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The Impact of Large Portfolio Insurers on Asset Prices

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ABSTRACT

We develop a simple model in which the presence of portfolio insurers in a market of risk-averse traders leads to multiple equilibria for the pricing of financial assets and can cause an increase in volatility, including insurance-induced price drops. We demonstrate, however, that centralized portfolio insurance firms may actually reduce, not increase, volatility, even if the existence of these firms increases the total amount of funds under insurance.

A PORTFOLIO INSURER TYPICALLY sells equity as stock prices fall, in order to limit the losses from holding equity in a declining market, and buys equity as prices rise. This can be accomplished directly by switching between stocks and cash, or indirectly by taking a position in derivative securities whose payoffs are tied to stock market performance.[1] The general effects of such dynamic hedging strategies have been studied by authors such as Grossman (1988), Brennan and Schwartz (1989), and Gennotte and Leland (1990). Gammill and Marsh (1988), Shiller (1989), Jacklin, Kleidon, and Pfleiderer (1992), and others have studied the role of portfolio insurance in the crash of 1987. Since these papers do not differentiate between an economy in which a group of atomistic investors follow their own individual insurance strategies and an economy in which large portfolio insurance firms act on behalf of investors who wish to insure, however, the specific price effects of large portfolio insurance firms are yet to be investigated. This is the purpose of our paper.

Section I develops a rudimentary asset-pricing model with two types of traders: atomistic portfolio insurers and atomistic ordinary traders. This model is essentially a simplified version of Gennotte and Leland (1990) in which we focus on the stop-loss basics of portfolio insurance. Like Gennotte and Leland, we demonstrate that the existence of atomistic portfolio insurers increases the variance of possible equilibrium prices (i.e., volatility) and can lead to situations in which there are many potential equilibrium prices for a single set of fundamentals. Section II extends the model by allowing some,

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[1] See, for example, Leland and Rubinstein (1981).

but not necessarily all, of the atomistic traders to employ the services of large portfolio insurance firms. We demonstrate that portfolio insurance firms may actually reduce, not increase, price volatility even if the existence of these firms increases the amount of funds under insurance. This occurs because the large insurance firms take the price impact of their own actions into account and thus internalize some of the externalities associated with portfolio insurance. Section III concludes.

I. A Simple Model

We posit a simple economy in which $i = 1, \dots, N$ atomistic agents live for two time periods, $t = 1, 2$, and trade two types of assets, stocks, and bonds. Bonds pay one unit of "the consumption good" with certainty in period 2. Stocks pay a random amount Z of the consumption good in period 2, as given by equation (1),

$$Z = \alpha + \beta + \gamma \tag{1}$$

in which α is a constant and β and γ are independent random variables with $\beta \sim N(0, \sigma_\beta^2)$ and $\gamma \sim N(0, \sigma_\gamma^2)$. The values of β and γ are revealed to all agents at the beginning of periods 1 and 2 respectively.[2] The price of stocks shall always be quoted in terms of bonds so that bonds are the numeraire. Each agent begins period 1 with an endowment of one stock and one bond and consumes only in period 2. Given the information revealed at the beginning of period 1, the agents are free to trade with each other; period 1 is the only period during which trade occurs.

Since our goal is to examine the effects of portfolio insurance on prices, we do not attempt to justify the existence of insurers. Instead, we simply assume that a fraction λ $(0 \leq \lambda \leq 1)$ of our agents follow an insurance-trading rule to be specified below. The remaining $(1 - \lambda)N$ agents are "ordinary traders" who follow no set trading rule but manipulate freely the contents of their asset portfolios. Each ordinary trader's objective is to maximize his expected utility function, $E(e^{-\delta C})$, in which C denotes consumption and $\delta > 0$ is the parameter of absolute risk aversion. This maximization occurs subject to the constraint that the total value of assets held by each trader cannot exceed his

[2] By assuming that all agents observe the same information regarding future cash flows, we have eliminated issues relating to asymmetric information. We make the assumption of perfect information to simplify the analysis and to more easily focus attention on the distinct effects of large portfolio insurance firms. As demonstrated by authors such as Gennotte and Leland (1990), imperfect information can lead to even more dramatic insurance-induced price movements than we produce in our analysis. This is because uninformed market observers cannot differentiate trades made by individuals with fundamental information from trades made for nonfundamental reasons and may therefore overreact to price moves that result from insurance-motivated buying or selling. This suggests that, had we allowed for asymmetric information, our analysis would produce even more dramatic effects than the ones we document. The nature of our conclusions regarding the price effects of large insurance firms relative to the price effects of atomistic portfolio insurers, however, would be unchanged.

endowed wealth. The stock price p must be such that, in equilibrium, the ordinary traders hold all the assets not held by the portfolio insurers.

