Increasing the capital income tax may lead to faster growth

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Abstract

According to conventional economic wisdom, capital income taxes should be low. The purpose of this paper is to cast doubt on this general conclusion and to show that theory can also point in the opposite direction. The paper shows that under rather mild conditions, higher capital income taxes lead to faster growth in an overlapping generations economy with endogenous growth. Government expenditures are fixed as a fraction of GNP and are financed with labor income taxes as well as capital income taxes. Since capital income accrues to the old, taxing it relieves the tax burden on the young and leaves them with more income out of which to save. The net effect on savings is positive, if the interest elasticity of savings is sufficiently low, which it seems to be according to several estimates found in the literature. The basic argument is not seriously challenged by a grandfather clause for initial capital or by the old receiving some labor income as well. Extending the model to allow for multiple periods of lives, however, can overturn our results and support the conventional wisdom instead.

JEL classification: H21; H23; E62; E21; E22; E25

Keywords: Capital income taxation; Overlapping generations; Endogenous growth

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1. Introduction

Economists often argue strongly in favor of a low or zero capital income tax, see e.g. Cline (1986), Sinn (1985), Lucas (1990) or Feldstein (1995a). Lucas, for example, states, that “eliminating capital income taxation would increase the capital stock by about 35 percent,” delivering “the largest genuinely free lunch I have seen in 25 years in this business”. A low capital income tax increases the private return to capital, thus encouraging savings, investment and growth, so the argument usually goes. The purpose of this paper is to cast doubt on this conventional wisdom.

The empirical support for low capital income taxes is much less clear cut than one would wish it to be. Examining the capital gains realizations elasticities with respect to the capital gains tax, Burman and Randolph (1994) recently reconciled the strong negative effect found in micro data with the time series evidence, that capital gains are relatively unresponsive. They find that the strong negative effect is essentially a temporary tax timing effect, while the permanent effect, which is relevant for the discussion about growth, is not significantly different from zero and can easily be positive in some regressions. These tax timing effects probably also explain the rather large responses to the tax reform act of 1986, documented by Feldstein (1995c). Turning to savings, the murky aggregate time series evidence can already be seen in Fig. 1, echoing the point raised in Minarik (1992): a plot of the U.S. personal savings rate versus the U.S. capital income tax rate shows, if anything, a remarkable degree of positively correlated movement of these two series. Contrary to the conventional wisdom, tax cuts seem to be accompanied by a falling rather than a rising personal savings rate. While the issue is certainly more complicated than shown by this simple figure, it nonetheless gives some taste for how difficult it is to convincingly support the conventional wisdom with aggregate data. Venti and Wise (1990) and others have thus turned to examine evidence based on individual retirement accounts, while Feldstein (1995b) examines the capital income tax inherent in college scholarship rules and its effect on savings, which he finds to be substantial. A cross-country study might be desirable, but does not seem to be available yet. Persson and Tabellini (1994) present some growth regressions to find support for their thesis that inequality and thus capital income tax hikes lead to lower growth. While their regressions show that transfers to the old decrease growth, they do not compare capital income tax

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1 There are also two arguments in favor of positive capital income taxation. The first stresses its progressiveness and the tradeoff between some kind of “fair” income distribution and efficiency. The second argues, that it may be sensible to highly tax capital already in place, since it as a fixed factor, but tax capital little or not at all in the more distant future, see Jones et al. (1993). Obviously, the issue of time consistency is not trivial here, see Chari et al. (1989).

2 The personal savings rate in percent of disposable income is taken from Citibase, while the capital income tax rates are taken from Mendoza et al., 1994.
Fig. 1. Personal savings versus capital income tax rates.

Note: This figure shows the personal rate as a fraction of disposable income vis-à-vis the capital income tax rate for the United States. The personal savings rate in percent of disposable income is taken from Citibase, while the capital income tax rates are taken from Mendoza et al. (1994).

rates across countries to examine their effect on growth, even though it would have lent strong support to their theory, if they had found a negative effect.

Thus, the case for a low capital income taxation is often based on theoretical rather than empirical arguments. The point of this paper is to demonstrate, that theory can point the other way too. Within the context of an overlapping endogenous growth model, the paper demonstrates, that raising the capital income tax leads to faster growth for plausible parameterizations. The argument builds on a simple life-cycle analysis. Consider choosing among different rates of labor income taxation and capital income taxation to raise a given amount of needed revenue. 3 If we think of labor income being paid mostly to the young and capital income accruing mostly to the old, a lower capital income tax and thus a higher labor income tax means that the younger people in an economy are left with less income out of which to save and to buy the capital stock. If savings decisions are not too elastic with respect to long term interest rates, this will lead to lower

