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# Sovereign Default Risk and Banks in a Monetary Union

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**Abstract.** *This study seeks to understand the interplay between banks, bank regulation, sovereign default risk and central bank guarantees in a monetary union. I assume that banks can use sovereign bonds for repurchase agreements with a common central bank, and that their sovereign partially backs up any losses should the banks not be able to repurchase the bonds. I argue that regulators in risky countries have an incentive to allow their banks to hold home risky bonds and risk defaults, whereas regulators in other 'safe' countries will impose tighter regulation. As a result, governments in risky countries get to borrow more cheaply, effectively shifting the risk of some of the potential sovereign default losses on the common central bank.*

**JEL classification:** E51, E58, E61, E62, E65, G21, G28, H63.

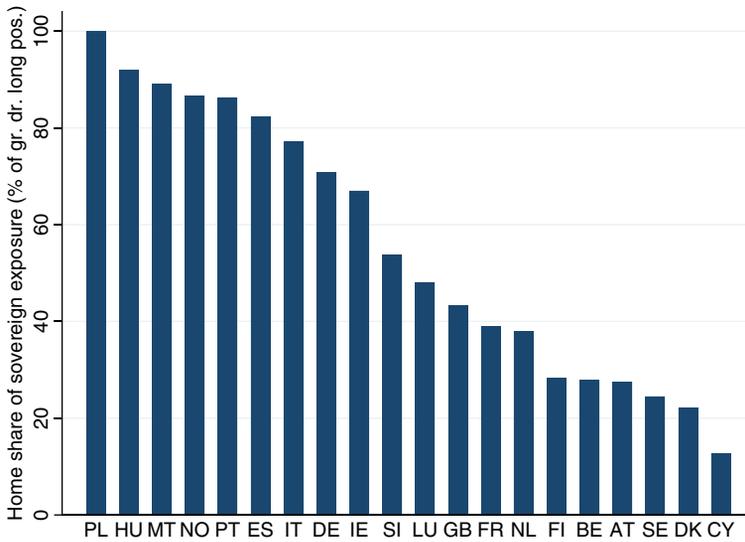
**Keywords:** Eurozone crisis; sovereign default risk; bank regulation; risk shifting; common central bank; European Central Bank; ECB; repurchase operations; haircuts.

## 1. INTRODUCTION

The European Monetary Union is in distress. Mechanisms that were meant to safeguard key institutions and to assure stability have become sources of balance sheet risk for these very institutions. Liquidity provision within the European Monetary Union rests upon repurchase agreements, by which banks guarantee the repurchase of assets deposited with the European Central Bank. If either the bank fails or the asset fails, but not both, this mechanism safeguards the repayment to the ECB, as it can either rely on the repurchase by the bank or sell the asset. However, when both fail as well as the bank home country fails, the ECB incurs a loss.

One symptom of these developments are the dramatic increases in Target 2 imbalances, as extensively documented and discussed by Sinn (2012). Indeed, banks in troubled eurozone countries appear to concentrate their portfolio of government bonds particularly on the bonds of their own country, rather than seeking safety by holding bonds of other countries (see Figure 1). This behavior appears to be encouraged or at least positively tolerated by the regulators in these countries, who may assert that purchases of the bonds of their countries by their own banks is necessary to be able to finance the government at all, thereby endangering the safety of their banks down the road in the case of a sovereign default.

This study is motivated by these developments. It seeks to understand the underlying forces and incentives more deeply. It seeks to understand the



**Figure 1** This graph uses data from the European bank stress test in 2011 to show the fraction of sovereign debt held in the form of domestic sovereign debt, aggregating across the banks in the data sample in each country. Country key: PL Poland, HU Hungary, MT Malta, NO Norway, PT Portugal, ES Spain, IT Italy, DE Germany, IE Ireland, SI Slovenia, LU Luxembourg, GB United Kingdom, FR France, NL Netherlands, FI Finland, BE Belgium, AT Austria, SE Sweden, DK Denmark, CY Cyprus

interplay between banks, bank regulation, sovereign default risk and central bank guarantees in a monetary union. I assume that banks can use sovereign bonds for repurchase agreements with a common central bank, and that their sovereign partially backs up any losses should the banks not be able to repurchase the bonds. I argue that regulators in risky countries have an incentive to allow their banks to hold home risky bonds and risk defaults, whereas regulators in other ‘safe’ countries will impose tighter regulation. As a result, governments in risky countries get to borrow more cheaply, effectively shifting the risk of some of the potential sovereign default losses on the common central bank. As a result, the monetary union has become a system engineered to deliver underpriced loans from country banks to their sovereigns, and to implicitly shift sovereign default risk onto the balance sheet of the ECB and the rest of the eurosystem.

In the model, banks pursue their investment strategy voluntarily: it is up to regulators to potentially constrain them. Other explanations are conceivable, of course. Useful, general perspectives on financial crises and government regulations are found in Reinhart (2012a) as well as Claessens and Kose (2013). A prominent alternative argument for why banks hold bonds of their home country is ‘financial repression’, see Reinhart (2012b), i.e. regulators make them hold the sovereign bonds, perhaps with strong-arm tactics, perhaps in exchange for ‘looking the other way’ concerning weak portfolios of commercial loans and mortgages, or simply as a ‘favor’ in a long, ongoing relationship. As the banks could potentially refuse, although at considerable cost, it still must ultimately be preferable to them to hold own-country bonds rather than invest elsewhere or to close: so, in some ways, this paper may also be understood as a model of

financial repression. A further alternative is a political economy argument: if sovereign bonds are held by home banks, it makes it politically harder to default on these bonds, as this will hurt domestic banks and savers. If so, then such a portfolio arrangement might serve as a commitment device for the government in trouble. Although this is an interesting angle, this study will be silent on this perspective.

