition to these packages that offer a comprehensive set of tools, there are a number of other available R packages that perform Bayesian inference for single classes of models. For a list of available packages, see the “Bayesian Inference” task view on the Comprehensive R Archive Network (Park, Martin, and Quinn 2010).

2.3. An overview of **MCMCpack** features

In addition to the model fitting functions mentioned above, **MCMCpack** has numerous features aimed at both researchers and instructors. For the sake of space we do not comprehensively demonstrate this functionality here. Rather, we highlight three features that may be of interest. We encourage interested users to consult the documentation for further details.

The **MCMCmetrop1R** function allows users to sample from a user-defined continuous density using a random walk Metropolis algorithm with a multivariate normal proposal distribution. This can be used to explore a posterior (or log-posterior) distribution, as well as any other target distribution of interest. See Gelman *et al.* (2003) and Robert and Casella (2004) for details of the random walk Metropolis algorithm. The sampler itself is coded in C++, so the iterations run quite fast. Users only have to provide the target density as an R function.

MCMCpack is sometimes used on large problems where parallel computation might afford

some advantages. While **MCMCpack** does not currently support parallelization within the Monte Carlo loop, for some problems it is useful to perform embarrassingly parallel simulations, e.g., sampling from the posterior density of the same model with twenty-five simultaneous processes. Doing so requires the availability of a random number generator that provides independent substreams of pseudo-random digits across processes. The default pseudo-random number generator in **MCMCpack** is the Mersenne twister (Matsumoto and Nishimura 1998). **MCMCpack** provides a default seed, which can be changed in any model-fitting function. The package also provides the generator of L’Ecuyer, Simard, Chen, and Kelton (2002), which provides independent substreams suitable for parallel computation. Again, the seed can be changed by the user, who only needs to provide a unique substream number to ensure independence. This generator works just fine with the powerful **snow** package (Tierney, Rossini, Li, and Sevcikova 2008).

To enhance the usability of **MCMCpack** in the classroom, the package also contains a number of “toy” instructional models. While the use of Monte Carlo simulation is not necessary to characterize the posterior density of these models, we find it useful to introduce Monte Carlo methods in contexts where analytical results are readily available. To wit, **MCMCpack** provides `MCmultinomialdirichlet` (Monte Carlo simulation from a multinomial likelihood with a Dirichlet prior), `MCnormalnormal` (Monte Carlo simulation from a normal likelihood with known variance and a normal prior), and `MCpoissongamma` (Monte Carlo simulation from a Poisson likelihood with a gamma prior).

3. An example of Bayesian linear regression

In this section, we look at how to fit a Bayesian linear model using `MCMCregress`. We use Wilkerson’s (1999) analysis of “killer” amendments in US Congress as an example. A killer amendment causes a bill, that would pass absent the amendment, to fail. According to Wilkerson (1999), a typical example of a killer amendment occurred in 1996,

...when the House was asked to approve an international shipbuilding trade agreement that had taken more than a decade to negotiate. During the debate, a majority party member, Herbert Bateman (R-VA), proposed adding 30 months of “transition assistance” for US shipbuilders to the agreement... Jennifer Dunn (R-WA) argued “Some individuals argue that no agreement is better than this agreement. In reality, if the Bateman amendment is adopted, that is exactly what we would have: No agreement” (Congressional Record, June 13 1996, H6321)... Ignoring Dunn’s warning, the House passed the amendment (278-149) and voted for the bill (325-100). Recognizing that the shipbuilding agreement was dead on arrival ... , the Senate never took it up. (pp. 544–545)

Thus, in order to achieve the preferred final result (an international shipbuilding trade agreement), legislative majorities are often forced to vote against an amendment which seems closer to their preferences (the Bateman amendment) than an original bill. Doing so is a classic example of strategic voting; i.e., voting for a less preferred choice at an early stage of a voting procedure to ultimately get a better outcome in the end.

