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What is the Real Story for Interest Rate Volatility?

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Abstract. *What is the source of interest rate volatility? Why do low interest rates precede business cycle booms? Most observers tend to assume that monetary policy is largely responsible for it. Indeed, a standard real business cycle model delivers rather small fluctuations in real interest rates. Here, however, we present two models of the real business cycle variety, in which the fluctuations of real rates are of similar magnitude as in the data, while simultaneously matching salient business cycle facts. The second model also replicates the cyclical behavior of real interest rates.*

The models build on recent work by Danthine and Donaldson, Jermann, and Boldrin, Christiano and Fisher. We assume that there are workers and capital owners. The first model posits habit formation and adjustment costs to the stock of capital. The second model assumes that it takes time to plan investment and time to build capital.

1. INTRODUCTION

Monetary policy is often perceived as working through changes in real interest rates. It is argued that monetary policy affects real rates through changes in nominal rates because nominal frictions make the adjustment of prices sluggish. How should optimal monetary policy be conducted, given this view on how monetary policy works? One suggestion is that monetary policy should move in a way to neutralize the effects of nominal frictions. According to this view, the monetary policy authority should try to find the real rate which would emerge in an economy without frictions – call this the natural rate – and then choose the nominal interest rate such that the implied real rate tracks the natural rate, given the inflation rate due to sluggish price adjustment.

To do so, one would need to calculate this natural real rate. Real business cycle theory offers an appealing framework to conduct these calculations, as this theory has been able to successfully match quantitative features of the business cycle despite its rather simple structure. In contrast to what is observed in the data, standard real business cycle models, however, also predict a very low volatility of the real rate. Moreover, the cyclical behavior of the real rate in these models is often judged to be at odds with the empirical evidence; see, for example, King and Watson (1996).

From these observations one could conclude that the real rate in an economy with nominal frictions behaves very differently than in an economy without nominal frictions. Furthermore, monetary policy apparently does not try to target the natural rate, and it is quite possible that a large part of the observed real rate volatility has been induced by monetary policy. Alternatively one might conclude that the monetary policy authority indeed tries to track the natural rate, but that basic real business cycle models miss out on the real rate movements, despite capturing other key quantity movements quite well.

The first conclusion appears unattractive to us for a number of reasons. First and foremost, it should be a cause of concern, because one would then have to argue that the large fluctuations induced by monetary policy are probably largely unnecessary and costly. The allocational effects of the large movements in the real rate could be substantial. This would be a high price to pay, if this was done only for the goal of pursuing price stability. Second, standard real business cycle models predict investment to be quite elastic with respect to interest rate movements: the fluctuations in the real rate should therefore lead to substantially larger swings in investments than those that we observe.

In this paper, we therefore pursue the second conclusion. We ask: is it possible to find an entirely 'real' explanation for the volatility of real interest rates? That is, is there a variant of the real business cycle model out there, which matches both the observed real interest rate volatility as well as other salient features of business cycles? We show that this is the case. We therefore call it the 'real' story for interest rate fluctuations. Indeed, we will provide two model variants that do the trick. It would then not be hard to extend these models with a rather passive monetary authority, which does nothing but trace out the developments of the real rates on the market in its policy decision.

We should emphasize that this paper only demonstrates the possibility for such a result, without taking a final and strong stand on the discussion at the beginning of this introduction. Even absent that discussion, it is interesting to push the real business cycle paradigm further in the direction of quantitatively reconciling macroeconomic behavior with financial market facts. There is a recent literature, focusing on these questions, often motivated by explaining the equity premium observation; see Mehra and Prescott (1985): a survey and further insights can be found in Rouwenhorst (1995). As explained in Lettau and Uhlig (1997), one needs sizeable fluctuations in the marginal utility of consumption in order to have any hope of explaining the equity premium. A similar observation lies at the heart of any attempt to explain the interest rate volatility, since the safe rate of interest is essentially equal to the expected intertemporal marginal rate of substitution, barring transaction costs. Thus, while we do not focus on the equity premium here, we can nonetheless fruitfully draw on that literature to make progress on our task at hand.

A substantial body of that literature has studied economies with exogenously evolving consumption streams and shown that 'habit formation' and 'catching-up with the Joneses' preferences can go a long way toward reconciling observed equity premia with theory; see, for example, Abel (1990, 1999), Constantinides (1990) and Campbell and Cochrane (1999). However, things become a lot more difficult, when trying to *endogenize* consumption, and introduce investment and production technologies as well as flexible labor. Jermann (1998), for example, holds labor supply fixed in order to obtain reasonable results. Lettau and Uhlig (1995) have shown that simply introducing 'catching-up' preferences into an otherwise standard real business cycle model leads to excessive smoothness of consumption, with other variables like leisure shouldering the burden of the productivity fluctuations.

The most successful attempt to date of marrying both financial market facts with business cycle fluctuations is due to Boldrin *et al.* (1999). They provide a 'preferred' model with habit formation in a two-sector economy, and an assumed inability to adjust either labor or capital for one period. As a result, a productivity shock on impact will only affect consumption, substantially moving marginal utility. Furthermore, because of habit formation, agents try to maintain the new consumption level in the future. They are able to match more or less both financial market facts as well as business cycle facts.

Our paper owes a substantial intellectual debt to their work. However, their assumption of infinitely high adjustment costs for one period and for both capital and labor, seems to generate the prediction that removing the problems with labor adjustments would also remove both the equity premium and the interest rate fluctuations. We judge this to be rather implausible. Instead, we pursue a route proposed by Danthine *et al.* (1992) and Danthine and Donaldson (1994) in a series of papers. They assume two rather than one 'class' of agents in their models: 'capital owners' and 'workers'. While the workers do not own any assets, the capital owners do not work. The latter feature makes it impossible for capital owners to use leisure as an adjustment channel for productivity shocks. We extend their work in two ways. In our first model, we additionally assume habit formation preferences for the capital owners, who furthermore face adjustment costs when investing in their capital stock, thus effectively turning them into Jermann-like agents. In our second model, we keep standard-looking preferences but assume that investment decisions take time to plan. In summary, our paper can be viewed as a synthesis of the work by Kydland and Prescott (1982), Danthine *et al.* (1992), Danthine and Donaldson (1994), Jermann (1998) and Boldrin *et al.* (1999). It turns out that both models are simple as well as able to match some of the key facts. Somewhat surprising to us, the second model actually appears to perform better than the first.

We view this paper not as the 'final answer' but rather as a stepping stone to a deeper understanding of the mechanism in real business cycle models. Surely, our models have some yet-to-be-discovered undesirable features, which future research may want to try to improve upon.[1]

The plan of the paper is as follows. In Section 2, we document some facts about the volatility and cyclical behavior of nominal and real interest rates. In Section 3, we show the inability of a benchmark real business cycle model to account for the fluctuations in real interest rates. In Section 4 we introduce our first model. Section 5 presents the second model. Finally, Section 6 concludes.