3 The needed revenue is assumed to be a fixed fraction of GNP in our model. Thus, the tax changes considered here do not drive a wedge between national savings and private savings. In particular, they do not simultaneously increase private savings and decrease national savings, as claimed by Feldstein (1995a).
savings and thereby to slower growth rather than faster growth. The issue becomes clearer when thinking about lump-sum taxes (or, alternatively, Leontieff preferences) instead: if a given amount of revenue has to be raised, taxing the old rather than the young will lead to faster growth, since agents compensate for the tax shift through higher savings. This is exactly in line with the evidence about the negative growth effects of transfers to the old found by Persson and Tabellini (1994) quoted above or with the results found by Feldstein (1995c). With proportional taxes, the question simply is whether the substitution effect on savings through lower interest rates is enough to undo the growth effect of the tax-burden shift towards the old. Most estimates of savings elasticities found in the literature are low enough for our result to hold, but some available estimates lead to the opposite conclusions. Excluding these high estimates, a higher capital income taxation leads to faster growth in the context of the theoretical analysis here.

Examining the welfare consequences should be important, since it is not savings or growth that enters the utility function, but consumption: capital income tax changes can have important welfare consequences even if their effect on savings is small (see Feldstein, 1995a). Since we are considering an endogenous growth model, the effect on welfare will be ambiguous in general, since the initially old will always prefer less to more capital income taxation and the generations in the far distant future will always prefer faster growth. For that reason, we focus on the positive analysis only.

The life cycle savings argument is essential to our story. What is apparently needed for our effect is that an increase in capital income taxation constitutes a shift in the tax burden to the relatively older agents. That this is so in practice can be seen from the calculations performed by Auerbach et al. (1994). While this tax-burden shifting argument has been raised before in the context of capital income taxation, see e.g. Feldstein (1978, 1995a) and Auerbach (1979), it often seems to be overlooked in the capital income tax debate and the growth literature, which typically focuses on infinitely lived agents. See e.g. Stokey and Rebelo (1995). But infinitely lived agents are in essence always young, unless particular time-dependent endowment patterns are assumed. Bertola (1994), following up on our analysis, thus provides for an elegant comparison of our results to the standard setup by analyzing the intermediate case of exponentially distributed lifetimes. He shows that our results can survive in such a model as well, provided for example, that labor supplied by an individual declines over its lifetime, i.e. that there is a reasonably strong need to save in order to provide for retirement.

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4 This is partly motivated by the argument raised by Barro (1974), that it may be better to think about finitely lived generations as infinitely lived dynasties linked by bequests. See however also Kotlikoff and Summers (1979) and Abel and Bernheim (1991).
Growth is endogenous in our model due to an externality in the individual production function. That feature has been kept very simple to keep the focus on the taxation issue at hand. The endogenous growth feature not only allows us to phrase the issue directly in the policy-relevant terms of capital income taxation versus growth, but it also simplifies the analysis considerably: regardless of initial conditions, the growth rate is obtained directly and there is no need to calculate adjustment paths to some steady state as in, say, Auerbach (1979). The most closely related paper may be Jones and Manuelli (1992), who also consider an overlapping generations economy with endogenous growth. However, in their analysis, the growth effects are asymptotic rather than immediate. More importantly and in contrast to their work, we do not require transfers to the young to sustain growth since our model structure implies that wages grow along with the economy. Furthermore and in addition to their paper, we show that reversing the positive effect of a capital income tax hike on growth often requires negative savings in a two-period OLG framework, see Section 4. Extending the analysis to a multiperiod overlapping generations model makes reestablishing the conventional wisdom easier.

Section 2 introduces the model. Section 3 considers the benchmark case, where only the young receive labor income. For Cobb–Douglas preferences, a capital income tax hike will always result in higher growth. In general, the savings elasticity needs to be examined. A rough calibration of the highly stylized model used here indicates, that savings decisions are sufficiently interest inelastic in the US economy for the positive effect of a capital income tax on growth to hold according to several estimates found in the literature. The range found in the literature also includes estimates, for which one obtains a negative effect and thus the conventional wisdom. Section 4 considers the case, where the old receive labor income as well. For the two-period overlapping generations case, we show, that the parameter range which yield a negative effect of a capital income tax hike on growth while keeping savings positive, is very thin. We examine the issue in the context of a multiple-period overlapping generations model in section five. Here, a negative impact of a capital income tax hike on growth can be obtained rather easily, providing support for the conventional wisdom and an important caveat to our two-period overlapping generations analysis. The final section concludes.

