WHAT ARE THE EFFECTS OF FISCAL POLICY SHOCKS?

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SUMMARY

We propose and apply a new approach for analyzing the effects of fiscal policy using vector autoregressions. Specifically, we use sign restrictions to identify a government revenue shock as well as a government spending shock, while controlling for a generic business cycle shock and a monetary policy shock. We explicitly allow for the possibility of announcement effects, i.e., that a current fiscal policy shock changes fiscal policy variables in the future, but not at present. We construct the impulse responses to three linear combinations of these fiscal shocks, corresponding to the three scenarios of deficit-spending, deficit-financed tax cuts and a balanced budget spending expansion. We apply the method to US quarterly data from 1955 to 2000. We find that deficit-financed tax cuts work best among these three scenarios to improve GDP, with a maximal present value multiplier of five dollars of total additional GDP per each dollar of the total cut in government revenue 5 years after the shock. Copyright © 2009 John Wiley & Sons, Ltd.

1. INTRODUCTION

What are the effects of tax cuts on the economy? How much does it matter whether they are financed by corresponding cuts of expenditure or by corresponding increases in government debt, compared to the no-tax-cut scenario? These questions are of key importance to the science of economics and the practice of policy alike. This paper aims to answer these questions by proposing and applying a new method of identifying fiscal policy surprises in vector autoregressions.

The identification method used in this paper is an extension of Uhlig (2005)’s agnostic identification method of imposing sign restrictions on impulse response functions. We extend this method to the identification of multiple fundamental shocks. More precisely, we identify a government revenue shock as well as a government spending shock by imposing sign restrictions on the fiscal variables themselves as well as imposing orthogonality to a generic business cycle shock and a monetary policy shock, which are also identified with sign restrictions. No sign restrictions are imposed on the responses of GDP, private consumption, private non-residential investment and real wages to fiscal policy shocks, and so the method remains agnostic with respect to the responses of the key variables of interest.

The identification method is thereby able to address three main difficulties which typically arise in the identification of fiscal policy shocks in vector autoregressions. Firstly, there is the difficulty of distinguishing movements in fiscal variables caused by fiscal policy shocks from those which are simply the automatic movements of fiscal variables in response to other shocks such as business cycle or monetary policy shocks. Secondly, there is the issue of what one means by a fiscal policy

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shock. While there is agreement that a monetary policy shock entails a surprise rise in interest rates, several competing definitions come to mind for fiscal policy shocks. Finally, one also needs to take account of the fact that there is often a lag between the announcement and the implementation of fiscal policy and that the announcement may cause movements in macroeconomic variables before there are movements in the fiscal variables.

For the first problem we identify a business cycle shock and a monetary policy shock and require that a fiscal shock be orthogonal to both of them. This filters out the automatic responses of fiscal variables to business cycle and monetary policy shocks.

To address the second problem, we argue that macroeconomic fiscal policy shocks exist in a two-dimensional space spanned by two basic shocks: a government revenue shock and a government spending shock. Different fiscal policies such as balanced budget expansions can then be described as different linear combinations of these two basic shocks. For example, a basic government spending shock is defined as a shock where government spending rises for a defined period after the shock, and which is orthogonal to the business cycle shock and the monetary policy shock. We choose to restrict responses for a year following the shock in order to rule out shocks where government spending rises on impact but then subsequently falls after one or two quarters. This provides additional identifying power.

For the third problem, we also identify fiscal policy shocks with the identifying restriction that the fiscal variable in question does not respond for four quarters, and then rises for a defined period afterwards. Restricting the responses of impulse responses as a means of identification is therefore particularly suitable for dealing with the announcement effect.

This paper therefore contributes to the recent and growing literature of employing vector autoregressions to analyze the impact of fiscal policy shocks, complementing the existing large literature analyzing monetary policy shocks; see, for example Leeper et al. (1996), Christiano et al. (1999) and Favero (2001) for excellent surveys. Most of the previous literature has identified fiscal shocks either by making assumptions about the sluggish reaction of some variables to fiscal policy shocks (see, for example, Blanchard and Perotti, 2002; Fatás and Mihov, 2001a,b; Favero, 2002; Gali et al., 2007) or by using additional information such as the timing of wars, detailed institutional information about the tax system and detailed historical study of policy decisions or elections (see, for example, Ramey and Shapiro, 1998; Edelberg et al., 1999; Blanchard and Perotti, 2002; Burnside et al., 2003; Eichenbaum and Fisher, 2004).  

By contrast, this paper relies on macroeconomic time series data alone for shock identification and does not rely on assumptions about the sluggish reaction of some variables to macroeconomic shocks. Indeed it imposes no restrictions on the signs of the responses of the key variables of interest—GDP, private consumption, private non-residential investment and real wages—to fiscal policy shocks. The approach of this paper thus sharply differentiates it from previous studies and provides an important complementary method of analysis which, being a purely vector autoregressive approach, is automatically systematic and can be universally applied.

The method of identifying policy shocks using sign restrictions on impulse responses has been introduced and applied to monetary policy in Uhlig (2005). Uhlig’s method is extended here by imposing orthogonality restrictions to the business cycle and monetary policy shocks as well as sign restrictions. Faust (1998) uses sign restrictions to identify monetary policy shocks, imposing them

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1 There are a wide variety of other empirical studies investigating the effects of fiscal policy. The focus of this paper is on the analysis of the effects of fiscal policy using vector autoregressions and so we do not attempt to summarize this literature here. For an excellent survey see Hemming et al. (2000).

We apply our new approach to US quarterly data, from 1955 to 2000. We use the same definitions of government expenditure and revenue as Blanchard and Perotti (2002) in order not to obscure the implications of our new methodological approach by using different data definitions. We show that controlling for the business cycle shock is important when analyzing the consequences of fiscal policy. By linearly combining our two base fiscal policy shocks—i.e., the government revenue shock and the government spending shock—we analyze three policy scenarios: deficit-spending, deficit-financed tax cuts and a balanced budget spending expansion. Comparing these three scenarios, we find that a surprise deficit-financed tax cut is the best fiscal policy to stimulate the economy, giving rise to a maximal present value multiplier of five dollars of total additional GDP per each dollar of the total cut in government revenue 5 years after the shock. Furthermore, we find that deficit spending weakly stimulates the economy, that it crowds out private investment without causing interest rates to rise, and that it does not cause a rise in real wages.

Despite the novel methodology developed in this paper, the results are reasonably similar to those of the existing literature. As with Blanchard and Perotti (2002), we find that investment falls in response to both tax increases and government spending increases and that the multipliers associated with a change in taxes to be much higher than those associated with changes in spending. This latter result also accords with the analysis of Romer and Romer (2007), who find large effects from exogenous tax changes. With regard to private consumption we find, in common with Blanchard and Perotti (2002) and Gali et al. (2007), that consumption does not fall in response to an unexpected increase in government spending. However, in contrast to these studies we do not find that consumption rises strongly. Our results show that the response of consumption is small and only significantly different from zero on impact and are thus more in line with those of Burnside et al. (2003), who find that private consumption does not change significantly in response to a positive spending shock. Finally, we find that real wages do not rise in response to an increase in government spending and have a negative response on impact and at longer horizons. Thus the responses of investment, consumption and real wages to a government spending shock are difficult to reconcile with the standard Keynesian approach, although they are also not the responses predicted by the benchmark real business cycle model either.

