Debt contracts and collapse as competition phenomena

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Abstract

We study financial intermediation in which sufficient sorting is impossible. We identify a new type of market failure that may occur even when returns of investing entrepreneurs are verifiable. Moreover, we suggest that the nature of competition determines the contracts banks offer. A monopoly bank will offer equity contracts. In any pure strategy equilibrium when lenders compete à la Bertrand, however, only debt contracts are offered.

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1. Introduction

This paper starts from two questions that are prevalent in the literature on financial intermediaries. First, is tough competition among banks socially desirable, or is an oligopolistic or even monopoly structure necessary to preserve appropriate incentives and to enhance the stability of the banking system? We will identify new types of market failures when there is Bertrand competition among banks. Hence, we suggest further tradeoffs between banking competition and market breakdowns. Second, an important empirical phenomenon is the prevalence of debt contracts. We examine in this paper whether such contracts are a result of banking competition and not a direct consequence of the bilateral contractual problem between lenders and borrowers as...
it is usually assumed in the literature. We suggest that the nature of competition may determine the type of contracts banks offer.

Our analysis is further motivated by the interaction of incentive and selection problems in financial markets that arise when verification of output of an entrepreneur is possible only if investment has taken place. This is a natural assumption when physical assets are created during investment. The interaction of incentive and selection problems will lead to new types of market failures when there is tough competition among banks.

To address these questions, we consider a credit market with different types of borrowers. Borrowers need external resources in order to invest in a project of fixed size, but of varying quality. The quality is the private information of the borrower. Lenders cannot observe whether borrowers invest their resources or whether they shirk by consuming them (moral hazard). Shirkers are assumed to mimic investing borrowers by distributing themselves across lenders in the same proportions. This creates a fixed cost for lending to investing borrowers.

Lenders can costlessly observe and verify the output after the investment has taken place. While this allows for the possibility of making the repayment contingent on the returns from the project realized by investing borrowers, lenders will not do so in equilibrium. Essentially, selecting the investing borrowers leads lenders into a Bertrand-like competition for the different types of investing borrowers. This makes it impossible for a lender to cross-subsidize between them. In any pure strategy equilibrium, only debt contracts will be offered. A debt contract is a menu of contracts where the repayment is independent of the type of borrower.

The intuition for this result is simple. Suppose there were a (pure-strategy) equilibrium in which a lender offered a non-debt contract, thus demanding higher repayments from investing borrowers of some particular quality than from others. A competing lender could offer these borrowers a better deal (cherry-picking), since the costs of lending are the same for each investing borrower. These borrowers would then self-select to contract with the competing lender. Debt contracts are the only ones immune to cherry-picking. A monopoly bank, however, offers equity contracts, where repayments are contingent on the returns of the project. We think that our explanation might be particularly well suited for transition economies. When there is no credit history for entrepreneurs and protection of property rights is poor, there are no devices available to sort out shirkers.

Our examination identifies three types of market failure. First, moral hazard, together with the selection of shirkers, requires different repayments from borrowers of different types in order to motivate them to invest. This is exactly what happens under monopoly banking. Competition among lenders forces equal repayments from borrowers of different types. Compared to a monopoly bank, average returns decrease since banks compete for good entrepreneurs but cannot avoid bad entrepreneurs. This may make lending altogether unprofitable. Second, competition of banks with debt and convertible debt contracts yields non-existence of equilibria in pure strategies, since banks compete with debt and convertible debt contracts that leads to insufficient cross-subsidization from high-quality borrowers. In these cases, a mixed-strategy equilibrium may exist, in which there is a mixing between pure debt and convertible debt contracts. Third, when credit contracts with random delivery are allowed, banks attempt to avoid shirkers by lowering their delivery probability. As a consequence, high-quality borrowers may not receive credits.

The paper proceeds as follows. After relating our analysis to the literature in Section 2 and describing our model in Section 3, we discuss the monopoly case in Section 4. In Section 5, we analyze the competitive credit market without random funding and characterize all pure-strategy equilibria and the conditions for their existence. If there is no pure strategy equilibrium, we
provide a mixed strategy equilibrium in Section 6, with banks randomizing between debt and convertible debt contracts. Section 7 concludes.

2. Relation to the literature

Our model is related to the literature on competition between principals in markets with adverse selection. This literature has stressed the role of sorting devices and identified the conditions for a breakdown of markets when separating and pooling are possible (see Hellwig, 1987, and Kreps, 1990, for surveys). In our model breakdown occurs because banks compete with different types of contracts that lead to insufficient cross-subsidization from high-quality borrowers. This market breakdown differs from the normal Rothschild–Stiglitz market failure (Rothschild and Stiglitz, 1976), which arises when separating contracts are possible (and dominate a pooling contract) but are again dominated by another pooling contract. The breakdown is also different from the information-based problem discussed by Sharpe (1990) and Fischer (1990) and recently rigorously formulated by von Thadden (2004).

In our context a debt contract is distinguished from an equity contract by the nature of repayments. Under a debt contract, the creditor receives the same repayments from all debtors who can pay it back. Under an equity contract, the bank receives the excess returns from all investing entrepreneurs. The distinction between equity and debt contracts in our model arises solely from the perspective of the banks because of asymmetric information. The investment projects are risk-free for entrepreneurs but risky for banks. The cross-sectional view of debt contracts taken in this paper differs from other recent explanations of the relevance of debt contracts, which are briefly summarized in the following.

The literature on incomplete contracting, e.g. Aghion and Bolton (1992), Hart and Moore (1988, 1994), Berglöf and von Thadden (1994) and the surveys by Hart (1993, 1995), explains why an investor enters into a debt contract with an entrepreneur who promises to make certain repayments. If the agent fulfills his obligations, he retains control of the asset. If he does not make the repayments, control shifts to the investor. In our model, a debt contract also contains the right to control the assets in the case of default. However, the shift of control in our model is always associated with zero repayments, and the return characteristics distinguish between debt and equity contracts.

The literature on costly state verification, developed originally by Townsend (1979), Diamond (1984), Gale and Hellwig (1985), Williamson (1986) and extended e.g. by Bernanke and Gertler (1989), Boyd and Smith (1993) and many others, shows that only debt contracts arise when revenues from projects are private information, and ex post monitoring is costly. In our model, there are no costs for monitoring the output when entrepreneurs have invested, but it is impossible to prevent the entrepreneur from consuming the funds he has obtained.