The issue of sovereign default risk, bank portfolios and the role of the central bank has received considerable attention in the recent literature. Acharya and Steffen (2013) is a careful empirical analysis of the ‘carry trade’ by banks, which fund themselves in the wholesale market and invest in risky sovereign bonds. They document that ‘over time, there is an increase in ‘home bias’ – greater exposure of domestic banks to its sovereign’s bonds – which is partly explained by the European Central Bank funding of these positions’, providing an in-depth analysis of the facts indicated in Figure 1 as well as solid empirical support for the theory provided below. Relatedly, Corradin and Rodriguez-Moreno (2013) show that USD-denominated sovereign bonds of eurozone countries became substantially cheaper (i.e., delivering a higher yield) than euro-denominated bonds during the eurozone crisis, and ascribe it to the usefulness to banks of euro-denominated bonds as collateral *vis-a-vis* the ECB, whereas USD-denominated bonds do not offer this advantage. Again, this supports the perspective in this study. Drechsler *et al.* (2013) document ‘a strong divergence among banks’ take-up of’ lender-of-last-resort assistance ‘during the financial crisis in the euro area, as banks which borrowed heavily also used increasingly risky collateral’. They test several hypothesis and argue that their ‘results strongly support the risk-shifting explanation’, in line with the theory below.

Like this study, Broner *et al.* (2013) is motivated by the shift toward home sovereign debt during the euro crisis. Their analysis is somewhat parallel to my model here, as they also show that ‘sovereign debt offers a higher expected return to domestic creditors than to foreign ones’. There are considerable differences in the modeling details, however. Roch and Uhlig (2012) analyze the role of actuarially fair bailouts to resolve multiple-equilibrium – driven sovereign default crises, thus shedding light on the OMT program of the ECB. There is considerably more, but let me stop here.

## 2. A MODEL

### 2.1. The actors and their interaction

There are three periods,  $t = 0, 1, 2$ . There is a monetary union of many countries and a common central bank. One ‘small’ country will be considered risky, whereas all other countries will be considered ‘safe’: we shall thus only distinguish between the risky country and the rest of the monetary union. All units are in terms of the common currency. I shall abstract from the issue of inflation, which does not appear to be central at this juncture in the European Monetary Union, although it may well be so in the future. I therefore assume that a unit of the currency can be used to purchase a unit of the consumption good in any given period, i.e. all the accounting could alternatively be done in terms of a numeraire consumption good.

1. There is a continuum of banks  $j \in [0,1]$  in each country with initial net worth  $^1n > 0$ . Therefore,  $n$  also denotes the total net worth available per country. The banks are each owned by a banker or bank owner, who maximizes expected consumption in period 2. Their consumption must be non-negative. I will use the word ‘bank’ and ‘banker’ interchangeably, but one may wish to think of the bank being regulated and carrying out investment strategies, whereas the bank owner or banker cares about the result.
2. There is large population of households in each country, holding in total a deposit of 1 unit with each bank at the beginning of period 0. Households withdraw their deposits in period 1.
3. In each country, there is a government, which has a spending need  $g_0$  in period 0, and otherwise seeks to maximize expected government spending in period 2. Each government is endowed with resources in period 2. I assume that all governments except one are ‘safe’, i.e. always have sufficiently large resources in all periods, including period 2. There is one ‘risky’ government (or one government in a ‘risky’ country), which has resources  $y \sim G$  in period 2,  $y \geq 0$  and  $p(y > 0) > 0$ . I assume that

$$E[y] > g_0 \tag{1}$$

Governments issue bonds in period 0 to finance their spending need, with the promise to repay them in period 2. The budget constraint of the risky government in period 0 is therefore

$$qB = g_0, \tag{2}$$

where  $B$  is the total outstanding face value of its debt and  $q$  is the market price per unit of the bond. The governments regulate the asset positions of banks in period 0, by imposing an upper bound  $\bar{b}$  with

$$\bar{b} \leq \frac{1+n}{q}$$

on their holding of risky bonds in period 0. Note that the same bound is imposed on all banks, by assumption.<sup>2</sup> Furthermore, the governments are nominally obliged to pay for a share  $0 < \alpha \leq 1$  of the losses of their banks in period 2, but will fractionally default on this obligation, if they lack the funds to do so. The regulators will impose the bounds on the bond positions optimally to maximize expected period 2 government consumption, and choose the most liberal regulation among all those achieving this maximum.

4. There is a common central bank. The central bank can issue cash and engage in repurchase agreements with the banks. Any shortfalls in period 2 are paid for by the governments in period 2: in equal proportions, up to the resources available to each government.

1. This can be interpreted as net worth inclusive of long-term deposits, which will not be withdrawn
2. One can easily generalize this model to the situation, where deposits as well as  $\bar{b}$  is proportional to  $n$ : for that reason, we shall often use a notation, expressing everything in proportion to  $n$ . It is considerably more challenging to allow heterogeneity in  $n$  as well as in the regulatory bounds. The assumption of homogeneity can be motivated by the general legal principle that the law should treat all market participants equitably.

## Sovereign Default Risk and Banks

I assume that these players interact as follows. The interactions place considerable restrictions on the available contracts: these restrictions shall be examined further below.

