

The Timing of Information in a General Equilibrium Framework*

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Traditional multiperiod models of asset markets assume that traders take the information filtration as given. This paper explores the effect of endogenizing the arrival of information on the dynamic completeness of markets. It is shown that if an agent is allowed to release information (at a sufficiently small cost) which prevents traders from dynamically completing the market, she might choose to do so. Furthermore we show that in economies with enough heterogeneity, it is always possible to find an agent who would choose to release information. Thus, with endogenous timing of information, markets are unlikely to be dynamically complete. *Journal of Economic Literature* Classification Numbers: D52, D82, G14.

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1. INTRODUCTION

Most economists agree that asset markets are probably not complete in the Arrow–Debreu sense. Of course, if agents are allowed to dynamically

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retrade, it may be possible for agents to achieve the complete markets equilibrium even if the number of states far exceeds the number of assets (see Arrow [1], Radner [11], Kreps [8] and Duffie and Huang [6]). Besides dynamic completeness, this result relies on two assumptions. Firstly, all agents agree on and can not influence how and when information is released. Secondly, all agents know all prices at all nodes (i.e., agents know the *conditional price path*).

Whether or not markets are dynamically complete for given assets depends on the information filtration. If agents are allowed to control the timing of the information arrival, market completeness becomes an endogenous feature of the economy. Suppose, agents initially face an information filtration that allows them to dynamically complete the market. Would any agent choose to change the filtration (i.e., alter the timing of the information arrival) if that results in dynamically incomplete markets? We show that in an economy with enough heterogeneity, the answer to this question is almost always yes. We therefore conclude that if the timing of information is endogenous then asset markets are unlikely to be complete.

The paper is organized as follows. In the next section we introduce a model which allows agents to control the resolution of the uncertainty and briefly outline the intuition behind our main theorem. In Section 3 we formally derive the result. The last section is the conclusion.

2. THE ACQUISITION OF INFORMATION AND ITS AFFECT ON THE MARKET STRUCTURE

Consider an economy with three time periods ($t=1, 2, 3$) and three states of nature. There is one consumption good, three identical agents, and two linearly independent assets in zero net supply. The agents consume in the final period but are allowed to trade the assets on spot markets in periods 1 and 2. The agents are endowed with period 3 state contingent consumption and are assumed to be price takers. There is no production. Uncertainty about the final state is gradually resolved over time according to the event tree in Fig. 1. Note that the market is (generically) dynamically complete. Let us now add an additional feature (see Fig. 2). Assume that at some cost an agent can commission (or "purchase") a report just before the first spot market opens. This report is publicly issued just before the second spot market in the upper branch opens. The report, it is assumed, will completely resolve the uncertainty as to whether the economy is in state 1 or state 2. Now, if at least one agent chooses to purchase the report, all the uncertainty in the economy is resolved at time 2 and thus no trade will occur at time 2. Of course agents will anticipate this at time 1 and will

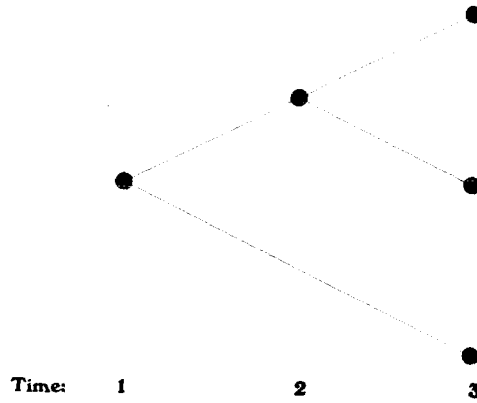


FIGURE 1

therefore conclude that the market is no longer dynamically complete! The market structure effectively becomes a one period incomplete market (Fig. 3). Now, if an agent prefers his equilibrium allocation in the economy with incomplete markets over his equilibrium allocation in the economy with complete markets, he would be willing to purchase the report provided the cost of the report is sufficiently small. Under what conditions then, will an agent prefer his allocation in the incomplete market economy?