2. The model

A new generation of agents is born every period. Agents live two periods. There is no population growth and that there is one representative agent per

5 See Eqs. (2) and (3).
6 There are also some similarities to the analysis in Bean and Pissarides (1993).
generation. When young, the agent is endowed with $0 < \lambda \leq 1$ units of time and when old, his or her time endowment is $1 - \lambda$. There is one consumption good per period and an agent born in period $t$ is assumed to enjoy consumption according to the utility function

$$u(c_{y,t}, c_{o,t} + 1),$$

where $c_{y,t} \geq 0$ is the consumption when young and $c_{o,t+1} \geq 0$ is the consumption when old. We assume that $u$ is homothetic and satisfies the usual list of conditions. In particular then, there is a continuously differentiable consumption rule $C(R)$ for $R > 0$, so that the utility function above, subject to the constraint

$$c_{y,t} + \frac{c_{o,t+1}}{R} \leq W,$$

is uniquely maximized at consumption

$$c_{y,t} = C(R)W$$

for any value of the endowment $W > 0$ in terms of consumption at date $t$ and any (after-tax) interest factor $R = 1 + r > 0$ ($r$ is the after-tax interest rate). Savings are

$$S_t = \left( R - \frac{W_y}{W} \right) W = (1 - C(R))W_y - C(R)W_o/R,$$

where $W_y$ is the value of the time endowment in consumption goods when young, $W_o$ is the value of the time endowment in consumption goods when old and $W = W_y + W_o/R$ is the total endowment in terms of present consumption. The agents supply their time endowment inelastically as labor, so that the total labor supply per period is unity.\(^7\) Below, it will turn out, that wages when young per unit of time are given by $wK_y/\alpha$, growing by some factor $g$ per period. We can then use the formulas above with $W_y = \lambda wK_y/\alpha$ and $W_o = g(1 - \lambda)wK_y/\alpha$.

There are many competing firms in this economy. The production function for the individual firm $i$ is given by

$$y_{i,t} = k_{i,t}^{\rho} \left( n_{i,t} \frac{K_t}{\alpha} \right)^{1-\rho},$$

where $k_{i,t}$ is the firm-specific capital, $n_{i,t}$ is the labor hired by that firm and $K_t = \sum k_{i,t}$ is the aggregate capital stock. The capital share is given by $0 < \rho < 1$.

\(^7\) We could assume a preference for leisure as well in the utility function. This would only strengthen our argument, since a lower labor income tax will mean less distortion in the labor market on top of simply leaving more income after taxes.
The labor input is augmented by the factor $K_i$, which generates a simple externality of the kind often used in theories of endogenous growth, see e.g. Romer (1986) or Grossman and Helpman (1991). Since all firms will have the same capital–labor ratio in equilibrium, dividends accruing to the holders of all capital in firm $i$ are given by $dk_{i,t} = \rho y_{i,t}$, whereas labor income paid to $n_{i,t}$ will equal $wn_{i,t} = (1 - \rho)y_{i,t}$. Aggregating, we find that total production is given by

$$Y_t = aK_t,$$

where $a = \alpha^p - 1$; a high value for $\alpha$ means a large spillover effect and thus higher output. Dividends $d$ per unit of capital are given by

$$d_i = \frac{Y_t}{K_i} = \rho a,$$

independently of $t$. Wages per unit of time are likewise given by $(1 - \rho)Y_t$, so that the wage rate $w_i$ per efficiency unit of labor, $n_i = K_i$, is given by

$$w_i = (1 - \rho)\alpha^p,$$

which is again independent of $t$. We will therefore omit the time index for $w_i$ and $d_i$ below.

We assume that capital depreciates at some rate $0 < \delta < 1$ and that output each period can be split into private consumption $C_t$, government consumption $H_t$, and investment $X_t$ to capital:

$$C_t + H_t + X_t = Y_t,$$

The capital stock thus evolves according to

$$K_{t+1} = (1 - \delta)K_t + X_t,$$

where we allow $X_t$ to be negative for simplicity. The total value of a unit of old capital at the beginning of period $t$ in terms of the present consumption good is now given by

$$v = d + (1 - \delta) = \rho a + 1 - \delta.$$

Note that $v$ is also the total return to a purchase of a unit of capital at $t - 1$.

Finally, we introduce the government which has to finance a given stream of expenditures $H_t$. Rather than fixing the level of these expenditures beforehand.

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8 Note that we normalized the aggregate labor supply N to equal unity. Without this normalization, we would have $a = (N/a)^{p-1} - 1$; and all calculations below still go through with the proper accounting for distinguishing individual from aggregate variables. The important point is that the constant $a$ is still the aggregate output to aggregate physical capital ratio.
irrespective of the growth rate, we assume that the government wants or needs to spend a certain fraction $\gamma$ of total output each period:  

$$H_t = \gamma Y_t.$$  

(9)

The sources of government revenue are capital income taxes and taxes on labor income. There is no debt.  