As an open issue for future research, we leave the question as to how far the responses calculated here are consistent with general equilibrium modeling, or whether additional restrictions from general equilibrium modeling ought to be imposed during estimation. Chung and Leeper (2007) have recently pointed out that long-run budget balance needs to hold and that this implies restrictions on the VAR coefficients. Put differently, it ought to be obvious that one cannot forever

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2 Theoretical explanations for why consumption does not fall in response to a government spending shock in an infinite horizon framework are given by Devereux et al. (1996), in a model with increasing returns to scale and Gali et al. (2007) in a model with both sticky prices and ‘Non-Ricardian’ agents.
stimulate the economy with deficit-financed tax cuts: they eventually need to be repaid. We agree with Chung and Leeper that this issue is important and merits further attention.

The paper is comprised of three main sections. Section 2 describes the identification procedure and the VAR. The empirically identified basic shocks are presented in Section 3, while Section 4 conducts policy analysis and compares the results with those of the existing literature in Section 4.6. The Appendix contains additional detail, in particular on the VAR framework and the sign restriction methodology, as well as describing the data sources.

2. IDENTIFYING FISCAL POLICY SHOCKS

A fiscal policy shock is a surprise change in fiscal policy. However, there is no such thing as a fiscal policy shock per se. Fiscal policy encompasses a wide variety of policies: there is an endless list of types of incomes, for which the tax rules could be changed, or categories of government spending, where changes could occur. In this paper we address the much broader and traditional ‘macro’-economic issue of the effects on the aggregate economy of aggregate fiscal variables. Even so, there still remain a large set of possible policies since changes in fiscal policy could, for example, be about changing the tax-debt mix for financing a given stream of government spending, or about changing the level of spending for a given level of debt.

In this paper we view fiscal policy shocks as existing in a two-dimensional space spanned by two basic impulse vectors: a government revenue shock and a government spending shock. We identify each of these basic fiscal policy shocks by imposing a positive reaction of the impulse response of the appropriate fiscal variable—i.e., government revenue or government spending—for quarters $k = 0, \ldots, 3$ following the shock and by requiring it to be orthogonal to a business cycle shock and a monetary policy shock, which in turn are also identified using sign restrictions. If we were not to control for the state of the business cycle, it would be easy to end up confusing, for example, an increase in government receipts due to a business cycle upturn with an upturn ‘caused’ by a tax increase. Note that we do not require the two fiscal policy shocks to be mutually orthogonal.

Rather than simultaneously identifying all three (or all four) shocks, subject to the orthogonality restrictions, we first identify the business cycle shock and monetary policy shock via a criterion function based on the sign restrictions, thus ascribing as much movement as possible to these shocks. The fiscal shocks are then identified via sign restrictions as well as the orthogonality restrictions. This procedure may be reminiscent of a causal ordering. If the criterion function was linear, one could linearly recombine the variables such that the first variable now corresponds to that linear combination of the criterion function: a Cholesky decomposition would then make the first shock explain as much as possible of the one-step-ahead prediction error in that first variable, thus maximizing the criterion function. Since our criterion function is nonlinear and involves the impulse responses for several periods, one could not rewrite the problem in this manner, but the analogy may still be helpful to understand the procedure and the results.

A further thorny and well-understood challenge when identifying fiscal policy shocks is the problem of the possible lag between the announcement and implementation of changes in fiscal policy. Considering the potentially lengthy debates in legislatures about, say, a reduction in tax rates, the change in government revenue is fairly predictable by the time the tax reduction actually takes effect. Forward-looking individuals and firms can adjust their economic choices before that date. While the tax change will happen eventually, the surprise of a change in fiscal policy occurs earlier. Our identification procedure is easily adapted to deal with this problem by directly
identifying a shock for which there is a lag between the announcement and the implementation of the change in fiscal policy. In particular, we shall identify announced fiscal policy shocks, where government spending only rises in the fourth quarter following the shock but shows no reaction beforehand.\(^3\)

Given the two basic fiscal shocks, different fiscal policy scenarios can be described as sequences of different linear combinations of these two basic shocks. For example, we will define a balanced budget expansionary fiscal scenario as a sequence of a linear combination of the two basic shocks such that the increase in government spending is matched by the increase in tax revenue for a sequence of five quarters, \(k = 0, \ldots, 4\), following the initial shock.

### 2.1. The VAR and Identifying Restrictions

We use a VAR in GDP, private consumption, total government expenditure, total government revenue, real wages, private non-residential investment, interest rate, adjusted reserves, the producer price index for crude materials and the GDP deflator. The VAR system consists of these 10 variables at a quarterly frequency from 1955 to 2000, has six lags, no constant or a time trend, and uses the logarithm for all variables except the interest rate, where we have used the level. The chosen approach largely dictates the choice of these variables. GDP, private consumption, private investment and real wages are included as the focus of interest. Private consumption is also included because the consumption–GDP ratio has predictive value for GDP, as Cochrane (1994) has shown. Real wages are also included as Neoclassical and New Keynesian models tend to predict different signs for the responses of real wages to deficit-spending shocks, with the former predicting negative and the latter positive responses (see Ramey and Shapiro, 1998). The monetary and price variables are there to identify monetary policy shocks. All the components of national income are in real per capita terms. A more detailed description can be found in Appendix B.

The two fiscal variables in the VAR are defined in the same way as in Blanchard and Perotti (2002). Thus total government expenditure is total government consumption plus total government investment and total government revenues is total government tax revenues minus transfers. Netting out transfer payments from the government revenue variable is a non-trivial decision, but we have chosen to use Blanchard and Perotti’s (2002) data definitions in order to emphasize the implications of the new identification technique rather than have the results obscured by using different data definitions.

### 2.2. The Identifying Assumptions in Detail

An overview of our identifying sign restrictions on the impulse responses is provided in Table I. A business cycle shock is defined as a shock which jointly moves output, consumption, non-residential investment and government revenue in the same direction for four quarters following the shock. Since we associate business cycles with the more substantial movements in these variables, we identify the business cycle shock by a criterion function, which rewards large impulse responses in the right directions more than small responses and penalizes responses of the wrong sign.

Such a co-movement is consistent with both demand and supply side shocks and hence the approach remains ‘agnostic’ on the issue of the determinants of business cycle fluctuations. The

\(^3\) In this respect the identified shocks resemble a type of ‘news shock’ about fiscal policy and so are related to the shocks identified by Beaudry and Portier (2003).

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restriction that government revenues increase with output in the business cycle shock should be emphasized. This is our crucial identifying assumption for fiscal policy shocks: when output and government revenues move in the same direction, we essentially assume that this must be due to some improvement in the business cycle generating the increase in government revenue, not the other way around. We regard this as a reasonable assumption and consistent with a number of theoretical views. Furthermore, our identifying assumptions are close to minimal: some assumptions are needed to say anything at all. The orthogonality assumption a priori excludes the view that positive co-movements of government revenues and output are caused by some form of short term ‘Laffer curve’ or ‘fiscal consolidation’ effect from a surprise rise in taxes. 4

A monetary policy shock moves interest rates up and reserves and prices down for four quarters after the shock. These identifying restrictions are close to those used in Uhlig (2005). We also require the monetary policy shock to be orthogonal to the business cycle shock. The main purpose of characterizing the business cycle and monetary shocks is to filter out the effects of these shocks on the fiscal variables. The additional orthogonalization among these two shocks has no effect on that.