Suppose, one agent does not trade in the initial complete markets filtration. Since asset prices will change in the new equilibrium, she will choose to trade and therefore she must be strictly better off in that economy by

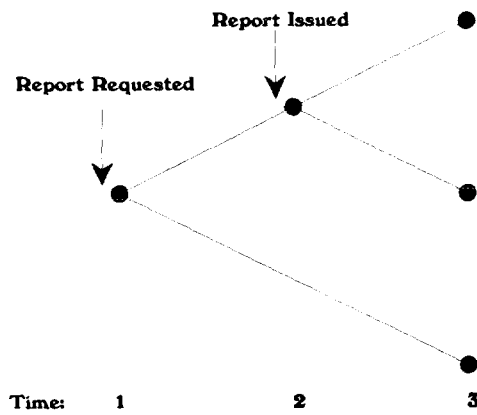


FIGURE 2

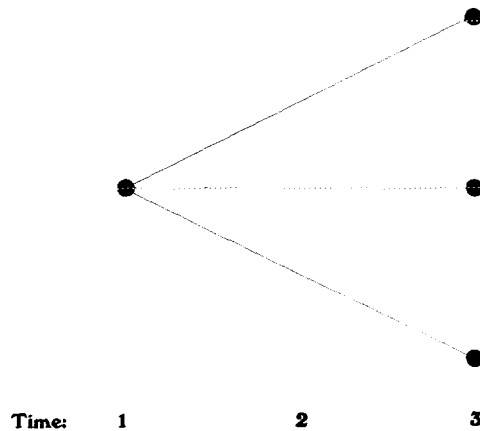


FIGURE 3

revealed preferences. Thus she is willing to pay a positive amount for the information (see Berk [2] for more details). By continuity, this is also true for agents whose net trades are sufficiently small in the initial filtration. Such agents exist in economies with enough heterogeneity. This is precisely the intuition behind our general result: under some conditions, some agents would always wish to “incomplete the market.”

The example above is, of course, very simple. Agents essentially choose between just two information filtrations. In a richer environment the influence agents have on the asset structure is much more complicated. Agents must correctly anticipate each other's strategies and they must react accordingly. The purpose of this work is not to model this complex interaction: we leave this problem as an interesting avenue for future research. All we show is that the dynamically complete market is not a feasible outcome of this process because given any complete markets equilibrium allocation, an agent will exist who is better off in *all* incomplete market equilibrium allocations that result if only that agent chooses to purchase information.

It is of course critical for our analysis to maintain the traditional assumption that all agents know the conditional price path in the complete markets equilibrium in the initial filtration. It is not necessary, however, for agents to know the conditional price path in every incomplete markets equilibrium that will result from her information purchase or even to know which, of possibly many, incomplete markets equilibria will be selected. Any agent who chooses not to trade in the initial filtration will know that in *any* incomplete markets equilibrium she will be better off. The same is true for an agent who barely trades in the initial filtration and who knows

that magnitude of the price difference will be greater than some lower bound.¹

Finally, it is important to address the question of market power. It is certainly true that agents will act strategically. However, the reason each agent can affect the market price has nothing to do with the *relative size* of the agent, indeed, our general results are derived in economies with a continuum of agents. An agent's ability to change the market structure emanates from the fact that she can divulge information to all market participants. We argue that this kind of market power is reasonable and must be modelled, if general equilibrium analysis is to be extended to include information acquisition. Examples abound of people, whose net asset positions are tiny in comparison to the market capitalization, who are nonetheless able to affect market prices through a timely announcement of information.

3. A GENERAL RESULT

We will consider economies

$$\mathcal{E} = (I, \mathcal{S}, (\pi_s)_{s \in \mathcal{S}}, (u_h)_{h \in I}, (e_h)_{h \in I}, A, T, \mathcal{F})$$