2. THE FACTS

In the postwar US economy real rates are quite volatile, and with respect to aggregate output low real rates tend to precede high future output, and high current output tends to precede high future real rates. In this section we document these facts for various definitions of the real rate, which is defined as the nominal rate minus the inflation rate. We provide a broad set of results for various measures of the nominal interest rate, the price index used for deflation, and the method used to calculate inflation rates. Additional discussion and documentation of the facts, partly employing alternative methods such as forecasted inflation rates, can be found in, for example, King and Watson (1996), Stock and Watson (1998) and Dotsey and Scholl (1999).

For nominal rates we use the Federal funds rate (FFR) and returns on three-month and one-year Treasury bills. Table 1 shows the volatility and the correlations with real output for nominal interest rates. As a measure of real aggregate output we use GNP in chained 1982 dollars. The data are for 1965 to 1996, using quarterly series, time-averaging monthly data for the interest rates. The data were obtained from the website of the Federal Reserve Bank of St Louis. We present two statistics for the volatility of rates of return. For the first measure, we calculate the standard deviation of annual rates of return obtained from averaging quarterly rates of return.[2] For the second measure we calculate the standard deviation of annualized quarterly rates of return after low-frequency movements have been removed with the Hodrick–Prescott filter; see, for example, Cooley and Prescott (1995). For the co-movement of rates of return with output we use Hodrick–Prescott filtered quarterly series. According to Table 1, nominal rates fluctuate 2.5 to 3 per cent annually, using unfiltered data. The correlations show that nominal interest rates are low when preceding a rise in output, and high following it. This feature in the data is typically interpreted as induced by monetary policy.

1. The analysis here complements other work which proceeds within the framework of aggregate economic models. There is a separate strand of literature which studies asset pricing questions in heterogenous agent economies with incomplete markets. See, for example, den Haan (1995, 1996), Krusell and Smith (1997) and Heaton and Lucas (1996).
2. Related work on the equity premium studies annual rates of return; see, for example, Cecchetti *et al*. (1993).

Table 1 The volatility and the correlations with real output for nominal interest rates

Asset	$\sigma_{r,\mathrm{unf.}}$	$\sigma_{r,\mathrm{HP}}$	−4.0	−3.0	−2.0	−1.0	0.0	1.0	2.0	3.0	4.0
FFR	3.11	1.87	−0.55	−0.40	−0.22	0.07	0.34	0.50	0.56	0.57	0.54
3mo Tbill	2.56	1.41	−0.53	−0.37	−0.19	0.08	0.33	0.46	0.51	0.51	0.49
1yr Tbill	2.66	1.37	−0.49	−0.36	−0.20	0.06	0.30	0.41	0.45	0.44	0.42

Notes: All series are quarterly and HP-filtered, except for the first column as described in the text. The column headed by '−4' contains the correlation of $r(t-4)$ and $y(t)$, i.e. for leading interest rates. The volatility measures here as well as in all other tables are in per cent.

Preferably one would like to calculate the real rate of an asset r_t by subtracting a measure of expected inflation over the asset's life span π_t from its nominal interest rate i_t. There are two issues that require some thought. First, which price index should be used to calculate inflation? And second, how do we calculate the expected inflation rate appropriate for calculating the real rate?

To deal with the first issue, we calculate inflation rates based on the consumer price index (CPI), the personal consumption expenditure price index (PCE), as well as the producer price index (PPI). Surely, the use of the CPI is standard in the literature. However, to the degree that there are differences using other price indices, they should be of interest.

To deal with the second issue, and since inflation rates are sluggish in the data and can be forecasted reasonably accurately for up to a year, we have used realized inflation rates rather than some forecast thereof: otherwise, an additional issue arises as to the choice of the forecasting method. We use three methods to calculate inflation rates. The first two methods calculate the realized inflation rate over the holding period of the asset, whereas the third method looks at the past inflation experience. For the first method we use the annualized, realized inflation rate over the life of the asset (denoted by 'equal' in the table), that is $\pi_t^{\mathrm{equal}} = (4/h) * (p_{t+h} - p_t)$, where p_t is the log of the price level and h denotes for many quarters the asset is held. For the second method we use the realization of the backward-looking one-year inflation rate (denoted by '1 yr' in the table), that is $\pi_t^{\mathrm{1year}} = p_{t+h} - p_{t+h-3}$. For the third method we use the one-year inflation rate which had just been realized at the start of the life of the asset, that is $\pi_t^{\mathrm{lagged}} = p_t - p_{t-4}$. This is appropriate if the current inflation rate is an excellent predictor of future inflation rates up to one year.

For the different nominal rates these methods are implemented as follows. We use the third method ('lagged' inflation rates) for all rates. For the one-year Treasury bill rate the first and second methods are equivalent. For assets with shorter maturity than one year, annualized inflation rates over the holding period are quite volatile ('equal'), so use of the realized one-year inflation rate seems perhaps more sensible ('1 year'). For the Federal funds rate, we have used the inflation rate at the highest frequency available to approximate the inflation rate over the life of the asset, i.e. monthly for the CPI and PPI, but

Table 2 Selected ‘reasonable’ numbers and their ranges and medians

p	π	$\sigma_{r,\text{unf.}}$	$\sigma_{r,\text{HP}}$	−4.0	−3.0	−2.0	−1.0	0.0	1.0	2.0	3.0	4.0
3mo Tbill:												
PCE	equal	2.68	1.46	−0.44	−0.46	−0.43	−0.30	−0.16	−0.12	−0.10	−0.07	0.00
CPI	equal	2.86	1.74	−0.24	−0.27	−0.35	−0.38	−0.30	−0.25	−0.25	−0.25	−0.16
PCE	lagged	2.17	1.15	−0.16	−0.01	0.11	0.28	0.38	0.33	0.22	0.07	−0.07
CPI	lagged	2.45	1.26	0.07	0.14	0.18	0.24	0.21	0.07	−0.10	−0.27	−0.37
1yr Tbill:												
PCE	equal	2.91	1.53	−0.46	−0.49	−0.48	−0.36	−0.24	−0.18	−0.15	−0.11	−0.02
CPI	equal	3.43	1.87	−0.46	−0.56	−0.60	−0.52	−0.41	−0.32	−0.26	−0.19	−0.08
PCE	lagged	2.31	1.22	−0.09	0.01	0.09	0.23	0.31	0.24	0.13	−0.03	−0.17
CPI	lagged	2.59	1.34	0.12	0.15	0.16	0.19	0.15	0.00	−0.17	−0.34	−0.44
Max:		3.43	1.87	0.07	0.14	0.18	0.28	0.38	0.33	0.22	0.07	−0.02
Median:		2.74	1.43	−0.43	−0.45	−0.43	−0.31	−0.19	−0.17	−0.15	−0.15	−0.07
Min:		2.17	1.15	−0.46	−0.56	−0.60	−0.52	−0.41	−0.32	−0.28	−0.36	−0.37

quarterly for the PCE. Table 10 in the Appendix shows the volatility and the correlations with real output for our entire variety of real interest rates, thus presenting a broad set of results.