Fiscal policy shocks are identified only through restricting the impulse responses of the fiscal variables and through the requirement that they are orthogonal to both business cycle shocks as well as monetary policy shocks. As stated above, we identify two basic fiscal shocks—a ‘government spending shock’ and a ‘government revenue shock’—employing tight identifying restrictions where the responses of fiscal variables are restricted for a defined period after the shock. For example, a basic government spending shock is defined as a shock where government spending rises for a year after the shock. These tight restrictions are designed to rule out very transitory shocks to fiscal variables where, for example, government spending rises on impact but falls after one or two quarters. Nonetheless we have checked that our results are robust to weaker identifying restrictions where responses are only restricted on impact. Finally, it should be noted that we do not restrict the behavior of government revenue when identifying the government spending shock or vice versa. This is not necessary since all that is required to describe the two-dimensional space of fiscal policy shocks are two linearly independent vectors. However, it is

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4 The ‘Laffer curve’ is a phenomenon which, if it exists, may be expected to operate over the medium term and so would not be ruled out by the short-run sign restrictions imposed. Indeed Figure 11 shows that the responses of government revenue in response to a tax cut can be positive in the medium term. See Trabandt and Uhlig (2006), Giavazzi et al. (1990, 2000) and Perotti (1999) for analysis on this issue.
possible to place restrictions on these shocks so that, for example, government revenue’s response to a government spending shock is initially zero. An example of such restrictions are the year delayed fiscal shocks in Figures 5 and 8, where fiscal responses are restricted to be zero for a year following the announcement of the shock.

Details on the estimation as well as on the implementation of the identification strategies, including these zero restrictions, are described in Appendix A and in Mountford and Uhlig (2002).

3. RESULTS

The identified fundamental shocks for each time period are displayed in Figure 1. The shocks are identified for each draw from the posterior and the 16th, 50th and 84th quantiles plotted. The impulse responses for these fundamental shocks are shown in Figures 2–9, where we have plotted the impulse responses of all our 10 variables to the shocks. The figures plot the 16th, 50th and 84th quantiles of these impulse responses, calculated at each horizon between 0 and 24 quarters after the shocks. The impulses restricted by the identifying sign restrictions are identified by the shaded area in the figures.

3.1. The Business Cycle Shock

The identified business cycle shocks are plotted in the first panel of Figure 1, where the NBER recession dates are shaded. The identified shocks correspond well with the NBER dates, with the only anomalies being the 1981–82 recession, which is not picked up, and the short 1990–91 recession, which does not appear to be very different in scale from other clusters of negative shocks.

The impulse responses of the business cycle shock are plotted in Figure 2. In response to the business cycle shock, output, consumption, non-residential investment and government revenue increase in the first four quarters by construction. Given that no restriction is placed on these responses after four periods, it is notable that all of these responses are persistent. Government revenues increase approximately twice as much in percentage terms as GDP. There is no contradiction here, provided marginal tax rates are approximately twice average tax rates. The persistence in the non-residential investment variable indicates that a business cycle shock may increase the steady-state capital to labor ratio and so generate a higher level of steady-state income, consumption and government revenue. It must be stressed that these responses are consistent with both demand and supply side explanations of the business cycle and this paper is agnostic on the issue of the relative importance and persistence of demand and supply shocks.

The responses of the monetary variables and the government spending variable to the business cycle shock were not restricted at all by the identification method, and their responses are quite interesting. The interest rate rises and the adjusted reserves fall in response to a positive business cycle shock. This could be caused by a systematic counter-cyclical response of monetary policy over the sample period, which fits with the description of monetary policy given by Romer and Romer (1994). The fall in adjusted reserves (compared to the no-business-cycle-shock scenario) would indicate that this counter-cyclical response is rather strong.

Government expenditures, in contrast, do not behave in a counter-cyclical fashion. Rather, they increase, slowly, with a positive business cycle shock. Thus, if a business cycle boom fills the...
government's coffers with cash, it will spend more eventually. Note again that, following Blanchard and Perotti (2002), we chose the government expenditure variable to be government consumption and investment in order to isolate changes in government expenditure from automatic changes over the business cycle. Thus the government expenditure variable does not include transfer payments which almost surely would automatically vary counter-cyclically.

Figure 1. The shocks identified by the VAR: the business cycle shocks are identified first, the monetary policy shocks are identified second and the government revenue and government spending shocks are identified third. The shaded areas represent NBER recessions in the business cycle plot and a change of president in the fiscal plots. This figure is available in color online at www.interscience.wiley.com/journal/jae
3.2. The Monetary Policy Shock

The identified monetary policy shocks are plotted in the second panel of Figure 1. Again these identified shocks correspond well with the existing literature, such as Bernanke and Mihov’s (1998) plot of monetary policy stance, and are on average negative (indicating a loose policy stance) in the late 1970s, positive (indicating a tight policy stance) in the early 1980s, as well as being more volatile in the Volcker experiment period, 1979–82.

The response to a monetary policy shock is shown in Figure 3. Note that we have constructed the monetary policy shock to be orthogonal to the business cycle shock shown in Figure 2. Thus
this shock represents that part of the unanticipated quarterly change in monetary policy that is not accounted for by systematic responses over the quarter to unanticipated business cycle shocks. A consequence of our identification strategy is that if, rather counter-intuitively, monetary policy shocks should be such that surprise rises in the interest rate cause short-term increases in output, consumption and investment, then these effects would be captured by the business cycle shock shown in Figure 2, not by the monetary shock shown here. Thus output, consumption and investment in Figure 3 have a propensity to fall in the short term almost by construction and they do, although interestingly by very little. Over the medium term, monetary policy shocks
Figure 4. The basic government revenue shock, identified by orthogonality to the business cycle shock and monetary policy shock as well as a positive impulse response function of government revenues for four quarters after the shock. The restriction is indicated by the shaded area on the graph. This figure is available in color online at www.interscience.wiley.com/journal/jae

are associated with a marginally lower interest rate and increases in income, consumption and wages. These results are thus not inconsistent with the findings in Uhlig (2005): there, without orthogonality to the business cycle shock, sign restriction methods do not deliver a clear direction for real GDP in response to a surprise rise in interest rates.

What is a little surprising is the rise in government revenue in response to the rise in interest rates. One plausible, although not the only, explanation for this is that over the sample period monetary and fiscal policy was coordinated so that a monetary tightening was accompanied by a
fiscal tightening via an increase in taxes. If this were the case then there would be a danger that requiring fiscal shocks to be orthogonal to monetary policy shocks will cause biases in the results. For this reason we have checked the robustness of our identified fiscal shocks by identifying them both second (orthogonal to only the business cycle shock) and third (orthogonal to both the business cycle and monetary policy shocks). We find that the responses of the real variables are very similar in both these specifications and hence any bias is small. This may be because
monetary policy shocks do not appear to have a large effect on real macroeconomic variables. We conclude from this that controlling for the monetary policy shock is not important when analyzing the consequences of fiscal policy; see Mountford and Uhlig (2002) for a greater discussion of this issue.

3.3. The Basic Government Revenue Shock

A basic government revenue shock is identified as a shock that is orthogonal to the business cycle and monetary policy shock and where government revenue rises for a year after the shock. The
Figure 7. The basic government expenditure shock, identified by orthogonality to the business cycle shock and monetary policy shock as well as a positive impulse response function of government expenditures for four quarters after the shock. The restriction is indicated by the shaded area on the graph. This figure is available in color online at www.interscience.wiley.com/journal/jae

identified government revenue shocks are plotted in the third panel of Figure 1, where the change from shaded to non-shaded areas denotes changes in presidential terms. These identified shocks correspond, in part at least, with Romer and Romer's (2007) measure of exogenous tax changes, in that the shocks are on average negative in the mid 1960s and mid 1980s and positive in the early 1990s. They also clearly pick up the large effects of the 1975 Tax Reduction Act.