described by a set of agents I which is either finite ($I = \{1, \dots, H\}$, $H > 1$) or a continuum ($I = [0, 1]$), a set of final states $\mathcal{S} = \{1, \dots, S\}$ with probability weights $\pi_s > 0$, utility functions $u_h: \mathbf{R}_+ \rightarrow \mathbf{R}$, which give rise to von-Neumann–Morgenstern utility functions $U_h = \sum_{s=1}^S \pi_s u_h(c_h(s))$ over final state consumption $c_h \in \mathbf{R}_+^S$, endowments $e_h \in \mathbf{R}_+^S$ in the final state consumption good, a $J \times S$ matrix A of asset payoffs a_j , $j = 1, \dots, J$, in the final state consumption good for J tradable assets, the number of periods T , and a list of increasingly finer partitions $\mathcal{F} = \{\mathcal{F}_0, \dots, \mathcal{F}_T\}$ of \mathcal{S} , where $\mathcal{F}_0 = \mathcal{S}$ and $\mathcal{F}_T = \{\{s\} | s \in \mathcal{S}\}$. The list of partitions \mathcal{F} defines a filtration on \mathcal{S} . Equilibrium is defined in the usual way (see, e.g., Duffie [5, Sect. 12]). We assume throughout, that the utility functions u_h are bounded, strictly increasing, strictly concave, twice continuously differentiable, and satisfy

¹ Indeed in deciding whether or not to purchase information all the barely trading agent really needs is a minimum bound on the probability that prices will differ by more than some (lower) bound. The matter could get more complicated, however, in some complete specification of the ensuing information purchase game. In that case, agents would choose their information acquisition strategies based on the price paths for the different filtrations. There will be agents for which the decision is more complicated than simply moving away from consuming their endowment. These agents would have to be "superrational" in the sense that they might indeed require knowledge not only about the prices in the current Walrasian equilibrium but also about all other filtration-dependent price paths.

$\lim_{c \rightarrow 0} (du_h(c)/dc) = \infty$. Note that agents are not endowed with assets, so that the assets are in zero net supply. The only nonstandard assumption is that we allow agents to receive at any trading period $t < T-1$ and node $\theta_t \in \mathcal{F}_t$ the information $\{\theta_{t+2} \in \mathcal{F}_{t+2} | \theta_{t+2} \subset \theta_t\}$ at $t+1$. For the remainder of the paper we will consider the decision to acquire information only at time $T-2$. If any agent decides to do so, then all remaining uncertainty is resolved at $T-1$ and thus no trade will occur at $T-1$.

To simplify our analysis without much loss of generality, assume that $T \geq 3$, $\mathcal{S} = 2^T$, and that the filtration is the standard binary filtration. That is, exactly two nodes θ_1 and θ_2 in \mathcal{F}_t , $t > 0$, have the same predecessor $\Theta(\theta_1) = \Theta(\theta_2)$ in \mathcal{F}_{t-1} . As usual the predecessor $\Theta(\theta) \in \mathcal{F}_{t-1}$ for some $\theta \in \mathcal{F}_t$, $t > 0$, is defined by $\theta \subseteq \Theta(\theta)$. We assume that there are exactly 2 assets $[a_1, a_2] = A$ with linearly independent, strictly positive payoffs given any node θ in any partition \mathcal{F}_t , $t < T$. We keep A , T , and thus the state space \mathcal{S} and the partition list \mathcal{F} fixed throughout.

For a given set of agents, economies are drawn at random in the following way. For each agent h , a utility function $u_h = u^k$ is drawn from some finite set of utilities $\mathcal{U} = \{u^1, \dots, u^K\}$ with probabilities $p_k > 0$ and, conditionally on that, the endowment e_h is drawn according to some probability measure μ^k on \mathbf{R}_{++}^S . μ^k is assumed to be absolutely continuous with respect to the Lebesgue measure and to have compact support $M^k \subset \mathbf{R}_{++}^S$ with nonempty interior. Utilities and endowments are drawn independently across agents.

For simplicity, we assume that all agents know each others' utility and endowments. Since this implies that any agent can solve for the equilibrium prices we assume that given any filtration, agents know the equilibrium prices. As discussed earlier this assumption can be weakened without significantly affecting the results.

After stating and discussing five assumptions, we will prove our main theorem. It states that a positive fraction of agents will want to incomplete the market in the continuum economy. The idea of the proof is to show that a positive fraction of agents will more or less just consume their endowment and thus almost not trade any assets. These agents are better off with incomplete markets by revealed preferences, since they now will trade assets due to the change in the relative asset prices. We also state a corollary to the main theorem, which says that the probability of some agent preferring to incomplete the market converges to one, as the number of agents diverges to infinity.