Another important strand of literature originating from the pecking-order hypothesis developed by Myers (1984) and Myers and Majluf (1984) explains the prevalence of debt by signaling if borrowers differ in their probability of success. A good borrower tries to signal good prospects by increasing the sensitivity of his own returns to the private information about the firm’s prospects. By offering debt contracts his utility is maximized, subject to the mimicking constraint that the good borrower’s terms will not be preferred by the bad borrower (see Innes, 1990,1 and also Innes, 1993a, and DeMarzo and Duffie, 1999, for general models on this and related issues).

1 Note that Innes (1990) focuses on moral hazard, and derives the conditions under which debt contracts are selected because they induce maximal effort.
The related work by Allen and Gale (1992), Innes (1993b) and Nachman and Noe (1994) shows that debt contracts can be an optimal response to adverse selection problems. In our model, the distinction between equity and debt contracts arises solely from the perspective of banks offering credit contracts to informed borrowers. Moreover, since a monopolist issues equity contracts, debt contracts arise solely from the competition of lenders.2

Finally, our paper is part of a rapidly burgeoning literature on security design that explores the role of standard securities such as debt and equity in the financing of real investments. Allen and Gale (1988) show how selling multiple financial claims that partition a firm’s total cash flow improves the allocation of risky wealth. Boot and Thakor (1993) show how selling multiple financial claims maximizes the informativeness of equilibrium prices and hence the issuers’ expected revenue. The role of debt contracts has been stressed e.g. in Chiesa (1992) and Garvey and Swan (1992). Chiesa (1992) shows that debt contracts with warrants for the lender and cash/equity settlement options for the borrower are optimal contractual arrangements in a setting with moral hazard and unverifiable states. Garvey and Swan (1992) show that a moderate level of debt with a high penalty of default and passive shareholders are optimal when top managers must provide incentive to a subordinate in addition to exerting directly efforts. Chowdhry et al. (2002) show that two distinct fixed-income securities to domestic and foreign investors allow investors to credibly transmit their private information to each other.3

The tractable structure of our model may be useful for future applications. As illustrated by Uhlig (1995) and Gersbach and Wenzelburger (2001), this model is a useful tool for incorporating financial intermediation with adverse selection, moral hazard and monitoring aspects into macroeconomic models in order to address regulatory issues of financial markets, or to examine fiscal and monetary policy.

3. Model

There are two periods—this period and the next period. We consider a finite number of entrepreneurs who have access to a project but do not have the funds to finance it. Entrepreneurs are of different types, denoted by \( j = 1, \ldots, n \) or by \( l = 1, \ldots, n \). Entrepreneurs of type \( j \) have quality \( q_j \geq 0 \), and constitute a fraction \( \gamma_j \) of the total entrepreneur population.4

Qualities are labeled such that \( 0 \leq q_1 < q_2 < \cdots < q_n \), i.e. qualities \( q_j \) are strictly increasing in \( j \). The projects are all of equal size. Suppose that the initial costs for each project are \( I + z \), but the entrepreneur’s initial wealth is only \( z \). Hence, an entrepreneur must borrow at least \( I \) for the project.

Given additional resources \( I > 0 \), he can choose to invest \((\delta_j = 1)\) or not \((\delta_j = 0)\). If he invests, he receives the cash flow

\[
(I + z) \cdot q_j
\]
in the next period. Otherwise, he has funds in the amount of $I + z$. Entrepreneurs cannot have negative wealth in the next period.

Entrepreneurs can borrow additional funds from banks. Banks face the following informational asymmetries. The quality $q_j$ is known to the entrepreneur of type $j$, but not to the banks. Moreover, banks cannot observe whether or not an entrepreneur invests. Thus, banks face a fixed pool of observationally identical borrowers. The banks, however, can only observe and verify realized cash flows in the next period if the entrepreneur invests. If the entrepreneur does not invest and simply consumes the funds granted to him, the banks cannot expect any repayment. In this respect, our model is a variation of the literature on incomplete contracts where it is assumed that the output is not or only partially verifiable, regardless of whether the entrepreneur invests.

It is important to discuss the main assumptions underlying our model. The non-verifiability of the investment decision is a standard scenario. Often, projects require specific human capital or may need blueprints for machinery, buildings or logistics, or an inventor may spend a lot of time on reading and designing. Whether the efforts are directed towards the project or whether blueprints are competently drafted is unlikely to be observed by the bank. Even when it becomes clear to the bank ex post whether the entrepreneur has invested or not, investment decisions are not verifiable in court.

The second assumption behind our model is that the verification of output conditional to investment is possible at low or zero costs, while entrepreneurs can have large private benefits if they do not invest. This assumption is a variant of the “take the money and run” problem. It arises most naturally when physical assets are created during investment. Suppose, for example, that investment involves the creation of physical buildings for private or commercial use. If the entrepreneur does not invest and simply consumes the funds, banks will not be able to secure repayments. Because of limited liability, entrepreneurs can always claim they were unsuccessful, since investment is non-verifiable.\(^5\) Although banks may be able to impose some bankruptcy costs on the entrepreneur,\(^6\) they will not be able to recover their loans. If the entrepreneur does invest and the building has been created, banks could seize the physical object and sell it if the entrepreneur does not want to pay back his loan.

The assumption can also be justified by the extensive possibilities that banks have to secure the repayments if entrepreneurs invest. Monitoring to secure repayments in the case of investment takes many forms, including inspection of a firm’s cash flow when customers pay, efforts to collateralize assets if these have been created in the process of investment and the sale of products to customers. Therefore, monitoring is easier when investment is undertaken. For simplicity, we assume that the cost of verifying cash flow is zero if the entrepreneur has invested. For the same reason, we assume that repayment will be zero if entrepreneurs do not invest and simply consume the funds. Both assumptions about monitoring possibilities can easily be relaxed.

Potentially there are infinitely many banks (free entry); let $m$ be the number of banks eventually entering. A bank is denoted by $i$ or $k$. Each bank $i$ offers loan contracts of size $I$ with a repayment schedule $R_i(y)$, where $y$ is the verifiable cash flow. We denote the verifiable cash flow of entrepreneurs of quality $j$ by $y_j$. Entrepreneurs who have obtained a contract receive $I$ and decide whether to invest or to shirk.