The impulse responses for this shock are displayed in Figure 4. Figure 4 shows that the responses of the real variables of interest to a standard government revenue shock are intuitive. GDP,
consumption and investment fall in response to an increase in revenue and real wages also fall, although with a lag. The responses of interest rates and prices are less intuitive as interest rates rise and reserves fall in response to a rise in revenue. Although no restriction is placed on the behavior of government spending for this shock, government spending follows the shape of the GDP and consumption responses and falls before recovering.
3.4. The Anticipated Government Revenue Shock

We also identify a year delayed shock where government revenue is restricted to rise only after a year. The responses of this shock are displayed in Figure 5. They show an intuitive ‘announcement
effect’ as the anticipated rise in revenues immediately depresses output and consumption. This accords with our intuition regarding the optimal smoothing behavior of individuals and firms. Interest rates also fall with this drop in output, which is also intuitive.6

The median responses of the unanticipated and anticipated shocks are compared in Figure 6, where the responses of the unanticipated shocks are shifted forward so that the implementation of the revenue shock is in the same time period. Figure 6 shows that the responses to an anticipated government revenue shock, while similar to those of the unanticipated shocks in terms of their sign, are somewhat smaller, perhaps because announced policies move fiscal variables by less than unanticipated changes.7

3.5. The Basic Government Spending Shock

A basic government revenue shock is identified as a shock that is orthogonal to the business cycle and monetary policy shock and where government spending rises for a year after the shock. The identified government spending shocks are plotted in the fourth panel of Figure 1, where the change from shaded to non-shaded areas denotes changes in presidential terms. These identified shocks show that government spending shocks were predominantly positive in the mid 1960s and early 1990s and predominantly negative around 1960 and in the early 1970s. In the the 1990s government spending appears to be more stable than average, with relatively few shocks reaching the 2.5% level in absolute terms.

The impulse responses for this shock are displayed in Figure 7. Figure 7 shows that the basic government spending shock stimulates output during the first four quarters, although only weakly, and has only a very weak effect on private consumption. It also reduces investment, although interestingly not via higher interest rates. Real wages do not respond positively to the government spending shock and indeed are negative on impact and in the medium term, which is more in accordance with neoclassical than New Keynesian models of government spending (see Ramey and Shapiro, 1998). Although no restriction is placed on the behavior of government revenue, this does not change very significantly and so the basic government spending shock will resemble a fiscal policy shock of deficit spending, whose responses are displayed in Figure 10 below. The response of prices to the increase in government spending is a little puzzling since both the GDP deflator and the producer price index for crude materials show a decline. Although this is a counter-intuitive result, it should be noted that this negative relationship between prices and government spending has also been found in other studies (see, for example Canova and Pappa, 2007; Edelberg, et al., 1999. Fatás and Mihov, 2001a).

6 When analyzing anticipated tax shocks, we implicitly assume that our VAR approach approximates a time series model, in which news about tax shocks are always about the next annual tax change and not about subsequent ones. Otherwise, overlapping news or a moving average process for taxes with two or more lags may result in problems with invertibility and an appropriate VAR representation, as Leeper et al. (2008) have shown.

7 Changes in fiscal policy may be the result of the systematic component of fiscal policy—unrelated to the business cycle—rather fiscal policy shocks, as Liu et al. (2007) have argued is the case for monetary policy shocks. However, we leave the joint identification of fiscal policy regime shifts and fiscal policy shock for future work.
3.6. The Anticipated Government Spending Shock

We also identify a year delayed shock where government spending is restricted to rise only after a year. The responses of this shock are displayed in Figure 8. They again show an intuitive ‘announcement’ effect on impact as the anticipated rise in spending immediately has a positive effect on output and interest rates. Again this is consistent with smoothing behavior by individuals and firms. The median responses of the unanticipated and anticipated shocks are compared in Figure 9, where the responses of the unanticipated shocks are shifted forward so that the implementation of the spending shock is in the same time period. Figure 9 shows that the responses to an anticipated government spending shock appear more persistent and stronger than for the unanticipated shocks and investment is no longer crowded out by the increase in government spending. This is an interesting and slightly puzzling result; however, it should be noted that the interpretation of these responses in the medium term is clouded by the marginal cut in taxes after a year. From Figure 5 this would have a significant expansionary effect and this may be the cause of the persistent positive effects on income and consumption. In Section 4.1 below we demonstrate a straightforward method for accounting for such changes when performing policy analysis.

4. POLICY ANALYSIS

We can use the basic shocks identified in the previous section to analyze the effects of different fiscal policies. We view different fiscal policy shocks as different linear combinations of the basic fiscal policy shocks. There are clearly a huge number of possible fiscal policies we could analyze, so here we restrict ourselves to comparing three popularly analyzed fiscal policies. A deficit-spending shock, a deficit financed tax cut and a balanced budget spending shock. We first detail how the impulse responses for these policies are generated.

4.1. CALCULATING THE IMPULSE RESPONSES FOR DIFFERENT FISCAL POLICY SCENARIOS

Our methodology regards different fiscal policy scenarios as being different combinations of the two basic shocks over a sequence of several quarters. For example, a government spending scenario where government spending is raised by 1% for four quarters while government revenue remains unchanged is the linear combination of the sequence of the two basic shocks that generates these responses in the fiscal variables as the combined impulse response. More formally, denoting $r_{j,a}(k)$ as the response at horizon $k$ of variable $j$ to the impulse vector $a$, then the above policy requires that

$$0.01 = \sum_{j=0}^{k} (r_{GS,BGS}(k-j)BGS_j + r_{GS,BGR}(k-j)BGR_j) \quad \text{for} \quad k = 0, \ldots, K$$

$$0 = \sum_{j=0}^{k} (r_{GR,BGS}(k-j)BGS_j + r_{GR,BGR}(k-j)BGR_j) \quad \text{for} \quad k = 0, \ldots, K$$

where $K = 4$, GS and GR stand for government expenditure and government revenue, and $BGS_j$ and $BGR_j$ are respectively the scale of the standard basic government spending and revenue shocks in period $j$. 

We analyze three key policy scenarios below: deficit spending, a deficit-financed tax cut and a balanced budget spending expansion. It should be clear, however, that other scenarios of interest can be analyzed in this manner as well.

4.2. A Deficit-Spending Fiscal Policy Scenario

The impulse responses for a deficit spending fiscal policy scenario are shown in Figure 10. The policy scenario is designed as a sequence of basic fiscal shocks such that government spending
rises by 1% and tax revenues remain unchanged for four quarters following the initial shock. As noted above, the standard basic government spending shock does not change tax revenues significantly, so the impulses in Figure 10 and are similar to those in Figure 7. Thus the deficit spending scenario stimulates output and consumption during the first four quarters although only weakly, it reduces non-residential investment and it produces a counter-intuitive response for prices.

4.3. A Deficit-Financed Tax Cut Fiscal Policy Scenario

The impulse responses for a deficit financed tax cut fiscal policy scenario are shown in Figure 11. The policy scenario is designed as a sequence of basic fiscal shocks such that tax revenues fall by 1% and government spending remains unchanged for four quarters (including the initial quarter) following the initial shock. The responses look very similar to a mirror image of the responses to the basic government revenue shock in Figure 4. Thus the tax cut stimulates output, consumption and investment significantly, with the effect peaking after about 3 years. The effect on prices is initially negative but subsequently positive following the rise in output.

4.4. The Balanced Budget Spending Policy Scenario

The balanced budget spending policy scenario is identified by requiring both government revenues and expenditures to increase in such a way that the increase in revenues and expenditure is equal for each period in the four-quarter window following the initial shock. For ease of comparison we choose a sequence of basic fiscal shocks such that government spending rises by 1% and government revenue rises by 1.28%. Government revenue rises by more than government spending since over the sample government revenue’s share of GDP is 0.162, while that of government spending is 0.208; thus we require government revenue to rise by \((0.208/0.162)\)%%. The results are shown in Figure 12. These show that on impact there is a small expansionary effect on GDP but thereafter the depressing effects of the tax increases dominate the spending effects, and GDP, consumption and investment fall.