Key building blocks for our analysis below are three demand functions. Let $C(p, u, e)$ be the demand function for final consumption $c \in \mathbf{R}^S$ for an agent with utility function u and endowment e in a complete market economy, given the price vector $p \in \mathbf{R}_{++}^S$ for final consumption. Secondly, define D to be the demand function $D(t, \theta, p, c - e)$ for the two assets at

time t and node θ , given the price vector $p \in \mathbf{R}_{++}^S$ for final consumption in order to finance consumption $c_{1\theta}$ in the complete markets case, starting from endowments e . Here, $c_{1\theta}$ is the vector with components $c(s)$, $s \in \theta$. Finally, $F(\theta_{T-2}, q, u, e, \alpha)$ is the individual demand for both assets at time $T-2$ and node $\theta_{T-2} \in \mathcal{F}_{T-2}$, assuming no trade at $T-1$, a relative price q for asset 1 in terms of asset 2 at that node, utility function u , endowment $e \in \mathbf{R}_+^S$, and portfolio $\alpha \in \mathbf{R}^2$, which that agent brings into period $T-2$. A portfolio $\alpha \in \mathbf{R}^2$ means that the agent holds $\alpha[1]$ units of asset 1 and $\alpha[2]$ units of asset 2. It is well known that C is a continuous function of p and e , that D is a continuous function in p and $c - e$, and that F is a continuous function in q , e , and α . The second claim is easy to see from the fact that there is a static complete markets equilibrium that implements C (Duffie [5]). For the third claim, observe that the choice F can simply be rephrased as a choice between two different commodities or as a demand function for numeraire assets (see Geanakoplos [7, Sect. 4.2] and the references therein).

We will repeatedly compute the integral of some function f from the unit interval into a space of random variables, where two different values of that function are independently distributed. We use the Pettis integral for that purpose (see Diestel and Uhl [4] for a definition). Uhlig [12] has shown that the Pettis integral delivers the law of large numbers without additional assumptions on the underlying measure space, avoiding the measurability problems raised in Judd [9].² We will therefore use the law of large numbers below without further comment.

An equilibrium for almost every random continuum economy, assuming complete markets, is given by some price vector $\bar{p} \in \mathbf{R}_{++}^S$ for final consumption so that

$$\bar{e} = \sum_{k=1}^K p_k \int e d\mu^k(e) = \bar{C}(\bar{p}),$$

where

$$\bar{C}(p) = \sum_{k=1}^K p_k \int C(p, u^k, e) d\mu^k(e).$$

² For simplicity, consider defining the random variable $I = \int_{[0,1]} f(x) \lambda(dx)$ via limits of Riemann sums, using some concept of metric or convergence for random variables. The Pettis integral corresponds to using the metric of mean squared difference and it is easy to see that the law of large numbers $\int_{[0,1]} f(x) \lambda(dx) = \int_{[0,1]} E[f(x)] \lambda(dx)$ a.e. comes about as long as the variances $\text{Var}[f(x)]$ are bounded. This idea is similar to the idea underlying the Ito integral which has found wide applications in finance (see Duffie [5]). The Pettis integral is one of the two standard generalizations of the Lebesgue integral to vector-valued functions. Judd's [9] approach corresponds to taking the limit pointwise almost everywhere, resulting in well-known measurability problems, and does not correspond to some standard approach for vector-valued integration.

$\bar{C}(p)$ is the aggregate demand in the continuum economy at prices p . Existence of an equilibrium \bar{p} follows via the standard excess demand function proof, see, e.g., Varian [13]. Because of our assumption about preferences and endowments, \bar{p} consists of strictly positive prices only. We assume w.l.o.g., that the state prices $\bar{p}(s)$ sum to 1. Fix \bar{p} now. Let $\bar{q}(p, t, \theta)$ be the corresponding relative price for asset 1, which we call the benchmark-complete-markets price.

Assumption A.1. With prices for final consumption \bar{p} , and corresponding benchmark-complete-markets prices $\bar{q}(p, t, \theta)$ of asset 1 in terms of asset 2, markets are dynamically complete.

This is a weak assumption, since with respect to asset payoffs, markets are generically dynamically complete (see, e.g., Duffie [5, Corollary 12 G] and the references therein).