Using a linear regression model estimated with ordinary least squares, Wilkerson shows that killer amendments are rare and hence legislative majorities seldom face situations that require

strategic voting. To investigate the existence of strategic voting, Wilkerson relies upon [Poole and Rosenthal's \(1997\)](#) measures that capture the fit of actual roll call voting positions to estimated unidimensional legislative preferences (NOMINATE, see also [Poole, Lewis, Lo, and Carroll 2011](#)). Among them, we focus on proportional reduction in error (PRE), which compares NOMINATE's success at predicting roll call voting positions with the success rate of simply assuming that everyone votes with the majority:

$$\text{PRE} = \frac{(\text{Minority vote} - \text{NOMINATE classification errors})}{(\text{Minority vote})}.$$

A high value of PRE for a particular roll call implies that the NOMINATE is accurately classifying the votes on that roll call compared to the prediction based on the majority voting. Wilkerson identifies five types of killer amendments and includes them as covariates: major weakening, strengthening, political cover, minor weakening, and new issue. Wilkerson also considers the sponsor. Amendments proposed by a member of the majority party are much more likely to be viewed as sincere attempts to improve the content of the legislation, rather than deliberate efforts to sabotage it. Lastly, Wilkerson controls for the chamber of introduction, in case there are chamber effects on sincerity of voting. The model to be estimated is:

$$y_i = \mathbf{x}'_i \boldsymbol{\beta} + \varepsilon_i \quad \varepsilon_i \sim \mathcal{N}(0, \sigma^2)$$

With priors:

$$b \sim \mathcal{N}_K(\mathbf{b}_0, \mathbf{B}_0^{-1}) \quad \sigma^2 \sim \mathcal{IG}(c_0/2, d_0/2)$$

To simulate from the posterior distribution of the model using `MCMCregress` we load Wilkerson's data and call `MCMCregress`.

```
R> library("MCMCpack")
R> load("killamdt.rda")
R> wilkerson <- MCMCregress(APRE1 ~ STRENGTH + COVER + WEAKMIN +
+   NEWISSUE + SPONSOR + CHAMBER, data = killamdt, b0 = 0,
+   B0 = 0.1, c0 = 2, d0 = 0.11, marginal.likelihood = "Chib95")
```

Note that the output from the call to `MCMCregress` has been placed in an `mcmc` object called `wilkerson`. By default, `MCMCregress` uses a noninformative prior for the coefficient parameters. To facilitate model comparison, here we use a weakly informative prior. The prior for $\boldsymbol{\beta}$ is specified using the `b0` and `B0` arguments, where `b0` is the prior mean and `B0` is the prior precision of $\boldsymbol{\beta}$. The prior for σ^2 is governed by two parameters `c0` and `d0`. We use a weakly informative prior (`c0 = 2` and `d0 = 0.11`) with a mean equal to the marginal variance of the APRE1 (0.11).

Before examining the results of our model, it is necessary to examine some diagnostics to assess whether the Markov chain has converged to its stationary distribution. The `plot` method for an `mcmc` object produces a trace plot and a density plot for each parameter. To save space, we only report these plots for first two parameters in [Figure 1](#). There are also a number of standard convergence diagnostics implemented in `coda` one should check before using the posterior density sample for inference.

The `coda summary` method can be used to summarize an the `mcmc` object produced by `MCMCregress`.

