PRE-COLLEGE HUMAN CAPITAL INVESTMENT AND AFFIRMATIVE ACTION: A STRUCTURAL POLICY ANALYSIS OF US COLLEGE ADMISSIONS*

BRENT R. HICKMAN

Abstract. I study a structural model of college admissions framed as a multi-object, asymmetric, all-pay auction with bid preferences and single-unit demands. Auction theory provides a parsimonious representation of endogenous investment incentives in matching markets. I establish semiparametric identification and show that the identifying assumptions employed are minimal, given the available data. I then propose an estimator based on a two-stage procedure pioneered by Guerre, Perrigne, and Vuong [19] for estimation of first-price bidding models with private information. Structural estimates produce counterfactual experiments to compare color-blind admissions and the current US form of affirmative action (AA) on investment and welfare. An AA ban would result in a large migration of minority students out of the best schools and into the lowest quality schools, as well as a significant drop in minority investment and college graduation rates. A more extreme form of AA, a representative race quota, induces more HC investment by minorities, but involves a larger welfare loss. However, if the social planner trading off racial equity and efficiency is indifferent between the actual US AA and a color-blind alternative, then counterfactual estimates imply that a quota is strictly preferred to both.

Date: **Original version: October 2009; This version: June 2013**.

Key words and phrases. Affirmative Action; all-pay auctions; admission preferences; quotas; racial achievement gap; approximate equilibrium

JEL subject classification: D44, C72, I20, I28, L53.

***NOTE: This version replaces a previous draft circulated under the title "Effort, Race Gaps, and Affirmative Action: A Structural Policy Analysis of US College Admissions".**

I am indebted to Srihari Govindan, Harry J. Paarsch, and Derek Neal for continual feedback and support. I am also grateful for helpful input from (in no particular order) Elena Pastorino, Timothy P. Hubbard, Isabelle Perrigne, Quang Vuong, Parag Pathak, Philipe Fevrier, Xavier D'Haultfoeuille, Steven Durlauf, Jimmy Roberts, Glen Weyl, Alessandra Voena, Suresh Naidu, B. Ravikumar, Guillame Vandenbroucke, and Gustavo Ventura, as well as conference/seminar participants at the University of Iowa, Johns Hopkins University, the University of Chicago, the University of Georgia, the Australian National University, the University of Wisconsin–Madison, the Milton Friedman Institute, the Stanford Institute for Theoretical Economics, Yale University, Brigham Young University, the University of Illinois at Urbana-Champaign, the Pennsylvania State University, Northwestern University, University of California–Berkeley, the Society for Economic Dynamics, Copenhagen Business School, Washington University in St. Louis, Ohio State University, and Cornell University. Any errors are my own.

The following people helped me to acquire the data for this project: Robert Ziomek and Justine Radunzel of ACT, Andrew Mary of the Integrated Postsecondary Education Data System of the US DOE, Wayne Camara and Sherby Jean-Leger of The College Board, and Kevin Brown of NORC. Ted Socha of the National Center for Education Statistics provided excellent support for the Baccalaureate and Beyond data. Brian Christensen provided research assistance in organizing the data.

Department of Economics, University of Chicago, 1126 E. 59th St., Chicago, IL 60637, USA;

E-mail address: HickmanBR@UChicago.Edu;

1. INTRODUCTION

Affirmative Action (AA) is the practice of giving preferential treatment based on race when allocating employment or educational opportunities. It was first mandated by the Kennedy Administration in the 1950s, and has since been widely implemented in public procurement, hiring, and education in the US. Today, AA is a pervasive fixture of American higher education, though it has generated much controversy. At the time of this writing, the US Supreme Court was deliberating on the legality of racial considerations in college admissions in the case of *Fisher v. Texas*. This was the fourth such case to be heard by the high court, being preceded by *Grutter v. Bollinger* (2003), *Gratz v. Bollinger* (2003), and *Regents of the University of California v. Bakke* (1978). Government-mandated AA is also implemented in a host of other countries, including Malaysia, Northern Ireland, South Africa, and India among others.