4.5. Measures of the Effects of Policy Scenarios

To compare the effects of one fiscal scenario with another it is useful to define summary measures of the effects of each fiscal scenario. One measure used in the literature is the ratio of the response of GDP at a given period to the initial movement of the fiscal variable (see, for example, Blanchard and Perotti, 2002; Canova and Pappa, 2007). We refer to this as the impact multiplier. We report the impact multipliers of the deficit financed tax cut and deficit-spending policy scenarios below in Tables III and IV.

However, we think a measure of the impact of a scenario along the entire path of the responses up to a given period is also useful. In Figure 13 we have therefore plotted the present value of the impulse responses of GDP and the fiscal variables for the deficit-financed tax cut and the deficit-spending policy scenarios. We have also calculated a present value multiplier for these scenarios, which we display in Table II. To calculate the present value multiplier we use the
The deficit-financed tax cut policy scenario where government spending remains unchanged and government revenue is reduced by 1% for four quarters. These impulses are linear combinations of a sequence of the basic shocks displayed in Figures 4 and 7. This figure is available in color online at www.interscience.wiley.com/journal/jae

Following formula:

\[
\text{Present value multiplier at lag } k = \frac{\sum_{j=0}^{k} (1 + i)^{-j} y_j}{\sum_{j=0}^{k} (1 + i)^{-j} f_j / y}
\]
where $y_j$ is the response of GDP at period $j$, $f_j$ is the response of the fiscal variable at period $j$, $i$ is the average interest rate over the sample, and $f/y$ is the average share of the fiscal variable in GDP over the sample. We use the median multiplier in all cases.

Table II and Figure 13 and tell the same story. They show that in present value terms tax cuts have a much greater effect on GDP than government spending. The present value of the GDP response to a deficit spending scenario becomes insignificant after 2 years, whereas that for the deficit-financed tax cut is significantly positive throughout. Figure 13 also shows that the standard

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DOI: 10.1002/jae
Figure 13. The present value of the impulses for GDP and the changing fiscal variable for the deficit-spending policy scenario and the deficit-financed tax cut policy scenario displayed in Figures 10 and 11. This figure is available in color online at www.interscience.wiley.com/journal/jae

Table II. Present value multipliers of the policy scenarios

<table>
<thead>
<tr>
<th>Policy Scenario</th>
<th>1 qrt</th>
<th>4 qrts</th>
<th>8 qrts</th>
<th>12 qrts</th>
<th>20 qrts</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deficit-financed tax cut</td>
<td>0.29</td>
<td>0.52</td>
<td>1.63</td>
<td>5.25</td>
<td>-4.55</td>
<td>5.25 (qrt 12)</td>
</tr>
<tr>
<td>Deficit spending</td>
<td>0.65</td>
<td>0.46</td>
<td>0.07</td>
<td>-0.26</td>
<td>-2.07</td>
<td>0.65 (qrt 1)</td>
</tr>
</tbody>
</table>

This table shows the present value multipliers for a deficit financed tax cut policy scenario and for a deficit-spending fiscal policy scenario. The multipliers given are the median multipliers in both cases.
error of the present value multiplier becomes very large for the deficit-financed tax cut at later time periods.\footnote{8}

\section*{4.6. Comparison of Results with the Existing Literature}

Despite the novel methodology used in this paper, there are many similarities to results obtained elsewhere in the existing literature. There are, however, also important differences, which we shall discuss below. As we have used Blanchard and Perotti’s (2002) data definitions for the fiscal variables and use a very similar sample period, it is natural to compare our results most closely with their paper. We do this in the following subsection. We compare our results to other studies in a further subsection.

\subsection*{4.6.1 Comparison with Blanchard and Perotti (2002)}

The impact multipliers, defined above, are compared with those of Blanchard and Perotti (2002) in Tables III and IV.

For the deficit-financed tax cut, Table III shows that our results are very comparable with Blanchard and Perotti’s. In both studies the effect on output of a change in tax revenues is persistent and large. The size of the impact multiplier is greater in our results than in Blanchard and Perotti’s, although in both studies the impact multipliers of the tax cut scenario are greater than those of the spending scenario.

For the spending scenario, Table IV shows that size of the impact multipliers associated with increased government spending is smaller than Blanchard and Perotti’s, but that the timing has a similar pattern in the sense that the largest impact multipliers are in the periods close to the impact of the initial shock and with the responses after a year being insignificant.

With respect to the responses of investment again the two studies are similar. In both Blanchard and Perotti’s and in our study investment falls in response to both tax increases and government

\begin{table}[h]
\centering
\begin{tabular}{lcccccc}
\hline
 & 1 qtr & 4 qtrs & 8 qtrs & 12 qtrs & 20 qtrs & Maximum \\
\hline
\textit{Mountford and Uhlig} & & & & & & \\
GDP & 0.28 & 0.93 & 2.05 & 3.41 & 2.59 & 3.57 (qrt 13) \\
Tax revenues & -1.00 & -1.00 & 0.06 & 1.05 & 1.03 & \\
Gov. spending & 0.00 & 0.00 & 0.27 & 0.43 & 0.48 & \\
\textit{Blanchard and Perotti} & & & & & & (2002) \\
GDP & 0.70 & 1.07 & 1.32 & 1.30 & 1.29 & 1.33 (qrt 7) \\
Tax revenues & -0.74 & -0.31 & -0.17 & -0.16 & -0.16 & \\
Gov. spending & 0.06 & 0.10 & 0.17 & 0.20 & 0.20 & \\
\hline
\end{tabular}
\caption{Impact multipliers of deficit-financed tax cut policy scenarios}
\end{table}

This table shows the impact multipliers for a deficit-financed tax cut fiscal scenario for various quarters after the initial shock and compares them to similar measures from Blanchard and Perotti (2002 Table III). The multiplier represents the effect in dollars of a one-dollar cut in taxes at the first quarter. For the Mountford and Uhlig results this is calculated with the formula: Multiplier for GDP = GDP response / initial fiscal shock \times (Average fiscal variable share of GDP), where the median responses are used in all cases. On the calculation of the Blanchard and Perotti (2002) multipliers see Blanchard and Perotti (2002 section V).

\footnote{8}The median multiplier becomes negative at later time periods for both the deficit spending and deficit-financed tax cut shocks but in both cases these negative multipliers are not significant.
Table IV. Impact multipliers of a deficit-spending policy scenario

<table>
<thead>
<tr>
<th></th>
<th>1 qrt</th>
<th>4 qrts</th>
<th>8 qrts</th>
<th>12 qrts</th>
<th>20 qrts</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mountford and Uhlig</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.65</td>
<td>0.27</td>
<td>−0.74</td>
<td>−1.19</td>
<td>−2.24</td>
<td>0.65 (qrt 1)</td>
</tr>
<tr>
<td>Gov. spending</td>
<td>1.00</td>
<td>1.00</td>
<td>0.90</td>
<td>0.37</td>
<td>−0.32</td>
<td></td>
</tr>
<tr>
<td>Tax revenues</td>
<td>0.00</td>
<td>0.00</td>
<td>−0.33</td>
<td>−0.87</td>
<td>−2.04</td>
<td></td>
</tr>
<tr>
<td><strong>Blanchard and Perotti (2002)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.90</td>
<td>0.55</td>
<td>0.65</td>
<td>0.66</td>
<td>0.66</td>
<td>0.90 (qrt 1)</td>
</tr>
<tr>
<td>Gov. spending</td>
<td>1.00</td>
<td>1.30</td>
<td>1.56</td>
<td>1.61</td>
<td>1.62</td>
<td></td>
</tr>
<tr>
<td>Tax revenues</td>
<td>0.10</td>
<td>0.18</td>
<td>0.33</td>
<td>0.36</td>
<td>0.37</td>
<td></td>
</tr>
</tbody>
</table>

This table shows the impact multipliers for a deficit-spending fiscal scenario for various quarters after the initial shock and compares them to similar measures from Blanchard and Perotti (2002 Table IV). The multiplier represents the effect in dollars of a one-dollar increase in spending at the first quarter. For the Mountford and Uhlig results this is calculated with the formula: Multiplier for GDP = Initial fiscal shock / (Average fiscal variable share of GDP), where the median responses are used in all cases. On the calculation of the Blanchard and Perotti (2002) multipliers see Blanchard and Perotti (2002 section V).

spending increases. With regard to consumption the results have some similarities to Blanchard and Perotti’s in that consumption does not fall in response to a spending shock. However, in Blanchard and Perotti (2002) consumption rises significantly in response to a spending shock, whereas in our analysis consumption does not move by very much.9 As Figure 10 shows, consumption’s response is only significantly positive on impact and then by only a small amount. Thereafter its response is insignificant. Finally, we find that real wages do not rise in response to an increase in government spending and have a negative response on impact and at longer horizons. Thus, taken together, these findings are difficult to reconcile with the standard Keynesian approach, although they are also not the responses predicted by the benchmark real business cycle model.