If an entrepreneur invests, returns are verifiable. Hence, an entrepreneur of type $j$ who has obtained a loan from bank $i$ receives $\max\{q_j(I + z) - R_i(q_j(I + z)), 0\}$, while bank $i$ has a

\(^5\) One could allow for an arbitrarily small probability that the return is zero even if the entrepreneur invests in order to further justify the non-verifiability assumption of investment decisions.

\(^6\) Such bankruptcy costs can easily be integrated into our analysis.
repayment of $\min\{q_j(I + z), R_i(q_j(I + z))\}$. If an entrepreneur does not invest, he can consume all the funds available to him and the bank receives nothing. Non-investing entrepreneurs cannot be forced to pay back.

An entrepreneur derives utility from terminal wealth. Entrepreneurs are risk neutral: their payoff, denoted by $u_j$, is expected wealth. Once entrepreneurs have received additional funds from banks they decide whether to invest. The payoff for an entrepreneur when he has obtained a credit from bank $i$ amounts to:

$$u_{ji} = \delta_j \left( \max\{0, q_j(I + z) - R_i(q_j(I + z))\} \right) + (1 - \delta_j)(z + I).$$

The entrepreneur chooses $\delta_j \in \{0, 1\}$ such that $u_{ji}(\delta_j)$ is maximized.

Banks are assumed to be risk-neutral. A bank can borrow unlimited funds at an interest rate of $v = 0$. Clearly, a bank can only obtain repayment if the entrepreneur invests, as otherwise the entrepreneur consumes the funds. Banks will not enter if the funds granted to entrepreneurs in this period exceed the expected repayments.

There are some contracts of a special type to which we refer repeatedly, and for which it is handy to have names. We call a contract an equity contract, denoted by $E_i$, if $R_i(y) = \max\{y - (I + z), 0\}$. Under an equity contract, the bank bears the entire risk of variations in the quality $q_j$.\(^7\) If an entrepreneur of type $j$ invests, the bank’s repayment under an equity contract is given by $\max\{(q_j - 1)(I + z), 0\}$. We call a contract a convertible debt at repayment $R$, denoted by $CD(R)$, if

$$R_i(y) = \max\{\min\{y - (I + z), R\}, 0\}.$$  

Hence, if the return of the project is below $R$, the entrepreneur can convert the contract into an equity contract with the associated repayment $(q_j - 1)(I + z)$ if the entrepreneur is of type $q_j$ and has invested. Note that there are two possible forms of convertible debt contracts depending on whether the right to convert the contract into equity is given to the debtor or the creditor. While the latter is often used in venture capital financing where the creditor is involved in business formation, the former will be helpful in our case to motivate the entrepreneur to invest.

We call a contract a pure debt contract at repayment $R$, denoted by $D(R)$, if $R_i(y) = \min\{R, \delta_j q_j(I + z)\}$. Note that under a pure debt contract, the bank obtains $q_j(I + z)$ if the entrepreneur has invested but cannot pay back the amount $R$, since control will shift to the creditor.\(^8\) It is important to stress that a debt contract is in fact a menu of contracts where the repayment is independent of the type of borrower. Similarly, a convertible debt contract is a menu of contracts where the threshold to convert the contract into equity is independent of the type of borrower.

The game unfolds as follows:

\(^7\) Note that the capital share of the bank amounts to $(q_j - 1)/q_j$, and thus varies with the return of the project in a similar way as a call option. The capital share is determined at the end of the investment which is different from the standard equity contract.

\(^8\) Hence, the debt contract contains the right of the lender to control the assets in case of default. In equilibrium, the shift of control will be associated with zero repayment, since the entrepreneur will not decide to invest if returns are below $R$. 

This period:

(1) Banks decide simultaneously whether or not to enter and which contracts to offer upon entering.
(2) Entrepreneurs choose banks simultaneously.
(3) Funded entrepreneurs make a decision whether to invest.

Next period:

(4) Payoffs are realized and repayments occur.

An equilibrium to this game is a perfect Bayesian Nash equilibrium. It is a self-selection model with three additional complications. First, there is an additional moral hazard problem because entrepreneurs cannot be forced to invest. Second, non-investing entrepreneurs cannot be separated from investing entrepreneurs in the selection process. Hence, standard sorting devices such as collateral (see Bester, 1985, 1987) cannot be used to separate the bad entrepreneurs from the good ones. Third, we allow for an arbitrary number of types of entrepreneurs with good projects. Competition between banks for such good types creates the distinction between debt and equity contracts, and leads to additional inefficiencies.

An equilibrium is a set of credit contracts, with non-negative profits for each bank entering and with creditors responding optimally in undertaking investments. Moreover, given a set of contracts, banks correctly assess the distribution of investing entrepreneurs and shirkers. Finally, there is no other contract for a bank that, when offered in addition, earns positive profits.

We additionally assume three tie-breakers in the case of indifference among the entrepreneurs. We briefly outline them here and describe them in greater detail in the analysis below. First, entrepreneurs who are indifferent between investing and not investing always choose to invest. Second, investing entrepreneurs who are indifferent between several banks will choose each bank with equal probability. Third, entrepreneurs who choose not to invest will randomize across their most preferred banks in order to mimic the investing entrepreneurs. The first two tie-breaker rules are standard and innocuous, while the third tie-breaker rule is critical for the analysis and will be discussed in more detail when we examine competition in the credit market.

Let \( j^* = \min \{ j \mid q_j \geq 1 \} \). Hence, \( j^* \) is the first index value for which the return of the investment project is greater than or equal to one. Then, by using a utilitarian welfare function, we obtain:

**Lemma 1.** The first-best solution is characterized by \( \delta_j = 1, \) iff \( q_j \geq 1, \) i.e., iff \( j \geq j^* \).

Hence, in a first-best allocation, investment in all those projects occur that at least meet the opportunity costs. The lemma is obvious. Next it is useful to examine briefly the entrepreneur’s problem. The payoff of any non-investing entrepreneur of type \( j \) who obtains funding, is given by \( z + I \).