In period 0, the households place 1 unit of deposits at the beginning of period 0 with each bank: thus,  $1/n$  can be understood as the leverage ratio of bank  $j$  in a country. Banks use their net worth and the deposited resources to buy a portfolio of bonds from the countries, subject to regulatory restrictions imposed by their respective governments. More specifically, they buy bonds  $b \geq 0$  at price  $q$  from the ‘risky’ government, where  $b$  is the face value (‘promise to pay in period 2’) of the bonds. They use the remainder of the funds to buy safe bonds, which are assumed to be like cash: Thus, their price is unity, and their quantity shall be denoted by  $m \geq 0$ . Taking into account the regulatory constraint, the budget constraint of a bank is

$$qb + m = 1 + n, b \leq \bar{b}, b \geq 0, m \geq 0. \quad (3)$$

In period 1, households withdraw their deposits.<sup>3</sup> To meet these liquidity needs, banks first sell safe bonds  $\tilde{m}$ , until they reach  $\tilde{m} = m$ , and then approach the common central bank for a repurchase contract for the remainder. In the repurchase contract, the bank sells  $\tilde{b} \geq 0$  bonds to the central bank at price  $\phi q$ , where  $0 < \phi \leq 1$  represents a haircut applied by the central bank, with the agreement of repurchasing them at the same price in period 2. I assume that the haircut parameter  $\phi$  is common across all banks. Thus, in period 1, a bank with portfolio  $(b, m)$  faces the budget constraint

$$\phi q \tilde{b} + \tilde{m} = 1, \tilde{m} = \min\{m, 1\}. \quad (4)$$

For the analysis here, I assume that the central bank chooses the common haircut parameter  $\phi$  in period 1, so that this constraint is always satisfied: otherwise, banks would need to default on their withdrawing households. This captures the impression that, up to 2013, the ESCB has largely provided liquidity to banks in affected countries, when needed.

In period 2, the bonds mature. The safe bonds (‘cash  $m'$ ) pay one unit per bond. The risky bonds pay  $0 \leq \theta \leq 1$  per unit of face value, with  $\theta < 1$  representing a default by the risky government. If a bank has entered a repo agreement with the central bank, it will purchase back as many of the bonds sold to the central bank in period 0 at the agreed-upon price  $\phi q$  per unit as possible, making use of its own resources as well as the maturing payments on the bonds it is repurchasing from the central bank. If the repurchase is not done in full, the bank is considered in default: The ensuing losses are then split between the central bank and the government of that bank, with the government bearing a share  $0 \leq \theta \alpha \leq 1$  of these losses<sup>4</sup> and the central bank bearing the remaining fraction  $1 - \theta \alpha$  of the losses. If the repurchase is done in full, the banks keep the excess resources and consume. Governments must first repay bonds and pay for their

3. This is motivated by the large withdrawals of depositors at banks in the south of the EMU in recent years. While the withdrawal is ‘hardwired’ here and assumed to be total, a more complete model might endogenize these withdrawal decisions.
4. Note that I assume that the government defaults on its obligations to ‘recapitalize’ its banks at the same rate  $\theta$  that it defaults on its bond repayments.

share of losses at their banks, before consuming the remainder  $g_2$ , with  $\alpha$  a parameter of the model, and  $\theta$  emerging endogenously from the budget constraint of the government

$$g_2 + \theta(B + \alpha L) = \gamma, 0 \leq \theta \leq 1, g_2 \geq 0 \text{ and } g_2 = 0 \text{ if } \theta < 1, \quad (5)$$

where  $B$  is the total face value of bonds issued in period 0 and where  $L$  is the total losses in their banks.

## 2.2. Analysis

Consider first the problem of a bank  $j$ , after purchase of the portfolio  $(b, m)$  per the budget constraint (3). Given the survival in period 1, as assumed, and the liabilities to the central bank in period 2, and substituting for  $m$  per (3), the banker owning this bank will enjoy period 2 consumption, written as a function of the value of the investment  $qb$ ,

$$\begin{aligned} c(qb) &= \max\{\theta b + m - 1; 0\} \\ &= \max\left\{\left(\frac{\theta}{q} - 1\right)\frac{qb}{n} + 1; 0\right\}n \end{aligned}$$

and produce losses of

$$\begin{aligned} l(qb) &= \max\{1 - \theta b - m; 0\} \\ &= \max\left\{\left(1 - \frac{\theta}{q}\right)\frac{qb}{n} - 1; 0\right\}n. \end{aligned} \quad (6)$$

Define

$$\beta = \min\left\{\frac{q\bar{b}}{n}, \frac{1+n}{n}\right\}. \quad (7)$$

Notice that  $0 \leq qb \leq \beta n$ , with the first entry denoting the regulatory bound and the second entry arising from  $m \geq 0$ . Therefore, if the banker invests the maximum amount possible in bonds (or, more precisely, instructs his bank to invest the maximum amount possible in bonds), he will receive expected consumption

$$E[c(\beta n)] = \chi(\beta)n,$$

where

$$\chi(\beta) = E[\max\left\{\left(\frac{\theta}{q} - 1\right)\beta + 1; 0\right\}]. \quad (8)$$

If the banker instructs his bank to invest only in cash, he will receive expected consumption equal to  $n$ .

**Proposition 1.** The results do not depend on the haircut parameter  $\phi$ .

*Proof:* None of the expressions above depends on  $\phi$ .

This may be a bit surprising, as it is often argued that by some appropriate haircut, the central bank can insulate itself against losses. The haircut parameter does not matter here, and therefore, it will not matter below either.