Assumption A.2. With prices for final consumption \bar{p} , there are some k_0 , an endowment point e_0 in the interior of the support M^{k_0} of μ^{k_0} , and a node $\theta_{T-3} \in \mathcal{F}_{T-3}$, so that an agent with endowment e_0 and utility function u^{k_0} would choose to consume his endowment in the states $s \in \theta_{T-3}$, given prices \bar{p} :

$$e_0(s) = C(\bar{p}, u^{k_0}, e_0)(s) \quad \text{for } s \in \theta_{T-3}.$$

Assumption A.2 is just an assumption on the support M^{k_0} of the probability measure μ^{k_0} with which endowments for agents with utility function u^{k_0} are drawn. Note that for any utility function u^k , the function $\lambda \mapsto C(\bar{p}, u^k, \lambda \mathbf{1}_S)$, $\lambda > 0$, traces out a one-dimensional submanifold of \mathbf{R}^S of consumption choices as wealth increases. The assumption above follows, if for at least one k_0 , the interior of the support M^{k_0} intersects this submanifold. This in turn can obviously be satisfied for some suitable affine transformation $e_0^k + M^{k_0}$ for some $e_0^k \in \mathbf{R}^S$, keeping the transformed set in \mathbf{R}_{++}^S .

Next, we need an assumption describing what happens in the deviation from the dynamically complete to the dynamically incomplete case at $T-2$, i.e., when agents arrive at some node $\theta_{T-2} \in \mathcal{F}_{T-2}$ with their portfolios for the dynamically complete market case only to find out that there will be no trade at $T-1$. Let

$$\delta(\theta_{T-2}, p, u^k, e) = D(T-3, \Theta(\theta_{T-2}), p, C(p, u^k, e) - e)$$

be the asset demands for the two assets at $T-3$ and $\Theta(\theta_{T-2})$ for an agent with utility u^k and endowment e , who wants to finance the consumption choice $C(p, u^k, e)$ at the complete market prices p for final consumption. In other words, this function describes the net asset holdings before trade at

$T-2$ in the benchmark complete markets case. The function δ is continuous in p and e , since it is the composition of continuous functions. In the random continuum economy all deviation-to-incomplete-markets prices $\tilde{q}(\theta_{T-2})$ can be found from the equation

$$0 = \bar{F}(\theta_{T-2}, \tilde{q}(\theta_{T-2})),$$

where

$$\bar{F}(\theta_{T-2}, q) = \sum_{k=1}^K p_k \int F(\theta_{T-2}, q, u^k, e, \delta(\theta_{T-2}, \bar{p}, u^k, e)) d\mu^k(e)$$

are the aggregate asset demands for the two assets at $T-2$ and θ_{T-2} for the deviation to incomplete markets.

It is important to ensure that the benchmark-complete-market asset prices are not also deviation-to-incomplete-markets prices.

Assumption A.3. For some node $\theta_{T-2} \in \mathcal{F}_{T-2}$ at $T-2$, whose predecessor node $\theta(\theta_{T-2}) \in \mathcal{F}_{T-3}$ satisfies Assumption A.2, the benchmark-complete-market asset price $\tilde{q}(\theta_{T-2})$ is not a deviation-to-incomplete-markets price, i.e., we have

$$0 \neq \bar{F}(\theta_{T-2}, \tilde{q}(\theta_{T-2})).$$

Again, this is a weak assumption, since Detemple, Gottardi, and Polemarchakis [3] have shown in a finite setting that relative asset prices are generically different in a complete and incomplete market. Furthermore, when the expected endowment is the same in all states, the assumption can be shown to hold in our context, if we allow arbitrarily small perturbations in the utility function of an arbitrarily small group of agents.³

Note that deviation-to-incomplete-markets prices exist as long as \bar{F} is defined on all of \mathbf{R}_{++} either by arguing directly with standard existence results for numeraire assets (see Geanakoplos [7, Sect. 4.2] and the references therein) or from the observations that \bar{F}_1 is continuous in q , that $\liminf_{q \rightarrow 0} \bar{F}_1(\theta_{T-2}, q) > 0$, and that $\limsup_{q \rightarrow \infty} \bar{F}_1(\theta_{T-2}, q) < 0$. Unfortunately, \bar{F} may not be defined on all of \mathbf{R}_{++} , since agents may come into period $T-2$, holding negative amounts of an asset and thus may end up with negative consumption at certain prices q . In order to guarantee existence, we need to make a further assumption.