Define y to be an agent's wealth, which is the sum of the value of the agent's stock endowment and bond endowment (i.e., $y = p + 1$). We then solve the period 1 utility maximization problem of an ordinary trader given the known values of α and β (the value of γ is still unknown). The resulting demands for stocks, subscript S, and bonds, subscript B, of an ordinary trader, superscript O, are given by equations (2) and (3), respectively.

$$D_S^O(p, y) = \frac{\alpha + \beta - p}{\delta \sigma_\gamma^2} \tag{2}$$

$$D_B^O(p, y) = y - pD_S^O(p, y) \tag{3}$$

The portfolio insurers follow a simple stop-loss trading rule which states that they will hold all their wealth in stocks so long as the stock price p is above the trigger level $\bar{p}$ ($\bar{p} \geq 0$); otherwise they will hold all their wealth in bonds. The demands for stock, subscript S, and bonds, subscript B, by an insurer, superscript I, with wealth y are therefore given by equations (4) and (5), respectively.[3]

$$D_S^I(p, y) = \begin{cases} y/p; & p > \bar{p} \\ 0; & \text{otherwise} \end{cases} \tag{4}$$

$$D_B^I(p, y) = \begin{cases} 0; & p > \bar{p} \\ y; & \text{otherwise} \end{cases} \tag{5}$$

Given the demands of the insurers and ordinary traders, and given that each of the N agents is endowed with one stock and one bond so that there is an aggregate supply of N stocks and N bonds, the market-clearing condi-

[3] We could alternatively—and perhaps more realistically—model the demands of portfolio insurers as continuous functions of the price, in the spirit of Gennotte and Leland (1990), or as discrete processes with incremental rebalancing at multiple trigger levels. We also could have assumed that each insurer has a different trigger level, series of trigger levels, or continuous rebalancing function. So long as over some range of p the aggregate stock demand function of the portfolio insurers remains upward sloping and more sensitive to price changes than the aggregate stock demand function of the ordinary traders, however, these modifications will not alter the paper's main conclusions regarding the effects of portfolio insurance and the existence of formal portfolio insurance firms. The only effect will be to smooth out the excess demand function in Figure 1. For example, if $D_S^I(p, y)$ in equation (4) was replaced by an appropriate continuous function of p, we could produce an excess demand function for our Figure 1 that displays the smooth S-shape seen in Figures 1 through 5 of Gennotte and Leland, instead our model's coarser zig-zag currently displayed in our Figure 1. However, our finding of multiple price equilibria in the presence of portfolio insurance and our results concerning the effects of large portfolio insurance firms would not change. With more complicated insurance strategies, however, these results would be significantly more difficult to obtain and less transparent. Since our goal is to understand the effects of portfolio insurance and formal portfolio insurance firms in the simplest possible terms, we therefore adopt the simplifying assumption that insurers follow the strategy formalized in equations (4) and (5).

tions for stocks and bonds are given by equations (6) and (7), respectively.

$$(1-\lambda)ND_S^O(p, p+1) + \lambda ND_S^I(p, p+1) = N \tag{6}$$

$$(1-\lambda)ND_B^O(p, p+1) + \lambda ND_B^I(p, p+1) = N \tag{7}$$

Since these two equations are linearly dependent by Walras's Law, we will work mostly with (6) alone. We now proceed to calculate equilibrium prices.

With no portfolio insurers in the market, $\lambda = 0$. Substituting (2) and (4) into (6) thus yields the market-clearing condition $(\alpha + \beta - p)/\sigma_\gamma^2\delta = 1$, which delivers the unique no-insurers (subscript n) market-clearing price of stocks, in terms of bonds, given in (8).

$$p_n = \alpha + \beta - \sigma_\gamma^2\delta \tag{8}$$

Conversely, with insurers in the market (i.e., $\lambda > 0$), there can be two Walrasian equilibrium prices: a price p_s, which is market clearing conditional on portfolio insurers holding only stocks and no bonds, and a price p_b which is market clearing conditional on insurers holding only bonds and no stocks. These prices are given in (9) and (10), respectively.

$$p_s = \frac{\alpha + \beta - \delta\sigma_\gamma^2}{2} + \sqrt{\left(\frac{\alpha + \beta - \delta\sigma_\gamma^2}{2}\right)^2 + \frac{\lambda\delta\sigma_\gamma^2}{1-\lambda}} \tag{9}$$

$$p_b = \alpha + \beta - \frac{\delta\sigma_\gamma^2}{1-\lambda} \tag{10}$$

Notice that $p_b < p_n < p_s$: even though the fundamentals of the stock ($\alpha + \beta$, and σ_γ^2) are the same in all cases, the ordinary traders, who are risk averse, must be induced through a lower price of stocks to hold the additional stock that the portfolio insurers dump on the market as the insurers move to hold only bonds. With insurers in the market and a trigger price $\bar{p}$, either (9) or (10), but not both, will hold: i.e., either $p = p_b \le \bar{p}$ or $p = p_s > \bar{p}$. The price p_s is an equilibrium if and only if $p_s > \bar{p}$. Similarly, p_b is an equilibrium price if and only if $p_b \le \bar{p}$.