### Proposition 2.

- If  $1 < \chi(\beta)$  the banker will invest the maximum amount possible in bonds,  $b = \beta n$ .
- If  $1 > \chi(\beta)$ , the banker will only hold cash.
- If  $1 = \chi(\beta)$ , the banker is indifferent between ‘all cash’ and ‘all bonds’ and
  - strictly prefers this to any interior solution, if  $p\left(\left(\frac{\theta}{q} - 1\right)\beta + 1 < 0\right) > 0$ .
  - is indifferent between all feasible portfolio allocations, if  $p\left(\left(\frac{\theta}{q} - 1\right)\beta + 1 < 0\right) = 0$ .

Furthermore,  $\chi$  is a convex function of  $\beta$ , with

$$\chi(\beta) \geq \left(\frac{E[\theta]}{q} - 1\right)\beta + 1, \quad (9)$$

and the inequality is strict, if  $p\left(\left(\frac{\theta}{q} - 1\right)\beta + 1 < 0\right) > 0$ .

*Proof:* Note that  $c(b)$  is a convex function of  $b$ , that  $E[c(b)]$  is a strictly convex function of  $b$ , if  $p\left(\left(\frac{\theta}{q} - 1\right)\beta + 1 < 0\right) > 0$ , and that  $E[c(b)]$  is a linear function of  $b$  in the feasible range, if  $p\left(\left(\frac{\theta}{q} - 1\right)\beta + 1 < 0\right) = 0$ . Equation (9) follows from dropping the ‘max’ operator in the definition of  $\chi$ .

### Proposition 3.

$$0 \leq q \leq 1. \quad (10)$$

*Proof:*  $q \geq 0$  is clear. As for  $q \leq 1$ , note that  $\theta \leq 1$ . Therefore, at  $q > 1$ ,

$$\left(\frac{\theta}{q} - 1\right)\beta + 1 < 1$$

and thus  $\chi(q) < 1$ . Per proposition 2, no banker would hold the risky bonds at  $q > 1$ , in contradiction to market clearing for the risky bonds.

Next, consider the problem facing regulators in safe countries. Given the distribution for default rates  $\theta$ , let

$$\underline{\theta} = \sup\{\tilde{\theta} | P(\theta \geq \tilde{\theta}) = 1\}$$

be the probabilistically smallest value for  $\theta$ .

**Proposition 4.** If  $q \leq \underline{\theta}$ , regulators in the safe countries will not impose any regulatory bounds. If  $q > \underline{\theta}$ , regulators in safe countries will impose

$$\bar{b}^{safe}(n) = \frac{1}{q - \underline{\theta}} n, \quad (11)$$

and thus assure that banks will avoid losses, with probability 1,

$$p\left(\left(\frac{\theta}{q} - 1\right)\beta^{safe} + 1 < 0\right) = 0.$$

Banks in safe countries will purchase risky bonds, if and only if

$$q \leq E[\theta], \quad (12)$$

with indifference between cash and bonds, in case of '='. That is, these banks only purchase risky bonds, if their price is less than or at most equal to the actuarially fair value.

*Proof:* This follows from the assumption that regulators maximize expected government consumption in period 2, and seek the most liberal portfolio regulation that achieves that. As banks never make losses,

$$\chi(\beta) = \left(\frac{E[\theta]}{q} - 1\right) \min\left\{\frac{q}{q - \underline{\theta}}, \frac{1 + n}{n}\right\} + 1.$$

Therefore,  $1 \leq \chi(\beta)$  is equivalent to equation (12). •

As an immediate consequence, we have

**Proposition 5.**

$$q \geq E[\theta]. \quad (13)$$

Furthermore, if  $\theta$  is not constant almost surely, then  $p(\theta < q) > 0$  and regulators in safe countries will impose the constraint (11).

*Proof:* The resources of the safe country banks are assumed large compared to the financing need of the risky country.

Therefore, from here on forward, we shall only refer to the risky country, when discussing the regulatory bound  $\bar{b}$  as well as

$$\beta = \min\left\{\frac{q\bar{b}}{n}, \frac{1 + n}{n}\right\}. \quad (14)$$

**Proposition 6.** If the initial financing needs of the risky country exceed the (regulated) resources of its own banks,  $g_0 > \beta n$ , then  $q = E[\theta]$ . Furthermore, at  $q = E[\theta]$ , the banks in the risky country are either indifferent between cash and bonds, or strictly prefer to hold bonds.

*Proof:* This is a consequence of the analysis above. Note, for example that (9) implies  $\chi(\beta) \geq 1$  at  $q = E[\theta]$ , regardless of  $\beta \geq 0$ . Proposition (2) therefore delivers the claim regarding the banks in the risky country. •

Therefore, if  $q = E[\theta]$  and  $\chi(\beta) = 1$  for banks in the risky country, the banks in the risky country might choose cash. But in that case, the regulators in the risky country might have already set  $\bar{b} = 0$  in the first place. Without loss of generality, we can therefore analyze the regulatory problem of the risky country as the problem of setting the bound  $\bar{b}$ , and assume that the banks in the risky country always hold the maximum amount of bonds allowed, provided the attractiveness condition

$$1 \leq \chi(\beta) \tag{15}$$

is satisfied, see proposition 2. If all bonds are sold to banks in the risky country (which obviously requires  $g_0 \leq n + 1$ ), then

$$g_0 = \beta n \tag{16}$$

is the market clearing condition. In that case,  $\bar{b}$  implies the market price  $q$  per

$$q = \frac{g_0}{\bar{b}} \tag{17}$$

and satisfies

$$q \leq E[\theta]$$

If some bonds are sold to safe countries, then

$$q = E[\theta] \tag{18}$$

and

$$\beta < \frac{g_0}{n}.$$

The distribution for  $\theta$  arises from taking into account the losses of risky country banks, at their maximum bond leverage  $\beta$ . They are

$$\begin{aligned} L(\beta) &= \max \left\{ \left( 1 - \frac{\theta}{q} \right) \beta - 1; 0 \right\} n \\ &\leq \max \left\{ \left( 1 - \frac{\theta}{q} \right) g_0 - n; 0 \right\} \end{aligned} \tag{19}$$

with equality in the second line if all bonds are sold to banks in the risky country. Non-zero losses occur, if and only if

$$1 < \left( 1 - \frac{\theta}{q} \right) \beta. \tag{20}$$

With these results, we can finally succinctly state the government's problem.

