Capital Income Taxation and the Sustainability of Permanent Primary Deficits*

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Abstract

If a government imposes a tax on capital income, it may, as a result, lower the private rate of return on capital below the growth rate of an economy, thereby giving rise to the possibility of running a permanent deficit. Since, however, the before-tax rate of return and not the after-tax rate of return is relevant for judging the dynamical efficiency of the economy, the possibility of a permanent deficit does not by itself imply a possibility for a Pareto-improving redistribution of income.

To examine this issue "step by step", we examine in general whether a government can run a deficit forever by rolling over its debt. Assuming the government to run a deficit in each period equal to a constant fraction of total output, we study several overlapping generations models, proceeding from endowment economies to neoclassical growth with a variable capital stock. We then introduce capital income taxation and show, for example, that permanent deficits are feasible in the case of a variable capital stock, provided the capital income tax is sufficiently high. We examine the welfare effects and discuss policy consequences.

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"I place economy among the first and most important virtues and public debt as the greatest of dangers to be feared. To perserve our independence, we must not let our rulers load us with perpetual debt. If we run into such debts, we must be taxed in our meat and drink, in our necessitites and in our comforts, in our labor and in our amusements. If we can prevent the government from wasting the labor of the people, under the pretense of caring for them, they will be happy." (Thomas Jefferson)

1 Introduction.

Arguably the most visible part of Reagans legacy is the budget deficit. The reasons for why it is of concern to many need not be repeated here. They have been discussed already by, say, Krugman, 1990, Buiter and Kletzer (1992b), Eisner (1992), Friedman (1992), Vickrey (1992). Auerbach, Gokhale and Kotlikoff (1994) use generational accounting to evaluate this issue, which in turn has been critized by Buiter (1993). Several of the arguments brought forward, however, state, that we need not worry. For example, some argue that a good part of the deficit corresponds to government investment and may be therefore a good thing after all: the deficit, corrected for this fact, is then actually quite a bit smaller (see Eisner, (1986)). Some claim, that because parents care about their children, it does not matter whether government expenditures are financed by taxes or by debt (Barro,1974).

Finally, it is sometimes heard that we may just grow out of the deficit over time. Suppose, there is a government deficit and the interest rate is not "too high". Then, over time, even though the real value of the debt rises, the output of the economy may rise even faster, trivializing the debt through the enlarged tax base. Indeed, in this case, there is even room for a Pareto-improving redistribution which makes everybody better off, since providing each generation with a "free", deficit-financed lunch and then simply rolling over this debt forever is feasible. The crucial issue then is, whether indeed a permanent deficit is sustainable. For a recent book-length contribution regarding this argument and the related literature, see Carlberg (1995). An empirical investigation is in Bovenberg and Petersen (1992).

We reexamine this last point of view in the context of four versions of a basic overlapping generations model, adding one by one three relevant features: capital, investment and depreciation, and capital income taxation. This paper is thus an exercise in model engineering: by moving from a simple to a complicated framework step by step, it becomes transparent how the individual parts in the final machinery interact and contribute to the analysis.

In the very basic endowment economy, the government can run a deficit forever if the deficit as a fraction of GNP is not too high. Intuitively, debt here fulfills the role of money in other overlapping generations models. The result here corresponds to standard results about seignorage (see Wallace, 1980). For the second of the four models, we add a fixed capital stock which is traded from generation to generation. Since the value of the capital stock rises with the growth rate, so too must the value of government bonds. The total, outstanding deficit then explodes, thereby eliminating the possibility of a sustainable deficit.

We then add the possibility for capital income taxation (or, equivalently, savings taxation) to the instruments of the government, where the returns on private capital are taxed. The emphasis here is on this distortionary aspect of taxation, in contrast to the analysis in, say, Buiter and Patel (1992a) or Buiter and Kletzer (1994). A higher capital income tax drives down the realized rate of return on capital, possibly rendering the deficit sustainable. The necessary tax rate for accomplishing this is typically quite sizeable: even as the deficit-to-GNP ratio becomes negligible, the capital income tax does not. The intuition behind this result is that the capital income tax needs to drive the interest rate down to at most the growth rate of the economy in order to have sustainable deficits at all. It turns out that for most tax rate, there is a "good" and a "bad" steady state equilibrium. Though there is some choice for the tax rate, the tax rate can only be chosen so that the good equilibrium becomes better when the bad equilibrium becomes worse at the same time.

Finally, we make the capital stock variable by introducing investment and depreciation. Since the model is a neoclassical growth model in nature, there will not be any growth effects. However, the level effects resulting from the different capital income taxes which make a government deficit sustainable, can be quite dramatic. The welfare effects are much less clear cut. Furthermore, a positive capital income tax may not be necessary for sustainability, if the economy without the government is already dynamically inefficient: this is demonstrated in a numerical example.

This paper is a variation of Diamonds (1965) celebrated analysis. Since rolling over the debt amounts to the creation of a bubble, this paper can also be viewed as an application of the bubble literature as in Tirole (1985) or Blanchard and Fisher (1989). However, while the focus there and in Diamond (1973) as well as Atkinson and Sandmo (1980) is on the normative aspects of government policy, the focus here is on the positive aspect. The question is not, whether government should run a deficit forever, but whether it can. Note that sustainability of a permanent deficit means that the interest rate is below the growth rate of the economy and that therefore the economy is dynamically inefficient: from a welfare perspective, there is always a Pareto improving redistribution, which, depending on the structure of the model, may or may not require a deficit (see Cass (1972), Balasko and Shell (1980), Sargent (1987) and Abel et al. (1989)). The sad fact is that permanent deficits seem to be politically attractive. The point of this paper is then to analyze what happens, given that a permanent deficit needs to be sustained.

This paper is related to Sargent and Wallace (1981), Darby (1984), Miller and Sargent (1984) and Aiyagari (1985). All of these papers, however, consider at most a savings technology with a fixed rate of return instead of a productive capital stock with a rate of return calculated from equilibrium conditions and taxes are lump sum, if introduced at all. In that respect, Chari (1988), Lucas (1990) and Bohn (1990) are more closely related, but they use an infinite-lived agent framework.

Finally, it should be emphasize, that the entire analysis proceeds in the context of a closed economy. Open economy issues make capital income taxation a much more tricky issue, and many additional problems may arise. For some of the related literature, see Bovenberg (1989, 1992) and Broer, Westerhout and Bovenberg (1994).

2 Model 1: No capital.

In each period t, $t = 1, 2, ..$, a new generation of N two-period lived agents is born. There also is a generation of N initially old agents alive at date 1. The effects of population growth, general excess demand functions or distributional issues are not examined here. N is chosen to equal one, keeping in mind that each agent is meant to be representative of his generation and therefore does not act strategically.

There is one consumption good each period. An agent born at t cares about consumption c_{1t} when young and c_{2t+1} when old according to the utility function

$$u(c_1, c_2) = \log(c_1) + \log(c_2).$$

The specific form has been chosen to make the results easy and tractable. Observe that a discount factor is not included: again, this keeps the algebra simple. The special form for the utility function implies a vertical savings line in Diamonds (1965) diagram 1, thereby ruling out his "perverse case".

The agent is endowed with one unit of labor when young, which he can use to produce the consumption good according to the production function

$$y_t = \zeta_t n_t,$$

where ζ_t is the productivity parameter at time t. The productivity parameters are assumed to be

$$\zeta_t = \zeta^t,$$

where

$$\zeta > 1$$

is some given constant. Since labor is supplied inelastically, it follows that the growth rate γ of the economy is given by $\gamma - 1$, where

$$\gamma = \zeta$$

(the symbol γ is introduced to keep the notation consistent throughout the paper).

There is a government, who tries to finance a deficit in each period by rolling over its debt. We assume that the governmental deficit is a constant fraction α of total output,

$$g_t = \alpha\ y_t,$$

where $\alpha > 0$. If R_t is the return (i.e. one plus the interest rate) from period $t-1$ to t, the government budget constraint is given by

$$b_t = g_t + R_t b_{t-1}, \tag{1}$$

where b_t are the one-period bonds issued by the government in time t. Note that the deficit is financed entirely by rolling over the debt. There are no income taxes and the like, since they are not the issue here (it is easy to append the model by having some kind of income tax, financing some government expenditures in excess of the deficit described above: in that case, the output y_t is to be read as the after — tax income). The results stay the same.

A **steady state equilibrium** is given by numbers $\beta > 0, \sigma > 0$ and $R > 0$, so that for

$$\begin{aligned} y_t &= \zeta^t \\ b_t &= \beta\ y_t \\ s_t &= \sigma\ y_t \\ R_t &= R, \end{aligned}$$

each agent maximizes its utility at savings s_t, given the gross return $R_{t+1} = R$, the government budget constraint is satisfied and markets clear:

1. the consumption goods market

$$c_{1t} + c_{2t} + g_t = y_t \tag{2}$$

2. the bond market

$$s_t = b_t. \tag{3}$$

It is easily shown that the savings of a young agent are given by

$$s_t = y_t / 2,$$

independently of the interest rate (which makes the logarithmic specification of the utility function so convenient for our purposes). Thus, the remaining constants β and R can be calculated from (1) or (2), given α : one equation suffices by Walras' law. The result is given by

$$\begin{aligned} R/\gamma &= 1 - 2\alpha \\ \beta &= 1/2. \end{aligned}$$

Since $R > 0$ is required for the steady state equilibrium, it follows, that $\alpha < 1/2$ is necessary and sufficient for a steady state equilibrium to exist. These results are summarized by

Proposition 1 *If there is no capital, any permanent deficit up to 50 % of total output each period is sustainable by rolling over the debt.*

Note, that the number of 50 % is simply the total savings of the agent in the model economy. This number is not meant to be interpreted as describing the actual situation in any particular country and depends critically on the specification of the utility function. A result of this type, however, probably holds for a wide variety of utility functions. The proposition seems like good news for politicians: optimality questions aside, it is at least possible to sustain a sizeable deficit forever. The question, of course, is, whether a crucial element is missing in deriving this answer to the sustainability question by making the model possibly too simple. That this is probably so should already be indicated by the following observation in the model.