AA is in general motivated by racial disparity in achievement and college placement. In 1996 18% of all new college freshmen were under-represented minorities (Black, Hispanic, or Native American), but they accounted for only 11% of new enrollees within the top fifth of US colleges, despite substantial considerations for race in the admissions process. This is in turn driven by gaps in academic achievement: in that same year the median minority SAT score was at the 19$^{\text{th}}$ percentile for Whites and Asians. Motivated by these facts, there is a substantial empirical literature studying the allocative and welfare implications of AA in US college admissions. Bowen and Bok [6], Arcidiacono [2] and Howell [23], have attempted estimation of counterfactual racial admissions profiles in a color-blind world. Loury and Garman [27], Sander [32], Long [26], Rothstein and Yoon [31], and Chambers, Clydesdale, Kidder, Lempert [10], Arcidiacono, Aucejo, Fang, and Spenner [4] and Arcidiacono, Aucejo, and Spenner [4] have estimated the impact of AA on graduation rates and/or major choice among Blacks. All of these papers find that AA plays a large role in the US college market.

One important concern that the literature has ignored, however, is the extent to which AA alters students' incentives to invest in pre-college human capital (HC). Since college candidates compete in HC investment for educational opportunities, AA may cause the marginal benefit of study time to diverge for similar-ability individuals of different races. Thus, computing counterfactual policy changes which hold SAT distributions fixed may be incomplete and misleading.

Recent theory developed by Hickman [21] explores this question by pointing out that in matching markets HC investment bears economic returns through two distinct channels. First, it is a productive asset because it prepares students for success and enables them to fully benefit from higher education; this is labeled the *productive channel* of incentives. Second, it bears an indirect return of determining access to high-quality match partners. In particular, colleges' preferences over applicants are largely based on HC investment. Standardized measures like SAT scores allow for efficient matching of better students with more thorough preparation to better schools with higher per-student spending and more accomplished faculty. This is labeled the *competitive channel* of investment incentives since it creates a strategic interaction between one's own investment and competitors' investments: if rivals never study mathematics and science, one might study less and consume more leisure time, and vice versa.

If school quality and HC are complementary inputs in production of value from a match, then AA alters both the productive channel—through interactions of HC with school quality—and the competitive channel—by re-ordering test scores of different race groups on the matching market. However, the overall sign of the incentive effect is unclear. Opponents of AA argue that by artificially lowering admission standards below levels dictated by the market, AA diminishes the need for minority students to compete and thus encourages them to fall behind. The proponents of AA argue just the opposite: by placing within reach better college placement outcomes, AA improves incentives in two ways. First, it "levels the playing field" for minorities who (on average) come from less advantageous socioeconomic backgrounds, but who must compete with other students with more resources. Second, AA provides access to higher-quality post-secondary environments which require more initial investment in order to fully exploit complementarity between student and college inputs. If a student who is headed for a mid-tier state school were given the opportunity to reach an elite private college instead, she might recognize that in order to fully benefit from the experience there she will have to study more during high school. While each of these arguments seems intuitively plausible, a formal model is needed in order to evaluate their relative merits.

Hickman [21] proposes an auction theoretic model of endogenous HC investment in competitive matching markets. He proposes a view of college admissions as a multi-object, asymmetric, all-pay auction, where students make costly investment and then a centralized allocation mechanism replicates the rank-order sorting of the college matching market. Admission rights for seats at colleges of varying quality are analogous to prizes being auctioned off, and SAT scores are analogous to bids, aside from also being a measure of productive human capital. Auction theory creates a parsimonious framework which decomposes investment by the productive and competitive channels, and it shows how arguments on both sides of the AA debate have some validity within different segments of the population. For the best and brightest minority students, if AA assists in attaining placement outcomes that were already in reach, a rational student can only respond by lowering investment somewhat. However, for medium- and low-ability minorities, AA can increase investment by effectively subsidizing their marginal costs of competing with relatively advantaged non-minority students.

Still, a detailed view of the magnitudes of these different effects is an empirical question; *e.g.*, what are the masses of students who increase/decrease HC investment, and by how much? Moreover, other issues raised in the AA debate are also empirical in nature. While AA is expected to diminish total market surplus by weakening the assortativity of the college match, many still argue that a higher degree of racial equality is a social good as well. This begs questions of, how much additional inequality would ensue if AA were banned? What is the price of lower inequality under AA in terms of market surplus? How much wealth is transfered between demographic groups as a result of AA? And finally, how might the answers to these questions change under alternative forms of AA which differ from the current US implementation?

