Fair Pensions

Financial Fairness in Collective Defined Contribution schemes

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Netspar Pension Day, October 31, 2019
Introduction and context

Two archetypal pension schemes:
- Collective Defined Benefit (DB)
- Individual Defined Contribution (DC)

Particularly DB based systems currently struggle:
- Low interest rates
- Increasing longevity
Motivation

DB sponsors are seeking ways to offload the risk.

- Obvious way: Abolish DB, switch to DC.

Several countries however consider or have implemented an alternative:

**Collective Defined Contribution (CDC)**
Motivation (2): CDC characterization and concern

CDC is like traditional DB, except that participants share in funding risk through benefit level adjustments.

CDC retirement income is risky, like DC, but the participants claim is expressed as a future benefit level paid from a collective asset pool, instead of a personal asset pool.

Important concern with CDC: (generational) fairness between participants in the collective pool.
Motivation (3): CDC seen in multiple countries

**UK:** government consultation on CDC earlier this year
  - Particular attention for generational fairness.

**The Netherlands:** CDC in this months’ ‘pension agreement’.
  - A lot of political debate on generational fairness and benefit valuation

**Canada:** Already features DB schemes with adjustable benefits
Objective: provide a formal analysis of fairness in CDC pension schemes

“Is it possible to design a fair CDC scheme?”
CDC schemes pool and reallocate risks of participants

Risks can be classified into two categories:

- Traded risk (i.e., equity risk, interest rate risk)
- Non-traded risk (i.e., longevity risk)

We study whether pooling and reallocating traded risk in a CDC pension scheme is fair using no-arbitrage arguments
Key finding

Fairness in CDC contracts is feasible but not straightforward:

- CDC schemes with benefit smoothing are fair only if they use a default-free market discount rate
- Fairness requires a specific horizon-dependent benefit adjustment process

Intuition: Using future target benefit levels as unit of account (as in DB), while benefits are stochastic (as in DC), there is a complex exposure to interest rate risk which cannot easily be ‘priced’
Agenda

- Assumptions and definitions
- Example of an unfair CDC scheme
- Configurations of fair CDC schemes
- Conclusion
Technical assumptions

- Time is continuous
- The asset market is arbitrage-free
- Ito-processes describe all variables
- Vector $dz$ of Brownian increments represents all sources of traded risk
- Default-free zero-coupon bonds are available for all maturities

So,

- No specific assumptions about risk-premia, volatilities, etc.
- Restriction: variables do not jump (results generalize though)
Definition 1: The CDC scheme

- There is a single collective pool of wealth with market value $A(t)$
- Each participant accrues a ‘target benefit’: $b_i(t)$

- An **adjustment rule** describes how target benefit levels change as a function of the **funding ratio**: $F(t)$
- $F(t)$ equals $A(t)$ divided by the **book value** of the target benefits: $L(t)$
- $L(t)$ is found by applying a **discount factor**, $D(t, \tau)$, to target benefits.

- Retirees receive an income flow equal to their target benefit: $b_i(t) dt$
- Premiums paid always equal the book value of new target benefit accrual.
- Value transfers always at book value of target benefit.
Definition 2: Completeness
The CDC scheme is complete if after benefit adjustment \( A(t) = L(t) \) for all \( t \)

We will impose completeness throughout the analysis.
Illustration: DB v.s. CDC benefit over the life-cycle

(a) DB benefit

(b) CDC target benefit
**Definition 3: Fairness**

A pension scheme is fair if all participants receive an arbitrage-free return on their contributions.

If the return is not arbitrage-free, this means that there are participants who could be strictly better off by not contributing to the scheme.
Actual return equals ‘return on book value’

Consider a participant that pays a one-time contribution \( C(t) \) at time \( t \). In exchange she gets target benefit \( b(t, \tau) \) which at time of accrual has book value \( V(t, \tau) \):

\[
C(t) = V(t, \tau) = b(t, \tau)D(t, \tau)
\]

Now, there are two options:

- she leaves the scheme early at time \( t < s < \tau \) and receives: \( V(s, \tau) \)
- she stays in the scheme until time \( s = \tau \) and receives: \( b(\tau, \tau) = V(\tau, \tau) \)

Total return during the period of participation equals the return on book value:

\[
\frac{V(s, \tau)}{C(t)} = \frac{V(s, \tau)}{V(t, \tau)}
\]
Fairness requires book returns to be market consistent

Fairness requires that the total return is market consistent:

$$\mathbb{E}^Q \left[ \frac{V(s, \tau)}{V(t, \tau)} \exp \left( - \int_t^s r(\nu) d\nu \right) \right] = 1 \quad \forall t, \forall s, \forall \tau$$

where $r(t)$ is the instantaneous risk-free rate.