Figure 1 plots the excess demand curve in equation (6) given the price functions in equations (9) and (10) and a particular choice of the parameters $\bar{p}$, α, σ_γ^2, λ, and β for which $p_b \le \bar{p} < p_s$.[4] In this case, both p_s and p_b are potential Walrasian outcomes: in equilibrium 1, all insurers hold only bonds and $p = p_b$ while, in equilibrium 2, all insurers hold only stocks and $p = p_s$. Of course, if the parameters were such that $\bar{p} \ge p_s$, then equilibrium 1 would be the unique equilibrium with the portfolio insurers holding only bonds and $p = p_b$. Likewise, if $\bar{p} < p_b$, then equilibrium 2 would be the unique outcome.

Given the values of α, σ_γ^2, $\bar{p}$, and λ, which are set prior to the beginning of period 1, it is useful to establish the relationship between the value of β,

[4] To draw Figure 1 we assumed that $\alpha = 1$, $\beta = 0$, $\sigma_\gamma^2 = 1$, $\delta = 0.4$, $\lambda = 0.2$, and $\bar{p} = 0.6$. It can be easily seen that the reason there are not the usual odd number of equilibria in Figure 1 is that the demand functions of insurers are discontinuous at $\bar{p}$.

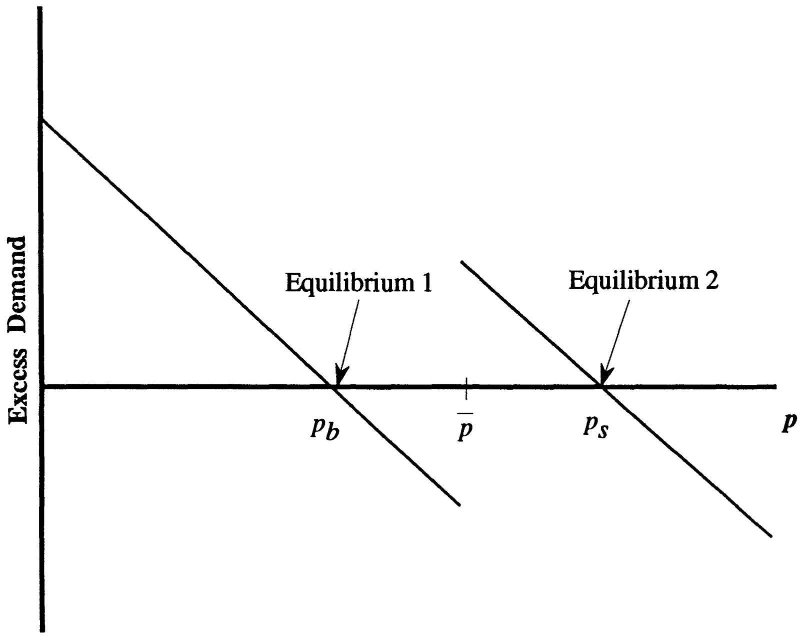

Figure 1. The excess demand for stocks with insurers in the market. This figure shows the excess demand for stocks as a function of the stock price p. If the stock price falls below $\bar{p}$, then insurers place all their wealth in bonds leaving only "ordinary traders" to demand stocks. In this case, the equilibrium stock price is p_b. If the stock price rises above $\bar{p}$, however, then insurers place all their wealth in stocks. This produces a discontinuous jump in excess demand and an equilibrium price of p_s.

which is revealed at the beginning of period 1, and the resulting equilibrium price. To do this, let β_s be the lowest value of β above which the equilibrium in which insurers hold stocks is the unique equilibrium, and let β_b be the highest value of β that enforces as unique the equilibrium in which insurers hold bonds. Thus, (1) if $\beta > \beta_s$ then $p = p_s$ is the unique equilibrium price, (2) if $\beta \leq \beta_b$ then $p = p_b$ is the unique equilibrium price, and (3) if $\beta_b < \beta \leq \beta_s$ then there are two Walrasian equilibrium prices: $p = p_s$ and $p = p_b$.

The relationship between p and β is shown in Figure 2. The top curve is p_s, insurers hold stock, the bottom line is p_b, insurers hold bonds, and the middle line is p_n, there are no insurers ($\lambda = 0$). The horizontal axis gives the value of β and the vertical axis the price. Although all prices are positive, for convenience the horizontal axis is drawn to intersect the vertical axis at the trigger price $\bar{p}$. The point where $p_s = \bar{p}$ (i.e., the point where the p_s curve intersects the horizontal axis) determines the value of β_b. The point where $p_b = \bar{p}$ determines the value of β_s.