Assumption A.4. For some node $\theta_{T-2} \in \mathcal{F}_{T-2}$ at $T-2$, which also satisfies Assumption A.3, agents can ensure themselves strictly positive consumption

³ A proof can be obtained from the authors on request.

by holding on to their assets acquired at $T-3$ under the assumption of complete markets, i.e.,

$$e(s) + (A \cdot \delta(\theta_{T-2}, \bar{p}, u^k, e))(s) > 0 \quad \text{for all } s \in \theta_{T-2}, \text{ all } k, e \in M^k.$$

This is perhaps the strongest assumption we need to make.⁴ It can be guaranteed for given assets, if the diameters of all endowment supports M^k are sufficiently small; in the extreme, where every agent receives the same endowment in all states, there is no trade in assets.

For the corollary, we need to make sure that the complete market equilibrium of the continuum economy does not vanish with small perturbations in the excess demand function.

Assumption A.5. \bar{p} is a regular equilibrium of the random continuum economy, i.e., the Jacobian matrix of the aggregate excess demand function

$$\bar{C}(p) - \bar{e}$$

with the last row and column deleted is non-singular at the equilibrium price \bar{p} .

Since regularity is a generic condition in many settings (see, e.g., Mas-Colell [10, Proposition 5.8.15]), this is a weak assumption as well.

Given an economy and a node $\theta_{T-2} \in \mathcal{F}_{T-2}$, we say that an agent $h \in I$ wants to dynamically incomplete the market, if, given portfolio allocations at θ_{T-2} resulting from trades up to and including $T-3$ in the complete market case, agent h is better off if there is no trade at $T-1$ than if there is. In the random continuum economies above, this amounts to comparing his lot facing the relative price $\tilde{q}(\theta_{T-2})$ and all markets closed at $T-1$ with facing the relative price $\bar{q}(\theta_{T-2})$ and all markets open at $T-1$, given that everybody holds the portfolio acquired at $T-3$ under the assumption of complete markets.

THEOREM 1 (Main Theorem). *Under Assumptions A.1, A.2, A.3, and A.4, there is almost surely a strictly positive fraction of agents in the continuum economy, who would want to dynamically incomplete the market in the equilibrium with price vector \bar{p} .*

The proof can be found in the Appendix.

⁴ This assumption can be relaxed if bankruptcy is allowed. The results in the theorems would remain unchanged however, since in some sensible specification of the strategic game that would ensue between the agents, bankruptcy will not occur in equilibrium because the Inada condition is assumed.

COROLLARY 1. *Under Assumptions A.1, A.2, A.3, A.4, and A.5, the probability P_H that there is an agent who would want to dynamically incomplete the market in a randomly drawn economy with H agents, converges to one, as the number of agents H converges to infinity:*

$$\lim_{H \rightarrow \infty} P_H = 1.$$

This corollary is not hard to prove, using continuity and a standard perturbation argument as in Mas-Colell [10, Proposition 5.4.3]. A complete proof is available from the authors on request.

The results above can probably be extended. In particular, it seems possible to prove a version of the corollary, in which the number of states converges to infinity together with the number of agents. The key observation here is that we made heavier use of Assumption A.2 in our proofs than was necessary. More precisely, in order to find an agent who would want to consume his endowments in some states $s \in \theta_{T-3}$, all we need to guarantee for a particular utility function is a particular endowment structure for these states and a particular wealth for the remaining states. In other words, it is not necessary to accomplish a precisely given endowment structure in all other states as well.

4. CONCLUSIONS

Traditional multiperiod models of asset markets assume that traders take the information filtration as given. This paper has explored the consequences of endogenizing the arrival of information. We considered continuum random economies with multiperiod asset markets, which are dynamically complete in the benchmark filtration. Agents may choose to have information released early, resulting in incompleteness of the markets. We list five rather mild assumptions which guarantee, among other things, that in the benchmark dynamically complete case, there is some agent who would not trade subsequently to some node three periods prior to the terminal date, since he is happy to consume just his endowment in the resulting final states. The main theorem states that under these assumptions, a positive fraction of agents would want to force the incompleteness of the market structure. As a corollary it can be shown that this will also be the case with high probability for finite but large random economies. We conclude that therefore markets are unlikely to be complete, if the timing of information is endogenous.