Proposition 2 *If there is no capital, the size of the total outstanding debt is independent from the government deficit, as long as it is sustainable.*

This proposition simply follows, because $\beta = 1/2$ is independent of α (or R, for that matter). This proposition runs counter to the intuition one usually has about the size of a government deficit: one would think that a larger yearly deficit implies a larger outstanding stock of debt. The reason that the model here does not deliver such a result is simple: government bonds are the only means of cross-generational trade in this model. Government bonds act like money and the government deficit like seignorage or an inflation tax: while these are disturbing the amount an old agent will receive, it will not change the amount a young agent wants to save due to the logarithmic specification of the utility function. Thus, savings and not the size of the budget deficit is what determines the amount of outstanding debt (compare to Sargent (1987)).

It can be concluded that this model is indeed too simple to give a reliable insight into the question of the sustainability of permanent deficits. Therefore, another element needs to be added: a different vehicle for saving. More precisely, a privately owned capital stock is added as a feature of the model in the next section.

3 Model 2: fixed capital stock, no capital income tax.

Let there be a fixed capital stock $k > 0$, which does not depreciate over time. Production is now given by the Cobb — Douglas production function

$$y_t = \zeta_t k^\rho n^{1-\rho},$$

where $\rho \in (0 \; ; \; 1 \;)$ is the share of capital, a constant. The capital is owned by the old, who sell it to the young for a total price of q_t. The young receive wage for their labor, spend part of it on consumption c_{1t}, part of it on saving in capital s_{kt} and part of it in saving in governmental bonds s_{bt}. When old, they receive the dividends from their capital holdings as well as the resale price and they are paid the interest on their bonds.

All markets are competitive. In particular, in order for any government bonds to be hold, it must be the case that the return on government bonds and on capital are the same in equilibrium. Furthermore, it is straightforward to calculate that the wage income is given by $(1-\rho)y_t$ and the dividend income by ρy_t, which we will substitute into the definition.

An **equilibrium** is given by sequences $(c_{1t}, c_{2t}, s_t, q_t, R_{t+}, b_t)$, so that for

$$y_t = \zeta^t k^\rho,$$

it is the case that

1. for each t, $t = 1, 2, ...$, the agent born at t, maximizes his utility at c_{1t} and $c_{2t+1} = R_{t+1} s_t$, given the budget constraint:

$$c_{1t} + s_t = (1-\rho) y_t,$$

2. the government budget constraint is satisfied:

$$b_t = g_t + R_t b_{t-1}$$

3. markets clear:

   (a) the consumption goods market

$$c_{1t} + c_{2t} + g_t = y_t$$

   (b) the capital market

$$s_t = q_t + b_t,$$

4. no arbitrage:

$$R_t = (\rho \; y_t + q_t)/q_{t-1}, \tag{4}$$

   where $q_t > 0$ and $R_t > 0$.

The condition $b_0 \geq 0$ ensures that the government cannot start up the economy by handing a liability to the old agents, which they may trade from generation to generation. The restrictions $q_t > 0$ and $R_t > 0$ are the usual positivity restrictions on prices. Finally, (4) is the restriction that the return on government bonds and capital must be equal (since the deficit is assumed to be strictly positive, this restriction must hold except for degenerate cases). This is called a no arbitrage condition, because that is its economic interpretation. It is of course possible to write the definition of an equilibrium without this condition and derive it from a more elaborate description of the maximization problem for the agents. Since this step is straightforward, the version of the definition above and in similar spirit everywhere below was chosen.

In contrast to the model without capital, the following result is obtained[1]

[1] A result of this type can already be found in Scheinkman (1980), see also Tirole (1985).

Proposition 3 *If there is fixed capital stock and no capital income taxation, the government cannot sustain a permanent deficit of a constant fraction of total output.*

Proof: *Suppose, there was an equilibrium. Market clearing in the consumption goods sector implies*

$$(1-\rho)y_t/2 + R_t(1-\rho)y_t/(2\gamma) + \alpha\ y_t = y_t,\ t \geq 2,$$

where $\gamma = \zeta$ is the growth rate of the economy, as before. Thus the return

$$R_t \equiv R,\ t \geq 2$$

has to be a constant. Define the fraction of saving which is capital by

$$\varphi_t = q_t/\ s_t,$$

and note that then $\varphi_t \in (0\ ;\ 1\ )$, since $q_t > 0$ and $b_t > 0$, $t \geq 1$. The condition (4), which guarantees an equal return on capital and government bonds can now be rewritten as

$$R/\gamma = \frac{\varphi_t + 2\frac{\rho}{1-\rho}}{\varphi_{t-1}}$$

or, equivalently,

$$\varphi_t = \frac{R}{\gamma}\varphi_{t-1} - 2\frac{\rho}{1-\rho}$$

for $t \geq 2$. Note that $\frac{R}{\gamma} > 0$. Consider the following three cases.

1. *Suppose, that $\frac{R}{\gamma} < 1$. Then*

   $$\varphi_t \to -2\frac{\rho}{1-\rho}\frac{1}{1-\frac{R}{\gamma}} < 0,$$

   in contradiction to the positivity of φ_t.

2. *Suppose, that $\frac{R}{\gamma} = 1$. Then*

   $$\varphi_t \to -\infty,$$

   in contradiction to the positivity of φ_t.

3. *Suppose then, that $\frac{R}{\gamma} > 1$. But then the outstanding government debt outgrows the economy and is therefore not sustainable: let*

   $$\beta_t = b_t/\ y_t$$

be the debt-to-GNP ratio. The government budget constraint can be rewritten as

$$\beta_t = \alpha + \frac{R}{\gamma}\beta_{t-1}.$$

Since $\beta_{t-1} \geq 0$, $t \geq 1$, $\alpha > 0$ *implies* $\beta_t \to \infty$. *This is impossible, since by capital market clearing and* $q_t > 0$, *we need to have* $\beta_t \leq 1$.

Since these three cases exhaust all possibilities, an equilibrium cannot exist. •

After some thought, the result is actually not that surprising: the value of the capital as well as the value of labor keep growing at the rate of the overall growth rate of the economy. But that means that the rate of return on capital must be even higher, *i.e.* it must be the case that

$$R > \gamma.$$

But then the outstanding debt grows faster than the economy and there is no way that output can catch up any more.

Given that intuition, the return on capital is somehow too high to make a deficit sustainable. So why not give the government some instrument to lower the return on capital. That will ease the debt problem as well! It is therefore natural to consider a capital income tax or savings tax.

4 Model 3: fixed capital stock and capital income tax

A capital income tax in this model is a tax on the net return on capital. In order to keep the notation simple, a tax rate τ on the entire return on capital is introduced. Both formulations are equivalent, if capital income tax rates are allowed to exceed 100 % (It turns out that they would need to for the numerical examples presented below. Whether this is reasonable will be discussed in the last section before the conclusion).

Since the focus in this paper is on the sustainability of a permanent deficit and therefore the effects of the elements in our model with respect to that, the tax is not used towards reducing the deficit, but simply increases government consumption. Also, the tax is not imposed on the return on government bonds for convenience. Otherwise, let the government use the tax on the bond returns to repay its bonds:

the result is equivalent to the economy below except that the return on the government bonds is simply higher by the tax rate. Taxing government bonds just amounts to rewriting the government bugdet constraint in another way by doing the accounting differently.

A **steady state equilibrium** is given by numbers $\beta > 0, \sigma > 0,\ R > 0,\ \theta > 0$ and a tax rate $\tau \geq 0$, so that for

$$
\begin{aligned}
y_t &= \zeta^t k^\rho \\
b_t &= \beta\ y_t, \\
s_t &= \sigma\ y_t, \\
R_t &= R, \\
q_t &= \theta\ y_t,
\end{aligned}
$$

it is the case that

1. for each t, $t = 1, 2, ...,$ the agent born at t, maximizes his utility at c_{1t} and $c_{2t+1} = R_{t+1} s_t$, given the budget constraint:

$$c_{1t} + s_t = (1-\rho) y_t,$$

2. the government budget constraint is satisfied:

$$b_t = g_t + R_t b_{t-1} \tag{5}$$

3. markets clear:

   (a) the consumption goods market

$$c_{1t} + c_{2t} + g_t + \tau(\rho y_t + q_t) = y_t \tag{6}$$

   (b) the capital market

$$s_t = q_t + b_t, \tag{7}$$

4. no arbitrage:

$$R_t = (1-\tau)(\rho\ y_t + q_t)/q_{t-1}. \tag{8}$$

Using the decision rules of the agent resulting from his maximization problem as well as substituting b_t by βy_t, etc., equations (5) through (8) can be rewritten as

$$\beta = \alpha + \frac{R}{\gamma}\beta \tag{9}$$

$$\frac{1-\rho}{2} + \frac{R}{\gamma}\frac{1-\rho}{2} + \alpha + \tau(\rho+\theta) = 1 \tag{10}$$

$$\frac{1-\rho}{2} = \theta + \beta, \tag{11}$$

$$\frac{R}{\gamma} = (1-\tau)(\frac{\rho}{\theta} + 1), \tag{12}$$

which, by Walras law, must be dependent. Therefore equations (9), (11) and (12) can be used to solve for the unknown parameters β, θ, R and τ under the positivity restrictions. It turns out that there is one degree of freedom: ideally, one would then fix the tax rate τ and solve for the other three variables. It is more convenient to fix the return R instead and solve for β, θ and τ. Because $\beta > 0$, it must be the case that

$$0 < \frac{R}{\gamma} < 1 - \frac{2\alpha}{1-\rho}$$

and for these values of R it follows that

$$\begin{aligned}
\beta &= \frac{\alpha}{1-\frac{R}{\gamma}}, \\
\theta &= \frac{1-\rho}{2} - \frac{\alpha}{1-\frac{R}{\gamma}}, \\
\tau &= 1 - \frac{\frac{R}{\gamma}}{1 + \frac{\rho}{\frac{1-\rho}{2} - \frac{\alpha}{1-\frac{R}{\gamma}}}}.
\end{aligned}$$

Substituting these three formulas into (10) and checking that it holds for any value of R in the range described above can be used to verify the calculations. The qualitative insight is summarized by

Proposition 4 *With a fixed capital stock and capital income taxation, there is a range of interest rates*

$$0 < \frac{R}{\gamma} < 1 - \frac{2\alpha}{1-\rho}$$

with corresponding capital income tax rates, so that the government deficit is sustainable forever.