**Proposition 7.** The risky government picks  $\bar{b}$  as well as  $\beta, \theta, g_2$ , to maximize  $E[g_2]$ , subject to

$$q = \max \left\{ E[\theta], \frac{g_0}{\bar{b}} \right\} \quad (21)$$

$$\beta = \frac{q\bar{b}}{n} \quad (22)$$

$$1 \leq \chi(\beta) \quad (23)$$

$$y = g_2 + \theta \left( \frac{g_0}{q} + \alpha n \max \left\{ \left(1 - \frac{\theta}{q}\right)\beta - 1; 0 \right\} \right) \quad (24)$$

$$\theta \leq 1 \quad (25)$$

$$0 = g_2(1 - \theta). \quad (26)$$

*Proof:* This collects equations and conditions from above.

This can be analyzed further as follows. Define

$$\theta^*(q) = q \left(1 - \frac{n}{g_0}\right). \quad (27)$$

Note that  $\theta^*(q)$  is that value for  $\theta$ , at which the expression for the losses in equation (19) equals zero, in case all bonds are sold to banks in the risky country. Furthermore, define

$$y^* = g_0 - n. \quad (28)$$

**Proposition 8.** Suppose, all bonds are sold to banks in the risky country. Then we have

1.  $0 < \theta^*(q) < 1$ .
2. The government budget constraint (24) can be rewritten as

$$y - g_2 = \begin{cases} \theta \left( \frac{g_0}{q} (1 - \alpha\theta) + \alpha(g_0 - n) \right) & \text{if } \theta \leq \theta^*(q) \\ \theta \left( \frac{g_0}{q} \right) & \text{if } \theta \geq \theta^*(q) \end{cases} \quad (29)$$

3. The right-hand side of equation (29) takes the value 0 at  $\theta = 0$  and the value  $B = g_0/q$  at  $\theta = 1$ , is continuous and strictly increasing in  $\theta \in [0, 1]$ . It has an upward-bending kink at  $\theta = \theta^*(q)$ .
4. Given  $q$  and for every  $y$ , there is a unique

$$\theta = \Theta(y, q) \in [0, 1]$$

and  $g_2$ , solving the government problem of proposition 7. For  $y < B$ , the solution is  $g_2 = 0$  and  $\theta < 1$ , solving equation (29) at  $g_2 = 0$ , whereas  $\theta = 1$  for  $y \geq B$  and  $g_2 = y - g_0/q$ . Given  $q$ , expected government consumption is

$$E[g_2](q) = \int_{y \geq \frac{g_0}{q}} \left( y - \frac{g_0}{q} \right) G(dy). \quad (30)$$

5. If  $y < y^*$ , then

$$\theta = \Theta(y, q) = \frac{1}{2} \left( \frac{1}{\alpha} + \theta^*(q) - \sqrt{\left( \frac{1}{\alpha} + \theta^*(q) \right)^2 - 4 \frac{yq}{\alpha g_0}} \right) \quad (31)$$

and  $\theta < \theta^*(q)$  as well as losses in the banking system,  $L > 0$ . If  $y^* \leq y \leq B = g_0/q$ , then

$$\theta = \Theta(y, q) = y \frac{q}{g_0}, \quad (32)$$

while the losses are zero,  $L = 0$ .

*Proof:*

1. This follows from  $n < g_0$ .
2. This should be clear.
3. The values at  $\theta = 0$  and  $\theta = 1$  as well as continuity are easy to verify. Given  $q$ , define

$$p(\theta) = \theta \left( \frac{g_0}{q} (1 - \alpha\theta) + \alpha(g_0 - n) \right), \quad (33)$$

which is a quadratic and concave polynomial in  $\theta$ . The derivative of  $p(\theta)$  is

$$\begin{aligned} p'(\theta) &= \frac{g_0}{q} (1 - 2\alpha\theta) + \alpha(g_0 - n) \\ &= \frac{g_0}{q} (1 - \alpha\theta) + \alpha \left( \frac{g_0}{q} (q - \theta) - n \right). \end{aligned} \quad (34)$$

Inspecting (19), notice that

$$0 = \frac{g_0}{q} (q - \theta^*(q)) - n. \quad (35)$$

Thus, while  $p(\theta)$  may reach its maximum for some  $\theta \in (0,1)$ , use the second line of (34) and the property (35) or brute-force algebra to calculate that

$$p'(\theta^*(q)) = \frac{g_0}{q} (1 - \alpha\theta^*(q)) > 0.$$

Hence,  $\theta^*(q)$  is to the left of the peak of  $p(\theta)$ . Furthermore,

$$p'(\theta^*(q)) < \frac{g_0}{q}$$

and  $\frac{g_0}{q}$  is the derivative of (29) to the right of  $\theta^*(q)$ , delivering the result about the nature of the kink.