Since this has to hold for all $t$ and $s$, this implies (taking differences):

$$\mathbb{E}^Q \left[ \frac{dV(t, \tau)}{V(t, \tau)} \right] = r(t) dt \quad \forall t, \forall \tau$$
The instantaneous book return has three components

Remember (suppressing all $t$’s):

$$V(\tau) = b(\tau)D(\tau)$$

So, by Ito’s lemma, the ‘book return’ on a target benefit is:

$$\frac{dV(\tau)}{V(\tau)} = \frac{db(\tau)}{b(\tau)} + \frac{dD(\tau)}{D(\tau)} + \frac{db_i}{b_i} \frac{dD(\tau)}{D(\tau)}$$

- ‘Book return’
- Benefit adjustment
- Discount factor change
- Covariance
A trivial example that turns out to be fair

Suppose we pick a fixed discount rate, $\lambda$:

$$D(t, \tau) = \exp \left( -\lambda \cdot (\tau - t) \right), \quad \frac{dD(\tau)}{D(\tau)} = \lambda dt$$

And set the benefit adjustment equal to:

$$\frac{db(\tau)}{b(\tau)} = \frac{dA}{A} - \lambda dt$$
A trivial example that turns out to be fair (2)

The instantaneous book return in this case is:

\[
\frac{dV(\tau)}{V(\tau)} = \frac{dA}{A} - \lambda dt + \lambda dt + 0
\]

‘Book return’ Benefit adjustment Discount factor change Covariance

All participants simply receive the return on assets, which is clearly fair:

\[
\mathbb{E}^Q \left[ \frac{dV(\tau)}{V(\tau)} \right] = \mathbb{E}^Q \left[ \frac{dA}{A} \right] = r \, dt
\]
A less trivial example that turns out to be unfair

Now suppose we use the default-free discount rate:

\[
D(t, \tau) = \underbrace{P(t, \tau)}_{\text{Price of ZC Bond}}, \quad \frac{dD(\tau)}{D(\tau)} = \frac{dP(\tau)}{P(\tau)} = \mu P(\tau) dt + \sigma(\tau)'d\mathbf{z}
\]

And target benefits are adjusted uniformly for everybody:

\[
\frac{db(\tau)}{b(\tau)} = \frac{d\tilde{F}}{F}
\]
A less trivial example that turns out to be unfair (2)

For illustration, assume:

- Interest rate dynamics: Vasicek model
- Duration of assets: 10
- Duration of liabilities: 20
- Interest rate volatility: 1 percent per annum
- Interest rate half-time: 20 years
Intergenerational unfairness can be significant
What causes this intergenerational transfer?

First some extra notation:

Return on assets:

\[
\frac{dA}{A} = \mu_A(t)dt + \sigma_A(t)\,dz
\]

Change in liabilities before benefit adjustment (return on matching portfolio):

\[
\frac{d\tilde{L}}{\tilde{L}} = \mu_L dt + \sigma'_L \,dz
\]
What causes this intergenerational transfer? (2)

Remember that the return is:

\[
\frac{dV(\tau)}{V(\tau)} = \frac{dF}{F} + \frac{dP(\tau)}{P(\tau)} + \frac{dF}{F} \frac{dD(\tau)}{D(\tau)}
\]

'Book return' Benefit adjustment Discount factor change Covariance

After solving for \(dF\), we find that in this case:

\[
\frac{dV(\tau)}{V(\tau)} = \frac{dA}{A} - \frac{dL}{L} + \frac{dP(\tau)}{P(\tau)} + \left(\sigma_A - \sigma_{\tilde{L}}\right)' \left(\sigma_P(\tau) - \sigma_{\tilde{L}}\right) dt
\]
What causes this intergenerational transfer? (3)

So, we have in this case that:

\[
\mathbb{E}^Q \left[ \frac{dV(\tau)}{V(\tau)} \right] = \mathbb{E}^Q \left[ \frac{dA}{A} - \frac{d\tilde{L}}{\tilde{L}} + \frac{dP(\tau)}{P(\tau)} + (\sigma_A - \sigma_{\tilde{L}})'(\sigma_P(\tau) - \sigma_{\tilde{L}})dt \right]
\]

Note that \( \frac{dA}{A} \), \( \frac{d\tilde{L}}{\tilde{L}} \) and \( \frac{dP(\tau)}{P(\tau)} \) are all traded assets, hence:

\[
= r(t)dt - r(t)dt + r(t)dt + \mathbb{E}^Q \left[ (\sigma_A - \sigma_{\tilde{L}})'(\sigma_P(\tau) - \sigma_{\tilde{L}})dt \right]
\]

\[
= r(t)dt + (\sigma_A - \sigma_{\tilde{L}})'(\sigma_P(\tau) - \sigma_{\tilde{L}})dt \neq r \, dt
\]

'Troublemaker'
Meet the ‘Troublemaker’

The scheme that uses a default-free discount rate and adjusts uniformly is only fair if:

\[
\left( \sigma_A - \sigma_{\tilde{L}} \right)' \cdot \left( \sigma_P(\tau) - \sigma_{\tilde{L}} \right) = 0
\]

A-L mismatch risk · Liability horizon mismatch

This condition is met if:

- Discount rate risk perfectly hedged \((\sigma_A = \sigma_{\tilde{L}} \rightarrow DB)\)
- All individuals have the same horizon \((\sigma_P(\tau) = \sigma_{\tilde{L}})\)
Under default-free market rate discounting, fairness requires benefit adjustments to be horizon-dependent.