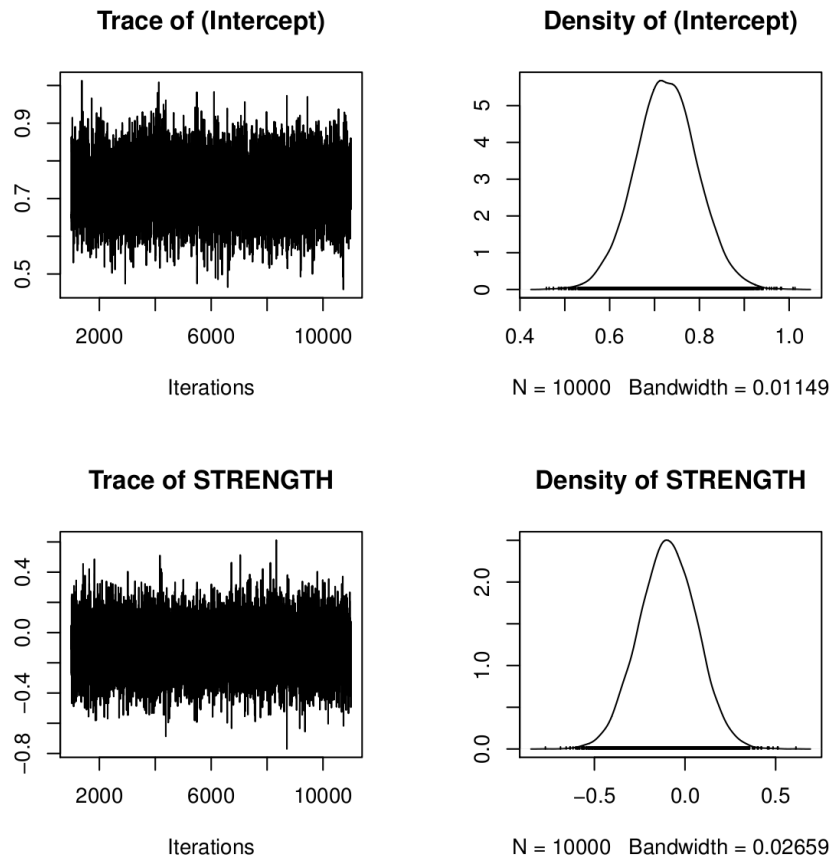


Figure 1: Trace and density plots for two coefficients in the `MCMCregress` example.

```
R> summary(wilkerson)
```

```
Iterations = 1001:11000
Thinning interval = 1
Number of chains = 1
Sample size per chain = 10000
```

1. Empirical mean and standard deviation for each variable, plus standard error of the mean:

	Mean	SD	Naive SE	Time-series SE
(Intercept)	0.72631	0.06934	0.0006934	0.0007085
STRENGTH	-0.09573	0.15828	0.0015828	0.0015134
COVER	-0.47822	0.15122	0.0015122	0.0015871
WEAKMIN	-0.28246	0.07989	0.0007989	0.0008861
NEWISSUE	-0.49685	0.11002	0.0011002	0.0009653
SPONSOR	-0.14988	0.07425	0.0007425	0.0007096
CHAMBER	0.18924	0.07666	0.0007666	0.0007562

	Mean	St. Dev	Lower	Upper
Constant	0.7263	0.0693	0.5902	0.8640
STRENGTH	-0.0957	0.1583	-0.4037	0.2115
COVER	-0.4782	0.1512	-0.7734	-0.1847
WEAKMIN	-0.2825	0.0799	-0.4395	-0.1274
NEWISSUE	-0.4968	0.1100	-0.7128	-0.2811
SPONSOR	-0.1499	0.0743	-0.2957	-0.0050
CHAMBER	0.1892	0.0767	0.0384	0.3422
σ^2	0.0749	0.0139	0.0523	0.1069

Table 1: Analysis of killer amendments (Wilkerson 1999). The dependent variable is proportional reduction in error (PRE) when the minority vote is used as benchmark. Lower and Upper indicate central 95 percent Bayesian credible intervals.

```
sigma2      0.07493 0.01387 0.0001387      0.0001681
```

2. Quantiles for each variable:

	2.5%	25%	50%	75%	97.5%
(Intercept)	0.59017	0.68042	0.72596	0.77207	0.864040
STRENGTH	-0.40370	-0.20210	-0.09570	0.01191	0.211530
COVER	-0.77342	-0.57890	-0.48005	-0.37593	-0.184722
WEAKMIN	-0.43952	-0.33595	-0.28221	-0.22865	-0.127368
NEWISSUE	-0.71283	-0.57028	-0.49699	-0.42332	-0.281150
SPONSOR	-0.29571	-0.19903	-0.14914	-0.10004	-0.005032
CHAMBER	0.03841	0.13804	0.18912	0.24031	0.342175
sigma2	0.05232	0.06512	0.07343	0.08294	0.106865

One of main advantage of Bayesian methods is that posterior inference is quite straightforward using MCMC output since the methods provide direct samples of parameters of interest. In this example, Wilkerson's primary quantity of interest is in whether PRE is higher for minority-sponsored major weakening amendments than for majority-sponsored major weakening amendments. Wilkerson considers a higher PRE for minority-sponsored major weakening amendments than for majority-sponsored major weakening amendments as major evidence for killer amendments.

We can answer this question using our posterior samples. In our data set, SPONSOR is coded 1 if majority-sponsored amendments and WEAKMIN is coded 1 if minor weakening amendments, and 0 otherwise. Thus, the probability that PRE is higher for minority-sponsored major weakening amendments than for majority-sponsored major weakening amendments can be shown by comparing a posterior distribution of Constant with a posterior distribution of Constant + SPONSOR.