The formula above allows for examining the behaviour of the capital income tax rate for various levels of α and R. As for the dependence on R, graphs are presented in figures 1 and 2 with $\alpha = .10$, $\rho = .3$ and $\zeta = (1.03)^{25}$ to get results which

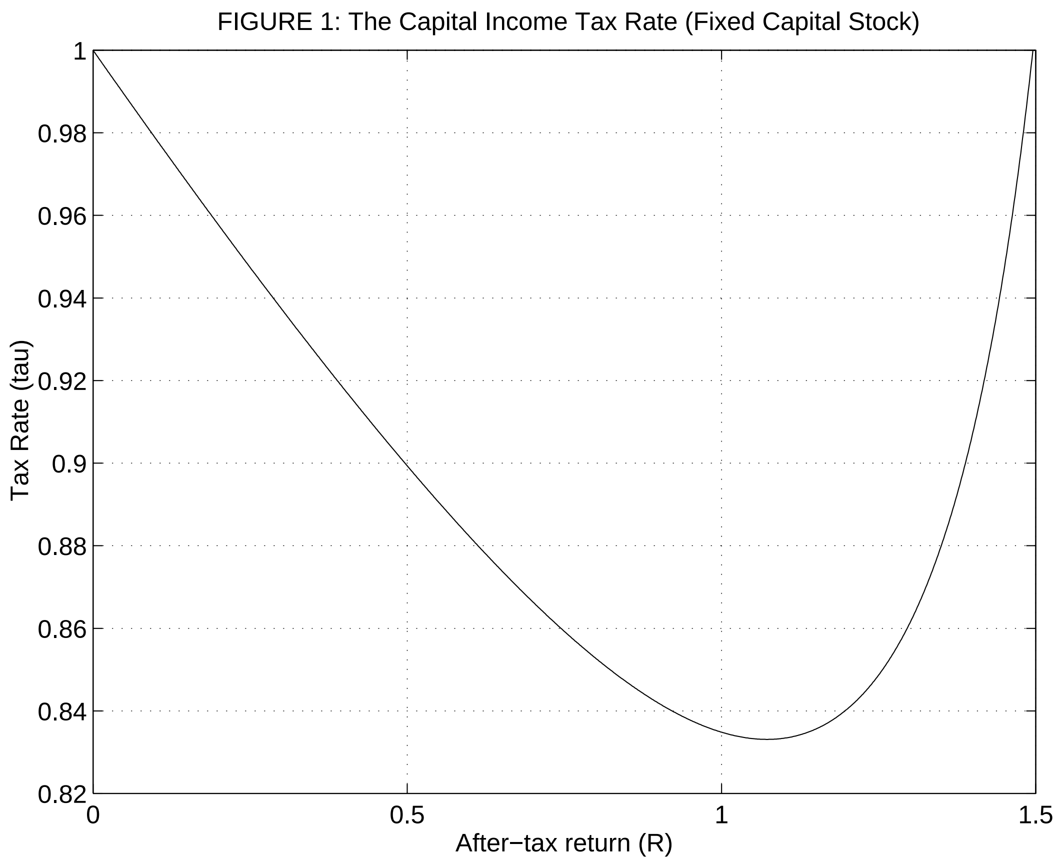

Figure 1: The capital income tax rate τ in the case of a fixed capital stock in dependence of the equilibrium return R. Parameters are $\alpha = .10$, $\rho = .3$ and $\zeta = (1.03)^{25}$. Note, how there are two equilibrium returns R for any given τ in the appropriate range.

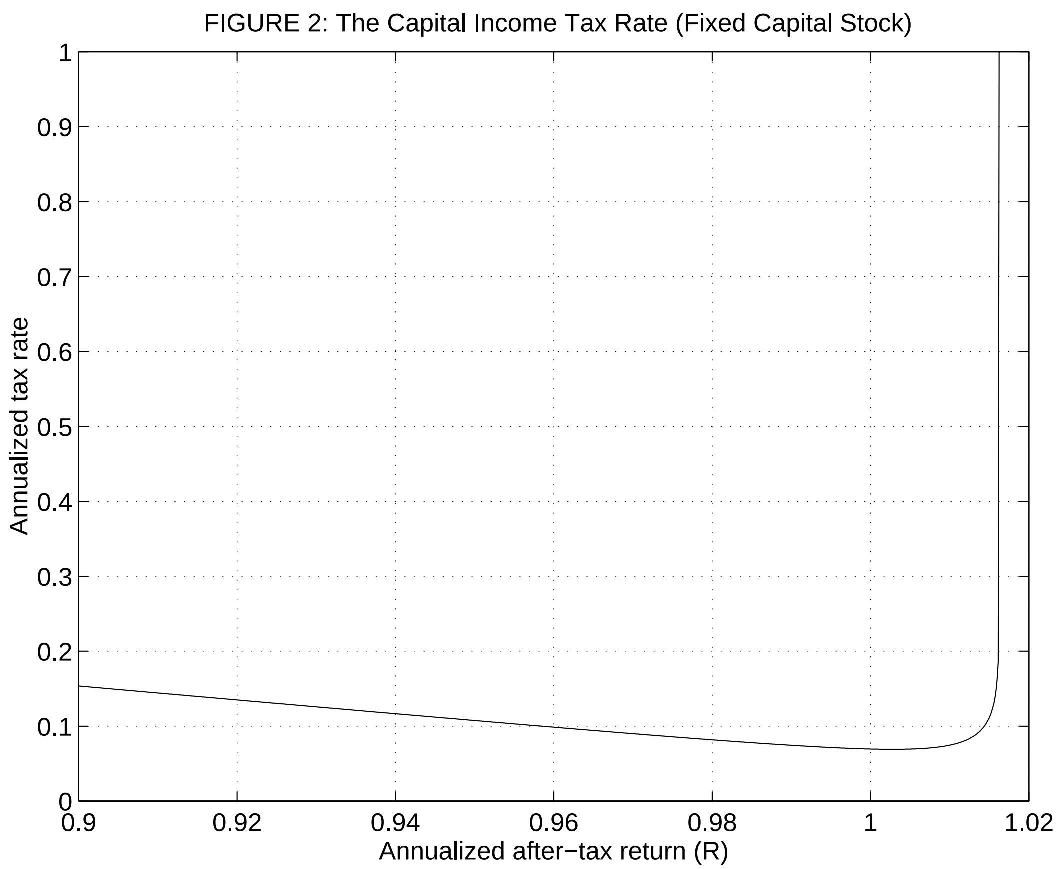

Figure 2: The annualized capital income tax rate $1-(1-\tau)^(1/25)$ in the case of a fixed capital stock in dependence of the annualized equilibrium return $R^{1/25}$. Parameters are $\alpha = .10$, $\rho = .3$ and $\zeta = (1.03)^{25}$. Note, how there are two equilibrium returns R for any given τ in the appropriate range.

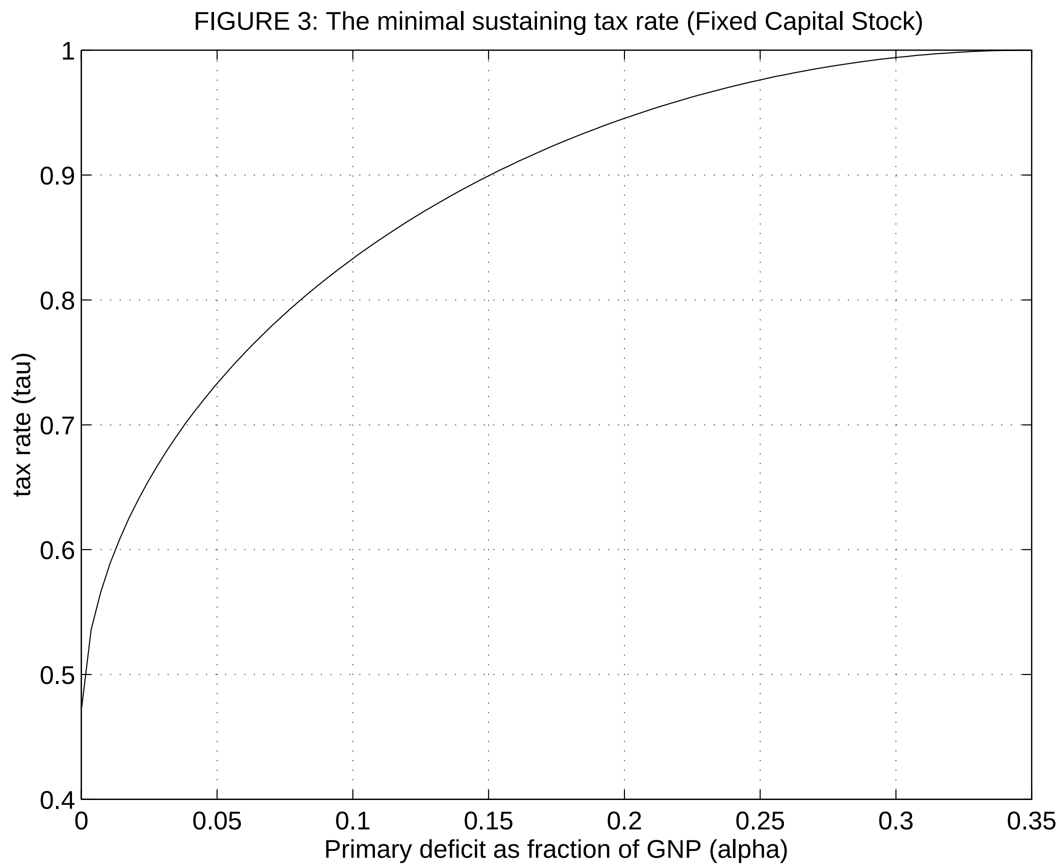

Figure 3: The minimal capital income tax rate sustaining the permanent primary deficit αy_t.