4.6.2 Comparison with Other Studies

With regard to other studies in the literature, not only are there differences in methodology but also in the definitions of the fiscal variables. There is thus no shortage of potential sources for differences in results. Nevertheless, there is still some common ground in the results of this paper and previous work. For example, consider the recent work by Burnside et al. (2003), which builds on the work of Ramey and Shapiro (1998) and Edelberg et al. (1999) in using changes in military purchases associated with various wars to identify government spending shock. They find that private consumption does not change significantly in response to a government spending shock. However, in contrast to our results they find that investment has an initial, transitory positive response to the spending shock.

4.7 Variance of the Policy Analysis

Clearly there is a considerable degree of uncertainty in the numbers displayed in Tables III and IV, as they are based on the maximum multipliers from the median impulse responses. An advantage of the Bayesian approach used in our analysis is that it naturally provides a measure of the standard

---

9 See also Gali et al. (2007), who also find a significantly positive consumption response.
errors for this policy analysis. Standard errors can easily be calculated for each policy scenario by taking the maximum and minimum multipliers of GDP and their corresponding lag for each of the draws from the posterior. These maxima and minima can then be ordered and the 16th, 50th and 84th percentiles reported. This is done in Table V.

Table V supports the conclusions above. The maximum expansionary effect of a deficit-spending scenario is much below that of the tax revenue scenario. Indeed the upper confidence limit of the deficit-spending scenario is below the lower confidence limit of the tax cut scenario. For the tax cut the maximum effects are significantly positive and the minimum effects are insignificantly different from zero.

The results in Table V are usefully related to the impulse responses in Figures 10–12. For the tax cut the maximum multipliers occur after two or more years, whereas for the balanced budget and deficit-spending scenarios the maximum effects occur at short lags and the minimum effects at longer lags. Since the variance of the impulse responses for these scenarios appears to increase at longer lag lengths, we also look at the maximum and minimum multipliers of the two spending scenarios in the first year after the initial shock. In this case we now get the result that the deficit-spending scenario’s minimum multiplier is insignificantly different from zero but that for the balanced budget spending scenario is still significantly negative.

These statistics relate to the distribution of the maximum and minimum impact multiplier effects of each fiscal scenario. For each draw the maximum and minimum fiscal multiplier is calculated and the 16th, 50th and 84th percentiles of these results are displayed. The multiplier statistic is calculated in terms of the initial, lag 0, fiscal shock as follows: multiplier for GDP = \( \frac{\text{GDP response}}{\text{Fiscal shock at Lag 0}} \times \frac{\text{Average fiscal variable share of GDP}}{\text{GDP multipliers}} \).

### 4.8. Policy Conclusions

An important lesson one can draw from the results is that while a deficit-financed expenditure stimulus is possible, the eventual costs are likely to be much higher than the immediate benefits. Suppose that government spending is increased by 2%, financed by increasing the deficit: this results, using the median values from Table V, at maximum, in less than a 2% increase in GDP. But the increased deficit needs to be repaid eventually with a hike in taxes. Even ignoring compounded

<table>
<thead>
<tr>
<th>Fiscal scenario</th>
<th>Maximum multiplier</th>
<th>Confidence interval 16th, 84th quantiles</th>
<th>Minimum multiplier</th>
<th>Confidence interval 16th, 84th quantiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deficit spending</td>
<td>0.91</td>
<td>0.68, 1.37</td>
<td>−2.88</td>
<td>−4.99, −0.80</td>
</tr>
<tr>
<td>Balanced budget</td>
<td>0.47</td>
<td>0.19, 0.84</td>
<td>−5.84</td>
<td>−9.46, −3.63</td>
</tr>
<tr>
<td>Tax cut</td>
<td>3.81</td>
<td>2.58, 5.49</td>
<td>0.19</td>
<td>−0.11, 0.36</td>
</tr>
<tr>
<td>in first year</td>
<td>lag 4</td>
<td>lag 2, lag 6</td>
<td>lag 24</td>
<td>lag 24, lag 8</td>
</tr>
<tr>
<td>Balanced budget</td>
<td>0.45</td>
<td>0.16, 0.79</td>
<td>−0.71</td>
<td>−1.11, −0.40</td>
</tr>
<tr>
<td>in first year</td>
<td>at lag 3</td>
<td>lag 1, lag 1</td>
<td>at lag 4</td>
<td>lag 1, lag 2</td>
</tr>
</tbody>
</table>

Table V. Maximum and minimum impact multipliers of fiscal policy scenarios

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\( J. \text{ Appl. Econ. 24: 960–992 (2009)} \)

DOI: 10.1002/jae
interest rates, this would require a tax hike of over 2%.\textsuperscript{10} This tax hike results in a greater than 7% drop in GDP. Thus, unless the policy maker’s discount rate is very high, the costs of the expansion will be much higher than the initial benefit.

This general line of reasoning is consistent with the balanced budget spending scenario whose impulses are shown in Figure 12. This shows that when government spending is financed contemporaneously the contractionary effects of the tax increases outweigh the expansionary effects of the increased expenditure after a very short time.

5. CONCLUSION

In this paper we have presented a new approach for distinguishing the effects of fiscal policy shocks by adapting the method of Uhlig (2005). This method uses only the information in the macroeconomic time series of the vector autoregression together with minimal assumptions to identify fiscal policy shocks. In particular, it imposes no restrictions on the signs of the responses of the key variables of interest—GDP, private consumption, real wages and private non-residential investment—to fiscal policy shocks. The paper applied this approach using post-war data on the US economy.

We have analyzed three types of policy scenarios: a deficit-financed spending increase, a balanced budget spending increase (financed with higher taxes) and a deficit-financed tax cut, in which revenues increase but government spending stays unchanged. We found that a deficit-spending scenario stimulates the economy for the first four quarters but only weakly compared to that for a deficit-financed tax cut. We also found that both types of spending scenario had the effect of crowding out investment.

Although the best fiscal policy for stimulating the economy appears to be deficit-financed tax cuts, we wish to point out that this should not be read as endorsing them. This paper only points out that unanticipated deficit-financed tax cuts work as a (short-lived) stimulus to the economy, not that they are sensible. The resulting higher debt burdens may have long-term consequences which are far worse than the short-term increase in GDP, and surprising the economy may not be good policy in any case. These normative judgements require theoretical models for which the empirical positive results in this paper can provide a useful starting point.