Let $\tau_L$ be the tax rate on labor income and let $\tau_K$ be the capital income tax rate. Capital income taxes are to be paid on the full amount of capital income, including the resale value of the capital and not just the capital gains and we assume that all savings are financed out of after-tax labor income. Thus, there usually will be double-taxation of savings. This is mostly a matter of accounting and notation: it is irrelevant for the individual agent, whether his or her savings are taxed twice or simply once at the appropriate sum of the two rates and there are many ways of writing down equivalent tax systems. All that matters for the individual is the tradeoff between consuming when young and consuming when old. With linear tax schedules, this tradeoff is constant and can be characterized by a relative price between the two relevant consumption goods, independently of the level of consumption.

This relevant relative price of the consumption good when young in terms of the consumption good when old is the private total return on capital or the after-tax interest factor on savings. It is given by

$$R = (1 - \tau_K) \nu = (1 - \tau_K)(\rho u + 1 - \delta)$$  

(10)

and independent of $t$. The after-tax interest rate per period is $r = R - 1$.

The government budget constraint requires that

$$\gamma Y_t = \tau_K vK_t + \tau_L wK_t/\alpha$$  

(11)

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9 Our assumption seems plausible based on casual empiricism. As an example, it certainly makes sense that a richer country typically builds a better road system than a poorer country. For an endogenous justification of this assumption, one may simply extend the Cobb–Douglas production function with another factor as in Barro (1990).

10 An earlier version of this paper (Uhlig and Yanagawa, 1994) considered a more general case, allowing for debt. Briefly, raising debt results in lower savings in the form of capital, reducing growth, see also Yanagawa and Grossman (1993). Hence, a capital tax increase can also be used to reduce the debt burden, strengthening our results here. An exception can be constructed with a grandfather clause for initial capital, see sect. 4.2 in Uhlig and Yanagawa (1994).

11 We assume limited liability throughout. That means, that capital owners cannot be forced to pay more taxes than their capital income and likewise, workers cannot be forced to pay more taxes than their labor income. This puts some mild restrictions on $\gamma$.

12 Furthermore, even though actual tax codes seem to avoid double taxation they are unsuccessful in doing so, since in practice, taxable capital gains are often mostly nominal gains due to inflation. Thus, our notation may not be far from describing tax practice.
This equation is the key to our argument: a raise in the capital income tax rate means a fall in the labor tax rate, since we keep the fraction of government expenditure $\gamma$ unchanged.

Let

$$g_t = K_t / K_{t-1} = Y_t / Y_{t-1}$$

be the growth factor from period $t-1$ to period $t$. Market clearing on the capital market requires $K_{t+1} = S_t$, where $S_t$ is aggregate savings from period $t$ to period $t+1$. Replacing aggregate savings by equation (12), the capital market clearing condition divided by $K_t$ can be rewritten as

$$g_{t+1} = \frac{1 - \tau_L}{\alpha} w \left( (1 - C(R)) \lambda - C(R) \frac{g_{t+1}}{R} (1 - \lambda) \right).$$  \hspace{1cm} (13)

Solving this equation for $g_{t+1}$ and making use of $w \alpha/ = a(1 - \rho)$ yields

$$g_{t+1} = g = \frac{(1 - C(R)) \lambda}{C(R) \left[ (1 - \lambda)/R \right] + 1/a(1 - \tau_L)(1 - \rho)}.$$  \hspace{1cm} (14)

The growth factor $g$ turns out to be constant in each period! This is a result of the capital externality assumed in (2) and simplifies the analysis in comparison to a neoclassical overlapping generations model as in Auerbach (1979).

### 3. The benchmark case: Only the young receive income

Assume now that only the young earn labor income, i.e. $\lambda = 1$. In that case, Eq. (14) simplifies to

$$g = a(1 - \rho)(1 - \tau_L) S(R; 1).$$  \hspace{1cm} (15)

The argument brought forward in the introduction can now formally be seen in Eq. (12), (10) and (15): a higher capital income tax rate leads to a lower after-tax interest factor $R$ and a lower labor income tax $c \tau_L$. If the decrease in the labor income tax overcompensates the possible decrease in the savings $S(R; 1)$, then a higher growth rate results.

As an example, consider the case, where the utility function for consumption is Cobb–Douglas,

$$u(c_{y,t}; c_{o,t+1}) = \log(c_{y,t}) + \beta \log(c_{o,t+1}).$$  \hspace{1cm} (16)
It is easy to see that the savings function \( S(R; 1) \) is constant:

\[
S(R; 1) = \frac{\beta}{i + \beta}.
\]

In this case, the only effect of a higher capital income tax is to lower the labor tax rate \( \tau_c \), thereby unambiguously increasing the growth rate \( g \) according to (15). In fact, the growth-rate maximizing capital income tax rate in this environment is to tax away practically all income to capital and use it to subsidize rather than tax labor income.