In the following we focus on the results for a subset of the interest rates and price indices. First, we only consider the two consumer price indices, PCE and CPI, because in view of our theory described below these are the relevant price deflators. Second, to exclude high-frequency movements, it is probably more sensible to calculate inflation rates as year-to-year changes in the price level, and to concentrate on three-month Treasury bills or one-year Treasury bills rather than the very short-term Federal funds rate. In Table 2 we document the volatility and the correlations with real output for all real interest rates. We find that the fluctuation of the (unfiltered) real rate is measured rather precisely to be somewhere between 2.17 and 2.91 per cent, excluding the one high number of 3.43 per cent for the *ex post* CPI inflation rate, matched to the one-year Treasury bill. Compared with other work our estimates of the real rate volatility are on the low side: Cecchetti *et al*. (1993), for example, estimate the volatility of the risk-free rate at 5.27 per cent. The cyclical behavior of real rates is less clear, and depends quite a bit on the exact way in which inflation rates are constructed and subtracted from the nominal returns. The range includes negative as well as positive numbers at all leads and lags. Nonetheless, sizeable negative correlations are more frequent for leading interest rates and small negative or slightly positive correlations are more frequent for lagging interest rates. Even if the leading correlations are positive, they tend to be smaller than the corresponding lagging ones. A similar comment applies to negative lagging correlations.

We therefore conclude that the row labelled ‘median’ gives a fairly good description of both the size of the fluctuations of real interest rates as well as their cyclical properties. The preceding discussions should provide for ample caution in interpreting this statement.

3. THE INABILITY OF THE BENCHMARK MODEL TO ACCOUNT FOR INTEREST RATE VOLATILITY

Before developing our own models, it is good to appreciate the difficulty of standard real business cycle models to replicate the features of interest rates documented in Section 2. We shall use the benchmark model of Hansen (1985) for this: many other real business cycle models in the literature exhibit similar features.

Hansen's model is easily stated. Time is discrete and the horizon is infinite. There is an infinitely lived representative agent with preferences over consumption c_t and labor n_t:

$$E\left[\sum_{t=0}^{\infty}\beta^t\frac{c_t^{1-\eta}-1}{1-\eta}-An_t\right] \tag{1}$$

where $0 < \beta < 1$ is the time preference parameter, and the curvature parameter $\eta > 0$ measures the degree of risk aversion. The linear disutility for working is interpreted in Hansen as the result of a labor lottery. Production is constant returns to scale and assumed to be given by a Cobb–Douglas production function:

$$y_t = \gamma_t k_{t-1}^{\theta} n_t^{1-\theta} \tag{2}$$

with $0 < \theta < 1$ the capital share parameter. Aggregate productivity γ_t evolves according to

$$z_t = \rho z_{t-1} + \epsilon_t \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2) \tag{3}$$

for $z_t = \log \gamma_t - \log \bar{\gamma}$. Output can be used for consumption or gross investment,

$$c_t + x_t = y_t \tag{4}$$

New capital is produced with the linear technology

$$k_t = (1 - \delta)k_{t-1} + x_t \tag{5}$$

This equation will be substantially altered in the models of Sections 4 and 5.

The competitive equilibrium of this economy is Pareto-optimal and an equilibrium allocation can be obtained as the solution to the social planner's problem, that is maximize (1) subject to the constraints (2), (3), (4) and (5) stated above. We obtain the risk-free real rate of return from the household's Euler equation which states that the marginal utility loss from consuming one unit less today has to equal the expected utility gain from the additional consumption when invested in a risk-free asset:

$$c_t^{-\eta} = \beta E_t\left[c_{t+1}^{-\eta}\right] R_t^f \tag{6}$$

We parameterize the model as follows. A period represents a quarter and we fix all parameters except the risk-aversion parameter η at their standard values, i.e.

$$\delta = 0.025 \quad \theta = 0.36 \quad \beta = 0.99 \quad \rho = 0.95 \quad \sigma = 0.712 \text{ per cent}$$

and we solve for A conditional on the normalization $n = 1$ for steady-state employment. In our experiments we vary η to study its effect on interest rates. Note that only logarithmic preferences, $\eta = 1$, are consistent with balanced growth, unless c_t is detrended appropriately: we shall ignore these issues for the purpose of our discussion.

To solve these and all subsequent models, we have used a log-linear Euler equation approach. More specifically, after obtaining the equations characterizing the equilibrium and asset prices for our economy from the usual first-order conditions and feasibility constraints, we have calculated the nonstochastic steady state, and then used a first-order linear approximation in the logarithm of all variables around that steady state to substitute our system of equations by a linear system of equations in the log-deviations away from the steady state. This system can then be solved by eliminating the unstable roots in a variety of essentially equivalent approaches. More details and an implementation of the solution using the undetermined coefficients method can be found in Uhlig (1999). The results here were calculated by the first author, using methods and algorithms posted by King and Watson (1997). Additional details on the programs can be obtained from that author on request.

The solution method relies on numerical approximation, as there is no closed-form solution available. Numerical approximation entails approximation errors. The solution literature has recently begun to investigate approximation errors of linearization methods in greater detail; see, for example, Dotsey and Mao (1992) or Judd (1998). In particular, the asset pricing implications obtained in this paper are unlikely to be very precise. In order to minimize approximation errors, we do not log-linearize the process for the risk-free rate but calculate the risk-free rate from an approximation of the Euler equation which takes risk aversion into account:

$$R_t^f = \beta^{-1} \exp\left(\hat{\lambda}_t - E_t\hat{\lambda}_{t+1} - \sigma_\lambda^2/2\right) \tag{7}$$

For this calculation $\hat{\lambda}_t$ denotes the log-deviation of marginal utility of consumption from its steady-state value, σ_λ^2 is the variance of the forecast error of the marginal utility of consumption, $\sigma_\lambda^2 = E_t[(\hat{\lambda}_{t+1} - E_t\hat{\lambda}_{t+1})^2]$, and we use the log-linear approximation for the stochastic process of the marginal utility of consumption (see Jermann, 1998).

We also want to obtain a measure of the market price for risk, and we shall do so, using a traded security: capital. The rate of return on capital is simply:

$$R_t^k = \left[\theta\gamma_{t+1}(k_{t1}/n_{t+1})^\theta + p_{0,t+1}^k\right]/p_{1,t}^k \tag{8}$$

where $p_{0,t}^k$ denotes the price of old capital in period t after production takes place and $p_{1,t}^k$ denotes the price of new capital at the end of period t.[3] In the basic growth model the price of new capital is fixed at one, $p_{1,t}^k = 1$, and the price of old capital is $p_{0,t}^k = 1 - \delta$.