ACKNOWLEDGEMENTS

This paper is a substantially revised version of the CEPR discussion paper No 3338. This research has been sponsored by the Deutsche Forschungsgemeinschaft through the SFB 649 ‘Economic Risk’ and by the RTN networks NASEF and MAPMU. We are grateful to comments by seminar audiences at Royal Holloway, Exeter University, Milan, an NASEF meeting on Hydra, an SFB 373 meeting in Wulkow, a conference in Alicante, Southampton University and the London Business School. Special thanks go to Greg Mankiw for pointing out an important issue in an earlier version of this paper, which we have now resolved.

\textsuperscript{10} For simplicity, we are assuming the tax hike to be a surprise, when it occurs, which allows us to use the results above.
REFERENCES


APPENDIX A: VARS AND IMPULSE MATRICES

A VAR in reduced form is given by

\[ Y_t = \sum_{i=1}^{L} B_i Y_{t-i} + u_t, \quad t = 1, \ldots, T, \quad E[u_t u'_t] = \Sigma \]

where \( Y_t \) are \( m \times 1 \) vectors, \( L \) is the lag length of the VAR, \( B_i \) are \( m \times m \) coefficient matrices and \( u_t \) is the one-step-ahead prediction error.

The problem of identification is to translate the one-step-ahead prediction errors, \( u_t \), into economically meaningful, or ‘fundamental’, shocks, \( v_t \). We adopt the common assumptions in the VAR literature that there are \( m \) fundamental shocks, which are mutually orthogonal and normalized to be of variance 1. Thus \( E[v_t v'_t] = I_m \). Identification of these shocks amounts to identifying a matrix \( A \), such that \( u_t = Av_t \) and \( AA' = \Sigma \). The \( j \)th column of \( A \) represents the immediate impact, or impulse vector, of a one standard error innovation to the \( j \)th fundamental innovation, which is the \( j \)th element of \( v \). The following definition is useful, and new.

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Definition 1  An impulse matrix of rank $n$ is a $n \times m$ sub-matrix of some $m \times m$ matrix $A$, such that $AA' = \Sigma$. An impulse vector $a$ is an impulse matrix of rank 1, i.e., a vector $a \in \mathbb{R}^m$ such that there exists some matrix $A$, where $a$ is a column of $A$, such that $AA' = \Sigma$.

One can show that the identification does not depend on the particular matrix $A$ chosen beyond a given impulse matrix, i.e., a given impulse matrix uniquely identifies the fundamental shocks corresponding to it.

Theorem 1  Suppose that $\Sigma$ is regular. Let a given impulse matrix $[a^{(1)}, \ldots, a^{(n)}]$ of size $n$ be a submatrix of two $m \times m$ matrices $A$, $\tilde{A}$ with $AA' = \tilde{A}\tilde{A}' = \Sigma$. Let $v_i = A^{-1}u_i$, $\tilde{v}_i = \tilde{A}^{-1}u_i$ and let $v^{(1)}_i, \ldots, v^{(n)}_i$ resp. $\tilde{v}^{(1)}_i, \ldots, \tilde{v}^{(n)}_i$ be the entries in $v_i$ resp. $\tilde{v}_i$ corresponding to $a^{(1)}, \ldots, a^{(n)}$, i.e., if, for example, $a^{(1)}$ is the third column of $A$, then $v^{(1)}_3$ is the third entry of $v_i$. Then, $v^{(1)}_3 = \tilde{v}^{(1)}_3$ for all $i = 1, \ldots, n$.

Proof  Without loss of generality, let $[a^{(1)}, \ldots, a^{(n)}]$ be the first $n$ columns of $A$ and $\tilde{A}$. If this is for example, not the case for $A$ and if, for example, $a^{(1)}$ is the third column of $A$, one can find a permutation matrix $P$ so that the given impulse matrix will be the first $n$ columns of $A = AP$ with $a^{(1)}$ the first column of $\tilde{A}$. Since $PP' = I$, $\tilde{A}\tilde{A}' = \Sigma$. Furthermore, the first column of $P$ is the vector $e_3$, which is zero except for a 1 in its third entry; hence the first entry of $A^{-1}u_i = P'v_i$ must be the third entry of $v_i$ and thus be $v^{(1)}_3$ corresponding to $a^{(1)}$.

With $[a^{(1)}, \ldots, a^{(n)}]$ as the first $n$ columns of $A$ and $\tilde{A}$, let $E_n = [I_n, 0_{n,m}]$. Note that $[v^{(1)}_i, \ldots, v^{(n)}_i]' = E_nA^{-1}u_i$. We need to show that $E_nA^{-1}u_i = E_n\tilde{A}^{-1}u_i$ for all $u_i$. It suffices to show that $E_nA^{-1}\Sigma = E_n\tilde{A}^{-1}\Sigma$, since $\Sigma$ is regular. But this follows from

$$
E_nA^{-1}\Sigma = E_nA^{-1}AA'
= E_nA'
= [a^{(1)}, \ldots, a^{(n)}]'
= E_n\tilde{A}^{-1}\tilde{A}\tilde{A}'
= E_n\tilde{A}^{-1}\Sigma
$$

In the VAR literature identification usually proceeds by identifying all $m$ fundamental shocks and so characterizing the entire $A$ matrix. This requires imposing $m(m - 1)/2$ restrictions on the $A$ matrix. This is done either by assuming a recursive ordering of variables in the VAR, so that a Cholesky decomposition of $A$ can be used (see Sims, 1986), or by imposing the $m(m - 1)/2$ restrictions via assumed short-run structural relationships as in Bernanke (1986) and Blanchard and Watson (1986), via assumed long-run structural relationships, as in Blanchard and Quah (1989) or via both assumed short-run and long-run structural relationships as in Galí (1992).

This paper instead extends the method of Uhlig (2005) and identifies at most three fundamental shocks and so needs to characterize an impulse matrix $[a^{(1)}, a^{(2)}, a^{(3)}]$ of rank 3 rather than all of $A$. This is accomplished by imposing sign restrictions on the impulse responses. Note that, by construction, the covariance between the fundamental shocks $v^{(1)}_i$, $v^{(2)}_i$ and $v^{(3)}_i$ corresponding to $a^{(1)}$, $a^{(2)}$ and $a^{(3)}$ is zero, i.e., that these fundamental shocks are orthogonal.

To that end, note that any impulse matrix \([a^{(1)}, \ldots, a^{(n)}]\) can be written as the product \([a^{(1)}, \ldots, a^{(n)}] = \tilde{A}Q\) of the lower triangular Cholesky factor \(\tilde{A}\) of \(\Sigma\) with an \(n \times m\) matrix \(Q = [q^{(1)}, \ldots, q^{(n)}]\) of orthonormal rows \(q^{(i)}\), i.e., \(QQ' = I_n\): this follows from noting that \(\tilde{A}^{-1}A\) must be an orthonormal matrix for any decomposition \(AA' = \Sigma\) of \(\Sigma\). Likewise, Let \(a = a^{(s)}, s \in \{1, \ldots, n\}\) be one of the columns of the impulse matrix and \(q = q^{(s)} = \tilde{A}^{-1}a^{(s)}\) be the corresponding column of \(Q\): note that \(q^{(i)}\) does not depend on the other \(a^{(p)}, p \neq s\). As in Uhlig (2005), it follows easily that the impulse responses for the impulse vector \(a\) can be written as a linear combination of the impulse responses to the Cholesky decomposition of \(\Sigma\) as follows. Define \(r_{ji}(k)\) as the impulse response of the \(j\)th variable at horizon \(k\) to the \(i\)th column of \(Q\), and the \(m\)-dimensional column vector \(r_i(k) = [r_{i1}(k), \ldots, r_{im}(k)]\). Then the \(m\)-dimensional impulse response \(ra(k)\) at horizon \(k\) to the impulse vector \(a^{(s)}\) is given by

\[
ra(k) = \sum_{i=1}^{m} q_ir_i(k)
\]

(1)

(where \(q_i\) is the \(i\)th entry of \(q = a^{(s)}\)).