An entrepreneur of type \( j \) who obtains a loan of bank \( i \) and invests has a payoff

\[
 u_{ji} = \max \{ q_j (I + z) - R_i (q_j (I + z)), 0 \}.
\]

Hence, an entrepreneur who applies at bank \( i \) invests only if

\[
 q_j (I + z) - R_i (q_j (I + z)) \geq I + z. \tag{2}
\]
Equation (2) is the incentive constraint (IC) that entrepreneurs invest.\(^9\)

We next introduce a useful notation. Let \( \gamma_{i,j} \) be the joint probability that an entrepreneur is of type \( j \) and chooses bank \( i \). Observe that

\[
\gamma_j = \sum_{i=1}^{m} \gamma_{i,j}.
\]

Consider an entrepreneur of type \( j \). If there is at least one contract for which \( u_{ji} \) reaches its maximum at \( \delta_j = 1 \), we assume that the entrepreneur will only choose to invest \( \delta_j = 1 \) (this is the first tie-breaker mentioned above), and will pick any of the banks at which \( u_{ij} \) is maximized for \( \delta_j = 1 \) with equal probability (this is the second tie-breaker mentioned above). These choices determine \( \gamma_{i,j} \) for these entrepreneurs. We call these entrepreneurs the investors. Let \( L \) be the set of all investor indices \( j \) (\( L = \{ j \mid \delta_j = 1 \} \)) and let their total mass be denoted by \( \lambda \), where

\[
\lambda = \sum_{j \in L} \gamma_j.
\]

Note that \( \lambda \) is endogenous since it is a function of the contracts offered. All other entrepreneurs, i.e., those entrepreneurs for which \( u_{ji} \) can only be maximized by setting \( \delta_j = 0 \), are called shirkers. They will not invest. Shirking means that they do not repay anything to the bank in the next period.

Shirkers will choose any bank that offers loan contracts. Let \( B \) be the set of all bank indices \( i \) that offer loan contracts. Shirkers are indifferent between banks in the set \( B \). In order to break that indifference, we are assuming our third tie-breaker rule. We assume that shirkers distribute themselves across the banks \( i \in B \) in exactly the same way as the investors do, i.e., that a shirker of type \( j \notin L \) will choose bank \( i \in B \) with the probability

\[
p_{i,j} = \frac{\sum_{l \in L} \gamma_{i,l}}{\sum_{i \in B} \sum_{l \in L} \gamma_{i,l}}.
\]

Note that \( \sum_{i \in B} p_{i,j} = 1 \). Obviously, \( \gamma_{i,j} = p_{i,j} \gamma_j \) for \( j \notin L \).\(^{10}\)

This is a critical assumption for the further analysis and is therefore worth some discussion. Imagine that investors distribute themselves across banks first and that the size of a bank is then given by the number of investors it has financed. If shirkers distribute themselves according to bank size, they end up mimicking the investors. An alternative story might run as follows. Indifferent borrowers are those who are of low quality and do not invest. Thus, they are indifferent because their terminal wealth does not depend upon different repayments offered in the various contracts, and the funds provided by banks under different contracts are the same. Shirkers may then want to mimic honest investors as best as they can, distributing themselves across lenders in the same proportion as honest investors. Suppose that a lender offers a new contract, and that this new contract offers high-quality entrepreneurs a better deal, while entrepreneurs of low quality are still indifferent since they do not invest. Certainly, investors would then redistribute themselves. If shirkers do not redistribute themselves likewise, they could be detected because they are not moving. The shirkers would be those that can never be attracted by such a new

\(^{9}\) Obviously, the participation constraint is automatically fulfilled, since the entrepreneurs do not risk losing anything by participating in the game. They can always take the credit and consume the total amount of funds.

\(^{10}\) If set \( L \) is empty, i.e., if all entrepreneurs are shirkers, we assume that shirkers distribute themselves arbitrarily across banks. Since this case does not occur in equilibrium or in any relevant deviation strategies, the assumption is harmless.
contract. Since they are indifferent about these contracts, they might as well move and thus avoid detection this way.

4. The monopoly case

Before analyzing the scenario of competing lenders, we first consider briefly the monopoly case where only one bank can offer contracts and further entry is impossible. Recall that \( j^* \) is the first index value for which \( q_j \geq 1 \).

It is useful to introduce \( \psi^* \) as

\[
\psi^* = \sum_{j \geq j^*} \gamma_j \cdot ((I + z)(q_j - 1) - I) - \sum_{j < j^*} \gamma_j \cdot I = \sum_{j \geq j^*} \gamma_j (I + z)(q_j - 1) - I.
\]

In other words, \( \psi^* \) is the expected profit for the bank if it obtains repayments above the invested funds from all entrepreneurs with quality levels of at least one, i.e., if it offers a pure equity contract with varying equity participation. From the entrepreneurs of lower quality, the bank suffers a loss equal to the credit granted. In the next proposition, we characterize the bank’s optimal credit policy using this profit expression.

**Proposition 1.** (1) If \( \psi^* \geq 0 \), then the optimal credit policy is given by the pure equity contract with varying equity participation. Entrepreneurs invest if \( j \geq j^* \). The optimal credit policy is efficient.

(2) If \( \psi^* < 0 \), a monopolistic bank does not offer credit.

The proof of Proposition 1 is not presented in detail because it is straightforward. A monopolist simply collects the maximum surplus under the IC condition. This immediately yields expected profits in Eq. (3) and the assertions in the proposition.

Credit policy by a monopolist is inefficient for \( \psi^* < 0 \), since credits for good projects are not granted in this case. This source of inefficiency exists because the contracts are pooled with respect to bad entrepreneurs. Shirkers cannot be distinguished from investing entrepreneurs. A good project for the monopoly bank is an entrepreneur with \( q_j (I + z) - (I + z) - I \geq 0 \). For \( \psi^* \geq 0 \), the mix of good and bad projects is sufficiently favorable for the bank to compensate losses in low-return projects by equity participation in projects with high returns. The distribution of funds involves a subsidy to bad entrepreneurs by the banks and, ultimately, by the good entrepreneurs. The monopolistic bank receives all rents from good entrepreneurs \( (j \geq j^*) \). However, the bank cannot separate bad entrepreneurs from good ones.

Proposition 1 implies that financing will take the form of equity if the provision of finance is monopolistic. Note that this is the perspective of banks, i.e. the issue of the contracts. Entrepreneurs who know exactly what they need to pay back when they sign the contract experience banks more as a price discriminating monopolist and not so much as an equity holder.