Note that our simple model exhibits features consistent with the results of previous models. Like Gennotte and Leland (1990), there is a range of fundamentals $\beta_b < \beta \leq \beta_s$ which can produce as an equilibrium outcome either a high or low stock price depending on whether insurers hold their

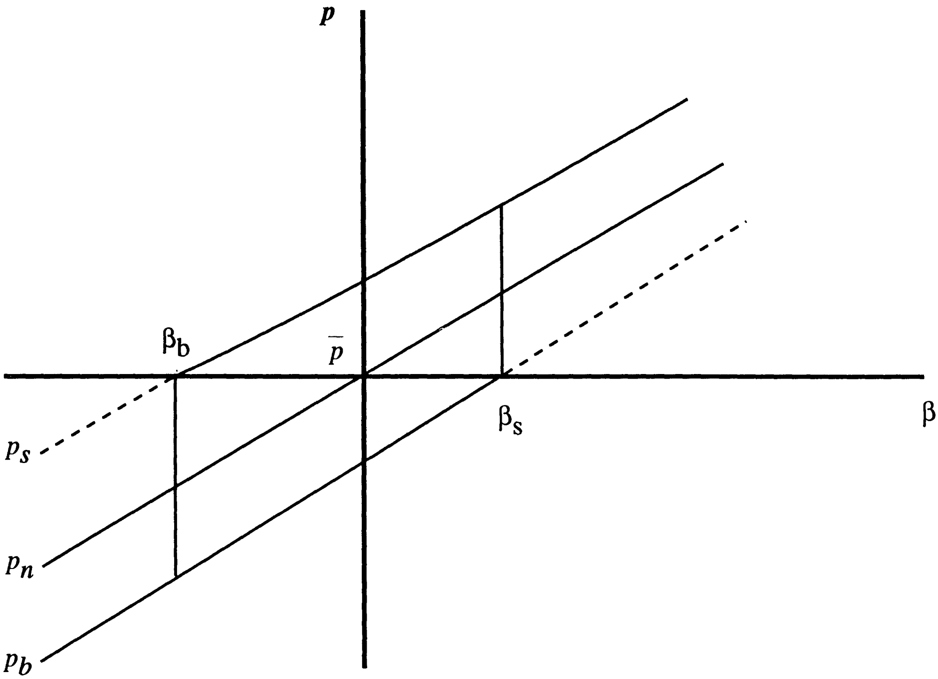

Figure 2. Stock prices and fundamentals with portfolio insurance. In this figure the vertical axis measures the stock price p. The horizontal axis measures β, a "fundamental" signal of future cash flows with higher values of β signalling higher cash flows. The p_n line shows the equilibrium stock price as a function of β in the absence of portfolio insurance. Prices along the p_s curve obtain if portfolio insurers hold all their wealth in stocks, while prices along the p_b line obtain if portfolio insurers hold all their wealth in bonds. $\bar{p}$ is the trigger price above which insurers hold only stocks and below which they hold only bonds (while all prices are positive, the horizontal axis has been drawn through $\bar{p}$ for convenience). Thus, if $\beta > \beta_s$ then the unique equilibrium price lies along the p_s curve, while if $\beta \leq \beta_b$ then the unique equilibrium price lies along the p_b line. However, if $\beta_b < \beta \leq \beta_s$ then there are two potential equilibria: p_s with insurers holding only stock, and p_b with insurers holding only bonds.

stocks or sell. Using the atemporal framework to gain some insight into the intertemporal behavior of asset prices, one might therefore think of a fall in stock prices from some initial price on the p_s curve as having two potential sources. The first is a price decline caused by successively worse realizations of the fundamental β and thus a movement down along the p_s curve. The second possibility, which can occur when β is inside the middle region of multiple equilibria in Figure 2, is a price decline caused by the selling of stocks by portfolio insurers and thus a drop from p_s to a new equilibrium on the p_b curve, even in the presence of a small (or no) change in fundamentals. One might view the first type of price decrease as a fundamental crash and the second type as a portfolio insurance-induced crash. Note from (9) and (10) that, as the fraction of insurers λ increases, the p_s and p_b curves move farther apart and the range of β over which multiple equilibria are possible increases. Thus, consistent with earlier studies, the severity of and possibility for insurance-induced price movements rises as a larger portion of traders follow portfolio insurance strategies.

II. The Impact of Large Portfolio Insurers

Recall that there are N traders in our model, λN of whom are portfolio insurers. To study the effects of large portfolio insurance companies on asset prices we now introduce M ($0 < \mathrm{M} \leq \lambda N$) firms, each of which acts on behalf of a subset of the λN individual insurers according to their demand schedules in equations (4) and (5). If all insurers use the same insurance firm then $M = 1$, while if each insurer is his own firm then $M = \lambda N$. Let μ be the fraction of the insurers who hold only stocks, with the rest holding only bonds. Thus, if all firms are of equal size and if M_s of these firms hold only stocks, then $\mu = M_s/M$. We will be most interested in four special cases: $\mu = 1$, all firms holds stocks, $\mu = 0$, all firms hold bonds, $\mu = (M - 1)/M$, all firms but one holds stocks, and $\mu = 1/M$, all firms but one holds bonds. The first two cases are equilibria; the second two are not. The usefulness of the nonequilibrium cases, $\mu = (M - 1)/M$ and $\mu = 1/M$, is made clear below.