In order to answer these questions and better inform the policy debate, I establish semiparametric identification results and propose an estimator for a structural econometric model of AA and HC based on Hickman [21]. Hickman's auction-theoretic framework has several empirical advantages. It makes available well-developed techniques pioneered by Guerre, Perrigne and Vuong [19, henceforth, GPV] for nonparametric estimation in models of private information. In so doing, it also provides a means to estimate incentives on the entire college market using aggregate data, rather than relying on matched student-college data, which has been a significant constraint on previous AA work. In a two-sided rank-order competition, the quantiles of each side are matched, and each agent chooses her strategy so as to best respond to the distribution of her opponents' actions given her type, all of which can be derived from aggregate, non-matched data.

The ultimate objective of the empirical exercise is to set up counterfactual comparisons between status-quo AA and alternative policies not observed in the data. Structural estimates enable synthetic policy experiments in which I compare investment, welfare, and inequality under three alternative college admission regimes: *admission preferences* or SAT score subsidies as implemented by American colleges, *race quotas*, or earmarking of seats in proportion to demographic shares as implemented in India, Malaysia, and South Africa, and *color-blind* admissions which ignore race.

The counterfactual results indicate that the American-style admission preference greatly improves both investment incentives and market outcomes for minority students. If AA were banned, minority enrollment in the top quintile of colleges would decrease by 52.4%. For most minority students there is a significant increase in investment due to AA, with the the gap between median SAT scores narrowing by 14%, relative to a color-blind mechanism. The reason for the improved incentives is that American AA is roughly an SAT markup with a positive slope that rewards students for higher investment with a larger markup. In other words, American AA effectively subsidizes *marginal costs* of HC production and induces most minorities to invest as if their costs were lower. American AA also discourages investment among the highest performing minorities, but this group is very small, amounting to only the top 1.6% of the group. A quota mechanism works similarly, but in fact produces even stronger incentives for minorities and even non-minorities too, leading to an increase in HC investment among 87% of all students in the market. Both forms of AA substantially improve minority projected graduation rates and post-college income, through a combination of higher investment and better college placement. Of course, these gains require wealth transfers away from non-minorities and a loss of total market surplus due to a weakening of the assortativity of the college match, but both are fairly small.

It is difficult to rank the US admission preference and a color-blind mechanism overall: the former results in lower total surplus but also lower racial inequality of market surplus. However, a system of racial quotas may be arguably superior. I consider various specifications of social preferences over total surplus and race inequality, and I compute the form of the social utility function by assuming that the planner is indifferent between the admission preference which was implemented and the foregone alternative color-blind system. For the derived social preferences it turns out that a quota is strictly preferred to both for achieving a more favorable mixture of inequality and surplus. In other words, the counterfactual study suggests that if society is at least as well off under the status-quo AA as under color-blind admissions in terms of the equity–efficiency trade-off alone, then a quota is strictly preferred to both. Though quotas are illegal in the US and cannot be implemented through explicit earmarking of college seats, it is possible to construct a simple equivalent admission preference scheme which mimics the incentives and outcome under a quota.[1] This alternative policy has another interesting property of being a self-adjusting AA rule that naturally phases itself out as fundamental racial asymmetry of investment costs diminishes.

The remainder of this paper has the following structure: I first briefly summarize the previous literature on AA and discus its relation to the current model. In Section 2, I briefly outline the theoretical model on which the econometric exercise is based. In Section 3, I describe the US college data that will be used to identify and estimate the model. In Section 4 I show that the model is nonparametrically non-identified, but that a parametric assumption on human capital production costs combined with a restriction

[1]The US Supreme Court Ruling in *Regents of the University of California v. Bakke*, 438 U.S. 265 (1978) established the unconstitutionality of explicit quotas in the US.

on a single quantile of the type distribution is enough to achieve semiparametric identification. The resulting empirical model also places testable restrictions on the observables. In Section 4.5, I outline a two-stage estimator for the structural model, based on methods pioneered by GPV in the empirical auctions literature. In Section 5, I discuss the results of estimation and in 6 I present the counterfactual exercise. Section 7 concludes and briefly describes directions for future research.

1.1. **Related Literature.** Bowen and Bok [6] were among the first to study how actual AA policies change college admissions. In their book *The Shape of the River* (Chapter 2), they estimate the effect of race on the probability of being admitted to a selective college, and then compute a counterfactual estimate of how many black students would not have been admitted if AA was absent. They find that AA plays a substantial role in allocating high-quality college seats to minorities. Chung and Espenshade [11] and Chung, Espenshade, and Walling [12] undertake a similar study using matched student-college applications data from a small set of elite private universities to similar results.