4. This is now obvious.
5. Calculate.

If all bonds are sold to banks in the risky country, the expected consumption of a banker there will be

$$\bar{c}(q) = E \left[ \max \left\{ \left( \frac{\Theta(\gamma; q)}{q} - 1 \right) \frac{g_0}{n} + 1; 0 \right\} \right]. \quad (36)$$

The attractiveness constraint (15) then demands that

$$1 \leq \bar{c}(q). \quad (37)$$

**Proposition 9.** Suppose, all bonds are sold to banks in the risky country. Then,

1. if  $\alpha < 1$ , then the solution to the problem of the government is to choose the maximum value of  $q$ , such that  $\bar{c}(q) \geq 1$ .
2. If  $\Theta(\gamma; q)$  is not constant almost surely as a function of  $\gamma$  and given  $q$ , then  $q < \sup_{\{\gamma \in \text{supp}(G)\}} \Theta(\gamma; q)$ .
3.  $\bar{c}(q) = 1$  implies that the expected return  $R^e$  on investing one unit in the risky government bond satisfies

$$R^e = \frac{E[\Theta(\gamma; q)]}{q} \leq 1. \quad (38)$$

Furthermore, this inequality is strict, if ‘max’ binds with positive probability in equation (36). In that case, banks in safe countries will not purchase any risky bonds.

*Proof:*

1. Note that expected government consumption  $E[g_2](q)$  given in equation (30) is an increasing function of  $q$ . The government will therefore pick the maximum  $q$  compatible with the attractiveness condition.
2. If  $q \geq \sup_{\{\gamma \in \text{supp}(G)\}} \Theta(\gamma; q)$  and thus  $q > \Theta(\gamma; q)$  with positive probability in realizations of  $\gamma$ , then  $\bar{c}(q) < 1$ , violating the attractiveness condition.
3. Note that

$$1 = \bar{c}(q) \geq \left( E \left[ \frac{\Theta(\gamma; q)}{q} \right] - 1 \right) \frac{g_0}{n} + 1, \quad (39)$$

which can be rewritten as

$$0 \geq \frac{E[\Theta(\gamma; q)]}{q} - 1.$$

Furthermore, the inequality in (39) is strict, if ‘max’ binds with positive probability in (36).

The second part of the proposition means that, to attract the banks to buy the bonds, they have to earn a profit from holding these bonds on occasions to

make it worth their while. The proposition now also gives us the key risk shifting insight of this study, and the insight that bonds of the risky country are held by their home banks, if feasible. Given that the risky government will drive  $\bar{c}(n)$  to (just above) unity, it follows that the expected return on investing one unit of resources in the debt of that country is below the safe return, if bank insolvencies occur at all in the defaulting country. As regulators in safe countries would strictly regulate their banks so that no losses are borne by the governments there, the risks are entirely borne by the banks, the expected return is what matters, and they would not invest in the bonds of the risky country.

As for the common central bank and the sovereign default risk shifted onto its balance books, note that its expected losses are given by

$$E[L_{CB}](q) = E \left[ (1 - \alpha \Theta(y; q)) \max \left\{ 0; \left( (q - \Theta(y; q)) \frac{g_0}{q} - n \right) \right\} \right] \quad (40)$$

as a function of  $q$ .

**Proposition 10.** If  $q = E[\theta]$ , then  $\bar{b} = 0$  solves the problem of the risky government, i.e. it is best to sell all bonds to foreign banks. In that case,

$$\theta = \min \left\{ 1; \frac{q}{g_0} \gamma \right\} \quad (41)$$

$$g_2 = \gamma - \theta \frac{g_0}{q} \quad (42)$$

With positive probability, the government will not default,  $p(\theta = 1) > 0$ .

*Proof:* This can be seen by examining (24): the left-hand side is constant or increasing in  $\beta$ , increasing in  $q$  and increasing in  $\theta$ , noting that  $\theta \alpha \beta n / q \leq \theta \alpha g_0 / q \leq g_0 / q$ . Finally, multiply (24) or

$$\gamma - g_2 = \theta \frac{g_0}{q}$$

with  $q$  and take expectations to find

$$E[\theta](E[\gamma] - E[g_2]) = E[\theta]g_0$$

or  $E[g_2] > 0$  due to (1). That, however, can only happen with  $p(\theta = 1) > 0$ .

Intuitively, if there is no price gain to be obtained from selling bonds to own-country banks, compared to the actuarially fair value  $E[\theta]$ , then it is best to get the best possible bond price by not also risking exposure to bank losses in the second period, as they will be 'priced in' by bond holders.

**Proposition 11.** The complete characterization of the solution to the government problem is as follows.

1. If  $g_0 > n + 1$ , then all bonds are sold to foreign banks as in proposition 10.
5. I exclude the case  $q = 0$ .

2. If  $g_0 \leq n + 1$ , then compare the solution of proposition 9 from selling all bonds to home country banks to the solution from selling all bonds to foreign banks given in proposition 10. Pick the solution, which offers the highest expected government consumption, i.e. the solution with the highest  $q$ .

*Proof:* This follows now from propositions 7, 9 and 10.

### 3. NUMERICAL ILLUSTRATIONS

#### 3.1. A two-point distribution

For an analytical case, consider a two-point distribution  $\{y_L, y_H\}$  for  $y$ , with

$$0 < y_L < g_0 < y_H$$

Let  $p_H = p(y = y_H)$ ,  $p_L = 1 - p_H$  and  $0 < p_H < 1$ . Suppose that  $\alpha = 1$ , so that the central bank only bears losses in the case that the country defaults. Finally, suppose that  $n$  is negligible compared to  $g_0$ : We shall equate it to zero, unless we need to divide by it. Assume that  $g_0 < 1$ , so that own-country resources at the home banks (per the initial deposits of households) are sufficient for purchasing all bonds.