For a given μ, the market-clearing condition for stocks from (6) can be written as (11).

$$(1 - \lambda)\frac{\alpha + \beta - p}{\delta\sigma_\gamma^{\,2}} + \lambda\mu\left(1 + \frac{1}{p}\right) = 1 \tag{11}$$

The solution for the market-clearing price, $p = p_\mu$, is then given by (12).

$$p_\mu = \left(\alpha + \beta - \frac{\delta\sigma_\gamma^{\,2}}{1 - \lambda}(1 - \mu\lambda) + \sqrt{\left(\alpha + \beta - \frac{\delta\sigma_\gamma^{\,2}}{1 - \lambda}(1 - \mu\lambda)\right)^2 + 4\frac{\lambda}{1 - \lambda}\mu\delta\sigma_\gamma^{\,2}}\right)\Big/2 \tag{12}$$

The results in the previous section can be generated as special cases of (12): note that $\mu = 1$ yields p_s as in equation (9) while $\mu = 0$ yields p_b as in (10).

It is important to emphasize that, even with insurance firms in the market, the equilibrium prices are still the prices given by (9) and (10), which are just (12) with $\mu = 0$ and $\mu = 1$. Prices for other values of μ are not equilibrium prices since, in equilibrium, all insurers will be completely in stocks or in bonds. However, the $0 < \mu < 1$ intermediate prices are useful in evaluating the actions available to the portfolio insurance firms. While price taking was the correct assumption in the previous section with a large number of marginal agents, the actions of a sizable portfolio insurance firm may have an appreciable impact on prices.[5] Recognizing this fact in their investigation of events that surrounded the crash of 1987, Gammill and Marsh (1988) suggest that, in studying the effects of portfolio insurance, "we need to develop further models in which investors account for the impact that their trading strategies have on prices" (p. 43).

With this in mind, consider the problem faced by the M insurance firms as they write their trading programs. Clearly if $\beta < \beta_b$ they will sell stocks and if $\beta > \beta_s$ they will not. But what if $\beta_b \leq \beta \leq \beta_s$; should the insurance firms respond to the Walrasian auctioneer's cry of the low price p_b by selling stocks or should they hold onto their stocks in spite of the fact that the proposed price is below $\bar{p}$? In the previous section with many small agents the answer would clearly have been to sell, since the individual's decision to buy or sell would not affect the ability of the Walrasian auctioneer to eventually produce a market-clearing equilibrium with a price below $\bar{p}$. A large agent's actions, however, can help to determine which one of the two potential equilibrium prices will eventually obtain as the market-clearing price.

Suppose, for example, that $M - 1$ firms sold stocks but one did not. The market-clearing stock price would then be given by (12) with $\mu = 1/M$. This price, which we denote $p_b^{\mu=1/M}$, exceeds the price given in equation (10) for the case when all insurers sell stocks (i.e., $p_b^{\mu=1/M} > p_b$). In fact, for some range of β such that $\beta < \beta_s$, it may be that $p_b < \bar{p} < p_b^{\mu=1/M}$. In this case, the other insurers do not want to sell stocks if the price is $p^{\mu=1/M}$ and hence, although p_b is a feasible equilibrium price, p_s is the only equilibrium price consistent with the refusal of our firm to sell into a falling market. Thus, in contrast to Section I's analysis with atomistic agents, when there are a few large firms and $\beta_s^{\mu=1/M} < \beta \leq \beta_s$ (where the value of $\beta_s^{\mu=1/M}$ is determined where $p_b^{\mu=1/M} = \bar{p}$), one of the firms can singlehandedly enforce the higher equilibrium price p_s by not selling stock.

The case described above is depicted in Figure 3 (for ease of notation, we have labelled p_b^{μ} as p_{μ} and similarly $\beta_s^{\mu=1/M}$ as β_{μ}). In drawing this figure we assume that the trigger price $\bar{p}$ is less than the benchmark no-insurers price p_n at the expected value of β, $\bar{\beta} = 0$, so that $\beta_b < \beta_s < 0$.[6] Thus, in Figure 3, the expected value of β yields an outcome in which insurance firms possess the types of portfolios with which we are usually concerned: portfolios consisting of stock.[7] Only for less likely realizations of β in which $\beta < \beta_s < \bar{\beta}$ are there potential equilibria in which insurance firms sell. This resembles

[5] Formally, even a small investor can have some very small effect on prices given our demand specifications. In the limit, however, this effect will be minute, which is what we rely on for the assumption of price-taking behavior in the previous section. Conversely, a very large investor or firm may have an appreciable effect on prices, which is what we rely on in the following discussion of the price impact of large insurers.

[6] The parameter values used to produce Figure 3 are $\delta = 0.4$, $\sigma^2 = 1.0$, $\lambda = 0.3$, $\alpha = 1.0 + \delta\sigma^2$, so that $p_n = 1.0$ at $\beta = 0$, $\bar{p} = 1.0 - 0.32$ and $M = 5$.

[7] One is usually concerned with these types of portfolios because it is their liquidation that can lead to an insurance-activated drop in stock prices. The public rarely complains about a sudden surge in stock prices.