Several other papers have taken a structural approach to studying the impact of AA. Epple, Romano, and Sieg [17] model colleges setting tuition and admissions standards for different races. These decisions are based on tastes for racial diversity and quality of education, which depends on college inputs and quality of the student body. They calibrate the model to the US market and find that the data are broadly consistent with a model involving diversity preferences on the part of colleges. Two additional papers structurally estimate the college admissions process in order to control for changes in application behavior from counterfactual policy shifts. Arcidiacono [2] models applications, admissions, enrollment, major choice, and entrance into the labor market. He then estimates counterfactual comparisons of observed black market outcomes with those arising when AA is eliminated. Howell [23] performs a similar exercise with a more recent data set, but focusing solely on the admissions process. Both papers find that AA plays a significant role in shaping black educational outcomes, especially among the most selective institutions.

Another vein of empirical literature on AA focuses on *mismatching*, or the idea being that, AA may cause black students to be placed higher, but then graduate with lower probability or self-select into less lucrative (but less demanding) majors. While it is known that AA improves the profile of colleges where minority freshmen enroll, much less is known of the profile of minority graduates that a given AA policy will produce. Some empirical studies which have found evidence of mismatching are Loury and Garman [27], Sander [32], Arcidiacono, Aucejo, Fang, and Spenner [3], and Arcidiacono, Aucejo, and Spenner [4]. Other empirical work has argued that, while mismatching may occur, its magnitude is small and is outweighed by the benefits of higher-quality placement for blacks. Some Examples are Long [26], Rothstein and Yoon [31], and Chambers, Clydesdale, Kidder, and Lempert [10].

In most of the papers above SAT scores are used as a proxy for student ability, and assumed to be fixed with respect to variations in admissions criteria. However, if SATs are jointly determined by both ability and market-based incentives for investment, then this is problematic. Recent empirical work by Ferman and Assunção [18] suggests this scenario. They use data from Brazilian natural experiment when political forces abruptly imposed an admissions quota for two of Rio De Janeiro's top public universities. They estimate that the quota altered incentives so as to produce a 25% widening of the achievement gap. A field experiment study by Cotton, Hickman, and Price [14] seeks to replicate the Hickman [21] framework in a classroom setting in order to directly measure shifts in investment incentives. Cotton *et. al.* set up a math competition among middle school students, with AA in place to assist students from lower grades who have less math preparation. They directly monitor investment of time into the learning process and document an average increase under AA. These studies suggest that HC investment is influenced by AA, and therefore a model must take this into account when producing counterfactual estimates of college placement under race-neutral admissions. The current study takes a structural approach to control for behavioral shifts induced by changes to market allocation rules. This facilitates other market design questions as well—*e.g.*, how do American allocation rules compare to other simple mechanisms implemented elsewhere?—a task to which the auction-theoretic framework is well-suited.

At the same time, this work is subject to its own limitations. I abstract from the intricacies of the admissions process, and concentrate on the link between achievement and final college placement outcomes in terms of matriculation. Since I have non-matched data on characteristics of college applicants and colleges, the goal of the current exercise is to estimate the link between policy, HC distributions, and college placement *on the aggregate*, rather than focusing on individual-level decisions and outcomes. I also do not explicitly model "supply-side" concerns—*e.g.*, decisions concerning how many students to admit—instead modeling college seats as fixed objects of known quality in order to concentrate just on student HC investment. To the extent that factors such as matching frictions and supply-side competition play a role, relative to the costs and benefits of HC investment, this work can be seen as complementary to college admissions models such as Chade, Lewis, and Smith [9], Epple, *et. al.* [17], Arcidiacono [2], and Howell [23] who treat these forces explicitly.

A final related paper is Cestau, Epple, and Sieg [8], which studies AA in elementary-school gifted programs, but it differs from the current model in two ways. First, a primary focus is on *profiling*, or the task of proactively searching for promising minority candidates at the pre-admissions stage. Second, their focus on elementary education justifies an assumption that student types are fixed, whereas such a restriction is inappropriate for college admissions where students and parents have more time and latitude to respond to market incentives through investment.

2. THE THEORETICAL MODEL

Following Hickman [21], I model college admissions as a Bayesian game where high-school students are characterized by a privately-known type that governs the costliness of HC production. Students compete matching rights at colleges of differing quality, and colleges' preferences over candidates is based on their HC and race. The decentralized assortative match is replicated by a centralized mechanism allocating match rights by the rank ordering of HC, taking into account preferences for race. Students observe the set of colleges and the form of the sorting mechanism before acting, but they must incur a non-recoverable investment cost *before* entering the match market. A student's *ex-post* payoff is her match utility minus her investment cost.