5. Competitive credit markets: pure strategies

We next consider competitive credit markets and focus on pure strategy equilibria. Note that shirkers distribute themselves according to

\[
y_{t,j} = \frac{\gamma_j}{\lambda} \sum_{l \in L} \gamma_{l,i} \quad \text{for} \quad i \notin L.
\]
We now calculate the expected payoff $\psi_i$ for a bank $i$ from the contract $C_i = R_i(y)$. This payoff is obtained by calculating the sum of all the payments received from the different types of agent in the next period, subtracting the payments made to them in this period and weighting them with the joint probability of the particular type choosing this particular bank. A simple calculation yields:

$$\psi_i = \sum_{j \in L} \gamma_{i,j} (R_i(y_j - I) - \sum_{j \notin L} \gamma_{i,j} I = \sum_{j \in L} \gamma_{i,j} \left( R_i(q_j(I+z)) - \frac{I}{\lambda} \right).$$

Note that $R_i(q_j(I+z))$ is the repayment to a bank $i$, if $j \in L$ is an investing type applying to this bank. The term $-I/\lambda = -I(1+(1-\lambda)/\lambda)$ arises from the resources given to each investor, plus the resources lost to the $(1-\lambda)/\lambda$ shirkers accompanying each investor.

The above formula describes the expected profit when each investor brings a share of imitating shirkers with him. Of all the loans made by the bank, only the share $\lambda$ reaches investors, whereas the share $1-\lambda$ is embezzled by shirkers with no chance of repayment for the bank. These extra sunk costs of lending are independent of the type of the honest entrepreneur. A consequence is:

**Lemma 2.** If there is an equilibrium of the game in pure strategies, then it can be written as an equilibrium in pure debt contracts at repayment

$$R = \frac{I}{\lambda}.$$

**Proof.** Let $j \in L$ be an investor and let $i > 0$ be a bank with $\gamma_{i,j} > 0$. We first show that $R_i(q_j(I+z)) \leq I/\lambda$. If indeed $R_i(q_j(I+z)) > I/\lambda$, then a bank $k$ that decided not to enter could have instead chosen to enter offering a slightly better contract solely to agents of type $j$, i.e., to offer $R_k(q_j(I+z))$ with $I/\lambda < R_k(q_j(I+z)) < R_i(q_j(I+z))$ and $R_k(q_l(I+z))$ arbitrarily high for $q_l \neq q_j$. As a result, investors of type $j$ will choose this bank, bringing along their share of shirkers, while all other types of investors will remain with the banks they would have chosen otherwise. But now, the profits to this bank are positive, $\gamma_{j}(R_k(q_j(I+z)) - I/\lambda)$, in contradiction to free entry. This shows that $R_i(q_j(I+z)) \leq I/\lambda$.

Now note that the same free entry condition shows that $R_i(q_j(I+z)) = I/\lambda$ for any $i \in B$, $j \in L$ for which $\gamma_{i,j} > 0$, since otherwise the bank would make negative expected profits.

Several remarks are necessary here. The simple price formula for loans is the result of Bertrand competition and the underlying linear technology. The debt contract arises from the marginal cost pricing of banks under Bertrand competition. Such a contract entails some degree of shirking when $\lambda$ is not equal to 1. Note that Lemma 2 implies that it is impossible for two contracts asking for different repayments from investing entrepreneurs to be offered in equilibrium.

We next determine the value of $\lambda$. Provided that banks offer contracts at some repayment $R$, it is easy to separate investors from shirkers:

$$L = L(R) = \{ j \mid (q_j - 1)(I+z) \geq R \}.$$

Likewise, we can find the total fraction of the population that invests,

$$\lambda = \lambda(R) = \sum_{j \in L(R)} \gamma_{j}.$$
Hence, let

\[ G(R) = \sum_{j \in L(R)} \gamma_j \left( R - \frac{I}{\lambda(R)} \right) = \lambda(R) R - I \]  

(5)

be the sum of the banks’ expected profits, given that they all charge \( R \). Note that \( G(R) \) is piecewise continuous. Discontinuous jumps occur whenever \( L(R) \) changes. \( L(R) \) is a step function with \( n + 1 \) jumps.

Finally, let \( R^\ast \) be the lowest repayment for which \( G(R) \) is greater than or equal to zero:

\[ R^\ast = \min \{ R \mid G(R) \geq 0 \} . \]

Obviously, \( R^\ast \) may not exist and in this case we face a market breakdown problem. \( R^\ast \) exists if the share of investing entrepreneurs is sufficiently large and if the return of high-quality projects is sufficiently larger than refinancing costs. Note that \( \lambda(R) \) is maximal at \( R^\ast \) for all \( R \) with \( G(R) \geq 0 \). We obtain the following result.

**Lemma 3.** Suppose that \( R^\ast \) exists. In any equilibrium with a positive entrance, banks charge the repayment \( R^\ast \) given by

\[ R^\ast = \frac{I}{\lambda(R^\ast)} . \]

**Proof.** In Lemma 2 we have shown that, if there is a Nash equilibrium, then it can be written in the form of a pure debt contract at repayment \( R = I/\lambda \). Suppose that \( R > R^\ast \). If \( G(R) < 0 \), this can certainly not be an equilibrium. Therefore, suppose \( G(R) > 0 \). In this case, there is a value \( \tilde{R} \), \( R^\ast < \tilde{R} < R \) with \( G(\tilde{R}) > 0 \). A bank offering pure debt contracts at repayment \( \tilde{R} \) will draw the entire market (since investors are better off with lower repayments and shirkers imitate investors), and will make positive profits \( G(\tilde{R}) \), a contradiction to free entry. Thus, we must have \( R = R^\ast \). Either from Lemma 2 or by examining \( G(R^\ast) = 0 \), we see that

\[ R^\ast = \frac{I}{\lambda(R^\ast)} . \]

Summarizing, we can say that if there is an equilibrium with positive amounts of loans handed out, banks must be charging \( R^\ast \). So if there is an equilibrium, it is unique in the sense of yielding a unique allocation of resources. However, we have not provided sufficient conditions for existence. Indeed, we will show that the possibility of offering pure debt and convertible debt contracts introduces a substantial problem for the functioning of the credit market. To that end, we consider a particular deviation from the candidate equilibrium at \( R_i(y) \equiv R^\ast \). Recall that \( j^\ast \) is the first index value for which \( q_{j^\ast} \geq 1 \). We can now state our main proposition.