Likewise, if the intertemporal elasticity of substitution is some constant \( \sigma < 1 \) (or, equivalently, the relative risk aversion is constant at \( 1 \sigma/ > 1 \)), resulting in the utility function

\[
u(c_{y, t}, c_{o, t+1}) = \frac{c_{y, t}^{1-\sigma} - 1}{1 - 1/\sigma} + \beta \frac{c_{o, t+1}^{1-\sigma} - 1}{1 - 1/\sigma},
\]

it is easy to see that the savings function is given by \( S(R; 1) = x/(1 + x) \), where \( x = \beta R^{\sigma-1} \). Now, \( S(R; 1) \) is decreasing in \( R \), so that an increase in capital income taxation leads to an increase in growth even without the labor-income tradeoff, and certainly in our model as well. Let us summarize the results of these examples in the following proposition.

**Proposition 1.** If the overall utility is characterized by a constant intertemporal elasticity of substitution of unity or lower, \( \sigma \leq 1 \), then a higher capital income tax rate will unambiguously result in a higher growth rate.

It is interesting to note, that Hall (1988) has measured the intertemporal elasticity of substitution and concluded that its “value may even be zero and is probably not above 0.2”, giving some empirical credibility to the proposition above. \(^{15}\)

More generally, the direction of the marginal change in the growth rate due to a marginal change in the capital income tax at a particular equilibrium will depend on the interest factor elasticity of savings

\[
\eta(R) = \frac{\partial S(R; 1)}{\partial R} \frac{R}{S(R; 1)}
\]

\(^{15}\) In contrast to our result, Buiter (1991) finds \( \sigma \leq 0.04 \) as the necessary condition for a higher capital income tax to increase growth and concludes, that this bound is too low to be satisfied. The reason for the difference to our analysis is that he considers a very different continuous-time overlapping generations model with exponentially distributed lifetime. An elegant reconciliation of these models and further discussion is in Bertola (1994).
the after-tax interest factor of that equilibrium. E.g., for the constant intertemporal elasticity of substitution utility function used above, we have

\[ \eta(R) = \frac{\sigma - 1}{1 + \beta^\sigma R_{a-1}}. \]  \hspace{1cm} (18)

Thus, for \( \sigma \leq 1 \), the elasticity is zero or negative, leading to the unambiguous result stated in the previous section. But if the elasticity is positive, the relative strength of each effect – decreased savings due to a lower after-tax return or increased savings due to higher income when young – matters. The following result obtains.

**Proposition 2.** A marginally higher capital income tax leads to a marginally higher growth rate across equilibria if and only if the interest elasticity of savings is not too big:

\[ \eta(R) < \frac{R}{a(1 - \rho)(1 - \tau_L)}. \]  \hspace{1cm} (19)

Observe, that the ratio on the right-hand side of (19) equals

\[ \frac{I_K}{I_L} = \frac{(1 - \tau_K) v K_t}{(1 - \tau_L) w (K_t / \alpha)}, \]

which is simply the ratio of after-tax capital income to after-tax labor income in period \( t \).

**Proof.** Substituting (12) and (10) into (15), it follows in a straightforward manner, that \( \frac{\partial g}{\partial \tau_K} > 0 \) holds if and only if

\[ \frac{S(R; 1)}{(1 - \rho) a} - (1 - \tau_L) \frac{\partial S(R; 1)}{\partial R} > 0. \]

Rewriting this inequality yields the result. □

A simple calibration exercise for this highly stylized overlapping generations model may help in evaluating this inequality constraint. A period in this model should thought of lasting half the life of a generation, i.e. 30 years, say. For \( \rho \) and \( \tau_L \), \( \rho = 0.3 \) and \( \tau_L = 0.3 \) may be reasonable choices, so that, roughly,

\[ \eta(R) < \frac{2R}{a} \]  \hspace{1cm} (20)

is necessary and sufficient for the claimed effect. To make a conservative guess for \( R \) and \( a \), one should not overstate the interest factor \( R \) and should not understate the spillover parameter \( a \). Since long-term real rates are quite low, choose \( R = 1 \). To calibrate the parameter \( a \), note that Christiano (1988) has found,
that $K/Y = 10.59$ or $Y/K = 0.0944$ on a quarterly basis. To translate that into a value for the parameter $a$ on a 30-year or 120-quarter basis as required by Eq. (3), the latter number needs to be multiplied with 120, resulting in 11.33. To have a round number, use $a = 12$.