are somewhat suggestive: a generation is thought of living 25 years, while young and accumulating wealth through labor, and 25 years, while old and consuming the returns to their investment. Figure 1 (and all following figures except for figure 2) shows the relationship between the total tax rate on capital gains after 25 years and the return over 25 years. Figure 2 annualizes these numbers, determining the tax rate needed each year to compound to the total tax rate shown in figure 1 in 25 years, likewise for the return. In figure 2, we have shown only a part of the range of possible values for the annualized return $R^{1/25}$. Note in both figures, that the tax rate τ first falls and then rises again. This can be shown analytically to be correct. Furthermore, at a given tax rate, there will be typically two steady state equilibria (if at all), which are Pareto ordered[2]. These results are summarized in the next proposition.

Proposition 5 *1. The tax rate τ converges to 1 as the return R approaches its maximal or its minimal value supporting a steady state equilibrium with deficit α,*

2. The tax rate τ first falls and then rises again as the gross interest rate is increased from its minimal to its maximal value. There is a unique minimum tax rate $\tau_{\min}$.

3. For each tax rate τ between $\tau_{\min}$ and 1.0, there are two steady state equilibria, one with a lower return R than the other one.

4. The steady state allocations are Pareto ordered: Welfare increases as the return R increases.

The proof for this proposition is in the appendix. Note from the figures that the capital income tax necessary to sustain a government deficit forever is very large (especially taking into account that τ here is really the tax on total earnings on capital, not just on the gain). One might conjecture that the minimal capital income tax necessary to sustain a government deficit α converges to zero as α approaches zero. That this is not so is demonstrated in figure 3 (for the same parameters as for figure 1) and by the following proposition.

[2]Note, that this is a Pareto ordering of different steady states. In particular, the intially old agent is endowed differently in these different steady states, making this welfare comparison potentially misleading.

Proposition 6 *The minimal tax rate which sustains a permanent deficit is bound below by a number strictly bigger than zero, even as that deficit becomes arbitrarily small.*

Proof: *Define*

$$\begin{aligned}\tau^* &= \inf_{0<\alpha} \inf_{0<R/\gamma<1-\frac{2\alpha}{1-\rho}} \{\tau\} \\ &= \inf_{0<R/\gamma<1} \inf_{0<\alpha<(1-\rho)(1-R/\gamma)/2} \{\tau\}\end{aligned}$$

Since for fixed R, τ is increasing in α and defined for $\alpha = 0$, it follows that

$$\begin{aligned}\tau^* &= \inf_{0<R/\gamma<1} \left\{1 - \frac{1-\rho}{1+\rho}\frac{R}{\gamma}\right\} \\ &= \frac{2\rho}{1+\rho} > 0,\end{aligned}$$

proving the claim. •

The intuition, that $\tau \to 0$ and $\alpha \to 0$ is wrong because, without τ, no government deficit is sustainable: R/γ ends up being strictly bigger than 1. In order to get sustainability, R/γ has to be supressed strictly below one no matter how small the deficit is that is to be sustained.

The possibility for capital income taxation brings back the possibility for sustainable deficits by depressing the return on private capital sufficiently far. Capital income taxes are usually attacked by economists for their undesirable effects on the efficiency of an economic system (see, for example, Lucas (1990)), although they can potentially have beneficial effects in the context of overlapping generation models, see Uhlig and Yanagawa (1996). A closer look at this issue is taken in the next variation of our basic model in which now the temporary capital stock is the result of depreciation and investment.

5 Model 4: variable capital stock and capital income taxation.

Let it be the case that

$$k_t = (1-\delta)k_{t-1} + x_t,$$

where $\delta \in [0; 1)$ is the rate of depreciation, x_t is investment and k_t is the capital stock planted in period t and productive in period $t+1$:

$$y_t = \zeta^t k_{t-1}^{\rho} n_t^{1-\rho}.$$

The steady state growth rate is now no longer $\zeta - 1$, since the capital stock will be growing as well. Also, it is necessary to calculate the value of the entire capital stock after dividends and depreciation, but before investment: the symbol q_t^{ante} is introduced for that. As before, the capital income tax revenues are used for additional government spending and not towards reducing the deficit.

A **steady state equilibrium** is given by numbers $\gamma > 0$, ξ, $\kappa > 0$, $\beta > 0$, $\sigma > 0$, $R > 0$, $\theta > 0$, $\theta^{ante} > 0$ and a tax rate $\tau \geq 0$, so that for

$$\begin{aligned} y_t &= \zeta^t k_t^{\rho}, \\ x_t &= \xi \; y_t, \\ k_t &= \kappa \; y_t, \\ b_t &= \beta \; y_t, \\ s_t &= \sigma \; y_t, \\ R_t &= R, \\ q_t &= \theta \; y_t, \\ q_t^{ante} &= \theta^{ante} y_t, \end{aligned}$$

it is the case that

1. for each t, $t = 1, 2, ...,$ the agent born at t, maximizes his utility at c_{1t} and $c_{2t+1} = R_{t+1} s_t$, given the budget constraint:

$$c_{1t} + s_t = (1 - \rho) y_t,$$

2. the government budget constraint is satisfied:

$$b_t = g_t + R_t b_{t-1} \tag{13}$$

3. markets clear:

   (a) the consumption goods market

$$c_{1t} + c_{2t} + g_t + \tau(\rho y_t + q_t^{ante}) + x_t = y_t \tag{14}$$

(b) the capital market

$$s_t = q_t + b_t, \tag{15}$$

4. no arbitrage:

$$R_t = (1-\tau)(\rho\ y_t + q_t^{ante})/q_{t-1}. \tag{16}$$

$$q_t^{ante} + x_t = q_t \tag{17}$$

$$q_t = k_t \tag{18}$$

5. the production function for capital holds

$$k_t = (1-\delta)k_{t-1} + x_t \tag{19}$$

Equation (17) and (18) result from the definition of q_t^{ante} and the fact that the consumption good and the investment good are the same: both equations could be arrived at more fundamentally by focusing on the appropriate production technology, which transfers consumption goods into investment goods one for one and vice versa.

From the production function for output and the fact that capital is a constant fraction of output, the steady state growth rate $\gamma - 1$ is calculated as follows:

$$\gamma = \frac{y_t}{y_{t-1}} = \frac{\zeta^t(\kappa y_{t-1})^\rho}{\zeta^{t-1}(\kappa y_{t-2})^\rho} = \zeta\ \gamma^\rho$$

or

$$\gamma = \zeta^{\frac{1}{1-\rho}}.$$

That is, the growth rate in this economy is a function of ζ, the growth rate of the underlying productivity parameter, and ρ, the capital share, and nothing else. Neither the budget deficit nor the capital income tax nor the interest rate have an impact on steady state growth. This is not the result of the particular utility function we used, but rather a standard result within models of the neoclassical growth variety, as can easily be seen from the derivation above: we should not expect the growth rate being changed by the capital income tax rate here. This would, of course, change in a model with endogenous growth.

However, the level effects of the capital income tax rate can be sizeable. They are derived now. Using the decision rules of the agent resulting from his maximization problem as well as substituting b_t by βy_t, etc., we can rewrite equations (13) through

(18) as above as

$$
\begin{aligned}
\beta &= \alpha + \frac{R}{\gamma}\beta \\
1 &= \frac{1-\rho}{2} + \frac{R}{\gamma}\frac{1-\rho}{2} + \alpha + \tau(\rho + \theta^{ante}) + \xi && (20) \\
\frac{R}{\gamma} &= (1-\tau)(\frac{\rho}{\theta} + \frac{\theta^{ante}}{\theta}), && (21) \\
\theta^{ante} + \xi &= \theta \\
\theta &= \kappa \\
\kappa &= \frac{1-\delta}{\gamma}\kappa + \xi
\end{aligned}
$$

which, again, by Walras law, must be dependent. Thus, leaving away equation (20), the parameters ξ, κ, $\beta, R, \theta, \theta^{ante}$ and τ can be solved for under the positivity restrictions via the remaining equations. As before the solutions are parameterizable by the interest rate R, which can be chosen freely in a certain range:

$$
0 < \frac{R}{\gamma} < 1 - \frac{2\alpha}{1-\rho}.
$$

The formulas for the other variables are

$$
\begin{aligned}
\beta &= \frac{\alpha}{1-\frac{R}{\gamma}} \\
\kappa = \quad \theta &= \frac{1-\rho}{2} - \frac{\alpha}{1-\frac{R}{\gamma}}, \\
\xi &= (1 - \frac{1-\delta}{\gamma})\kappa, \\
\theta^{ante} &= \frac{1-\delta}{\gamma}\kappa, \\
\tau &= 1 - \frac{\frac{R}{\gamma}}{\frac{1-\delta}{\gamma} + \frac{\rho}{\frac{1-\rho}{2} - \frac{\alpha}{1-\frac{R}{\gamma}}}},
\end{aligned}
$$

which can be substituted into (20) to check the validity of the solution. The changes to the solution with the fixed capital stock are minor: *e.g.* the discount rate now enters the formula for the tax rate.