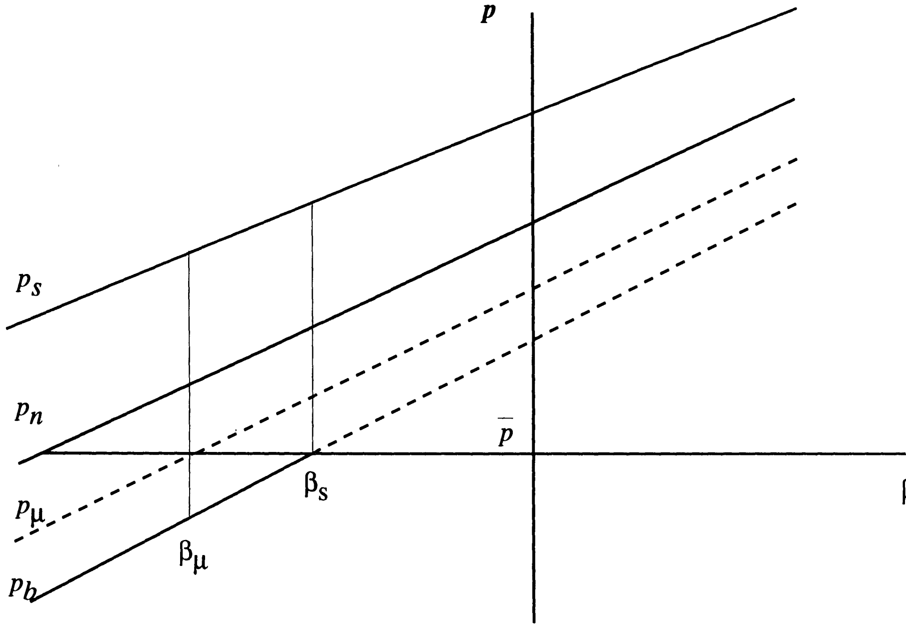

Figure 3. The effect of large portfolio insurers on prices. In this figure the vertical axis measures the stock price p. The horizontal axis measures β, a "fundamental" signal of future cash flows with higher values of β signalling higher cash flows. The p_n line shows the equilibrium stock price as a function of β in the absence of portfolio insurance. $\bar{p}$ is the trigger price above which insurers hold only stocks and below which they hold only bonds (while all prices are positive, the horizontal axis has been drawn through $\bar{p}$ for convenience). Prices along the p_s curve obtain as equilibria if portfolio insurers hold all their wealth in stocks, while prices along the p_b line obtain if portfolio insurers hold all their wealth in bonds. p_μ is a (nonequilibrium) market-clearing price conditional on all but one insurer selling stocks and holding bonds. If $\beta > \beta_s$ then the unique equilibrium price lies along the p_s curve. If $\beta \leq \beta_s$ then both p_s and p_b are potential equilibria. By refusing to sell stocks, however, one firm can singlehandedly enforce p_s as the only viable equilibrium for $\beta_\mu < \beta \leq \beta_s$.

the economy studied by Gennotte and Leland (1990) in that insurers usually hold stocks, but for especially bad realizations of the shock β may switch to bonds. Note, however, that while the bond-holding p_b equilibrium is possible below β_s, a single firm can enforce the higher-priced p_s outcome so long as $\beta > \beta_\mu$.

Recall the definitions of p_b^μ and $\beta_s^{\mu=1/M}$ above and define $p_s^{\mu=(M-1)/M}$ as the price if all firms but one hold stock and $\beta_b^{\mu=(M-1)/M}$ as the value of β such that $p_s^{\mu=(M-1)/M} = \bar{p}$. As demonstrated formally in Proposition 1 of the Appendix, the arguments above can then be extended to show that a decrease in the number of firms that act on behalf of a constant population of insurers (i.e., a decrease in μ for a given λ) reduces the range of β ($\beta_b^{\mu=(M-1)/M} < \beta \leq \beta_s^{\mu=1/M}$) over which a single firm cannot enforce one of the two equilibrium prices p_s and p_b. We therefore conclude that a small number of large portfolio insurance firms, acting on behalf of a constant population of individual insurance traders, can avoid insurance-induced price movements more easily than can the individual traders acting on their own. Indeed, taken to the extreme, a large insurance firm could potentially force prices to be continuous over the intermediate range of β while, with atomistic insurers, prices could potentially bounce up and down between the two equilibria.

Consider now the more difficult possibility that the introduction of portfolio insurance firms and the services they provide may entice some of the ordinary traders to become insurers and thus increase λ, the fraction of funds under insurance. This case is depicted in Figure 4 in which μ is fixed but λ increases from λ_0 to λ_1.[8] The observation that $\beta_s(\lambda_1) > \beta_s(\lambda_0)$ in Figure 4 reveals that the increase in the fraction of funds under portfolio insurance increases the range of β over which multiple equilibria are possible. The observation that $\beta_\mu(\lambda_1) < \beta_\mu(\lambda_0)$, however, also reveals that the increase in the fraction of all funds controlled by insurance firms decreases the range of β over which a single firm cannot enforce market stability by enforcing the upper price p_s. Thus we see that the introduction of insurance firms has the potential to reduce volatility by reducing the probability of an insurance-caused price drop even if more agents choose to be insurers. This result is in stark contrast to those reported in Section I for the case of atomistic agents.