2.1. **Agents.** The set of high-school students is denoted $\mathcal{K} = \{1, 2, \ldots, K\}$, but there are two demographic subgroups, $\mathcal{M} = \{1, 2, \ldots, K_{\mathcal{M}}\}$ (minorities) and $\mathcal{N} = \{1, 2, \ldots, K_{\mathcal{N}}\}$ (non-minorities), where $K_{\mathcal{M}} + K_{\mathcal{N}} = K$ and demographic class is observable. Each student has a privately-known cost type $\theta \in [\underline{\theta}, \overline{\theta}]$, and they view their opponents' independent random variables Θ whose realizations follow group-specific distributions, or $\Theta \sim F_i(\theta)$, $i = \mathcal{M}, \mathcal{N}$. For convenience, I denote the unconditional type distribution by $F_{\mathcal{K}}(\theta) \equiv \frac{K_{\mathcal{M}}}{K_{\mathcal{M}}+K_{\mathcal{N}}} F_{\mathcal{M}}(\theta) + \frac{K_{\mathcal{N}}}{K_{\mathcal{M}}+K_{\mathcal{N}}} F_{\mathcal{N}}(\theta)$, and the asymptotic mass of the minority group will be denoted μ.[2] Each agent's strategy space, $\mathcal{S} = [\underline{s}, \infty)$, is the set of attainable HC levels. These are observable (say, through a standardized exam like the SAT) and $\underline{s}$ is the minimum required to attend college. In order to produce s units of HC, one incurs cost $C(s; \theta)$ which is increasing in both s and θ.[3]

2.2. **Payoffs.** On the other side of the market there is a fixed and observable set of match partners (colleges) $\mathcal{P}_{\mathcal{K}} = \{p_1, p_2, \ldots, p_K\}$, where $p_k \in [\underline{p}, \overline{p}]$ is the quality level of the k^{th} college, and $p_k \neq p_l$, $\forall k \neq l$. Although agents observe $\mathcal{P}_{\mathcal{K}}$, it will be useful to model the rank ordering as being generated by a "prize distribution", or $P_k \sim F_P(p)$ $\forall k$. I denote the

[2] That is, when nature creates the k^{th} agent, she assigns her to the minority group with probability μ and then draws an *iid* type θ_k from the appropriate distribution, so that $\lim_{K \to \infty} K_{\mathcal{M}} / (K_{\mathcal{M}} + K_{\mathcal{N}}) = \mu$.

[3] Costs can arise in various ways. They could come from a consumption–investment tradeoff, where θ indexes preferences for foregone leisure time or consumption. Alternatively, θ could represent idiosyncratic psychic disutility from exerting effort to learn. For the purpose of the current study, I shall remain agnostic on the exact interpretation of θ: types are simply the exogenous portion of HC production costs.

k^{th} order statistic by $p(k:K)$ so that $\min_k\{\mathcal{P}_\mathcal{K}\} = p(1:K)$ and $\max_k\{\mathcal{P}_\mathcal{K}\} = p(K:K)$. By assuming the number of competitors and college seats are the same I abstract from the extensive margin of college attendance, focusing only on the intensive margin of competition for admission to the best colleges, *conditional on entering the market*.[4]

Both college quality and HC are valued: utility $U(p,s)$ results from having human capital s and being matched to a college with quality p.[5] Letting $\mathbf{s}$ denote the vector of all players' strategies, the *ex post* payoff to agent i in group $j = \mathcal{M}, \mathcal{N}$ is the match utility minus the cost of achievement, or $\Pi_j(s_i, \mathbf{s}_{-i}; \theta) = U\left[P_j(s_i, \mathbf{s}_{-i}), s_i\right] - C(s_i; \theta_i)$, where $P_j(s_i, \mathbf{s}_{-i})$ is an allocation mechanism that distributes seats according to achievement (and possibly race as well), and $\mathbf{s}_{-i}$ denotes the strategy profile of of i's opponents.