Figure 2 demonstrates that minority-sponsored major weakening amendments are better predicted by NOMINATE scores than majority-sponsored major weakening amendments. This result is consistent with Wilkerson's finding that members of the majority party are more likely to give serious consideration to the substance and strategic implications of amendments

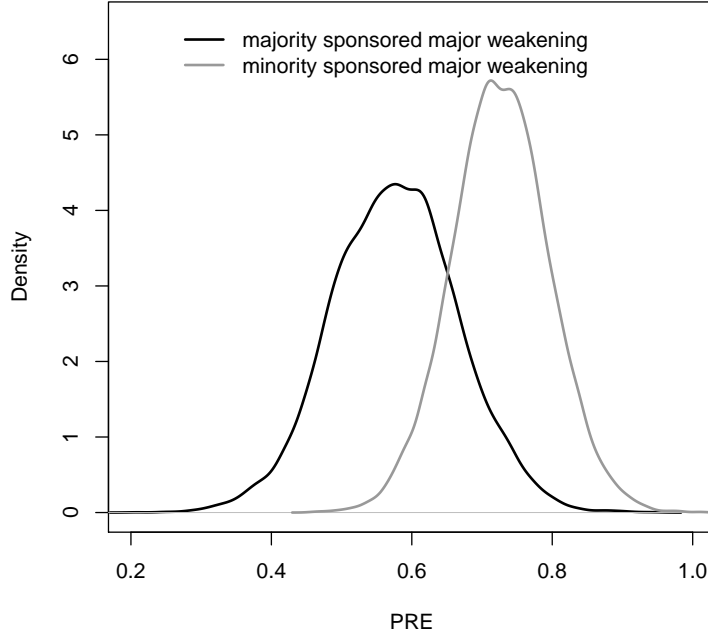


Figure 2: PRE for various types of amendments and sponsors.

offered by fellow party members.

MCMCpack also provides a tool for Bayesian model comparison (for some of its models): Bayes factors. Bayes factors can be used to compare any model of the same data. Suppose that the observed data y could have been generated under one of two models \mathcal{A}_1 and \mathcal{A}_2 . A natural thing to ask from the Bayesian perspective is: “What is the posterior probability that \mathcal{A}_1 is true assuming either \mathcal{A}_1 or \mathcal{A}_2 is true?” Using Bayes theorem we can write:

$$\Pr(\mathcal{A}_1|y) = \frac{p(y|\mathcal{A}_1) \Pr(\mathcal{A}_1)}{p(y|\mathcal{A}_1) \Pr(\mathcal{A}_1) + p(y|\mathcal{A}_2) \Pr(\mathcal{A}_2)}$$

It is instructive to look at the posterior odds in favor of one model (say \mathcal{A}_1):

$$\frac{\Pr(\mathcal{A}_1|y)}{\Pr(\mathcal{A}_2|y)} = \frac{p(y|\mathcal{A}_1)}{p(y|\mathcal{A}_2)} \times \frac{\Pr(\mathcal{A}_1)}{\Pr(\mathcal{A}_2)}.$$

What this means is that if we want to move from the prior odds in favor of \mathcal{A}_1 to the posterior odds in favor of \mathcal{A}_1 we simply multiply the prior odds by:

$$B_{12} = \frac{p(y|\mathcal{A}_1)}{p(y|\mathcal{A}_2)}$$

which is called the *Bayes factor* for \mathcal{A}_1 relative to \mathcal{A}_2 . B_{12} provides a measure of the strength of evidence for \mathcal{A}_1 over \mathcal{A}_2 . For a thorough discussion of Bayes factors see [Kass and Raftery \(1995\)](#).

	Model 1	Model 2	Model 3	Wilkerson
Model 1	0.00	-3.11	-1.39	-0.52
Model 2	3.10	0.00	1.71	2.58
Model 3	1.39	-1.72	0.00	0.86
Wilkerson	0.52	-2.58	-0.86	0.00

Table 2: A matrix of Bayes factors (on the natural log scale) for the [Wilkerson \(1999\)](#) replication. The dependent variable is PRE.

	Mean	St. Dev	Lower	Upper
(Intercept)	0.7888	0.0591	0.6720	0.9042
STRENGTH	-0.2681	0.1572	-0.5765	0.0385
COVER	-0.4741	0.1583	-0.7849	-0.1676
WEAKMIN	-0.3059	0.0839	-0.4687	-0.1411
NEWISSUE	-0.4518	0.1060	-0.6607	-0.2404
σ^2	0.0848	0.0157	0.0593	0.1210

Table 3: Killer amendments results from model 2. Lower and Upper indicate central 95 percent Bayesian credible intervals.