The market price for risk or the Sharpe ratio can then be calculated by dividing the excess return on holding capital over the risk-free rate divided by the standard deviation of the return on holding capital. We view this calculation as representing the Sharpe ratio of a capital-based stock market rather than the theoretically maximal obtainable Sharpe ratio. Since the returns to holding capital may not be perfectly correlated with consumption, it may be

3. The notation follows Boldrin *et al.* (1999).

Table 3 Key business cycle statistics for Hansen's model

$\eta =$	0.5	1	2	10	US data
σ_y	2.20	1.72	1.44	1.08	1.72
σ_c/σ_y	0.29	0.52	0.24	0.08	0.74
σ_x/σ_y	3.40	2.46	3.29	3.69	4.79
σ_n/σ_y	0.91	0.51	0.60	0.39	0.98
$E[R^f]$	4.10	4.11	4.10	4.11	
σ_{R^f}	0.46	0.39	0.35	0.33	
$E[R^k - R^f]/\sigma_{R^k}$	0.02	0.02	0.03	0.04	0.5

Notes: Volatilities σ here and all other tables are measured in per cent. The numbers for US data were calculated from Cooley and Prescott (1995, Table 1.1, p. 32). For consumption, investment and labor, the rows entitled 'CONS', 'INV' and 'ES-HOURS' have been used. The value of 0.5 for the Sharpe ratio of investing in capital is from Cochrane (1997, Table 2).

possible to construct assets with a higher Sharpe ratio in this model. On the other hand, the same may be true for the Sharpe ratio typically calculated from, say, the returns on holding an index. We therefore chose this approach rather than directly calculating the Sharpe ratio from the process of marginal utilities. In short, the Sharpe ratio in all our tables should be understood as an interesting lower bound for the market price of risk, constructed similarly to the way the Sharpe ratio is typically calculated in the data.

The statistics we use to characterize an economy are obtained by generating 200 samples with 200 observations each. For each sample we calculate realizations for quantity variables based on the log-linearization and realizations of the quarterly risk-free rate and the quarterly return on capital using equations (7) and (8). We define the annual rate of return on the risk-free asset or capital as the average rate of return over a year. This definition also corresponds to the realized rate of return from holding the asset for four quarters. We do not detrend the annualized risk-free rate or the return on capital when we calculate their volatility. Business cycle statistics for quarterly series are obtained by detrending each series with the Hodrick–Prescott filter. For the calculation of the leading and lagging correlations of the quarterly risk-free rate with output, the quarterly risk-free rate is also detrended.

One might think that higher values for η will result in higher fluctuations in marginal utilities and thus of the risk-free rate. This is actually not true: as one can see from Table 3, increasing η merely results in smoother consumption and employment; see also the discussion in Lettau and Uhlig (1995). Overall the

Table 4 Correlations of interests and outputs at leads and lags for Hansen's model

Lead/lag =	−4.0	−3.0	−2.0	−1.0	0.0	1.0	2.0	3.0	4.0
$\eta = 2.0$	0.22	0.36	0.52	0.73	0.95	0.58	0.27	0.05	−0.11

volatility of the risk-free rate is of an order of magnitude too low compared with the data. Finally, Table 4 clearly shows the strong pro-cyclicality of the risk-free rate in the standard growth model, which is not consistent with the observed lead–lag pattern for real rate–output correlations.[4]

The last row of Table 3 (and similar tables below) also lists the Sharpe ratio as a measure of the market return for risk: that ratio would need to be around 0.5 for annual data. It will not be near that for any of our models, indicating that our models would fail to match the observed equity premium; see Lettau and Uhlig (1997) for a more extensive discussion. However, as emphasized in the Introduction, our focus here is on the interest rate volatility rather than the equity premium, which is relatively easier to explain. As explained above, the calculated Sharpe ratio should furthermore be understood as a lower bound.

4. MODEL 1: HABIT FORMATION AND ADJUSTMENT COSTS

We now consider an economy with two types of agents: workers and capital owners. Workers do not have access to capital markets and consume their labor income. Their choice involves only how much to work. Capital owners do not work but they have access to capital markets, and they can smooth their consumption over time by choosing how much to save. Capital owners' preferences over consumption across time periods are not additively separable, rather they are subject to a 'habit formation' feature which has been introduced in recent work on asset pricing in dynamic general equilibrium models. The well-being of an agent with these preferences does not only depend on the agent's current consumption, but it also depends on the agent's past consumption levels. This feature of preferences has the potential to make the real rate of return more volatile because it makes the marginal rate of intertemporal substitution more volatile. In the equilibrium of the economy capital owners' savings are invested in the capital stock of the economy. For our economy we assume that changes of the capital stock are subject to an increasing marginal adjustment cost. Together with the 'habit' preferences this feature tends to increase the real-rate volatility, since it makes it more costly to smooth consumption over time which in turn increases the volatility of the intertemporal marginal rate of substitution.

Preferences of workers and capital owners are different in our model. One can view this as a convenient description of a more complicated preference specification, where the capital owners' preferences are an approximation, if consumption is high, and the worker preferences are an approximation if consumption is low: this would also justify, why we leave labor supply out of the capital owners' choice set, since the market wage earned here could be considered as trivial compared to the dividend income. Alternatively, one can

4. We only report the correlations for $\eta = 2$ since the correlations for other values of η are essentially the same.

view this as modelling the result of self-selection: agents who easily get used to a high consumption level, that is have habit formation preferences, may be more likely to build up a large capital stock over time than agents who do not. We now describe the economy in more detail, in particular in its deviation from Hansen's model.

4.1. The economy

There are now two infinitely lived representative agents, a capital owner and a worker. The capital owner does not supply labor and her intertemporal preferences over consumption are

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\,\frac{(c_{ct}-q_{t-1})^{1-\eta}-1}{1-\eta}\right] \tag{9}$$

where q_{t-1} is the habit stock, evolving according to

$$q_t = \phi q_{t-1} + (1-\phi)\alpha c_{ct} \tag{10}$$

where $0 \leq \alpha, \phi \leq 1$. This preference specification implies the creation of 'habits': high consumption in this period implies a high q_t next period, and therefore induces the capital owner next period to try to stick to this higher level of consumption. This intertemporal dependence is recognized by the capital owner when solving her maximization problem to be described below.

The infinitely lived worker has preferences over consumption c_w and labor n:

$$E_0\left[\sum_{t=0}^{\infty}\beta_w^t V\left(c_{wt} - An_t^{\varphi}\right)\right] \tag{11}$$

where $\varphi > 1$, $0 < \beta_w < 1$, $A > 0$, and V is an increasing and concave function. The exact specification of the function V is irrelevant since we assume that the worker has no access to the capital market and therefore no opportunities for intertemporal consumption smoothing.[5] To justify the abstinence of workers from capital markets, we appeal to unmodelled transaction costs or borrowing limits. For example, if β_w is smaller than β, the worker would typically want to borrow from the capital owner against future income: a borrowing constraint would prevent him from doing so. Likewise, if $V(\cdot)$ has a curvature similar to

5. The preferences of the worker imply a labor supply function which depends on real wages only; that is, there are only substitution effects but no wealth effects (see Greenwood *et al.*, 1988). This applies even if the worker had access to a capital market. More precisely, suppose that the worker could accumulate wealth a_t according to $a_t = R_t a_{t-1} - c_{wt} + n_t w_t$. It is straightforward to see that the two first-order conditions from differentiating the objective function with respect to c_{wt} and n_t subject to this constraint imply that $An_t^{\rho-1} = w_t$, i.e. that the labor supply only depends on wages and not on a_t. Consumption, of course, would depend on a_t.

the utility function of the capital owner, there may not be much gain from trade between these two groups of agents in order to cross-insure each other against shocks: if transaction costs are sizeable enough, these trades will not take place. Indeed, Lusardi (1998) documents that 50 per cent of all households with household heads between 51 and 61 years of age have a net wealth less than 2.5 times their income *including* retirement savings in the form of IRAs, home equity, vehicles, etc. Mulligan and Sala-i-Martin (1996) show that the vast majority of households do not directly participate in equity and bond markets. Therefore, while our procedure to exclude these workers from asset markets per assumption is rather crude, it seems a useful step to make in light of these facts.