We consider two options:

1. Proportional sharing in mismatch risk
2. Smoothing of mismatch risk
Proportional sharing in mismatch risk

The CDC scheme will be fair if we add a correction term to the adjustment process:

\[
\frac{db(\tau)}{b(\tau)} = \frac{dA}{A} - \frac{d\tilde{L}}{\tilde{L}} - \left(\sigma_A - \sigma_{\tilde{L}}\right)' \cdot \sigma_D(\tau)dt
\]

ALM - individual liability covariance

The return in this case becomes:

\[
\frac{dV(\tau)}{V(\tau)} = \frac{dA}{A} - \frac{d\tilde{L}}{\tilde{L}} + \frac{dP(\tau)}{P(\tau)}
\]
Smoothing mismatch risk

The CDC scheme will be fair if we add a correction term to the adjustment process:

\[
\frac{db_i(\tau)}{b_i(\tau)} = \alpha(\tau) \left[ \frac{dA}{A} - \frac{d\tilde{L}}{\tilde{L}} - (\sigma_A - \sigma_{\tilde{L}})' \cdot \sigma_P(\tau) dt \right]
\]

Smoothing parameter

Discrete time equivalent

The return in this case becomes:

\[
\frac{dV(\tau)}{V(\tau)} = \alpha(\tau) \left[ \frac{dA}{A} - \frac{d\tilde{L}}{\tilde{L}} \right] + \frac{dP(\tau)}{P(\tau)}
\]
Default free market rate discounting: conclusions

- Fairness requires **horizon-dependent** benefit adjustment
- Default free market rate discounting allows for **fair smoothing**
Could a CDC also use a different discount rate?

Some argue that, since the benefit cash-flows are not guaranteed, it is appropriate to use a discount rate that is not default-free.

The problem with a ‘non-default-free’ market interest rate:
- Value of benefit payment on due date: $b(\tau)D(\tau, \tau)$
- Default risk implies: $D(\tau, \tau)$ may be smaller than one.
- Actual payment is $b(\tau)$, which implies that book and market value may diverge.

So, we need that $D(\tau, \tau) = 1$ with certainty.
Some alternatives consistent with $D(\tau, \tau) = 1$:

1. Default-free market interest rate with deterministic spread:

   $$D(\tau) = P(\tau) \times e^{-\int_t^\tau \lambda(s, \tau) ds}$$

2. Non-stochastic discount rate:

   $$D(\tau) = e^{-\int_t^\tau \lambda(s, \tau) ds}$$
Both alternatives are fair only if:
- No smoothing, $\alpha(\tau) = 1$
- and the spread/rate is a constant, $\lambda(t, \tau) = \bar{\lambda}$

If the scheme does smooth (the only relevant case?):
- Fairness requires that the benefit adjustment process is appended with additional correction terms.
Final consideration: Fairness ≠ Optimality

This analysis focused on fairness:
- CDC scheme design without ex-ante redistribution of wealth

Policy makers should also care about optimality:
- How to achieve an optimal exposure to risk for all individuals?

From an optimality perspective, CDC is seems to be a unnecessary restriction on the life-cycle investment strategy.
Implied life-cycle: Stock exposure
Implied life-cycle: Duration

The graph illustrates the implied duration (years) versus age for assets. The lines represent different scenarios:

- Blue line: with smoothing
- Red line: without smoothing
- Dashed line: Duration of assets

Current age is indicated on the x-axis, with age increasing from left to right. The y-axis shows the implied duration in years, decreasing as age increases.
Appendix: Fair benefit adjustment in discrete time

\[ \% \Delta b_{\tau,t+1} = \left( \frac{A_{t+1}}{A_t} - \frac{\tilde{L}_{t+1}}{L_t} \right) \frac{P_{\tau,t}}{P_{\tau,t+1}} \]

- \( P_{\tau,t} \): Value of zero-coupon bond with maturity \( \tau \)

Horizon correction
Appendix: Fair smoothing in discrete time

\[ \% \Delta b_{\tau,t+1} = \alpha_{\tau} \left( \frac{A_{t+1}}{A_t} - \frac{\bar{L}_{t+1}}{L_t} \right) \frac{P_{\tau,t}}{P_{\tau,t+1}} \]

- \( P_{\tau,t} \): Value of zero-coupon bond with maturity \( \tau \)

Horizon correction