The key ingredients necessary for calculating Bayes factors (and posterior model probabilities) are the marginal likelihoods. There are several efficient methods ([Tierney and Kadane 1986](#); [Chib 1995](#); [Chib and Jeliazkov 2001](#)) for these calculations. `MCMCregress` computes marginal likelihoods using either the Laplace approximation ([Tierney and Kadane 1986](#)) or [Chib's \(1995\)](#) method to compute the marginal likelihoods. Using [Chib's](#) method, we compare three alternative models with [Wilkerson's](#) model using comparable prior distributions.

```
R> model1 <- MCMCregress(APRE1 ~ STRENGTH + COVER, data = killamdt,
+   mcmc = 10000, b0 = 0, B0 = 0.1, c0 = 2, d0 = 0.11,
+   marginal.likelihood = "Chib95")
R> model2 <- MCMCregress(APRE1 ~ STRENGTH + COVER + WEAKMIN +
+   NEWISSUE, data = killamdt, mcmc = 10000, b0 = 0,
+   B0 = 0.1, c0 = 2, d0 = 0.11, marginal.likelihood = "Chib95")
R> model3 <- MCMCregress(APRE1 ~ SPONSOR + CHAMBER, data = killamdt,
+   mcmc = 10000, b0 = 0, B0 = 0.1, c0 = 2, d0 = 0.11,
+   marginal.likelihood = "Chib95")
R> BF <- BayesFactor(model1, model2, model3, wilkerson)
R> summary(BF)
```

The output from `BayesFactor` shows a matrix of Bayes Factors for all pairs of models specified in the argument. [Table 2](#) shows that there is positive evidence or better to support model 2 over all other models considered. That is, information about the sponsorship and the chamber of introduction does not help explain the variation of PRE given the information on five types of killer amendments. We report a posterior summary of model 2 in [Table 3](#) for interested readers.

Framework	Cycle phases			
Goldstein	Stagnation	Rebirth	Expansion	War
	1816	1831	1849	1861
	1873	1885	1894	1911
	1921	1934	1941	1969
Level of conflict	Low conflict	High conflict	High conflict	Highest conflict
Wallerstein	Victory	Maturity	Decline	Ascent
	1816	1850	1873	1897
	1913	1945	1967	
Level of conflict	Scattered conflict	Lowest conflict	Competitiveness	Highest conflict
Modelski and Thompson	World Power	Delegitimation	Deconcentration	Global war
	1816	1850	1874	1915
	1946	1974		
Level of conflict	Lowest conflict	High conflict	Low conflict	Highest conflict
Gilpin	UK hegemony	No hegemony	US hegemony	
	1815	1873	1945	
Level of conflict	Low conflict	High conflict	Low conflict	

Table 4: Starting dates of cycle phases and predicted level of conflict. Source: [Pollins \(1996\)](#).

4. A Bayesian Poisson changepoint model example

We show how to fit a Poisson change-point model using `MCMCpoissonChange` in this section. Generally speaking, a change-point model is a time series model in which data are generated from multiple regimes or states. Main quantities of interests in the change-point analysis are the number, timing, and effects of regime changes. [Chib \(1998\)](#) reinterprets the change-point problem in the context of Markov mixture model in which observations are assumed to be drawn from latent states. The sampling algorithm of latent states in `MCMCpoissonChange` is based on [Chib \(1998\)](#).

Using conjugate prior distributions, we can write a Bayesian Poisson change-point model without covariates as follows:

$$\begin{aligned}
 y_t &\sim \mathcal{Poisson}(\lambda_i), \quad i = 1, \dots, k \\
 \lambda_i &\sim \mathcal{G}(c_0, d_0), \quad i = 1, \dots, k \\
 p_{ii} &\sim \mathcal{Beta}(\alpha, \beta), \quad i = 1, \dots, k.
 \end{aligned}$$

When there is no covariate, all parameters can be sampled using the Gibbs sampler as shown in [Chib \(1998\)](#). When users have covariates to model the expected count, `MCMCpoissonChange` uses the auxiliary mixture sampling method suggested by [Frühwirth-Schnatter and Wagner \(2006\)](#) to sample regression parameters. The details of the algorithm are discussed in [Park \(2010\)](#).