APPENDIX

Proof of the Main Theorem. First, define the utility gain function

$$g(q, e, k, \theta_{T-2}) = \sum_{s=1}^4 \pi_s u^k(A \cdot F(\theta_{T-2}, q, u^k, e, \delta(\theta_{T-2}, p, u^k, e))(s)) \\ - \sum_{s=1}^4 \pi_s u^k(C(p, u^k, e)(s)).$$

$g(q, e, k, \theta_{T-2})$ is the gain in utility from incompleting the market at $\theta_{T-2} \in \mathcal{F}_{T-2}$ (or the loss, if it is negative) for an agent with utility function u^k , endowment e , and the deviation-to-incomplete-market price q , given complete market prices \bar{p} . An agent would want to incomplete the market at θ_{T-2} and $T-2$, if $g(q, e, k, \theta_{T-2}) > 0$. Note that g is continuous in q and e , since it is a composition of continuous functions. Furthermore note that g can be extended in a uniformly continuous way to the compact set $\bar{\mathbf{R}}_+ \times M^{k_0}$, where $\bar{\mathbf{R}}_+$ is the usual compactification $\mathbf{R}_+ \cup \{\infty\}$. This continuity is easy to see, observing that $\lim_{q \rightarrow \infty} g(q, e, k, \theta)$ and $\lim_{q \rightarrow 0} g(q, e, k, \theta)$ exist for any $e \in M^{k_0}$ by the boundedness of the utility functions.

Pick a node $\theta_{T-2} \in \mathcal{F}_{T-2}$ with predecessor $\theta_{T-3} = \Theta(\theta_{T-2})$ for which Assumptions A.2, A.3, and A.4 are satisfied. Recall, that according to A.2, an agent with e_0 and u^{k_0} will just consume his endowment at prices \bar{p} in the states $s \in \theta_{T-3}$. It follows that $\delta(\theta_{T-2}, \bar{p}, u^k, e) = 0$ and $D(T-2, \theta_{T-2}, \bar{p}, e) = 0$, i.e., the agent is not trading assets at θ_{T-2} . For this agent, the ratio of expected marginal utilities for asset 1 versus asset 2 equals \bar{q} at θ_{T-2} . Thus, this agent will trade assets at any other price $q \neq \bar{q}$. Since not trading is still feasible, we have $g(q, e_0, k_0, \theta_{T-2}) > 0$ by revealed preferences. It follows from the compactness of $\bar{\mathbf{R}}_+ \times M^{k_0}$ and the uniform continuity of g on this set, that for any distance $\Delta > 0$, there is some positive number $g_{\min} > 0$ and an open neighborhood E of e_0 such that $g(q, e, k_0, \theta_{T-2}) \geq g_{\min} > 0$ for any q with $|q - \bar{q}| \geq \Delta$ and $e \in E$.

By Assumptions A.3 and A.4 and continuity, there is at least one deviation-to-incomplete-markets price $\tilde{q}(\theta_{T-2}) \in \mathbf{R}_{++}$. Furthermore, every deviation-to-incomplete-markets price differs from $\bar{q} = \bar{q}(\theta_{T-2})$ by some fixed positive amount $\bar{\Delta} > 0$. With the properties of the function g stated above, there is an open neighborhood E of e_0 such that $g(q, e, k_0, \theta_{T-2})$ is strictly positive for any deviation-to-incomplete-markets price $\tilde{q}(\theta_{T-2}) \in \mathbf{R}_{++}$. Since e_0 is in the interior of the support M^{k_0} of μ^{k_0} by Assumption A.2, the probability of drawing agents with utility u^{k_0} and an endowment e in this neighborhood E is strictly positive. As usual, the fraction of agents having this characteristic in some state of nature ω is

defined as $I(\omega)$, where $I = \int_{[0,1]} \chi_{f(x)} \lambda(dx)$, where χ is the appropriate characteristic function and where \int is the Pettis integral. By the law of large numbers then, this fraction equals almost everywhere the positive probability for an agent with these characteristics. Those agents would want to incomplete the market. ■

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