Thus, if the elasticity of savings over long horizons like 30 years with respect to the after-tax interest factor $R$ over the same horizon is less than $1/6$, a higher capital income tax on these savings should lead to faster growth. For the constant intertemporal elasticity of substitution utility function, this translates into $\sigma < 1.333$ (or $1/\sigma > 0.75$) at $\beta = 1$, $R = 1$ via Eq. (18). Or, to state the required elasticity $\eta(R) < 1/6$ on a more intuitive, annual basis: the elasticity $\eta_i(R_i)$ of ‘retirement’ savings with respect to the yearly after-tax interest factor $R_i = R^{1/30}$ on these savings must be less than 5 in order to get the claimed effect. In other words, suppose the yearly interest rate on savings for retirement or long-term purposes rises from 0% to 1%. As long as that doesn’t raise these savings by 5% or more, taxing these savings more will lead to faster growth as claimed in the context of this model.

Most of the empirical work states savings elasticities $\epsilon(r_i)$ with respect to the yearly interest rate $r_i = R_i - 1$ rather than the elasticity $\eta_i(R_i)$ with respect to the yearly interest factor $R_i$. For some fixed $r_i = R_i - 1$, these elasticities translate into each other via

$$\epsilon(r_i) = \eta_i(R_i) \frac{r_i}{R_i},$$

so that for $r_i = 0.04$, say, an interest factor elasticity of 5 corresponds to an interest rate elasticity of about 0.2. Translating estimated elasticities is more problematic due to the stochastic nature of interest rates and since real yearly interest rates are notoriously low.

Empirical estimates for the interest rate elasticity range from negative, insignificant or trivially small (see e.g. Blinder, 1975, 1981; Blinder and Deaton, 1985; Bosworth and Burtless, 1992; Hall, 1988; Skinner and Feenberg, 1990) to quite large: Boskin (1978) found the elasticity to be around 0.4 (which Summers (1981) even considers to be low on theoretical grounds). Large effects are also found by Feldstein (1995b), based on examining the capital income tax implicit in college scholarship rules. The range of elasticity estimates found in the literature can thus support positive as well as negative effects on savings and growth due to an increase of a capital income tax, although most estimates seem to be in the range where the inequality of the proposition above is satisfied and the effect is positive.

4. The old receive labor income too

Let us relax the condition that it is only the young who receive labor income (cmp. Summers, 1981). Consider again the logarithmic example, where the utility
Table 1
The effect of marginally raising the capital income tax rate on the growth rate together with savings. \( \beta = 1.0 \)

<table>
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<th>( \lambda )</th>
<th>( \tau_K = -10% )</th>
<th>( \tau_L = 41% )</th>
<th>( \tau_K = 0% )</th>
<th>( \tau_L = 36% )</th>
<th>( \tau_K = 10% )</th>
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<td>( S = 0.14 )</td>
<td>( \frac{d g}{d \tau_K} = 0.13 )</td>
<td>( S = 0.12 )</td>
<td>( \frac{d g}{d \tau_K} = 0.00 )</td>
<td>( S = 0.11 )</td>
<td>( \frac{d g}{d \tau_K} = -0.15 )</td>
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<td>( \frac{d g}{d \tau_K} = 0.02 )</td>
<td>( S = 0.01 )</td>
<td>( \frac{d g}{d \tau_K} = -0.05 )</td>
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<td>( \frac{d g}{d \tau_K} = -0.12 )</td>
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<tr>
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<td>( \frac{d g}{d \tau_K} = 0 )</td>
<td>( S = -0.11 )</td>
<td>( \frac{d g}{d \tau_K} = 0 )</td>
<td>( S = -0.12 )</td>
<td>( \frac{d g}{d \tau_K} = 0 )</td>
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<td>( \frac{d g}{d \tau_K} = 0 )</td>
<td>( S = -0.17 )</td>
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</table>

function is given by (16). Unfortunately general results look rather messy. Consider the case, where the issue is whether to marginally tax capital income or to marginally subsidize capital income. We have the following result.

Proposition 3. Suppose, the utility function is given by Eq. (16) and \( b = 0 \). Consider the equilibrium, where \( \tau_K = 0 \). A marginal increase in the capital income tax rate will marginally increase the growth rate if and only if

\[
\frac{1}{1 + \beta} \frac{1 - \lambda}{R} I_L < \frac{I_K}{I_L},
\]

(22)

where \( I_L = (1 - \rho) a - \rho y \) is the after-tax labor income per unit of capital (or the after-tax labor share) and where \( I_K = R = a + 1 - \delta \) is the after-tax capital income per unit of capital.