The interesting result from Figure 4 obtains because, for the chosen parameter values, $\beta_\mu(\lambda_0) > \beta_\mu(\lambda_1)$. The conditions under which this occurs is formalized as case (i) of Proposition 2, contained in the Appendix. In short, the result obtains when the insurance firms are "sufficiently large" relative to the size of the market. If instead case (ii) of Proposition 2 obtains, however, then $\beta_\mu(\lambda_1) > \beta_\mu(\lambda_0)$ and the introduction of insurance firms does not necessarily reduce volatility although, at least in the eyes of agents who use the firms, they may still provide a valued service. Our goal in this paper is not to argue for one case over the other, but to simply point out that the introduction of portfolio insurance firms need not increase volatility, even if they increase the amount of funds under insurance.

III. Conclusions

This paper develops a simple model of portfolio insurance and asset prices and then uses the model to investigate the impact of large portfolio insurers, such as formal portfolio insurance firms, on the behavior of asset prices. We demonstrate that, in general, the presence of portfolio insurers in a market of

[8] In drawing Figure 4 we assume that one insurance firm represents 40 percent of insurers both before and after the increase in λ. If insurance firms enter a market without any firms to begin with, the results of Figure 4 would highlight our point even more strongly. We also assume that the market value of stocks is three times that of the bonds portfolio insurers would purchase if they sold their stocks. This ratio of stock to bond value is in accordance with the ratio of the value of stocks to that of cash instruments in April 1987 (source: *Federal Reserve Bulletin*). We use cash instruments (i.e., currency plus money market funds plus checkable and demand deposits) since insurers typically switch between stocks and cash (see Leland and Rubinstein (1981)).

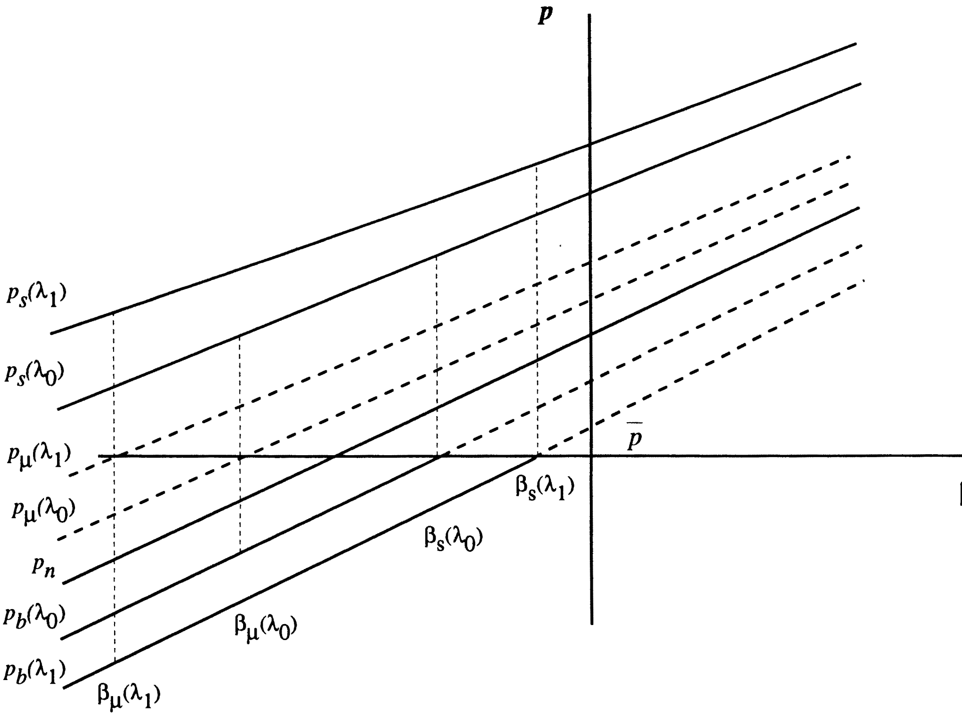

Figure 4. The effects of increasing portfolio insurance. In this figure the vertical axis measures the stock price p, which is a function of expected future cash flows and the fraction of investors, λ, who follow a portfolio insurance strategy. The horizontal axis measures β, a "fundamental" signal of future cash flows with higher values of β signalling higher cash flows. The p_n line shows the equilibrium stock price as a function of β in the absence of portfolio insurance. $\bar{p}$ is the trigger price above which insurers hold only stocks and below which they hold only bonds (while all prices are positive, the horizontal axis has been drawn through $\bar{p}$ for convenience). For a given λ_i, prices along the $p_s(\lambda_i)$ curve obtain as equilibria if portfolio insurers hold all their wealth in stocks, while prices along the $p_b(\lambda_i)$ line obtain if portfolio insurers hold all their wealth in bonds. $p_\mu(\lambda_i)$ is a (nonequilibrium) market-clearing price conditional on all but one insurer selling stocks and holding bonds. If $\beta > \beta_s(\lambda_i)$ then the unique equilibrium price lies along the $p_s(\lambda_i)$ curve. If $\beta \leq \beta_s(\lambda_i)$ then both $p_s(\lambda_i)$ and $p_b(\lambda_i)$ are potential equilibria but, by refusing to sell stocks, one firm can singlehandedly enforce $p_s(\lambda_i)$ as the only viable equilibrium for $\beta_\mu(\lambda_i) < \beta \leq \beta_s(\lambda_i)$. We see that increasing the fraction of the population that insures increases the spread between the two possible equilibrium prices, p_s and p_b, but also increases the range of β over which a single large insurer can enforce the higher price p_s.