Production is the same as in Hansen's model; see equations (2) and (3). The output good can be used for consumption c_c, c_w and investment x:

$$c_{ct} + c_{wt} + x_t = y_t \tag{12}$$

In contrast to equation (5) in Hansen's model, we assume that there are increasing marginal adjustment costs to investment such that higher investment yields lower marginal additions to the capital stock:

$$k_t = (1-\delta)k_{t-1} + g(x_t/k_{t-1})k_{t-1} \tag{13}$$

where $0 < \delta < 1$ is the depreciation rate and g is an increasing concave function which satisfies $g(\delta) = \delta$ and $g'(\delta) = 1$.[6]

A key parameter is the curvature of $g(\cdot)$, which we summarize locally around the steady state by the parameter $\xi \geq 0$,

$$\xi = -\frac{1}{\delta g''(\delta)}$$

The larger the value the smaller is the effect of increasing marginal adjustment costs. For the limiting case of $\xi = \infty$, there are no adjustment costs and the capital accumulation equation (5) applies. We will experiment with ξ in the numerical simulations below.

For the competitive equilibrium we assume that a representative firm operates the production technology and behaves competitively in all markets. The firm rents capital services from the capital owner for a rate d_t and hires labor services for a wage rate w_t from the worker. The capital owner increases her capital stock by making investments according to the capital accumulation equation (13). The capital owner then optimizes utility (9) subject to the habit formation equation (10), the capital accumulation equation (13) and the budget constraint:

6. With this specification of the adjustment costs g the steady state of the economy with adjustment costs is the same as the one of an economy without adjustment costs and standard capital accumulation equation $k_t = (1-\delta)k_{t-1} + x_t$.

$$c_{ct} + x_t = d_t k_{t-1} \tag{14}$$

The worker maximizes utility (11) subject to the budget constraint

$$c_{wt} = w_t n_t \tag{15}$$

which implies the labor supply function

$$\varphi A n_t^{\varphi-1} = w_t \tag{16}$$

and therefore a labor supply elasticity of $1/(\varphi - 1)$. For values of φ close to unity, the labor supply elasticity on the aggregate level becomes quite high and may appear to be at odds with microeconomic observations. In the Appendix we provide an interpretation for a high labor supply elasticity in a non-representative agent framework.

A competitive equilibrium is a process for $\{c_{ct}, c_{wt}, x_t, y_t, k_t, n_t, w_t, d_t, q_t\}$ such that (i) given $\{d_t\}$, $\{c_{ct}, x_t, k_t, q_t\}$ solves the capital owner's utility maximization problem; (ii) given $\{w_t\}$, $\{c_{wt}, n_t\}$ solves the worker's utility maximization problem; (iii) given $\{d_t, w_t\}$, $\{y_t, k_t, n_t\}$ solves the firm's profit maximization problem; (iv) all markets clear.

The risk-free rate and the return on capital are still as defined in equations (7) and (8). For the risk-free rate we have to modify the capital owner's marginal utility of consumption to account for habit formation. Because of adjustment costs the price of capital in terms of the consumption good is no longer fixed but satisfies

$$p_{0,t}^k = \left[1 - \delta + g\left(\frac{x_t}{k_t}\right) - g'\left(\frac{x_t}{k_t}\right)\frac{x_t}{k_t}\right] p_{1,t} \quad \text{and} \quad p_{1,t} = g'\left(\frac{x_t}{k_t}\right)^{-1}$$

(See Boldrin *et al.*, 1999.)

4.2. Results

We calibrate the model the same way the Hansen model is calibrated in Section 3 to the extent that is possible; that is, we use the same value for the time discount factor β, the capital income share θ, and the productivity process ρ and σ_ϵ. For the labor elasticity, we use $\varphi = 1.4$ in order to approximately achieve a ratio of the volatility of labor to output of three quarters. For the specification of the utility function of the capital owners, we have experimented with two choices. The first is due to Jermann (1998) and has also been used as well as estimated in Boldrin *et al.* (1999): here, we set $\phi = 0$ and $\alpha = 0.87$. The second is motivated by Campbell and Cochrane (1999), who have suggested to fix $\phi = 0.97$ and $\alpha = 1 - \bar{S}$, where $\bar{S}$ is a parameter given in their paper, fixed at $\bar{S} = 0.057$.

Two points have to be made concerning our use of Campbell and Cochrane's specification. First, for this specification habit represents an externality. In particular this means that for the capital owner's utility maximization problem the habit stock is not treated as a choice variable, rather it is treated as an exogenous process. Second, our habit accumulation equation (10) is a linearized version of the highly non-linear specification used by Campbell and Cochrane. Instead of writing down a law of motion for the habit level, they postulate a process for the surplus consumption ratio $S_t \equiv (c_{ct} - q_{t-1})/c_{ct}$.[7] Using tildes to indicate logs, they assume that $\tilde{s}_t$ evolves according to:

$$\tilde{s}_t = (1 - \phi)\log(\bar{S}) + \phi\tilde{s}_{t-1} + \kappa(\tilde{s}_{t-1})(\tilde{c}_t - \tilde{c}_{t-1} - g) \tag{17}$$

where ϕ, $\bar{S}$ and g are parameters, and where the function $\kappa(\tilde{s})$ is given by:

$$\kappa(\tilde{s}) = \begin{cases} \bar{S}^{-1}\sqrt{1 - 2(\tilde{s} - \log(\bar{S}))} - 1, & \tilde{s} \leq \tilde{s}_{\max} \\ 0 & \tilde{s} \geq \tilde{s}_{\max} \end{cases} \tag{18}$$

with $\tilde{s}_{\max} = \log(\bar{S}) + \left(1 - \bar{S}^2\right)/2$. Equation (17) can then be used to back out the implied habit level q_t. For our purposes, we want to use a linearized version of (17). To do so, one needs to replace $\kappa(\tilde{s}_{t-1})$ with its steady-state value $\bar{\kappa} \equiv \bar{S}^{-1} - 1$ in equation (17). Furthermore, to a first-order log-linear approximation, $\tilde{s}_t = \bar{\kappa}(\tilde{c}_t - \tilde{q}_{t-1} + \log(1 - \bar{S}))$ by definition of S_t. Using these approximations in equation (17) yields

$$\tilde{q}_{t-1} = \text{const.} + \phi\tilde{q}_{t-2} + (1 - \phi)\tilde{c}_{c,t-1}$$

But this is the same form as a (lagged) log-linear approximation of (10)! Comparing steady states finally demonstrates that the latter equation can indeed be viewed as the linearized version of equation (17). We should emphasize, that we used the work of Campbell and Cochrane (1999) merely as a guide to pick reasonable parameters α and ϕ, rather than doing justice to their preference specification. One reason for their non-linear specification is to achieve a constant risk-free rate, if consumption is a random walk. This gets lost in the linear approximation used here: indeed it is the very volatility in that risk-free rate which is the focus of our investigation. Furthermore, the full consequences of the original non-linear preference specification in a dynamic economy are likely to be hard to analyze, since, for example, consumption bunching may be preferable to a smooth consumption path (see Ljungqvist and Uhlig, 1999). For all these reasons, we have focused on the linearized version of their preference specification here.