Per proposition 11, calculate the solution given in proposition 9, when all bonds are sold to home country banks. With  $n = 0$ , it follows that

$$\begin{aligned} y^* &= g_0 \\ \theta^*(q) &= q \end{aligned}$$

It follows from proposition 8, that bank losses will occur for  $y = y_L$ , but not for  $y = y_H$ . Furthermore, the government will be in default for  $y = y_L$  and  $g_{2,L} = 0$  will be zero in that state. The government budget constraint for  $y = y_H$  reads

$$y_H = g_{2,H} + \theta_H \frac{g_0}{q}$$

and the government will be able to obtain positive consumption  $g_{2,H} > 0$  for this state only if  $\theta_H = 1$ : we shall thus proceed with the assumption that the government can feasibly choose  $\theta_H = 1$  and verify it. With these results, one can now examine the attractiveness condition 37, restated as

$$\begin{aligned} n &\leq \bar{c}(q)n \\ &= p_H \left( \left( \frac{1}{q} - 1 \right) g_0 + n \right) \end{aligned}$$

or as

$$q \leq 1 \tag{43}$$

with  $n = 0$ . Thus, the government chooses  $q = 1$  with the appropriate date-0 regulation, and consumes

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$$g_{2,H} = \gamma_H - g_0$$

in the high-income state. It follows from proposition 11, that the government cannot possibly do better than this, as  $q \leq 1$ . Thus, selling all bonds to home country banks is a solution to the government problem. As the government must necessarily be in default in the  $\gamma_L$ -state, if  $q = 1$ , it also follows that this is the unique solution.

In the low-income state, the repayment fraction is given by equation (31) and thus by

$$\theta_L = 1 - \sqrt{1 - \frac{\gamma_L}{g_0}}. \quad (44)$$

Note that the expected return on investing one unit of resources in the government bond is given by

$$R^e = 1 - p_L \sqrt{1 - \frac{\gamma_L}{g_0}} < 1 \quad (45)$$

and that therefore banks in solvent countries will be regulated to invest in these only at their own peril, and then choose not to do so, verifying our initial conjecture. Finally, note that the losses to the common central bank occur due to the bankruptcy of the country in the  $\gamma = \gamma_L$  state. The expected losses are given by (40) and calculate to

$$\begin{aligned} E[L_{CB}](q = 1) &= p_L(1 - \theta_L)^2 g_0 \\ &= p_L(g_0 - \gamma_L), \end{aligned} \quad (46)$$

as the central bank bears the entire default burden of the difference between the promised repayment  $g_0$  on the bonds minus the actual repayment  $\gamma_L$  in the default state.

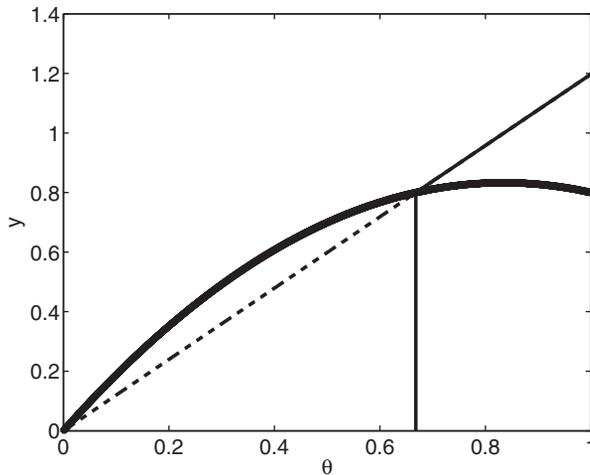
### 3.2. A numerical example

To gain some more insights, it is useful to examine some numerical cases.

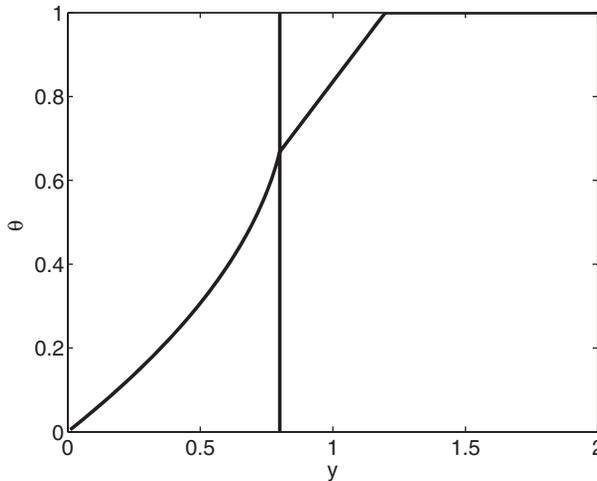
Suppose that  $\alpha = 1$ ,  $g_0 = 1$ ,  $n = 0.2$  and that  $\gamma$  is uniformly distributed on  $[\varepsilon, 2]$  for some tiny  $\varepsilon > 0$ . Note that (1) holds 'nearly' with equality, which implies that selling only to foreign banks as in proposition 10 yields a value very close to  $q = 0.5$ , as well as  $E[g_2]$  very close to zero.

When selling only to home banks as in proposition 9, the numerical calculation yields  $q = 0.835$ ,  $E[g_2] = 0.163$  and the expected losses to the central bank are  $E[L_{CB}] = 0.198$ . As the bond price as well as expected second period consumption of the government is higher than in the solution, when selling only to foreign banks, the government will prefer this solution.

For the equilibrium  $q$ , Figure 2 shows the relation given by the right-hand side of equation (29) between the default rate  $\theta$  and the income  $\gamma$ . Notice how the relationship first follows a quadratic, when taking into account the losses to insolvent banks, and then follows a linear relationship, when banks remain solvent.