risk-averse traders leads to multiple equilibria for the pricing of financial assets and can cause an increase in volatility, including insurance-induced price crashes. We demonstrate, however, that centralized portfolio insurance firms may actually reduce, not increase, volatility, even if the existence of these firms increases the total amount of funds under insurance. This result obtains in our model, as it would in reality, when large insurers take the price impact of their own actions into account. The equilibrium outcome is then one in which the effects of portfolio insurance strategies on prices are reduced.

Appendix

PROPOSITION 1: *Given a constant* $\lambda > 0$, *the price* p *in* (12) *is a strictly increasing function of* μ.

Proof of Proposition 1: The proof follows immediately from inspection of equation (12), noting the effect of a rising μ at each place where the variable appears and keeping in mind that, to ensure positive stock prices, $\alpha + \beta - \frac{\delta\sigma_\gamma^2}{1-\lambda}(1 - \mu\lambda) > 0$. Q.E.D.

PROPOSITION 2: *Let* $\bar{\mu} \equiv \frac{p_n}{p_n + 1}$ *be the fraction of the endowment held in the form of stocks for the benchmark no-insurer economy. Then*:

(*i*) *if* $\mu > \bar{\mu}$ *then* p *is a strictly increasing function of* λ,
(*ii*) *if* $\mu < \bar{\mu}$, *then* p *is a strictly decreasing function of* λ,
(*iii*) *if* $\mu = \bar{\mu}$, *then* p *is a constant function of* λ.

Proof of Proposition 2: Recall from (8) that the no-insurance benchmark price is $p_n = \alpha + \beta - \delta\sigma_\gamma^2$, and define $\varphi = \varphi(\lambda) \equiv \frac{\lambda}{1-\lambda}\delta\sigma_\gamma^2$. Note that $\varphi(\lambda)$ is a strictly increasing function of λ. Equation (12) can then be rewritten as

$$p = \left(p_n - \varphi(1-\mu) + \sqrt{(p_n + \varphi(1-\mu))^2 - 4\varphi(p_n(1-\mu) - \mu)}\right)/2.$$

To gain some intuition, note that $p_n = p_{\bar{\mu}}$. For our claim, consider first the special case in which $\mu = 0$. We then have via direct calculation that $p = p_n - \varphi(\lambda)$, which is decreasing in λ. For all other cases, apply Lemma 1 below with $a = p_n$, $b = 1 - \mu$ and $c = -4(p_n(1-\mu) - \mu)$, noting that $4ab + c = \mu > 0$. Q.E.D.

LEMMA 1: *Let* $a > 0$, $b > 0$, *and* $4ab + c > 0$ *and let*

$$f(\varphi) = a - b\varphi + \sqrt{(a + b\varphi)^2 + c\varphi}.$$

Then (*i*) $f'(\varphi) > 0$, *if* $c > 0$, (*ii*) $f'(\varphi) < 0$, *if* $c < 0$, (*iii*) $f'(\varphi) = 0$, *if* $c = 0$.

Proof of Lemma 1: Note, that

$$f'(\varphi) = \frac{2b(a + b\varphi) + c - 2b\sqrt{(a + b\varphi)^2 + c\varphi}}{2\sqrt{(a + b\varphi)^2 + c\varphi}}.$$

Hence, the sign of the derivative is the sign of the numerator. It is shown after some calculation that this numerator is strictly positive if and only if $c(4ab + c) > 0$. Since $4ab + c > 0$ by assumption, the result follows. Q.E.D.

REFERENCES

Brennan, Michael J., and Eduardo S. Schwartz, 1989, Portfolio insurance and financial market equilibrium, *Journal of Business* 62, 455–476.

Board of Governors of the Federal Reserve System, 1989, *Federal Reserve Bulletin 75.*

Gammill, James F., and Terry A. Marsh, 1988, Trading activity and price behavior in the stock and stock index futures markets in October 1987, *Journal of Economic Perspectives* 6, 25–44.

Gennotte, Gerard, and Hayne Leland, 1990, Market liquidity, hedging and crashes, *American Economic Review* 80, 999–1021.

Grossman, Sanford J., 1988, An analysis of the implications for stock and futures price volatility of program trading and dynamic hedging strategies, *Journal of Business* 61, 275–298.

Jacklin, Charles J., Allan W. Kleidon, and Paul Pfleiderer, 1992, Underestimation of portfolio insurance and the crash of October 1987, *Review of Financial Studies* 5, 35–64.

Leland, Hayne, and Mark Rubinstein, 1981, Replicating options with positions in stock and cash, *Financial Analysts Journal* 12, 63–72.

Shiller, Robert, 1989, *Market Volatility* (MIT Press, Cambridge, Mass.).