Habit formation increases the volatility of the marginal rate of intertemporal substitution and therefore also the volatility of the risk-free rate for a given consumption process. On the other hand, habit formation also

7. Note that our time index convention for the habit stock is different from the one used in Campbell and Cochrane (1999): there is no substantive difference.

Table 5 Key business cycle statistics for the Jermann/Boldrin–Christiano–Fisher specification of habit formation

$\xi =$	0.15	0.23	0.30	1.00	US data
σ_y	1.67	1.70	1.69	1.67	1.72
σ_c/σ_y	0.95	0.93	0.92	0.88	0.74
σ_{c_c}/σ_y	0.91	0.70	0.59	0.26	
σ_{c_w}/σ_y	1.00	1.00	1.00	1.00	
σ_x/σ_y	1.18	1.23	1.26	1.35	4.79
σ_n/σ_y	0.71	0.71	0.71	0.71	0.98
$E[R^f]$	1.13	3.12	2.96	4.00	
σ_{R^f}	10.11	6.41	4.80	0.98	
$E[R^k - R^f]/\sigma_{R^k}$	0.24	0.14	0.15	0.04	0.50

Note: For explanations see Table 3.

increases the incentive to smooth consumption over time. In this section's experiments we study the effects on the risk-free rate when we vary the adjustment cost parameter ξ; that is, we make it more or less difficult for the capital owner to smooth consumption over time. For our benchmark scenario, we use the risk-aversion parameter $\eta = 2$ and the adjustment cost curvature parameter $\xi = 0.23$. Boldrin *et al.* (1999) argue that, based on empirical evidence, $\xi = 0.23$ represents an upper bound for adjustment costs, and that is why we chose that value for our benchmark specification. However, this parameter is hard to pin down decisively by empirical observations. This, together with our aim at showing consistency of our model with observed real rate fluctuations, made us experiment with the following range of choices:

$$\xi \in \{0.15, 0.23, 0.30, 1\}$$

All other parameters are the same as for the Hansen specification in the previous section.

The business cycle statistics are presented in Table 5 for the Jermann/Boldrin–Christiano–Fisher specification, and in Table 6 for the linearized Campbell–Cochrane specification. The cross-correlations of the risk-free rate with output are presented in Table 7 for the baseline parameter values of both specifications.[8] These tables show that it is indeed possible to increase the real-rate fluctuations substantially above the values generated by the baseline Hansen specification. Note that due to the higher volatility of the marginal utility of consumption, the average risk-free rate declines substantially with higher adjustment costs; see equation (7). This simply suggests that we have to

8. We only report results for the baseline parameter value $\xi = 0.23$ because, for the variations in ξ which we consider, the quantitative effect on the correlation of the risk-free rate with output is minimal for each specification.

Table 6 Key business cycle statistics for the Campbell–Cochrane specification of habit formation

$\xi =$	0.15	0.23	0.30	1.00	US data
σ_y	1.66	1.67	1.69	1.68	1.72
σ_c/σ_y	0.90	0.89	0.88	0.87	0.74
σ_{c_c}/σ_y	0.28	0.15	0.19	0.06	
σ_{c_w}/σ_y	1.00	1.00	1.00	1.00	
σ_x/σ_y	1.29	1.33	1.34	1.38	4.79
σ_n/σ_y	0.71	0.71	0.71	0.71	0.98
$E[R^f]$	1.40	2.38	2.95	3.73	
σ_{R^f}	7.28	5.01	3.79	0.89	
$E[R^k - R^f]/\sigma_{R^k}$	0.26	0.19	0.16	0.09	0.50

Note: For explanations see Table 3.

adjust the time discount factor to match the average risk-free rate. The models do not match the cyclical behavior of the real rate in the data; see Section 2. Essentially the real rate switches from contemporaneously pro-cyclical in the Hansen model to contemporaneously counter-cyclical in our habit formation models; see Table 7. Still, the leading correlations are somewhat more decisively negative than the lagging ones, in line with the observed cyclical behavior of the real rate. Finally, note that for high enough adjustment costs the Sharpe ratios for the habit formation models are about half of what we observe in the data.

The model does replicate most of the salient business cycle features, such as the pro-cyclicality of investment, labor and consumption as well as the ordering of volatilities: investment is more volatile than output, which in turn is more volatile than consumption. The investment-to-output volatility is too low compared to the data, but this is a feature of our modelling strategy. Log-linearization of the budget constraint of the capital owner yields:

$$\bar{c}_c \hat{c}_{ct} + \bar{x}\hat{x}_t = \theta \bar{y}\hat{y}_t$$

where bars denote steady-state values and hats denote log-deviations. As long as consumption of the capital owner is pro-cyclical, it must therefore be the case that:

Table 7 Correlations of interests and outputs at leads and lags for the Jermann/Boldrin–Christiano–Fisher (JBCF) and the Campbell–Cochrane (CC) specification of habit formation

Lead/lag =	–4.0	–3.0	–2.0	–1.0	0.0	1.0	2.0	3.0	4.0
JBCF	–0.19	–0.34	–0.52	–0.71	–0.95	–0.53	–0.22	0.01	0.17
CC	–0.14	–0.30	–0.49	–0.72	–1.00	–0.68	–0.43	–0.22	–0.05

$$\frac{\sigma_x}{\sigma_y} \leq \frac{\theta \bar{y}}{\bar{x}} \tag{19}$$

Since the steady-state investment-to-output ratio is set equal to approximately 0.26 as one of the key observable quantities, to which the model is calibrated, and since the capital share is likewise set to $\theta = 0.36$, the ratio of these volatilities cannot exceed 1.4. As a flip-side, the volatility of consumption cannot be too low either. One could aim at rectifying this, but this, in any case, is not the main focus of this paper.

Likewise, the volatility of capital owner consumption is a multiple of worker consumption in the data (see Mankiw and Zeldes, 1991), whereas that rarely will be the case in any of the models we consider in this paper. One can see this as follows. Since $c_{wt} = w_t n_t = (1 - \theta) y_t$, it follows that c_{wt} must be perfectly correlated with y_t and must have the same volatility, measured in per cent. Since total consumption is $c_t = c_{ct} + c_{wt}$, one can rewrite this in log-deviation form as

$$\hat{c}_t = \frac{\bar{c}_c}{\bar{c}} \hat{c}_{ct} + \frac{\bar{c}_w}{\bar{c}} \hat{c}_{wt}$$

Suppose now for simplicity that the consumption of the capital owner c_{ct} and output y_t are perfectly correlated, i.e. that $\hat{c}_{ct} = \kappa \hat{y}_t$ for some κ. The equation now implies:

$$\sigma_c^2 = \left(\frac{\bar{c}_c \kappa + \bar{c}_w}{\bar{c}} \right)^2 \sigma_y^2$$

and one can see that $\kappa < 1$, i.e. capital owner consumption fluctuates less than worker consumption iff $\sigma_c^2 / \sigma_y^2 < 1$. The numbers in the tables indicate that this calculation is not exact, since indeed $\hat{c}_{ct}$ is not perfectly correlated with $\hat{y}_t$, but nonetheless this logic is compelling. A possible avenue may be to allow the worker some possibility to smooth consumption, but this would obviously also alter all other statistics in this model. In any case, this feature is surely an undesirable property of the class of models studied in this paper, and suggests one dimension in which future research can improve on the results obtained here.