**Proposition 2.** Suppose that there exists a debt contract which yields non-negative profits for banks (and thus \( R^\ast \) exists). If the set of intermediate quality shirkers given by \( S :=

\[ 11 \text{ Here we switch from the symbol } \psi \text{ to } G \text{ because we are now considering situations where all banks charge the same } R. \]

\[ 12 \text{ Note that } G(R) \text{ is continuous from below.} \]

\[ 13 \text{ Here the proof of this proposition has similarities with an argument proposed by Mankiw (1986).} \]
\{j^*, \ldots, n\} \setminus L(R^*) is empty, the candidate equilibrium of Lemma 3 is an equilibrium. If \( S \) is not empty, no equilibrium in pure strategies exists.

**Proof.** Consider the candidate equilibrium in which all banks chose the debt contract with repayment \( R^* \). Let \( \lambda = \lambda(R^*) \), let \( L = L(R^*) \) and let \( \tilde{\lambda} = \lambda(0) \). Note that \( \lambda(0) = \{ j \mid q_j \geq 1 \} \).

Suppose that \( S \) is empty. Entrepreneurs either invest or are of such low quality that no bank can induce them to invest without incurring losses beyond those incurred due to shirking. A newly entering bank cannot profitably change the composition of investors and shirkers, i.e., \( \lambda \) cannot be changed by offering additional contracts. The proposition above shows that a deviation cannot be profitable, since charging less than \( I/\lambda \) means losing money. Hence, if \( S \) is empty, no profitable deviation is possible.

Suppose now that \( S \) is not empty. Choose some sufficiently small \( \epsilon > 0 \). A new bank could offer a convertible debt contract at repayment \( R^* - \epsilon \). According to our first tie-breaking rule, this turns all entrepreneurs with \( j \in S \) into investors, raising the fraction of investors from \( \lambda \) to \( \tilde{\lambda} \). Furthermore, all previous investors \( j \in L \) will strictly prefer this contract. The shirkers will mimic the investors. The bank will therefore capture the entire market. Hence, the profits of the new entrant are given by:

\[
\sum_{j \in S} \gamma_j \left( (q_j - 1)(I + z) - \frac{1}{\lambda} \right) + \sum_{j \in L} \gamma_j \left( R^* - \frac{I}{\lambda} \right) - \epsilon \lambda
= \sum_{j \in S} \gamma_j ((q_j - 1)(I + z)) + G(R^*) - \epsilon \lambda
> G(R^*) = 0
\]

for \( \epsilon > 0 \) sufficiently small. Hence the bank makes positive expected profits and this deviation destroys the candidate equilibrium. Therefore, no equilibrium exists.

Note that if the competitive equilibrium in pure strategies exists, the allocation is efficient since all entrepreneurs with good projects are granted credits and invest accordingly. Since it is certain that banks will offer a credit contract with repayment \( R^* \) or lower, the credit market provides funds for all good projects. It is worth discussing why an equilibrium in pure strategies may fail to exist. If intermediate quality levels exist, i.e. if the set \( S \) is not empty, then the debt contract in the candidate equilibrium is not attractive enough for entrepreneurs \( j \in S \) to induce them to invest. But then a bank could offer a convertible debt contract at repayment \( R^* - \epsilon \). This new contract yields higher profits than the original debt contract. However, this contract cannot be an equilibrium either. This follows directly from Lemma 2. More intuitively, it follows that such a contract is dominated by a simple debt contract for high quality entrepreneurs with slightly better conditions. Additionally, assume that the bank offering this debt contract just breaks even, given the enlarged set of investors. This leaves only entrepreneurs with intermediate and bad quality levels for the convertible debt contract. The cross-subsidization from high quality entrepreneurs disappears. The convertible debt contract now generates a loss, since it recoups less from each investing entrepreneur than the new break-even debt contract (via the self-selection of entrepreneurs), but loses the same resources to the accompanying shirkers. Overall, no equilibrium in pure strategies exists.

On the regulatory side, one might ask which regulatory schemes can be used to avoid the non-existence of equilibria. From our preceding discussion we can infer that a regulation that forces banks to offer either debt or equity (or convertible debt) contracts would restore equilibrium.
with pure debt contracts. An alternative scheme for avoiding the non-existence of equilibria is the coordination of banks on the decision not to lend to some easily demarcated group of intermediate quality borrowers. This might explain redlining, i.e., the refusal of banks to provide credit to some particular neighborhoods.

If one considers mixed strategies as a viable description of bank behavior, regulation limiting of the number of contracts banks can issue may not be necessary. As we will show in the next section, mixed strategy equilibria imply that banks randomize over debt and convertible debt contracts if intermediate quality borrowers exist.

6. Competitive credit markets: mixed strategies

The non-existence result for a pure strategy subgame perfect equilibria in Proposition 2 raises the question of what mixed strategy equilibria might look like. This is the subject of this section. In a mixed strategy equilibrium, banks that enter can choose a lottery over the set of contracts they offer. As we will show, mixed strategy equilibria imply that banks randomize over debt and convertible debt contracts if intermediate quality borrowers exist.

We assume that there are exactly two banks that can enter. The features of the analysis generalize to any finite number of banks.

To develop the equilibrium in mixed strategies, we introduce the following notation. $G^{CD}(R)$ and $G^{D}(R)$ denote the expected profit associated with a convertible debt contract and a pure debt contract at repayment $R$, respectively, if only one of these contracts is offered. $G^{CD}(R | D(R'))$ denotes the expected profit from a convertible debt contract at repayment $R$, when a debt contract at repayment $R'$ is simultaneously offered. Clearly, $G^{CD}(R | D(R' > R)) = G^{CD}(R)$ since the presence of the debt contract is irrelevant. Similarly, $G^{D}(R | CD(R'))$ denotes the expected profit from a debt contract at repayment $R$, if $CD(R')$ is offered simultaneously. Note that $G^{D}(R | CD(R' < R)) = 0$. Finally, we define $S(R) = \{j^* \ldots, n\} \setminus L(R)$ as the set of intermediate shirkers when debt contracts at repayment $R$ are offered. For the remaining combinations of debt and convertible debt contracts, we obtain:

$$G^{D}(R) = \lambda(R)R - I,$$

$$G^{CD}(R) = \sum_{j \in S(R)} \gamma_j \left((q_j - 1)(I + z)\right) + \lambda(R)R - I,$$

$$G^{CD}(R | D(R' < R)) = \sum_{j \in S(R')} \gamma_j \left((q_j - 1)(I + z)\right) - \left(\frac{\lambda(0) - \lambda(R')}{\lambda(0)}\right)I,$$

$$G^{D}(R | CD(R' > R)) = \lambda(R)R - \left(\frac{\lambda(R)}{\lambda(0)}\right)I.$$  

Note that only $G^{CD}(R)$ is continuous everywhere.