Thus, the inequality (22) compares the presently consumed fraction of future, discounted labor income (when capital is normalized to one unit) with the ratio of capital income to labor income after taxes: as long as that fraction is not too high, a higher capital income tax will still lead to faster growth.

Proof. Note that \( C(R) = 1/(1 + \beta) \) is constant. Substituting Eq. (12) and (10) into Eq. (14) and some algebra reveals, that \( \frac{dg}{d\tau_K} > 0 \) if and only if

\[
\frac{1}{I_L/I_K + \tau_K} + \frac{C(R)(1 - \lambda)}{1 - \tau_K}
\]
The effect of marginally raising the capital income tax on the growth rate together with savings, $\beta = 0.5$

<table>
<thead>
<tr>
<th>$\beta = 0.5$</th>
<th>$\tau_K = -10%$</th>
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<td>1.43</td>
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<td>-0.17</td>
<td>-0.19</td>
<td>-0.22</td>
</tr>
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</table>

has a negative derivative with respect to $\tau_K$. It is easy to see that this is the case at $\tau_K = 0$ if and only if

$$C(R)(1 - \lambda)\frac{I_L}{I_K} < \frac{I_L}{I_K}.$$  

Rewriting this yields the result. $\square$

To evaluate the issue more directly, consider Tables 1 and 2. Each entry in these tables lists firstly the derivative $d g/\partial \tau_K$ and secondly the savings rate $S(R; A)$. We chose log-utilities. For the parameters in our model we chose $a = 12$, $\rho = 0.3$, $\gamma = 0.2$, $\delta = 0.3$. For the first table we chose $\beta = 1$, whereas we chose $\beta = 0.5$ for the second table to evaluate the effect of a change in the discount factor. We varied both the parameter $A$ and the parameter $\tau_K$ in each table. Note that the parameter $\lambda$ here corresponds closely to the redistribution parameter $\eta$ in Jones and Manuelli (1992, scnt. 2), since in their model wage income is negligible asymptotically. The parameter $\tau_K$ implies a value for $\tau_L$ via Eq. (12), which is given as well.

It is possible to find parameter combinations in these tables that look reasonable and produce a decrease in the growth rate due to an increase in the capital income tax, while at the same time keeping a positive savings rate. For example, for $\beta = 1.0$, $\lambda = 0.4$ and $\tau_K = 0.3$, the derivative has the value $-0.15$, while the savings rate is equal to 0.10. It is important to note, that the parameter ranges for which this occurs are somewhat extreme in that they require either a rather high
capital-income tax to begin with or a rather low fraction \( \lambda \) of earned income when young. More importantly, perhaps, these ranges are also rather thin in the sense that savings rates are extremely low and more often negative rather than positive for those table entries, where the derivative of the growth rate with respect to the capital income tax rate is negative.

5. Multiple periods of life

Interestingly and importantly, the issue changes, once a multiple-period overlapping generations model is considered. I.e. assume, that agents live \( n \) rather than two periods and that they work the first \( k \) periods of their life, rather than just in the first period. We proceed with simple assumptions which allow for an easy extension of the analysis above. Assume that agents supply the same amount of labor each period. Keeping aggregate labor supply at unity, agents in their first \( k \) periods of life thus receive wages \( W_t/k \) in period \( t \). Further, assume that an agent born at date \( i \) maximizes \( \sum_{j=1}^{n} \log c_{j,t} \), where \( c_{j,t} \) denotes the consumption of an individual in his \( j \)th period of life in period \( s \). Concentrate on a steady state analysis, where the interest factor \( R \) and the growth factor \( g \) between two periods is independent of time. The net present value of labor income at birth is thus given by

\[
NPV_i = \frac{1 - (g/R)^k}{k(1 - (g/R))} W_t,
\]

and consumption growth with the factor \( R \) each period, starting at \( c_{1,t} = NPV_i \). Aggregating over all individuals alive at date \( t \), one finds aggregate consumption to be

\[
C_t = \sum_{j=1}^{n} c_{j,t} = \frac{1 - (R/g)^n}{n(1 - (R/g))} NPV_i.
\]

Let \( S_t \) denote aggregate savings. Since

\[
S_t + C_t = W_t + RS_{t-1}
\]

\[\text{Remember that } \tau_k \text{ is the tax on the total capital income and that savings are out of after-tax labor income.}\]

\[\text{It is not hard to introduce a discount factor different from unity, but it makes the algebra a bit more tedious.}\]

\[\text{In all of these equations, the functions are well-defined at } g = R \text{ per continuation.}\]
Table 3
The marginal effect of a rise of the capital income tax, $d\gamma/dK$, in an OLG model where agents live $n$ periods and supply equal amounts of time as labor during their first $k$ periods. Parameters: $\rho = 0.3$, $\gamma = 0.2$, $\delta = 0.3$, $\kappa = 0$ and $a = 24/n$