5. MODEL 2: PREDETERMINED INVESTMENT

For the second model, we shall assume that it takes time to implement investment plans, a feature which has been emphasized by Boldrin *et al.* (1999). We keep the structure of the previous model of distinguishing between capital owners and workers. It turns out that habit formation is no longer needed to match the key facts: the delayed implementation feature alone makes it sufficiently difficult for the capital owner to swiftly react to changing economic circumstances. Since she does not supply labor, her consumption shoulders much of the burden of the productivity fluctuations, implying large variations in the intertemporal rate of substitution and therefore in real interest rates.

5.1. The environment

The description of the model is essentially the same as the model in Section 4 above: we will concentrate on stating the differences. The preferences of the capital owner are as in (9) and (10) with the restriction that there is no habit formation; that is, we set $\alpha = \phi = 0$. The utility function of the worker is given by (11), as before. Production is the same as in Hansen's model; see equations (2) and (3). The goods market-clearing condition is equation (12).

The critical difference to the model in the previous section is that we now assume that it takes time to implement investment. More specifically, to complete an investment project of size s_t started in period t requires as inputs ψs_t units of the output good in period t and $(1 - \psi)s_t$ units in period $t + 1$. Thus total expenditures on investment in period t are:

$$x_t = \psi s_t + (1 - \psi)s_{t-1} \tag{20}$$

We assume that an investment project adds to the capital stock according to its stage of completion:

$$k_t = (1 - \delta)k_{t-1} + [\psi s_t + (1 - \psi)s_{t-1}] = (1 - \delta)k_{t-1} + x_t \tag{21}$$

Our specification differs from the standard time-to-build specification (Kydland and Prescott, 1982), in that an investment project immediately adds to the capital stock and not only after it is completed. The time-to-plan specification considered by Boldrin *et al.* (1999) corresponds to $\psi = 0$, and essentially means that investment is predetermined.

The competitive equilibrium is defined in a similar fashion as in the previous section. The return on capital is calculated based on the price of new capital $p^k_{1t} = \mu_t/\lambda_t$ and the price of old capital $p^k_{0t} = (1 - \delta)\mu_t/\lambda_t$, where μ_t is the Lagrange multiplier on the capital owner's capital accumulation equation (21) and λ_t is the capital owner's marginal utility of consumption.

5.2. Results

To calibrate the model, we have used $\eta = 2$, $\theta = 0.36$, $\beta = 0.99$, $\delta = 0.025$, $\varphi = 1.4$, $\rho = 0.95$ and $\sigma_\epsilon = 0.712$ per cent, as before. The critical issue here is now the parameter ψ which determines to what extent investment is predetermined. We have selected the following range:

$$\psi \in \{0.4, 0.425, 0.45, 0.5\}$$

Table 8 Key business cycle statistics for the model with predetermined investment

$\psi =$	0.40	0.425	0.45	0.50	US data
σ_y	1.68	1.66	1.68	1.68	1.72
σ_c/σ_y	0.92	0.91	0.90	0.88	0.74
σ_{c_c}/σ_y	1.21	1.04	0.88	0.15	
σ_{c_w}/σ_y	1.00	1.00	1.00	1.00	
σ_x/σ_y	1.34	1.35	1.35	1.35	4.79
σ_n/σ_y	0.71	0.71	0.71	0.71	0.98
$E[R^f]$	4.07	4.02	4.16	4.04	
σ_{R^f}	7.41	5.88	4.30	0.46	
$E[R^k - E^f]/\sigma_{R^k}$	0.04	0.04	0.01	0.03	0.50

Note: For explanations see Table 3.

The model does substantially better than the habit formation models of the previous section for the key business cycle statistics and the risk-free rate statistics (see Tables 8 and 9). For the case that about 55 per cent of expenditures on investment projects are predetermined, $\psi = 0.45$, the volatility of the risk-free rate is close to the empirically observed values. Furthermore, the risk-free rate is leading counter-cyclical and lagging pro-cyclical. When more than 55 per cent of investment expenditures are pre-committed, the volatility of the risk-free rate is quite high and the risk-free rate is strongly counter-cyclical leading and pro-cyclical lagging. The other key business cycle facts are roughly in line with the data, subject to the upper bound on the investment-to-output volatility; see equation (19) above and the discussion that follows it. Notice that for low values of ψ consumption of capital owners is now more volatile than consumption of workers. For low values of ψ (that is, high pre-commitment of investment expenditures), both consumption and investment expenditures are more volatile than output. This occurs because investment and consumption are only weakly correlated with each other.

The intuition for the correlation pattern between the risk-free rate and output is simple: upon impact of the productivity shock, the capital owners wish to postpone consumption and instead buy more of the capital stock or the investment goods, as they are now quite productive. However, in aggregate, the supply of capital cannot easily be increased because most investment is

Table 9 Correlations of interest rate and output at leads and lags for the model with predetermined investment

Lead/lag =	−4.0	−3.0	−2.0	−1.0	0.0	1.0	2.0	3.0	4.0
$\psi = 0.40$	−0.08	−0.10	−0.13	−0.15	−0.19	0.39	−0.03	0.21	0.01
$\psi = 0.425$	−0.07	−0.09	−0.11	−0.14	−0.16	0.36	−0.06	0.22	−0.02
$\psi = 0.45$	−0.06	−0.07	−0.09	−0.10	−0.13	0.31	−0.08	0.22	−0.06
$\psi = 0.50$	0.15	0.31	0.50	0.72	0.99	0.67	0.40	0.19	0.02

predetermined. To make these aggregate feasibility conditions consistent with the desires of the capital owners, the interest rate must fall sufficiently far to discourage savings. The fall in the interest rate is particularly large in the first period, and then is more modest in the subsequent periods, that is adjusts back to steady state fairly quickly. The technology process, on the other hand, shows a lot more persistence. Therefore, the fall in the interest rate happens right at the impact of the productivity shock: in total, this gives the 'appearance' of interest rate declines leading the cycle.

6. CONCLUSIONS

We have presented two models of the real business cycle variety, in which the fluctuations of real rates are substantially higher than in the baseline Hansen real business cycle model while simultaneously matching salient business cycle facts. For the model with predetermined investment we match or exceed the observed volatility of the risk-free rate and we replicate the cyclical features of interest rate movements.