Define the function \(f\) on the real line per \(f(x) = 100x\) if \(x \geq 0\) and \(f(x) = x\) if \(x \leq 0\). Let \(s_j\) be the the standard error of variable \(j\). Let \(J_{S1+}\) be the index set of variables for which identification of a given shock restricts the impulse response to be positive and let \(J_{S1-}\) be the index set of variables for which identification restricts the impulse response to be negative. To impose the additional identifying inequality sign restrictions beyond the zero restrictions of equation (3), we solve

\[
a = \arg\min_{a=\tilde{A}q} \Psi(a)
\]

(2)

where the criterion function \(\Psi(a)\) is given by

\[
\Psi(a) = \sum_{j \in J_{S1+}} \sum_{k=0}^{3} f\left(-\frac{r_{ja}(k)}{s_j}\right) + \sum_{j \in J_{S1-}} \sum_{k=0}^{3} f\left(\frac{r_{ja}(k)}{s_j}\right)
\]

Computationally, we implement this minimization using a simplex algorithm: it is available on many statistical packages, such as MATLAB and RATS; for this paper we use the version of the algorithm written in GAUSS by Bo Honore and Ekaterini Kyriazidou, available from http://webware.princeton.edu/econometrics/programs/gaussmax/

Note that ‘zero’ restrictions, where the impulse responses of the \(j\)th variable to an impulse vector \(a\) for, say, the first four periods are set to zero, can be incorporated into the analysis fairly easily. These restrictions can be written as a restriction on the vector \(q\) that

\[
0 = Rq
\]

(3)

where \(R\) is a \(4 \times m\) matrix of the form

\[
R = \begin{bmatrix}
  r_{j1}(0) & \cdots & r_{jm}(0) \\
  \vdots & \ddots & \vdots \\
  r_{j1}(3) & \cdots & r_{jm}(3)
\end{bmatrix}
\]

To identify an impulse matrix \([a^{(1)}, a^{(2)}]\), where the first shock is a business cycle shock and the second shock is a fiscal policy shock, first identify the business cycle shock \(a^{(1)} = \tilde{A}q^{(1)}\) in the manner described above and then identify the second shock \(a^{(2)}\) by replacing the minimization problem (2) with

\[
\alpha = \arg\min_{\alpha=\tilde{A}q,Rq=0,q'q^{(1)}=0} \Psi(\alpha)
\]

i.e., by additionally imposing orthogonality to the first shock. The two restrictions \(Rq = 0, q'q^{(1)} = 0\) can jointly be written as

\[
0 = \tilde{R}q
\]

where \(\tilde{R}' = [q^{(1)}, R']\). Likewise, if orthogonality to two shocks—the business cycle shock and the monetary policy shock—is required, identify the business cycle shock \(a^{(1)} = \tilde{A}q^{(1)}\) and identify the monetary policy shock \(a^{(2)} = \tilde{A}q^{(2)}\) and solve

\[
\alpha = \arg\min_{\alpha=\tilde{A}q,Rq=0,q'q^{(1)},q'q^{(2)}=0} \Psi(\alpha)
\]

Given the above we can now state our identification restrictions more formally. We only provide two: the others follow the same pattern.

**Definition 2** A business cycle shock impulse vector is an impulse vector \(\alpha\) that minimizes a criterion function \(\Psi(\alpha)\), which penalizes negative impulse responses of GDP, private consumption, non-residential investment and government revenue at horizons \(k = 0, 1, 2,\) and 3.

**Definition 3** A basic government revenue shock impulse vector is an impulse vector \(\alpha\) minimizing a criterion function \(\Psi(\alpha)\), which penalizes negative impulse responses to the vector \(\alpha\) of government revenue at horizons \(k = 0, 1, 2,\) and 3.

The computations are performed using a Bayesian approach as in Uhlig (2005) (see also Sims and Zha, 1998). We take a number of draws from the posterior. For each draw from the posterior of the VAR coefficients and the variance–covariance matrix \(\Sigma\), the shocks are identified using the criteria described above. Given the sample of draws for the impulse responses, confidence bands can be plotted.

**APPENDIX B: THE DATA**

All the data we use are freely available from the World Wide Web. The data on components of US national income is taken from the *National Income and Product Accounts* (NIPA), which are made publically available by the Bureau of Economic Analysis on their website http://www.bea.gov/national/nipaweb/index.asp/. The monetary data—the interest rate, producer commodity price index and adjusted reserves—and the real wage data are taken from the Federal Reserve Board of St Louis website http://www.stls.frb.org/fred/.

**7.1. B.1. Definitions of Variables in the VAR**

All the components of national income are in real per capita terms and are transformed from their nominal values by dividing them by the gdp deflator (NIPA Table VII.1, row 4) and the population
measure (NIPA Table II.1, row 35). The table and row numbers refer to the organization of the data by the Bureau of Economic Analysis.

**GDP:** This is NIPA Table I.1, row 1.

**Private Consumption:** This is NIPA Table I.1, row 2.

**Total Government Expenditure:** This is ‘Federal Defense Consumption Expenditures’, NIPA Table III.7 row 4, plus ‘Federal Non Defense Consumption Expenditures’, NIPA Table III.7, row 15, plus ‘State and Local Consumption Expenditures’, NIPA Table III.7, row 28, plus ‘Federal Defense Gross Investment’, NIPA Table III.7, row 11, plus ‘Federal Non Defense Gross Investment’, NIPA Table III.7, row 24, plus ‘State and Local Gross Investment’, NIPA Table III.7, row 35.

**Total Government Revenue:** This is ‘Total Government Receipts’, NIPA Table III.1, row 1, minus ‘Net Transfers Payments’, NIPA Table III.1, row 8, and ‘Net Interest Paid’, NIPA Table III.1 row 11.

**Real Wages:** This is ‘Nonfarm Business Sector: Real Compensation Per Hour’ Series COMPRNFB from the US Department of Labor: Bureau of Labor Statistics.

**Private Non-Residential Investment:** This is ‘Nominal Gross Private Domestic Investment’, NIPA Table I.1, row 6, minus private residential investment, NIPA Table I.1, row 11.

**Interest Rate:** This is the Federal Funds rate which is the series fedfunds at the Federal Reserve Board of St Louis website http://www.stls.frb.org/fred/. We take the arithmetic average of the monthly figures for the Federal Funds Rate.

**Adjusted Reserves:** This is the Adjusted Monetary Base given by the series adjressl at the Federal Reserve Board of St Louis website http://www.stls.frb.org/fred/. We take the arithmetic average of the monthly figures to get a quarterly figure.

**PPIC:** This is the Producer Price Index of Crude Materials given by the series ppicrm at the Federal Reserve Board of St Louis website http://www.stls.frb.org/fred/. We take the arithmetic average of the monthly figures to get a quarterly figure.

The **GDP Deflator:** This is NIPA Table VII.1, row 4.

The VAR system consists of these 10 variables at quarterly frequency from 1955 (Q1) to 2000 (Q4), has six lags, no constant or time trend, and uses the logarithm for all variables except the interest rate where we have used the level.

The fiscal variables are chosen so that they will have different responses to business cycle movements and fiscal policy shocks. The government expenditure variable is chosen so as to exclude expenditures which will vary over the business cycle such as transfer payments; see, for example, Blanchard (1997, p. 600) on this. The government receipts variable should clearly respond positively to a business cycle shock; an increase in output should increase tax receipts and reduce transfer payments.

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11 This definition follows Blanchard and Perotti (2002) regarding transfer payments as negative taxes. We use this definition in order not to obscure the implications of the new identification technique used in this paper, by using different data.