For $\alpha = \beta = \tau = 0$, we obtain a benchmark version of this model, in which there is no government. For the equilibrium return, we get from $\tau = 0$

$$
\frac{R^*}{\gamma} = \frac{2\rho}{1-\rho} + \frac{1-\delta}{\gamma}.
$$

and the steady state capital is given by $\kappa^* = \frac{1-\rho}{2}$. This benchmark economy is already dynamically inefficient, if the benchmark return R^* is smaller than the growth factor γ.

One immediate implication of this analysis is to figure out the effect of the capital income tax rate, which corresponds to a certain return, on the steady state path of output. For that, the solution above can simply be plugged into the formula for output in period 1

$$y_1 = \zeta(\kappa\, y_1/\gamma)^\rho$$

to find

$$y_1 = \zeta^{\frac{1}{1-\rho}}(\frac{1-\rho}{2\gamma} - \frac{\alpha}{\gamma - R})^{\frac{\rho}{1-\rho}}.$$

One can easily see the level effect: a higher α depresses the output in the first period and thus in all subsequent periods, because a higher α depresses the capital-to-output ratio κ.

Figure 4 repeats figure 1 with the same parameters and additionally the parameter $\delta = 1 - (1 - .1)^{25}$ corresponding to a yearly depreciation of the capital stock of 10 %, where the capital stock is now variable (and the growth rate higher). It turns out, that the benchmark economy without a government is dynamically efficient for these parameters: $R^* > \gamma$. Note, that the solution for the supporting capital income tax did not change too much when compared to figure 1. Figure 5 plots y_1 (in percent of the level of first period output in the benchmark version) as a function of the return R in the relevant range. Figure 5 clearly shows, that output is a decreasing function of the return R, which can also be easily derived from looking at the equations above.

Thus, with variable capital, output is not maximized at the minimum capital income tax which makes the deficit sustainable, but rather at a capital income tax, which approaches 1 and a return which approaches zero. The intuition for this result is clear by looking at the algebra of how it is derived. A higher interest rate drives up the debt-to-GNP ratio β. This in turn drives down the amount of savings in the form of capital as a fraction of GNP, which is given by the parameter κ or θ : since total savings as a fraction of GNP remain constant, government debt crowds out capital. Finally, the level of output is increasing with the capital-output ratio κ : it is here, where the ratios expressing everything in terms of total GNP affect total GNP itself. To sum up, higher interest rates let government debt crowd out capital as a means of savings, thus lowering total output which needs capital as a productive factor. It is important to realize, that this is a steady state comparison. Nothing is said here about what will happen in these economies if the capital income tax is unexpectedly

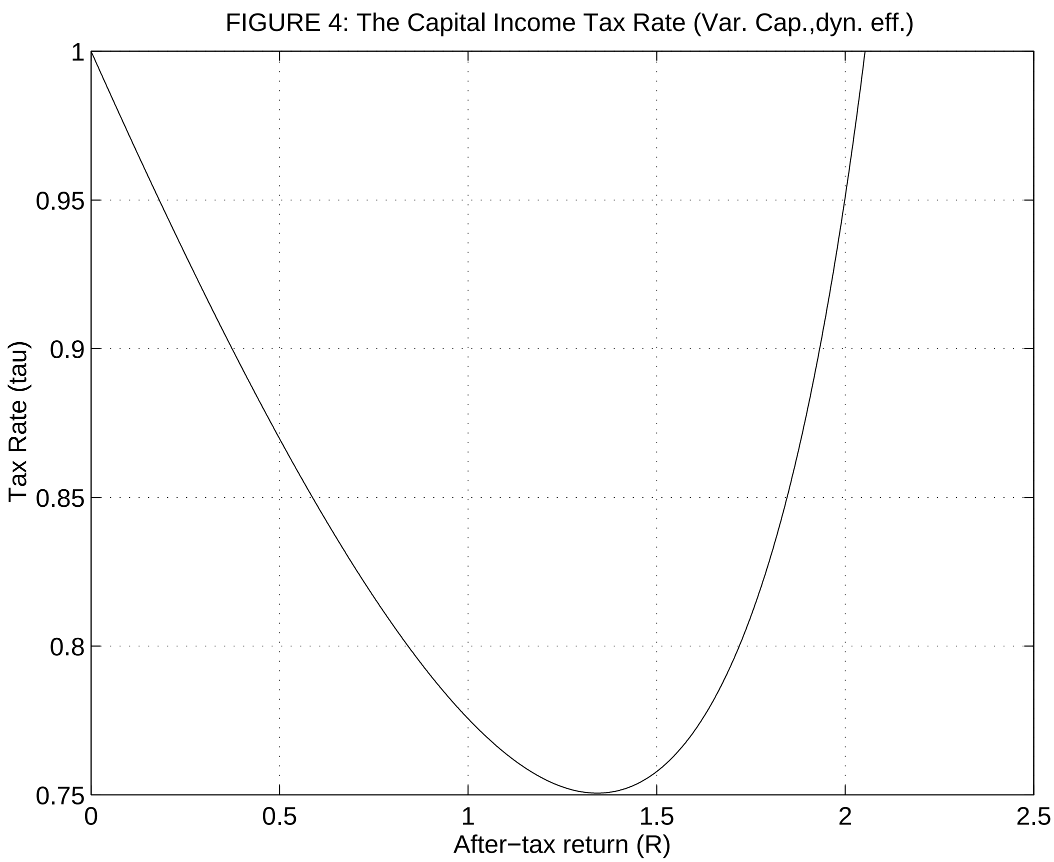

Figure 4: The capital income tax rate τ in the case of a variable capital stock in dependence of the annualized equilibrium return R. The economy is dynamically efficient. Parameters are $\alpha = .10$, $\rho = .3$, $\delta = 1 - (1 - .1)^{25}$ and $\zeta = (1.03)^{25}$. Note, how there are again two equilibrium returns R for any given τ in the appropriate range.

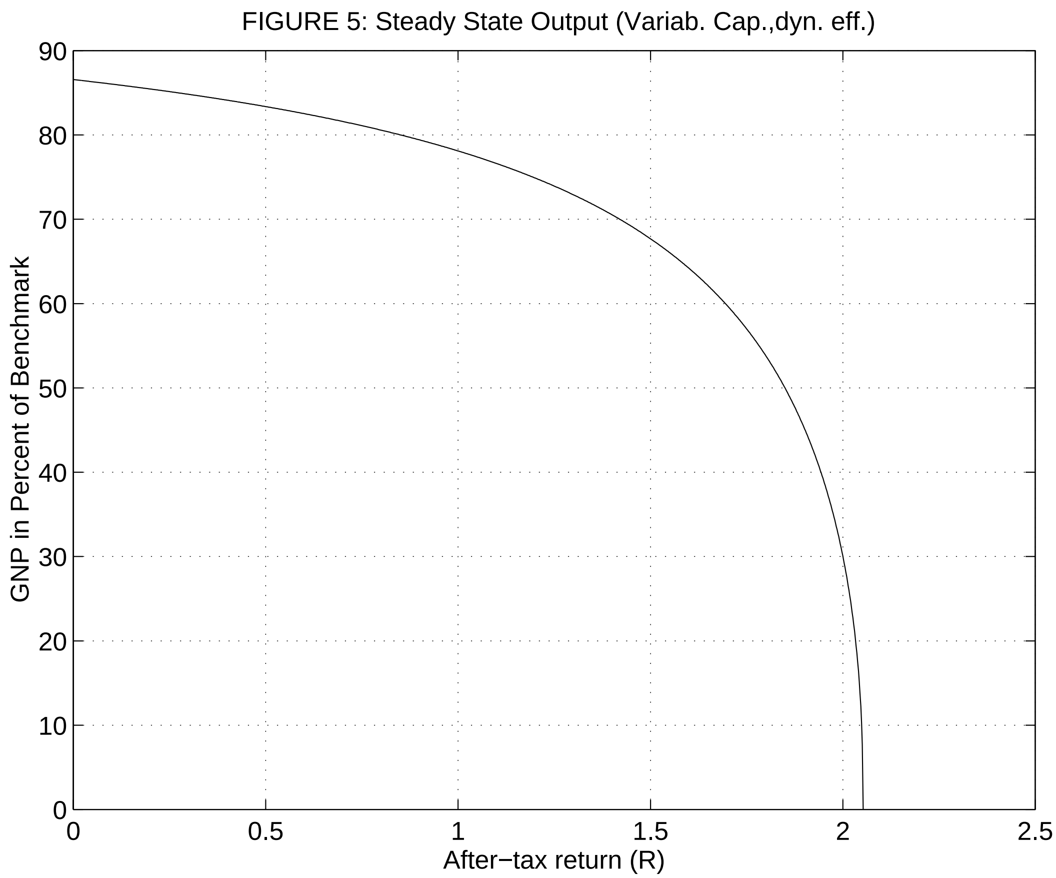

Figure 5: Equilibrium output in dependence of the annualized equilibrium return R in case of a variable capital stock. The economy is dynamically efficient. Parameters, as above, are $\alpha = .10$, $\rho = .3$, $\delta = 1 - (1 - .1)^{25}$ and $\zeta = (1.03)^{25}$.

raised forever to a new level. The conclusion that a high capital income tax leads to maximal output is dangerous for another reason in this model too, of course: for a given capital income tax, there are typically two equilibria, one with a low and one with a high return, and the high return equilibrium for a capital income tax approaching one delivers the worst steady state output of all.