We next define

$$R^* = \min \{ R \mid G^{D}(R) \geq 0 \},$$

14 Such regulation limiting the type of contracts a financial institution can offer is broadly connected to the widely known Glass–Steagall Act of 1933, which has prohibited commercial banks from underwriting and holding corporate securities.

15 We are grateful to a referee for this insight.
\[ R^\circ = \min \{ R \mid G^{CD}(R) \geq 0 \} , \]
\[ \bar{R} = \min \{ R \mid G^D(R \mid CD(R') > R) \geq 0 \} . \]

It is obvious that \( R^* \geq R^\circ \geq \bar{R} \). Moreover, \( R^* \neq R^\circ \) if and only if there are intermediate shirkers, i.e., \( S(R^*) \) is not empty.

For simplicity of presentation, we assume that \( S(R^*) = S(R) \) and hence that there are no intermediate shirkers with net returns \((q_j - 1)(I + z)\) in \([\bar{R}, R^*]\).\(^{16}\)

We obtain:

**Proposition 3.** Suppose that \( R^* \) and \( \bar{R} \) exist. Suppose that the set of intermediate shirkers \( S(R^*) \) is not empty and that \( S(R^*) = S(R) \). Then there exists an equilibrium in mixed strategies, characterized by a tuple \( \{ w, F(R) \} \), where \( F(R) \) is a continuous distribution function on \( \mathbb{R} \), with support on \( [R^u, R^*] \), for some \( R^u > \bar{R} \). Each of the two banks offers

(i) a debt contract \( D(R) \) with probability \( w \) (0 < \( w < 1 \)) at repayment \( R^* \);
(ii) a convertible debt contract with probability \( 1 - w \) at repayment \( R \in [R^u, R^*] \), where \( R \) is randomly selected according to \( F(R) \).

Each bank makes zero expected profits.

The proof is given in Appendix A. The assumptions of Proposition 3 require that banks can break even with either debt contracts or convertible debt contracts alone. Otherwise we face a market breakdown problem since the share of shirkers is too large. The other assumption \( S(R^*) = S(R) \) avoids having shirkers in the support of the convertible debt contract repayments. This assumption is only made for technical reasons, since it avoids the occurrence of mass points and allows us to work with a continuously differentiable distribution function. If \( S(R^*) \neq S(R) \) does not hold, a mixed strategy equilibrium, much more cumbersome to characterize, may be established using the Dasgupta–Maskin theorem (Dasgupta and Maskin, 1986).

Propositions 2 and 3 can be used to address regulatory issues. First, competition between lenders can create a new type of market failure compared to monopoly banking, and may thus justify less competition in banking. Moral hazard, together with adverse selection, potentially require different repayments to be demanded from borrowers of different types in order to motivate them to invest. This is precisely what happens under monopoly banking. Competition among lenders forces equal repayments from borrowers of different types, at least with a certain probability. Compared to a monopoly bank, average returns decrease as banks compete for good entrepreneurs while being unable to avoid bad entrepreneurs. This may make lending altogether unprofitable, and therefore \( R^* \) does not exist. This is distinct from the normal Rothschild–Stiglitz market failure, which arises when separating contracts are possible (and thus make pooling impossible in a competitive situation) but are again dominated by pooling contracts.

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\(^{16}\) The assumption is made in order to avoid the occurrence of mass points in the mixed strategies of banks when they offer CD contracts. If the assumption does not hold, the analysis becomes very cumbersome, but does not provide new insights about the nature of the mixed strategy equilibria.
7. Empirical implications and conclusion

In this paper, we have offered a simple explanation for the prevalence of debt contracts, and we have also identified new types of market failures which might raise regulatory concerns. Our model has some empirical implications. First, in transition economies when there is little or no credit history, a competitive banking system faces high risks of a breakdown. Second, if two countries have the same bank-based financial system we should observe an inverse relationship between the volatility of repayments from firms that do not default and the intensity of competition among banks.

There are a variety of extensions that can be pursued. First, one could allow for random delivery. Random delivery is a particular form of credit rationing, since borrowers only receive funds with a certain probability. This can be achieved by being clear about the terms of the contract but opaque about the procedure for the delivery. One can show that no subgame perfect Bayesian equilibrium in pure strategies exists, since banks attempt to avoid shirkers by lowering their delivery probability. Hence, there can be no equilibrium in which debt credit contracts are offered with probability 1. The credit market can break down partially since high-quality borrowers may not receive credits. It is, however, still the case that, in any possible mixed strategy equilibria where the set of intermediate borrowers is empty, only debt contracts will be offered. Banks will also compete until any rents from investing entrepreneurs under random delivery are gone. Hence, allowing random delivery does not destroy the basic result from the last section, i.e., that debt contracts occur in equilibrium.

Second, one could introduce monitoring technologies. For instance, by screening entrepreneurs, banks can obtain information about the quality $q_j$. By monitoring entrepreneurs over the course of investments, banks can reduce the private benefits of entrepreneurs who want to consume their funds. Thus, shirking becomes less attractive. How the competition between banks works under such a combination of monitoring technologies is largely unknown. The present framework may be a suitable starting point for such a more comprehensive analysis of banking competition.

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Appendix A

Proof of Proposition 3. We first establish the following simple relationships among the profits associated with different menus of contracts. The relationships will be helpful in the proof. To develop the equilibrium, we use three variables $R, R'$ and $R''$ to describe repayments. Throughout the proof, we assume that the variables $R, R'$ and $R''$ are in $[R, R^*]$ to ensure that there are no intermediate shirkers in any of the intervals considered. We rely on our assumption that investing

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17 A detailed analysis is given in Gersbach and Uhlig (2000). This argument is a twist on the classical insight by Jaffee and Russell (1976) that banks attempt to make credit conditions for dishonest borrower more unattractive than other banks which, in turn, can lead to a collapse of the market.
borrowers will distribute themselves evenly when indifferent.

\[ G_{CD}(R \mid CD(R' < R)) = \frac{1}{2} G_{CD}(R \mid D(R'' < R)), \]  
(A.1)

\[ G_{CD}(R \mid CD(R' > R)) = G_{CD}(R) - \frac{1}{2} G_{CD}(R \mid D(R'' < R)) \]  
(A.2)

\[ = G^D(R \mid CD(R' > R)) + \frac{1}{2} G_{CD}(R \mid D(R'' < R)) \]  
(A.3)

\[ G_{CD}(R) = G^D(R \mid CD(R' > R)) + G_{CD}(R \mid D(R'' < R)). \]  
(A.4)

We next show that the proposed strategies are an equilibrium with two active banks.