In this section, we employ `MCMCpoissonChange` to revisit the empirical test of cyclical theories of great-power war by [Pollins \(1996\)](#). Pollins identifies four core frameworks in the study of cyclical patterns of international conflicts based on “(1) the relative importance of *global political versus economic subsystems*, (2) the *dating of the phases* they identify in the cycle, and (3) predictions regarding the *level of conflict* within particular phases of their respective

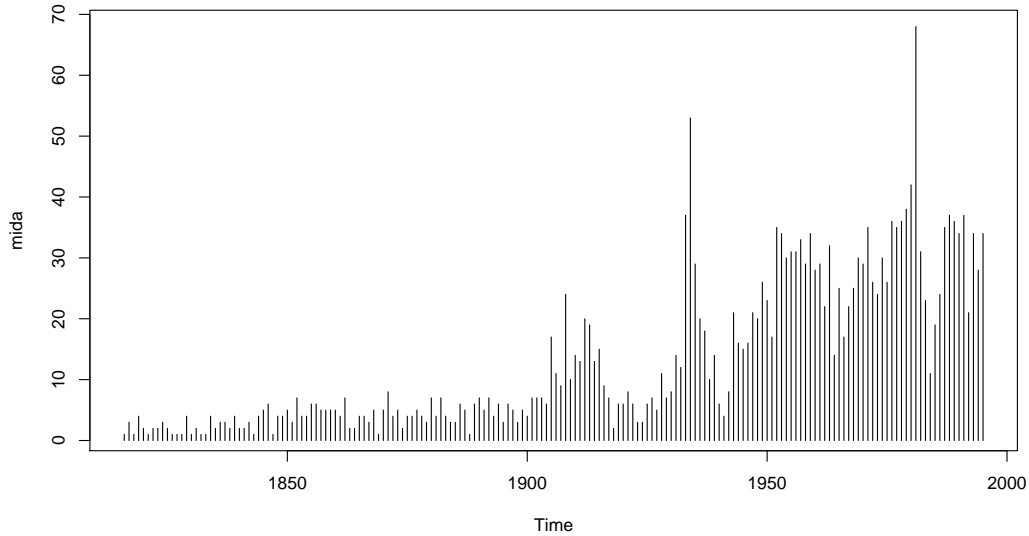


Figure 3: Militarized interstate disputes data.

	Mod. 1	Mod. 2	Mod. 3	Mod. 4	Mod. 5	Mod. 6	Mod. 7	Mod. 8
Mod. 1	0.0	-60.5	-80.0	-93.3	-96.0	-111.61	-109.02	-106.46
Mod. 2	60.5	0.0	-19.5	-32.9	-35.6	-51.12	-48.53	-45.96
Mod. 3	80.0	19.5	0.0	-13.3	-16.0	-31.59	-29.00	-26.44
Mod. 4	93.3	32.9	13.3	0.0	-2.7	-18.26	-15.67	-13.11
Mod. 5	96.0	35.6	16.0	2.7	0.0	-15.56	-12.97	-10.41
Mod. 6	111.6	51.1	31.6	18.3	15.6	0.00	2.59	5.15
Mod. 7	109.0	48.5	29.0	15.7	13.0	-2.59	0.00	2.56
Mod. 8	106.5	46.0	26.4	13.1	10.4	-5.15	-2.56	0.00

Table 5: A matrix of Bayes factors (on the natural log scale) for the Poisson change-point analysis of militarized interstate disputes between 1816 and 1995.

cycles” (Pollins 1996, 106). These four core frameworks are a theory of hegemony by Gilpin (1981), a theory of the world system by Wallerstein (1983), the long wave in the global economy by Goldstein (1988), and the world leadership cycle by Modelski and Thompson (1988).

These four theories give different weights on the role of global economic order and global political order in explaining the patterns of international conflicts. For example, while the theory of Long Wave and the theory of world system argue that the growth and stagnation of global economy (Goldstein 1988) or economic hegemon in the system (Wallerstein 1983) generate different levels of international conflicts, Modelski and Thompson (1988) and Gilpin (1981) consider the rise and decline of global leadership or a hegemonic power to play a central role in producing international conflicts. These four theories are further diverged in their predictions on the specific cycle phases of international conflicts as shown in Table 4. Following Pollins (1996), we use the annual number of military conflicts from the militarized interstate disputes (MID) data (Jones, Bremer, and Singer 1996) to detect the number and