For welfare calculations, output is not the relevant measure, but rather utility, in which the return is of relevance. Up to a factor, which depends only on the time t, steady state welfare for each two-period lived generation is given by

$$W = 2\log(\frac{1-\rho}{2}y_1) + \log(R).$$

The welfare for the initally old generation is calculated as

$$W_0 = \log(R\frac{1-\rho}{2\gamma}y_1)$$

by stationarity. Both functions are plotted in figure 6. It turns out, that the equilibria are no longer as nicely Pareto ordered as in the situation without investment. Anticipating proposition 7, we can also plot the welfare-maximizing capital income tax rate as a function of the primary deficit parameter α, see figure 7. As one can see, that tax rate can become quite substantial.

We now chose parameters so that the benchmark economy is dynamically inefficient: we chose $\rho = .15$. For small α, it then turns out, that a permanent deficit is sustainable even for a negative capital income tax (*i.e.* for a savings subsidy) and furthermore, that welfare can improve due the deficit. This is shown in figures 8, 9, 10 and 11, which correspond to the figures 4, 5, 6 and 7 described above. Note, in particular, that welfare can even improve when compared to the benchmark economy with a government and without a government deficit. This is of course just a restatement of Diamonds (1965) insight.

Some of the theoretical facts are stated in the next proposition. It is important to keep in mind for these figures as well as for the following proposition, that this is a comparison across steady states: in particular, the endowment of the initial old is changing in this comparison. Furthermore, it is important to note, that different capital income taxes mean different total government expenditures. A complete welfare analysis will have to take that into account, if the goods purchased by the government enter the utility function of the agent.

Proposition 7 1. *The welfare of the initial old has a global maximum in the range of returns sustaining a deficit at a return* $R_{\max,1}$. *The welfare of the initial old*

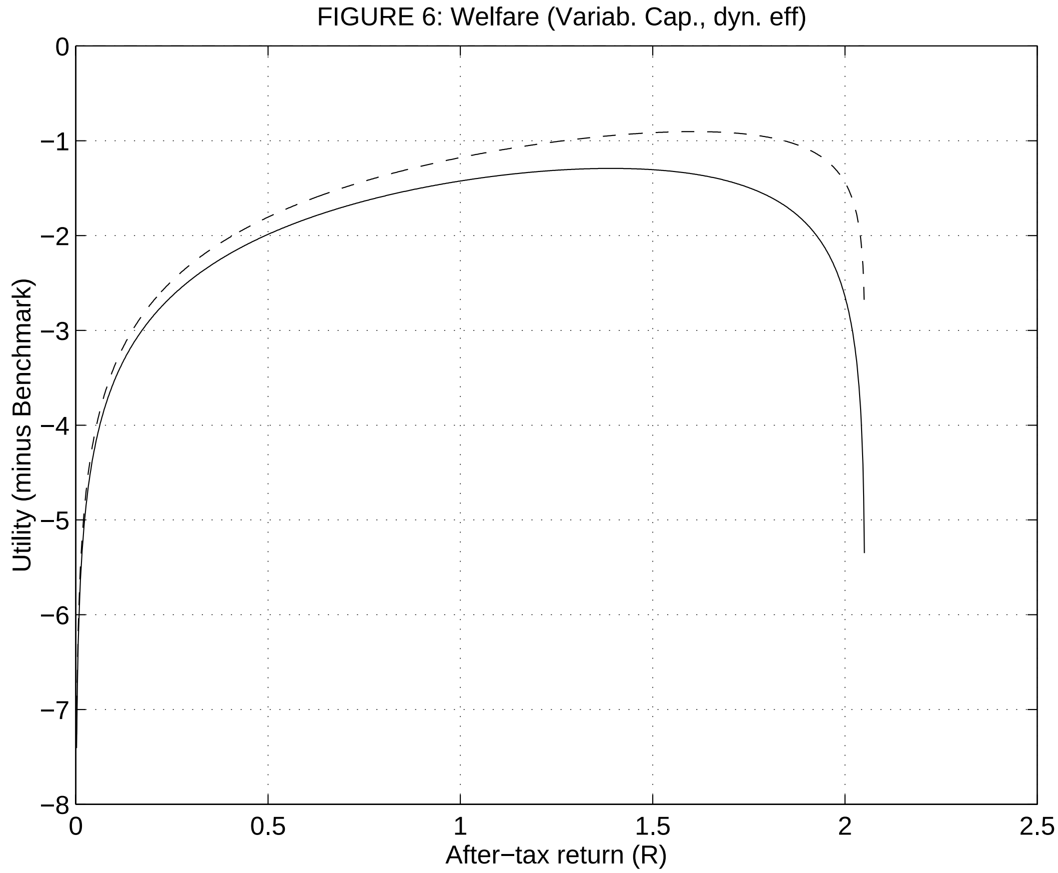

Figure 6: Equilibrium welfare of the two-period lived as well as the initially old in dependence of the annualized equilibrium return R in case of a variable capital stock. The economy is dynamically efficient. Parameters, as above, are $\alpha = .10$, $\rho = .3$, $\delta = 1 - (1 - .1)^{25}$ and $\zeta = (1.03)^{25}$.

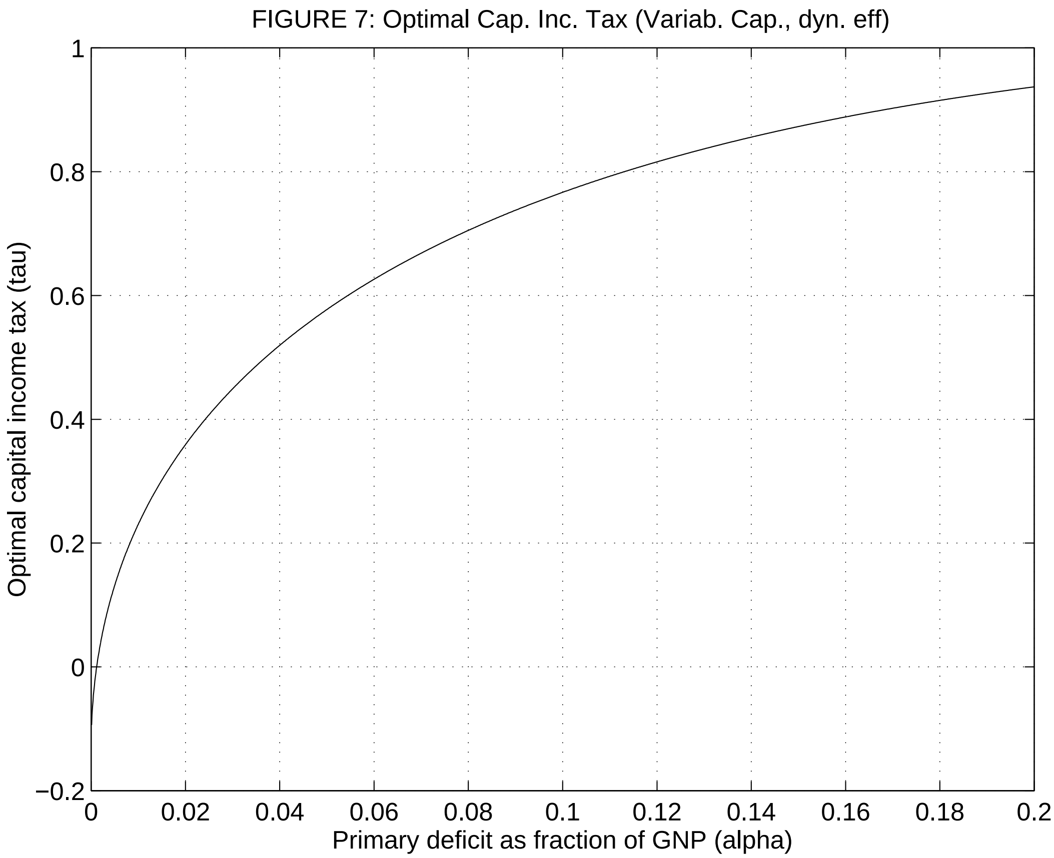

Figure 7: Welfare-maximizing capital income tax rate in dependence of the primary deficit αy_t in the case of a variable capital stock. The economy is dynamically efficient, except for very small values of α, where the optimal capital income tax rate is negative. Parameters, as above, are $\rho = .3$, $\delta = 1 - (1 - .1)^{25}$ and $\zeta = (1.03)^{25}$.