**Figure 2** For the equilibrium  $q$ , this figure shows the relation given by the right-hand side of equation (29) between the default rate  $\theta$  and the income  $\gamma$ . Notice how the relationship first follows a quadratic, when taking into account the losses to insolvent banks, and then follows a linear relationship, when banks remain solvent

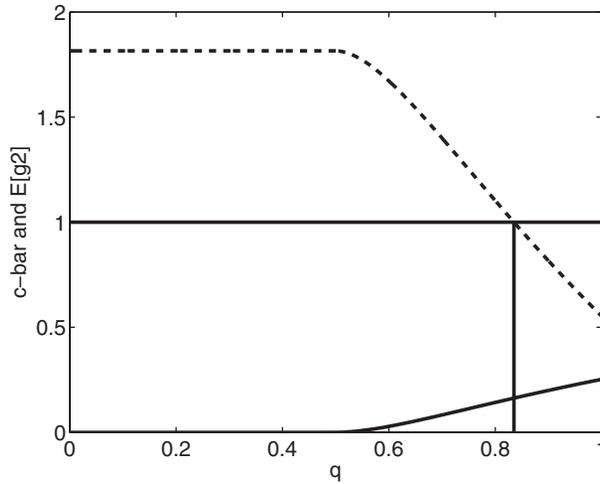


**Figure 3** For the equilibrium  $q$ , this figure flips the relationship shown in (2) and shows the solution for  $\theta$ , given the income realization  $\gamma$ . Notice how the solution first follows the square root law of equation (31) and then follows a linear relationship for  $\gamma \geq \gamma^*$ , until reaching the full repayment point  $\theta = 1$

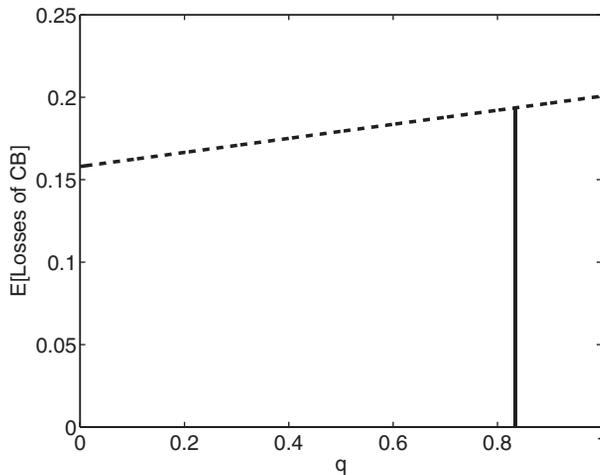
For the equilibrium  $q$ , Figure 3 flips the relationship shown in (2) and shows the solution for  $\theta$ , given the income realization  $\gamma$ . Notice how the solution first follows the square root law of equation (31) and then follows a linear relationship for  $\gamma \geq \gamma^*$ , until reaching the full repayment point  $\theta = 1$ .

In Figure 4, the dashed line shows  $\bar{c}(q)$ , whereas the solid line starting at zero and then sloping upward near  $q = 0.6$  shows  $E[g_2](q)$ . The government chooses  $q$

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**Figure 4** The dashed line shows  $\bar{c}(q)$ , whereas the solid line starting at zero and then sloping upward near  $q = 0.6$  shows  $E[g_2](q)$ . The government chooses  $q$  as high as possible, subject to the attractiveness constraint, i.e. subject to the constraint that the dashed line is just above unity, indicated by a horizontal solid line



**Figure 5** The expected losses to the common central bank as a function of  $q$  are shown: They are increasing with  $q$ , as higher  $q$  means larger losses in the default states

as high as possible, subject to the attractiveness constraint, i.e. subject to the constraint that the dashed line is just above unity, indicated by the horizontal solid line.

The expected losses to the common central bank as a function of  $q$  are shown in Figure 5. They are increasing with  $q$ , as higher  $q$  means larger losses in the default states.

Judged entirely from the perspective of the expected losses, the common central bank would prefer the country to regulate the banks such that a lower  $q$

results: This makes the banks more profitable, induces fewer losses on the central bank, but implies lower expected consumption for the risky country in the second period.

#### 4. CONCLUSIONS

This paper seeks to understand the interplay between banks, bank regulation, sovereign default risk and central bank guarantees in a monetary union. The empirical literature has documented an increasing exposure and shift toward home sovereign bonds, in particular in troubled countries in the eurozone during the eurozone crisis. It has found support for the risk-shifting hypothesis, i.e. that these portfolio strategies enable banks and sovereign regulators to shift some of the sovereign default risks onto the common central bank. The study at hand has provided a model of these developments. I assume that risk neutral bankers can use sovereign bonds for repurchase agreements with a common central bank, and that their sovereign partially backs up any losses should the banks not be able to repurchase the bonds. I argued that regulators in risky countries have an incentive to allow their banks to hold home risky bonds and risk defaults, whereas regulators in other 'safe' countries will impose tighter regulation, and indeed, regulate their banks such that all losses have to be borne privately. Consequently, banks in safe countries will hold these bonds only if their price is at least actuarially fair, whereas banks in the risky country may be allowed by their regulators to risk defaults, as these defaults happen precisely when the country is in default itself, and therefore no longer can and will pay for their share of the bank losses. As a result, governments in risky countries get to borrow more cheaply, effectively shifting some of the potential sovereign default losses on the common central bank. I have analyzed the interplay between the various agencies, and illustrated the result with a numerical example.

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