Both models assume two classes of agents: capital owners and workers. Workers are prevented from participating on asset markets per assumption. In the first model, capital owners have habit-formation preferences. Accumulation of the capital stock is subject to adjustment costs. In the second model, it takes time to implement investment projects. Both environments lead to fairly large fluctuations in the intertemporal marginal rate of substitution of capital owners and therefore to real interest rate fluctuations commensurate with the data. In the second model, the largest movement happens on impact with the technology shock. Since the technology process is highly persistent, but the interest fluctuations return to steady state quickly, the model predicts interest rate declines to lead the cycle. In sum, the paper demonstrates the possibility that there is an entirely real explanation for the fluctuations in real interest rates.

This exercise sheds doubt on the interpretation of most observers, that monetary policy is responsible for these real rate fluctuations. Instead, it may well be that monetary policy behaves rather passively, following rather than distorting the real-rate fluctuations dictated by the market, and indeed concentrating on the task of maintaining price stability instead.

APPENDIX 1: REAL RATES IN THE US

Table 10 The volatility and the correlations with real output for real interest rates

p	π	$\sigma_{r,\text{unf.}}$	$\sigma_{r,\text{HP}}$	−4.0	−3.0	−2.0	−1.0	0.0	1.0	2.0	3.0	4.0
FFR:												
PCE	equal	2.72	1.69	−0.43	−0.39	−0.32	−0.15	0.05	0.15	0.15	0.09	0.14
CPI	equal	2.88	1.97	−0.30	−0.31	−0.29	−0.27	−0.14	−0.06	−0.03	−0.07	0.01
PPI	equal	4.12	3.30	−0.19	−0.27	−0.28	−0.25	−0.22	−0.15	−0.15	−0.17	−0.12
PCE	1yr	2.50	1.38	−0.38	−0.26	−0.15	0.08	0.28	0.34	0.30	0.21	0.13
CPI	1yr	2.58	1.33	−0.18	−0.09	−0.00	0.14	0.24	0.21	0.09	−0.04	−0.12
PPI	1yr	3.79	1.75	−0.11	−0.09	−0.08	−0.01	0.01	−0.05	−0.15	−0.25	−0.32
PCE	lagged	2.43	1.42	−0.33	−0.17	−0.01	0.24	0.43	0.48	0.41	0.30	0.17
CPI	lagged	2.55	1.37	−0.15	−0.04	0.06	0.23	0.32	0.28	0.14	−0.00	−0.11
PPI	lagged	3.74	1.76	−0.07	−0.04	−0.01	0.08	0.10	0.02	−0.10	−0.21	−0.31
3mo Tbill:												
PCE	equal	2.68	1.46	−0.44	−0.46	−0.43	−0.30	−0.16	−0.12	−0.10	−0.07	0.00
CPI	equal	2.86	1.74	−0.24	−0.27	−0.35	−0.38	−0.30	−0.25	−0.25	−0.25	−0.16
PPI	equal	4.18	3.17	−0.15	−0.23	−0.30	−0.30	−0.31	−0.25	−0.26	−0.28	−0.21
PCE	1yr	2.57	1.33	−0.40	−0.41	−0.39	−0.25	−0.13	−0.10	−0.08	−0.09	−0.05
CPI	1yr	2.57	1.22	−0.04	−0.03	−0.07	−0.07	−0.07	−0.17	−0.28	−0.36	−0.37
PPI	1yr	3.96	1.90	−0.03	−0.07	−0.13	−0.17	−0.23	−0.31	−0.39	−0.46	−0.45
PCE	lagged	2.17	1.15	−0.16	−0.01	0.11	0.28	0.38	0.33	0.22	0.07	−0.07
CPI	lagged	2.45	1.26	0.07	0.14	0.18	0.24	0.21	0.07	−0.10	−0.27	−0.37
PPI	lagged	3.80	1.84	0.09	0.09	0.07	0.06	0.00	−0.14	−0.27	−0.39	−0.47
1yr Tbill:												
PCE	equal	2.91	1.53	−0.46	−0.49	−0.48	−0.36	−0.24	−0.18	−0.15	−0.11	−0.02
CPI	equal	3.43	1.87	−0.46	−0.56	−0.60	−0.52	−0.41	−0.32	−0.26	−0.19	−0.08
PPI	equal	4.82	2.54	−0.44	−0.55	−0.59	−0.54	−0.47	−0.40	−0.34	−0.25	−0.10
PCE	lagged	2.31	1.22	−0.09	0.01	0.09	0.23	0.31	0.24	0.13	−0.03	−0.17
CPI	lagged	2.59	1.34	0.12	0.15	0.16	0.19	0.15	0.00	−0.17	−0.34	−0.44
PPI	lagged	3.94	1.95	0.12	0.10	0.05	0.04	−0.02	−0.18	−0.31	−0.43	−0.51

Note: Explanations and comments are in the text.

APPENDIX 2: AN ALTERNATIVE INTERPRETATION OF LABOR SUPPLY

This section provides an interpretation of the labor supply elasticity in a non-representative agent framework. Suppose that there is a continuum of workers $i \in [0; 1]$ with preferences

$$\max\{u(c_i, n_i); \bar{u}_i\} \tag{22}$$

where $\bar{u}_i$ is the individual reservation utility of worker i, reflecting, for example, home production, family support or unemployment insurance, and where $c_i = wn_i$, as before. Suppose, furthermore, that work only comes in contracts of fixed length, i.e. $n_i = \bar{N}$ or $n_i = 0$ are the only choices. In other words, suppose the worker faces the choice of either taking up a full-time job or staying unemployed, and receiving his reservation utility instead.

Let $F(\cdot)$ be the population distribution function for $\bar{u}_i$. Equation (22) and our other assumptions imply that aggregate labor n is given by

$$n(w) = \bar{N}F(u(w\bar{N}, \bar{N})) \tag{23}$$

Given a utility function $u(\cdot, \cdot)$, write $w = u^{-1}(\bar{u})$ for the solution to the equation $\bar{u} = u(w\bar{N}, \bar{N})$, provided it exists. The distribution function $F(\cdot)$, which delivers the equilibrium relationship (16), is then given by

$$F(\bar{u}) = \frac{1}{\bar{N}}\left(\frac{u^{-1}(\bar{u})}{A\varphi}\right)^{\frac{1}{\varphi-1}}$$

wherever $u^{-1}(\cdot)$ exists. Alternatively, given a distribution over reservation utilities, one can use the equations above to back out the utility function $u(\cdot, \bar{N})$.

In sum, given a fixed-length work day and the 'right' population distribution of reservation utilities, one can generate the same labor supply as a function of wage as in the model with equation (11). We find this little aggregation result useful for the interpretation of our model, in particular since rather high labor supply elasticities are needed to match the observed fluctuations in labor to fluctuations in GDP ratio. The result is, of course, rather similar to the lottery justification in Hansen (1985) or Rogerson (1988). We have more liberty here in the choices, since there are no consequences for intertemporal decisions by assumption. Finally, it should be pointed out that one needs to be careful in evaluating welfare consequences of, say, imposing labor taxes: aggregate welfare for the workers is now given by:

$$\text{Welfare} = u(w\bar{N}, \bar{N})F(u(w\bar{N}, \bar{N}) + \int_{u(w\bar{N},\bar{N})}^{\infty} \bar{u}\; dF(\bar{u})$$

rather than (11).

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