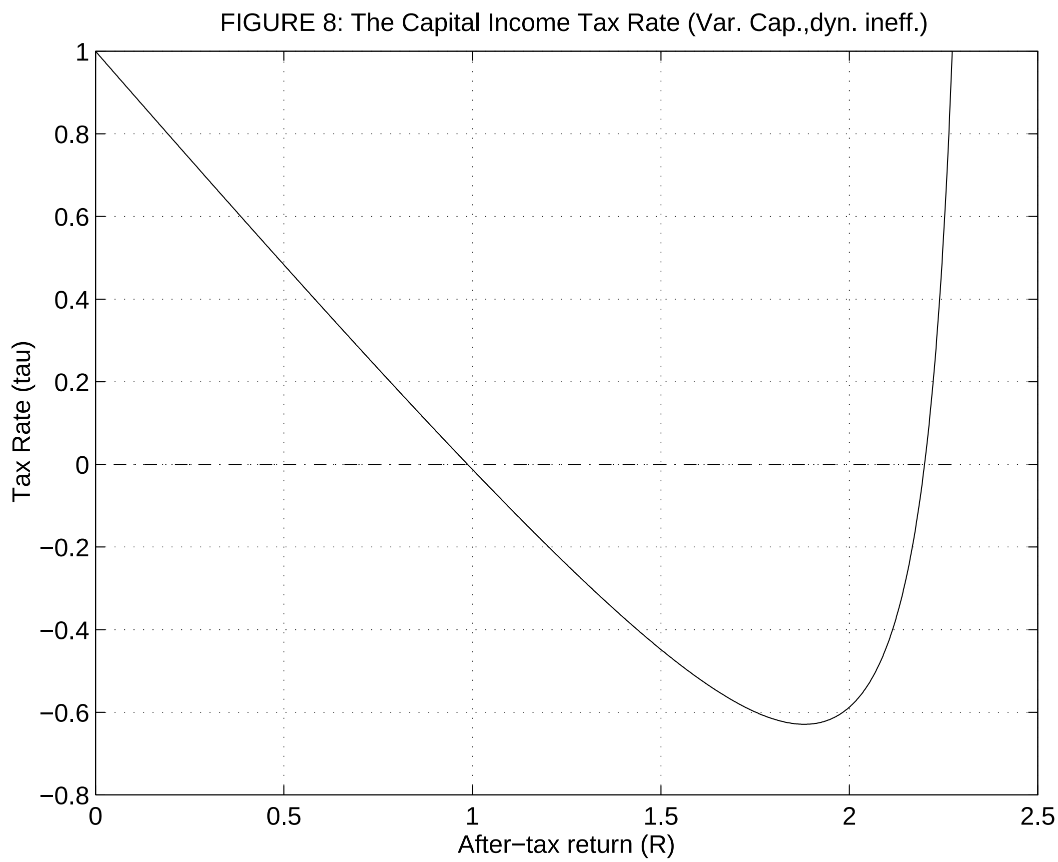

Figure 8: The capital income tax rate τ in the case of a variable capital stock in dependence of the annualized equilibrium return R. The economy is dynamically inefficient. Parameters are $\alpha = .10$, $\rho = .15$, $\delta = 1 - (1 - .1)^{25}$ and $\zeta = (1.03)^{25}$. Note, how there are again two equilibrium returns R for any given τ in the appropriate range.

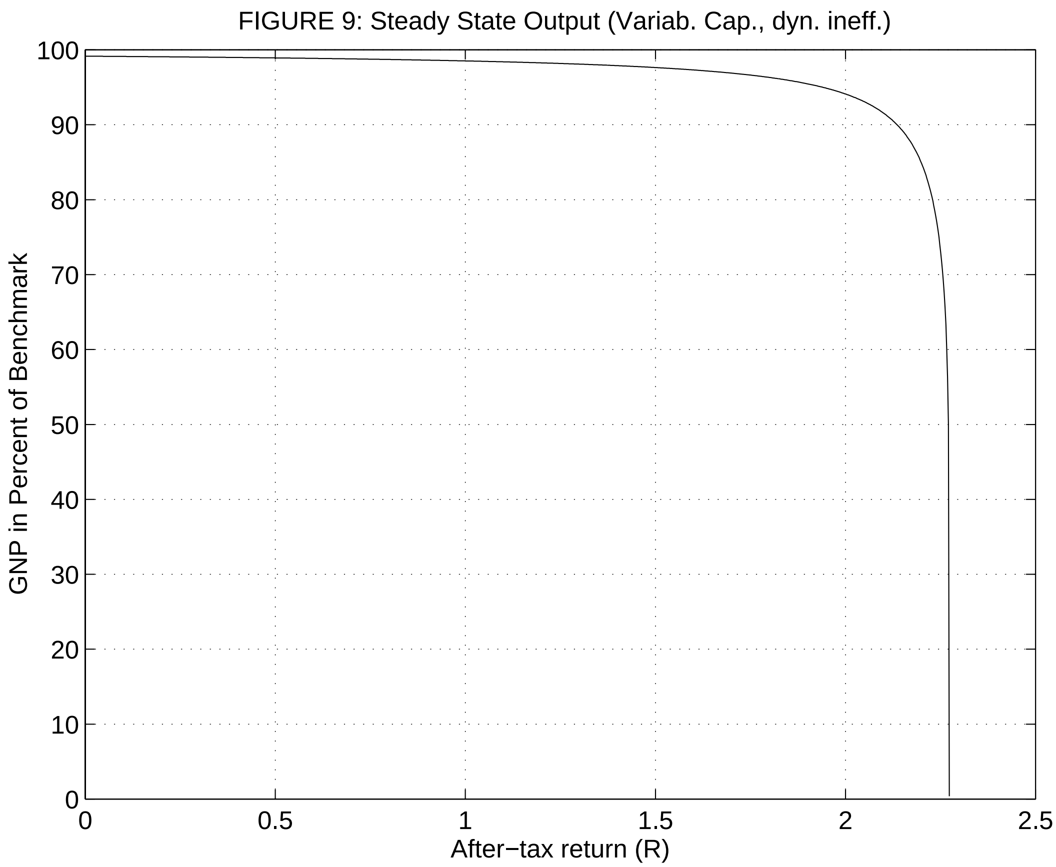

Figure 9: Equilibrium output in dependence of the annualized equilibrium return R in case of a variable capital stock. The economy is dynamically inefficient. Parameters, as above, are $\alpha = .10$, $\rho = .15$, $\delta = 1 - (1 - .1)^{25}$ and $\zeta = (1.03)^{25}$.

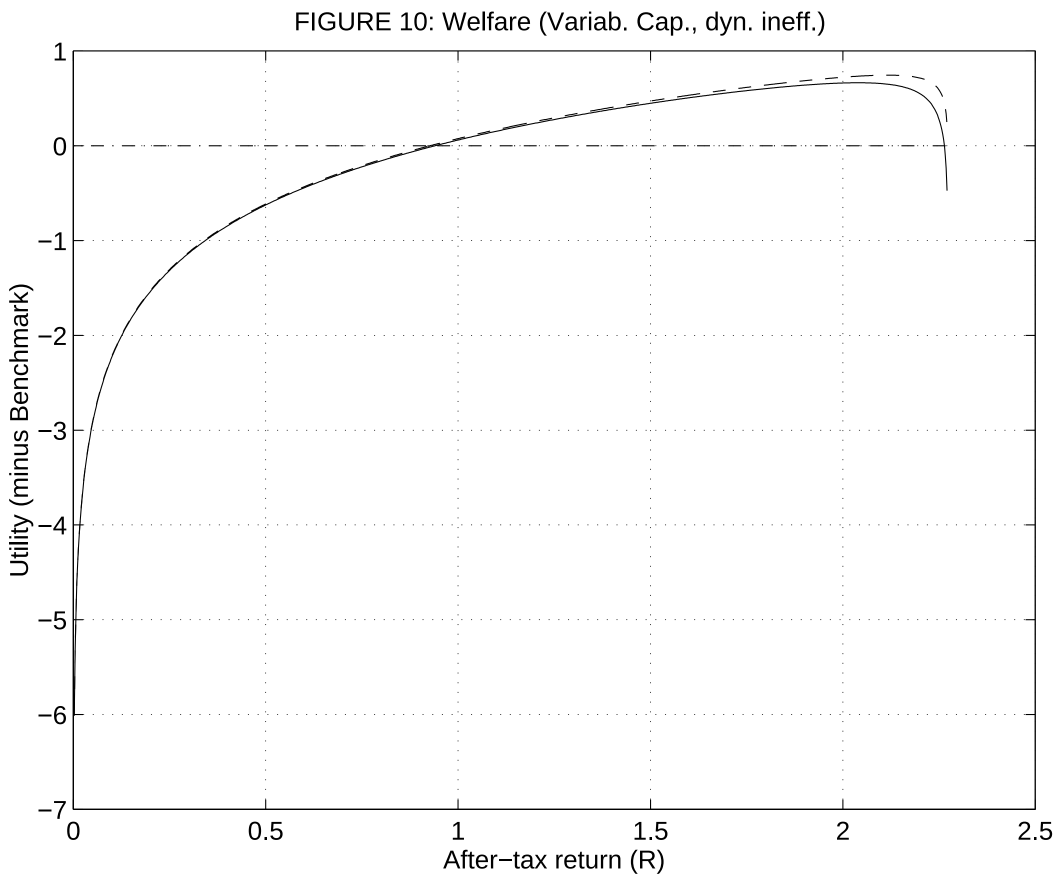

Figure 10: Equilibrium welfare of the two-period lived as well as the initially old in dependence of the annualized equilibrium return R in case of a variable capital stock. The economy is dynamically inefficient. Parameters, as above, are $\alpha = .10$, $\rho = .15$, $\delta = 1 - (1 - .1)^{25}$ and $\zeta = (1.03)^{25}$.