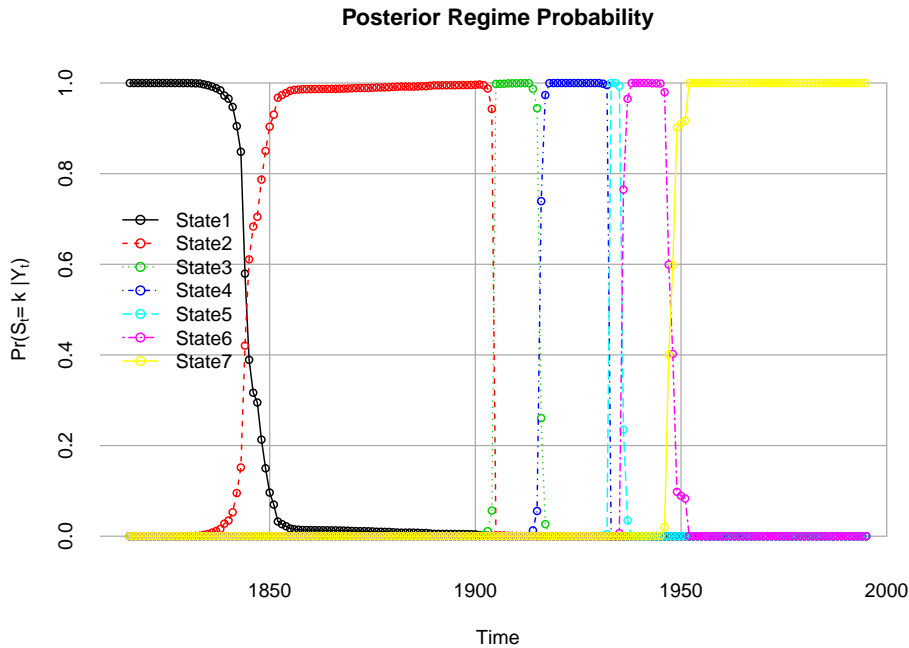


Figure 4: Posterior probabilities of states: each circled line shows the time-varying probabilities of each regime. For example, the probability of Regime 1 (the solid circled line) starts from 1 in the beginning of the sample period and drops to 0 around the 1840s. Estimates are from the Poisson change-point analysis of militarized interstate disputes from 1816 to 1995.

timing of cycle phases in international conflicts.

To implement `MCMCpoissonChange`, a user needs to specify data, the number of states (m), and hyperparameters of λ ($c0$ and $d0$). Users must provide hyperparameters of λ ($c0$ and $d0$) for model comparison using Bayes factors. Marginal likelihoods are not identified under diffuse priors. Thus, users should avoid diffuse priors for model comparison. In this specific example, we set $c0 = 13$ and $d0 = 1$ since the mean of MID is about 13. The following R commands implement eight Poisson change-point models for the militarized interstate disputes data.

```
R> model1 <- MCMCpoissonChange(mida, m = 1, c0 = 13, d0 = 1,
+   marginal.likelihood = c("Chib95"))
R> model2 <- MCMCpoissonChange(mida, m = 2, c0 = 13, d0 = 1,
+   marginal.likelihood = c("Chib95"))
R> model3 <- MCMCpoissonChange(mida, m = 3, c0 = 13, d0 = 1,
+   marginal.likelihood = c("Chib95"))
R> model4 <- MCMCpoissonChange(mida, m = 4, c0 = 13, d0 = 1,
+   marginal.likelihood = c("Chib95"))
R> model5 <- MCMCpoissonChange(mida, m = 5, c0 = 13, d0 = 1,
+   marginal.likelihood = c("Chib95"))
R> model6 <- MCMCpoissonChange(mida, m = 6, c0 = 13, d0 = 1,
+   marginal.likelihood = c("Chib95"))
R> model7 <- MCMCpoissonChange(mida, m = 7, c0 = 13, d0 = 1,
+   marginal.likelihood = c("Chib95"))
```