(i) In equilibrium, a bank must be indifferent between any contract offered in the support of the mixed strategy, given the randomized strategy of the opponent. If a bank offers a debt contract at repayment \( R^* \), we denote by \( G_{D^1}(R^* \mid eq.) \) its expected profit, given the other bank’s equilibrium strategy. It is easy to see that the debt contract generates zero profits, regardless of the contract drawn by the other bank from its support of the mixed strategy, unless the other bank offers \( CD(R^*) \). But \( CD(R^*) \) occurs with probability zero, since \( F(R) \) is a continuous distribution function, and thus contains no mass point. Therefore,

\[ G_{D^1}(R^* \mid eq.) = w \cdot \frac{G^D(R^*)}{2} = 0. \]  
(A.5)

If a bank offers a convertible debt contract at repayment \( R \), its expected profit when the other bank follows its equilibrium strategy is denoted by \( G_{CD^1}(R \mid eq.) \). It is given by

\[ G_{CD^1}(R \mid eq.) = w \cdot G_{CD}(R) + (1 - w) \{(1 - F(R)) \cdot G_{CD}(R \mid CD(R' \geq R)) \}
+ F(R)G_{CD}(R \mid CD(R' < R)) \}. \]  
(A.6)

Using Eqs. (A.1), (A.3) and (A.4), we obtain for \( R, R', R'' \) in \( [\bar{R}, R^*] \)

\[ G_{CD^1}(R \mid eq.) = G^D(R \mid CD(R' > R)) \cdot \{1 - F(R)(1 - w)\}
+ \frac{1}{2} G_{CD}(R \mid D(R'' < R)) \cdot (1 + w) \}.
\]

Let \( a(R) = G^D(R \mid CD(R' > R)) \), \( b(R) = G_{CD}(R \mid D(R'' < R)) \); setting \( G_{CD^1}(R \mid eq.) = 0 \) yields

\[ F(R) = \frac{a(R) + \frac{1}{2} b(R)(1 + w)}{a(R)(1 - w)} = \frac{1}{1 - w} + \frac{\frac{1}{2} b(R)(1 + w)}{a(R)(1 - w)}. \]  
(A.7)

For \( R \in [\bar{R}, R^*] \) note that \( a(R) \) is continuously differentiable with \( a(R) > 0 \) and \( a'(R) > 0 \) and that \( b(R) \equiv b(R^*) < 0 \). Therefore, \( F(R) \) is continuously differentiable and \( dF/dR > 0 \). Setting \( F(R^*) = 1 \) yields

\[ w = \frac{-b(R^*)}{2a(R^*) + b(R^*)}. \]  
(A.8)

It is straightforward to show that \( 0 \leq w \leq 1 \). To find the lower boundary for \( F(R) \), we set \( F(R) = 0 \) and obtain

\[ 2a(R^u) = -b(R^u)(1 + w). \]
Using Eq. (A.8) yields an implicit equation for $R_u$:

$$
\frac{2a(R_u)}{-b(R)} = \frac{-b(R^*)}{2a(R^*) + b(R^*)} + 1. 
$$

(A.9)

$b(R)$ is constant in $[\tilde{R}, R^*]$ and $b(R) < 0$, $a(\tilde{R}) = 0$ and $a(R)$ is monotonically increasing in $R$. Moreover, since $f(R) = dF/dR > 0$, there is a unique value $R_u$, $\tilde{R} < R_u < R^*$, such that $F(R_u) = 0$. Hence, each bank is indifferent between offering a debt contract at $R^*$ and a CD contract at repayment between $[R^*, R^u]$. 

(ii) We need to ensure that banks cannot offer any other profitable contract. In particular, banks may want to offer a debt contract at a lower repayment than $R^*$ to benefit from CD contracts offered by other banks. Suppose that a bank offers a debt contract at repayment $R$. Its expected profit is given by

$$
G^D_1(R | \text{eq.}) = w \cdot G^D(R) + (1 - w) \cdot \{(1 - F(R))G^D(R | CD(R' > R))\}.
$$

Hence

$$
G^D_1(R | \text{eq.}) = w \cdot \{G^D(R) - G^D(R | CD(R' > R))\} \\
+ G^D(R | CD(R' > R)) \cdot \{1 - F(R)(1 - w)\}.
$$

If we define

$$
\alpha := \sum_{j \in S(R^*)} \gamma_j ((q_j - 1)(I + z))
$$

and use

$$
G^D(R) = G^{CD}(R) - \alpha = G^D(R | CD(R' > R)) + G^{CD}(R | D(R'' < R)) - \alpha,
$$

we obtain

$$
G^{CD}_1(R | \text{eq.}) - G^D_1(R | \text{eq.}) = G^{CD}(R | D(R'' < R)) \cdot \{\frac{1}{2}(1 + w) - w\} + \alpha w \\
= \frac{1}{2}b(R)(1 - w) + \alpha w \\
= \frac{b(R^*)}{2a(R^*) + b(R^*)} \cdot \{a(R^*) + b(R^*) - \alpha\}.
$$

Note that we have used $b(R) = b(R^*)$. Since

$$
a(R^*) + b(R^*) - \alpha = G^D(R^*) = 0,
$$

introducing a debt contract is not profitable.

(iii) A last possibility could be to offer convertible debt contracts of the type

$$
R_i(y) = \min\{R; \delta(q_j - 1)(I + z)\}
$$

with $0 < \delta < 1$. However, such CD contracts would lower profits in the case of $G^{CD}(R)$, as well as in $G^{CD}(R | CD(R' > R))$ and $G^{CD}(R | CD(R'' < R))$. Therefore no bank will want to introduce such CD contracts. \[\Box\]
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