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<th>4</th>
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<tr>
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<td>-0.67</td>
<td>-0.84</td>
<td>-0.92</td>
<td>-0.94</td>
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it follows in steady state, that $S_t = S(R, g)W_t$, where

$$S(R, g) = \left(1 - \left(\frac{1 - (R/g)}{n(1 - (R/g))}\right)\left(\frac{1 - (g/R)^k}{k(1 - (g/R))}\right)\right)/(1 - R/g).$$

As before for Eq. (13), one has that $gK_t = K_t + 1 = S_t = S(R, g)W_t = (1 - \tau_L)a(1 - \rho)K_t$, and hence the equation

$$g = (1 - \tau_L)a(1 - \rho)(R, g),$$

(23)

where $R$ is given by Eq. (10) and $\tau_L$ is given by Eq. (12). Thus, given the parameters as well as a value for $\tau_k$, Eq. (23) implicitly determines the growth factor $g$. This equation does not seem solvable analytically, but is not hard to analyze numerically. In Tables 3 and 4, we have therefore numerically calculated the marginal effect of a tax rate increase on the steady state growth rate.\footnote{Savings are always positive in these examples, although one needs to check that they are, if $g > R$. They are thus not shown in these tables. Note, that we are making steady state comparisons: thus, the usual caveats apply.} The parameters for both tables are $\rho = 0.3$, $\gamma = 0.2$, $\delta = 0.3$ and $\kappa = 0$. The discount factor has been assumed to be $\beta = 1$ already. In the first table, $a$ was calibrated to yield the value used above to yield the same capital-to-GNP ratio used above on a ‘yearly’ basis, if an agent is always thought to have an economically active life of 60 years, i.e. we set $a = 24/n$. For the second table, the interest factor has been chosen to equal unity, $R = 1$, which implies that $a = \delta/\rho = 1$, a rather small value.

The case considered in the previous section, where only the young receive income, is the case where $n = 2$ and $k = 1$: as claimed before, the effect of tax raise on growth is positive. However, as $n$ increases, the effect can quickly become negative even for rather small values of $k$. Precisely when this happens is
Table 4
The marginal effect of a rise of the capital income tax, \( \frac{d g}{d \tau_K} \), in an OLG model where agents live \( n \) periods and supply equal amounts of time as labor during their first \( k \) periods. Parameters: \( \rho = 0.3, \gamma = 0.2, \delta = 0.3, \tau_K = 0 \) and \( a = \rho / \delta = 1 \)

<table>
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<tr>
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<td>-0.15</td>
<td>-0.23</td>
<td>-0.28</td>
</tr>
</tbody>
</table>

Sensitive to parameter choices. For ‘reasonable’ capital-income ratios, i.e. in the first table, the effect is practically always negative for large values of \( n \), unless the individual receives income only in the first period of his life, an unrealistic assumption. If \( R \) is set to the conservatively low value of unity, the negative effect sets in later, but still seems to require only that the retirement period is somewhat shorter than the active working life. This issue merits further and deeper investigation. Again, the reader is pointed to the continuous-time analysis in Bertola (1994).

6. Conclusion

The purpose of this paper has been to cast doubt on the conventional wisdom, that capital income taxes should be low. We have shown that a higher capital income tax rate means faster growth in two-period overlapping generations model with endogenous growth, where government expenditures are a fixed fraction of total GNP. In this model, a higher capital income tax means a lower labor income tax, which leaves the presently young with more net income out of which to save. Higher savings lead in turn to faster growth.

To obtain a positive net effect on savings, savings need to be sufficiently interest-inelastic, which they seem to be according to several estimates in the literature. The effect can be undone, if the old earn labor income too, but the range of parameters for which the effect can be undone, while keeping savings in the positive range, is quite thin in the context of a two-period overlapping generations model. Extending the analysis to a multiple overlapping-generations model, a negative impact of a capital income tax hike on growth can be obtained quite easily, however, adding support to the conventional wisdom and an important
caveat to our two-period analysis. It will be important to reconsider the issue carefully in richer models similar to those in Auerbach and Kotlikoff (1987).

Should the conventional wisdom thus be overturned based on the analysis presented here? Probably not. But perhaps, that wisdom should be taken to be a bit more doubtful than it often has been by economists and perhaps it should be rethought. The empirical support for lowering capital income taxation as a way to boost economic performance has always been much less clear-cut than one would wish it to be. The case for a low capital income tax has thus often been based on theoretical arguments. This paper has demonstrated that theory can also point the other way.

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