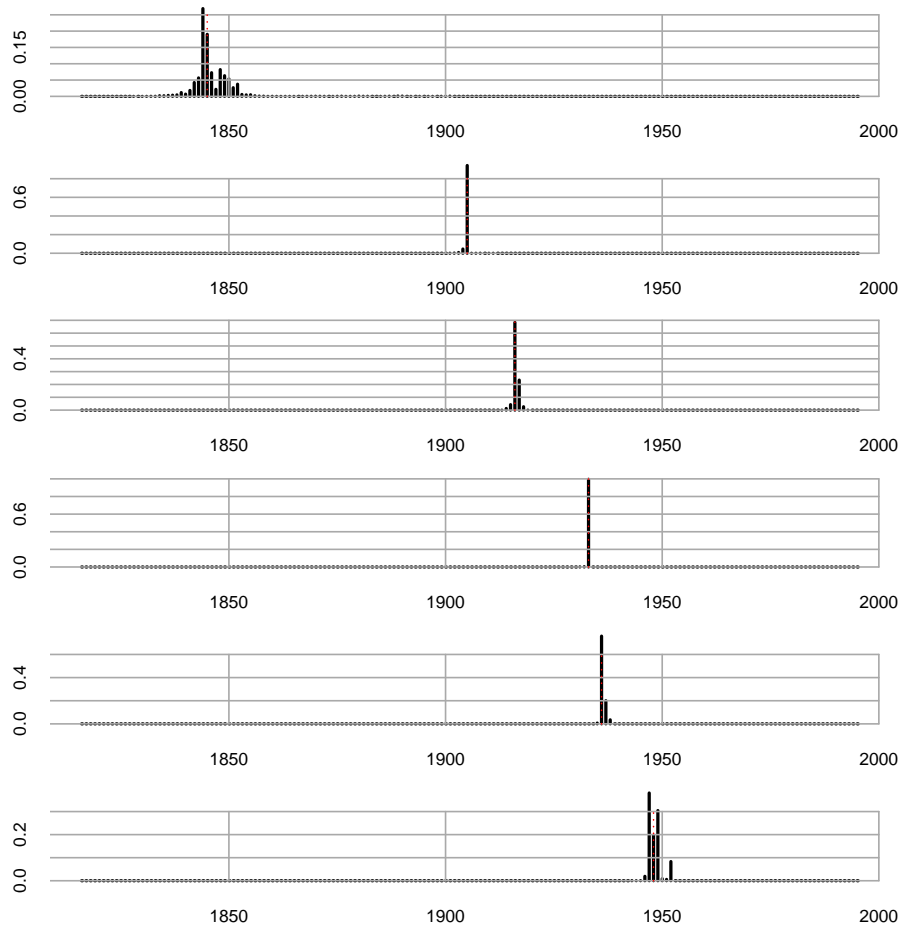


Figure 5: Posterior density of regime change probability: each panel shows the posterior distribution of regime change probabilities. For example, the top panel shows the probability density of the regime change from Regime 1 to Regime 2.

```
R> model8 <- MCMCpoissonChange(mida, m = 8, c0 = 13, d0 = 1,
+   marginal.likelihood = c("Chib95"))
```

The results of the model comparison are summarized in Table 5. The ratio of marginal likelihoods shows that the sixth change-point model is favored over the alternatives. Using Jeffrey's rule, which provides guidelines as to the strength of a Bayes factor, we can conclude that there is positive evidence for the sixth change-point model over the alternatives considered.

`plotState()` generates a plot of posterior state probabilities and `plotChangepoint()` plots of posterior probabilities of six change-points as shown in Figures 4 and 5, respectively. Both graphs indicate that critical shifts in the level of international conflicts are concentrated in the first half of the twentieth century and cyclical patterns predicted by the four theoretical models are not supported by the evidence. However, the picture provided by the Poisson change-point analysis is closest to Gilpin's theory of hegemony which predicts high levels of conflicts in the absence of a world hegemon between 1873 and 1945.

5. Conclusion

MCMCpack remains a work in progress. While it is difficult to commit to specific future development efforts, we hope to implement additional models others will find useful in their research and teaching. The ability to estimate marginal likelihoods and compute Bayes factors is a new feature in **MCMCpack**, and is only currently available for a handful of models. We hope to implement this for a wider range of models in the future. Of course, since **MCMCpack** is open source, others can build on this code-base to implement models of their choice.

In the next few years Jong Hee Park will develop a number of models to be used for Bayesian time series analysis, including change-point models and state space models. He intends to expand some of these models to include covariates, and to provide various methods to enhance usability for this class of models. Finally, while the **coda** package provides a great deal of power for convergence checking and posterior density summarization, it does not provide adequate methods for formal model assessment, including prior and predictive checks, Bayes factors, and Bayesian model averaging (Hoeting, Madigan, Raftery, and Volinsky 1999). Our hope is develop software to make these tools as easy to use as possible.

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