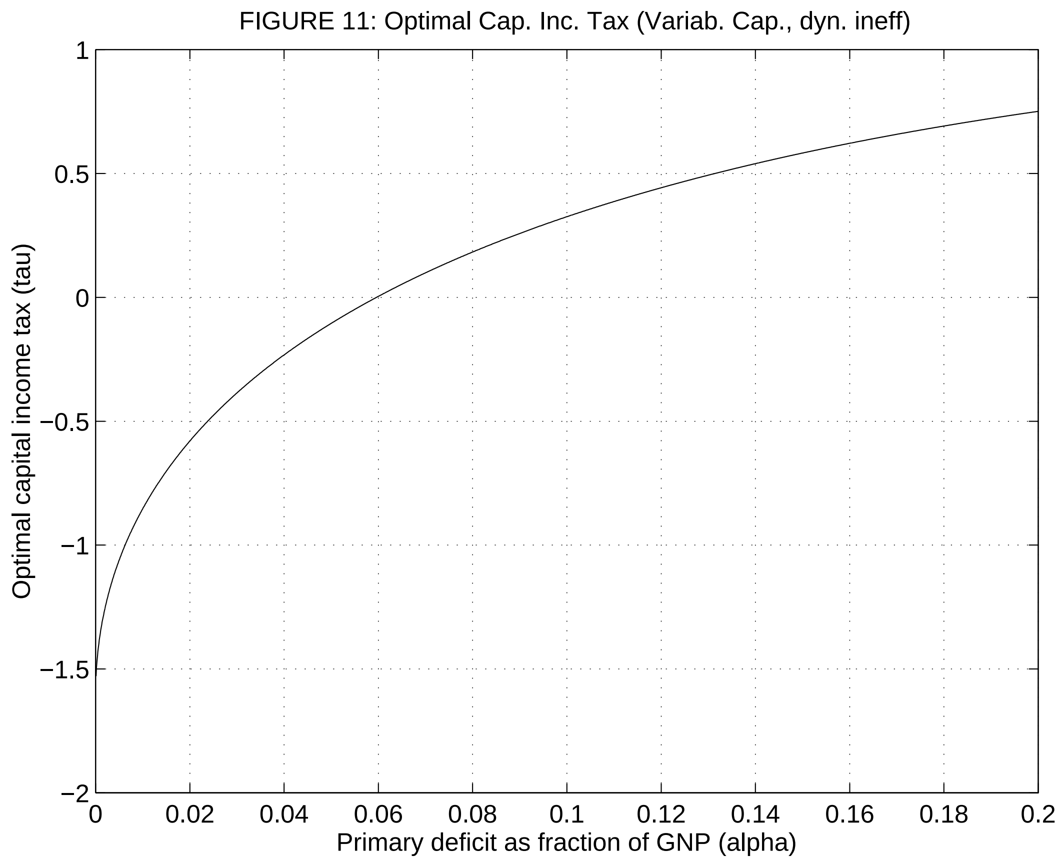

Figure 11: Welfare-maximizing capital income tax rate in dependence of the primary deficit αy_t in the case of a variable capital stock. The economy is dynamically inefficient, except for larger values of α, which imply an optimal positive capital income tax rate. Parameters, as above, are $\rho = .15$, $\delta = 1 - (1 - .1)^{25}$ and $\zeta = (1.03)^{25}$.

is increasing in R for R smaller than this maximizing return and decreasing in R for R bigger than $R_{\max,1}$.

2. *The welfare of the two period lived agents has a global maximum in the range of returns sustaining a deficit at a return $R_{\max,2}$, which is smaller than the return $R_{\max,1}$. The welfare of the two period lived agents is increasing in R for R smaller than this maximizing return and decreasing in R for R bigger than $R_{\max,2}$.*

The proof is in the appendix. The intuition behind this result should be clear, however. Consider again the model with a fixed capital stock and no investment. There, welfare was unambigously increasing in the return, because total output does not change, consumption and savings in the first period of the live stay the same, but second period consumption increases with the return on the constant savings. In the model here, where the capital stock is variable, this effect is counteracted by the decrease of output at higher returns for the reasons explained above. Overall then, the first effect is more important for very small returns: in essence, second period consumption rises faster than output declines. Eventually, however, the crowding-out effect starts to take over, resulting in declining welfare, as the return becomes too big. Associated with the return $R_{\max,2}$, which makes all the two-period lived agents best off, is a capital income tax rate which can be calculated from the formulas above and the formulas given in the proof of proposition 7. Leaving out the initial old (which could be motivated with a social welfare function which aims at assigning the same weight to all generations), this capital income tax may be considered optimal, given that the deficit needs to be sustained forever. Note, that this tax will in general be quite substantial. This conclusion still holds true, if a social welfare function assigns weight to the initial old generation as well: in general, the maximizing tax rate will correspond to a return somewhere between $R_{\max,2}$ and $R_{\max,1}$. The welfare-maximizing tax rate has been plotted in figures 7 and 11 as a function of the primary deficit parameter α for the two parameterizations used: note, how the tax rate can vary between a negative number in the case of a dynamically inefficient economy to a quite substantial positive number (with $\tau = 1$ corresponding to confiscation).

6 Interpretation.

To what extent are the arguments brought forward in this paper relevant for the current situation of the Netherlands, say? It should be noted, that even though the numbers were picked to be suggestive, a more careful calibration would be necessary before drawing reliable conclusions. One insight from the numerical experiments is that the capital income tax necessary to sustain a deficit is very sensitive in particular to the capital share and to the deficit that needs to be sustained, when the benchmark economy is dynamically inefficient. The more reasonable choices for the parameters, however, seem to imply capital income taxes which are very high. This may give rise to worry about the applicability of the results presented here.

However, a few remarks may be in order in defense of these numbers. First of all, it actually may be the case that the realized capital income taxes in the Netherlands are very high indeed, simply because the taxation is not indexed by a price-deflator: if, say, inflation and not real appreciation has trippled the price of some asset, a capital gains tax of, say, 50 percent amounts to a tax of 33 percent on the original value of the asset. Furthermore, even if the government does not impose the extremely high capital income tax rates implied by the model, it certainly depresses the market interest rate, thereby easing the burden of debt, even though the government may not be able to sustain its deficit level forever. This point is possibly well understood by governments, which seem to have great difficulties in reducing the debt and at the same time are often reluctant to remove the capital income tax.

7 Conclusions

It was examined, whether a government can run a deficit forever as a fraction of total output in several overlapping generations models with growth by rolling over its debt. The answer is "yes" for the model without capital, "no" for the model with capital, but "yes" for the model with capital and capital income taxation. The impact on steady state output and welfare by the capital income taxes that make a permanent government deficit sustainable was analyzed. It was shown that for a certain range of capital income taxes, there are two equilibria: one with a low return on savings and one with a high return on savings. For the low return equilibria, output is an increasing function of the capital income tax, whereas for the high interest rate equilibria, output is a decreasing function of the capital income tax. It was demonstrated, that output in the version of the model with investment was maximized at a capital income tax rate approaching 100 %, if the economy can be in the low return equilibrium. While welfare is not maximized at this point, it is still true that the welfare maximizing capital income tax may be very substantial, given that a government deficit needs to be sustained forever.

Appendix

Proof: Proof of Proposition 5.

1. *is clear from looking at the formula for* τ *and letting* R/γ *approach 0 and* $1-\frac{2\alpha}{1-\rho}$ *respectively*

2. *It is clear, that* $\tau < 1$ *for the range of admissable gross interest rates* R*. It thus remains to show that the derivative* $\frac{d\tau}{dR}$ *has a unique zero in this range. In order to prove this, it is convenient to change notation. Let*

$$\begin{aligned} \nu &= \frac{1-\rho}{2\rho} \\ \mu &= \frac{\alpha}{\rho} \\ \lambda &= (\nu - \mu + 1) * \nu/\mu \\ \eta &= (1 - \mu/\nu) * (\nu - \mu) * \nu/\mu \\ z &= \frac{R/\gamma}{1 - \mu/\nu} \end{aligned}$$

noting that $\mu < \nu$ *(and thus* $\lambda > 1$*) in order to have a nonempty range of interest rates to sustain a deficit in equilibrium to begin with. Then*

$$\begin{aligned} \tau(z) &= 1 - \frac{z(1-\mu/\nu)}{1 + \frac{1}{\nu - \frac{\mu}{1 - z(1-\mu/\nu)}}} \\ &= 1 - \frac{\eta}{\frac{\lambda}{z} + \frac{1}{1-z}}, \end{aligned}$$

where $z \in (0\ ;\ 1\ )$*: it suffices to show, that* $\tau'(z)$ *has a unique zero in the unit interval. Note, that* $\tau'(z) = 0$ *on this interval is equivalent to*

$$0 = -\lambda(1-z)^2 + z^2,\ z \in (0\ ;\ 1)$$

or

$$z_{min} = \frac{\sqrt{\lambda}}{1+\sqrt{\lambda}},$$

proving the claim. Note that this z *now corresponds to*

$$R/\gamma = (1 - \mu/\nu)\frac{\sqrt{\lambda}}{1+\sqrt{\lambda}},$$

and that one can calculate the location of the minimum to be

$$\tau(z_{min}) = 1 - \frac{\eta}{(1+\sqrt{\lambda})^2}$$

3. follows immediately from part 2. (d)

4. follows from observing that

$$u(c_{1t}) + u(c_{2t+1}) = 2\log(y_t/2) + \log(R)$$

for $t \geq 1$ and

$$u(c_{20}) = \log(y_1/(2\gamma)) + \log(R)$$

for the initial old.

•

Proof: Proof of Proposition 7.

Note, that the welfare function for both, the initially old and the two-period lived agents, is given by

$$W(R) = \log(R) + \varphi\ \log(y_1)$$

up to an additive term, where φ equals 1 for the inital old and φ equals two for the two-period lived agents (for a social welfare function, which weighs together all utility functions, φ will be somewhere between one and two). Taking the derivative with respect to R and using the formula for y_1 in the text, it follows that

$$\begin{aligned} W'(R) &= \frac{\gamma}{R/\gamma} - \frac{2\varphi\alpha/\gamma}{(1-\frac{R}{\gamma})^2} \frac{1-\frac{R}{\gamma}}{(1-\rho)(1-\frac{R}{\gamma}) - 2\alpha} \\ &= \frac{\frac{R^2}{\gamma^2} - (2 + (\varphi - 1)\frac{2\alpha}{1-\rho})\frac{R}{\gamma} + 1 - \frac{2\alpha}{1-\rho}}{R(1-\frac{R}{\gamma})(1-\frac{2\alpha}{1-\rho} - \frac{R}{\gamma})}, \end{aligned}$$

where the quadratic in the numerator has the two roots

$$\frac{R}{\gamma} = 1 + (\varphi - 1)\frac{\alpha}{1-\rho} \pm \sqrt{(1 + (\varphi - 1)\frac{\alpha}{1-\rho})^2 - 1 + \frac{2\alpha}{1-\rho}}.$$

Some more algebra then reveals that only the lower root lies within the admissable range, that the quadratic in the numerator positive in this range for R/γ below this root and negative above it and that the lower root is a decreasing function of φ. This finishes the proof. •

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