2.2.1. *Allocation Mechanisms.* Strategies are mapped into payoffs within a frictionless matching market whose pairwise stable equilibrium is positively assortative on HC and college quality. Such an equilibrium can be implemented by a centralized auction mechanism which uses HC $\mathbf{s} = \{s_{\mathcal{M}1}, \ldots, s_{\mathcal{M}K_\mathcal{M}}, s_{\mathcal{N}1}, \ldots, s_{\mathcal{N}K_\mathcal{N}}\}$ to allocate matching rights among students in rank-order fashion. I allow for the decentralized market to involve a preference for racial diversity (on the part of colleges), such that deviations from assortative matching across demographic groups are possible, but within groups matching remains assortative. The baseline *color-blind* mechanism is $P_j^{cb}(s_i, \mathbf{s}_{-i}) = P^{cb}(s_i, \mathbf{s}_{-i}) = \sum_{k=1}^{K} p(k:K)\mathbb{1}\left[s_i = s(k:K)\right]$, where $\mathbb{1}$ is an indicator function.

As for AA, I concentrate on two canonical forms that have received attention due to wide implementation: *quotas* and *admission preferences*. A quota means earmarking seats for each group in proportion to that group's mass, and then asortatively allocating seats within each group. It's defining characteristics are a split of the competition into two separate race-specific games, with both groups of students matching to sets colleges of comparable quality.[6] Let $\mathcal{P}_\mathcal{M} = \{p_{\mathcal{M}1}, p_{\mathcal{M}2}, \ldots, p_{\mathcal{M}K_\mathcal{M}}\}$ and $\mathcal{P}_\mathcal{N} = \{p_{\mathcal{N}1}, p_{\mathcal{N}2}, \ldots, p_{\mathcal{N}K_\mathcal{N}}\}$ denote the sets of earmarked seats, and let $\mathbf{s}_\mathcal{M} = \{s_{\mathcal{M}1}, s_{\mathcal{M}2}, \ldots, s_{\mathcal{M}K_\mathcal{M}}\}$ and $\mathbf{s}_\mathcal{N} = \{s_{\mathcal{N}1}, s_{\mathcal{N}2}, \ldots, s_{\mathcal{N}K_\mathcal{N}}\}$ denote the group-specific HC profiles. Then a quota mechanism is $P_j^q(s_i, \mathbf{s}_{-i}) = \sum_{m=1}^{K_j} p_j(m:K_j)\mathbb{1}\left[s_i = s_j(m:K_j)\right]$, $j = \mathcal{M}, \mathcal{N}$.

Admission preferences are a more flexible class of mechanisms where minority SAT scores are given more weight than non-minority counterparts. Formally, an admission

[4]As I argue later, given the US college data, my identifying assumptions, and the counterfactuals I consider, this abstraction is inconsequential, but for the sake of clarity I defer discussion to Section 4.4.

[5]Note that this student-centric view of college admissions corresponds to a setting of matching with *non-transferable utility*.

[6]For large K, the "comparable" property can be accomplished by randomly selecting $K_\mathcal{M}$ of them or by ordering and then selecting out each $K\left(K_\mathcal{M}/(K_\mathcal{M}+K_\mathcal{N})\right)^{\text{th}}$ seat for minority candidates.

preference is a markup function $\widetilde{S} : \mathcal{S} \to \mathbb{R}_+$ which produces a set of transformed human capital levels, $\widetilde{\mathbf{s}} = \{s_{\mathcal{N}1}, \ldots, s_{\mathcal{N}K_{\mathcal{N}}}, \widetilde{S}(s_{\mathcal{M}1}), \ldots, \widetilde{S}(s_{\mathcal{M}K_{\mathcal{M}}})\}$, and allocations follow $P^{ap}_{\mathcal{M}}(s_i, \widetilde{\mathbf{s}}_{-i}) = \sum_{k=1}^{K} p(k:K)\mathbb{1}\left[\widetilde{S}(s_i) = \widetilde{s}(k:K)\right]$ for minorities, and $P^{ap}_{\mathcal{N}}(s_i, \widetilde{\mathbf{s}}_{-i}) = \sum_{k=1}^{K} p(k:K)\mathbb{1}\left[s_i = \widetilde{s}(k:K)\right]$ for non-minorities. This class is empirically attractive for its generality: it nests color-blind admissions as a special case (*i.e.*, when $\widetilde{S}(s) = s$), and if $\widetilde{S}$ maps the minority HC quantiles into the non-minority quantiles, it can even behave like a quota. Regardless of the mechanism, ties are assumed to be broken randomly.

Before investing, agents observe the set of seats $\mathcal{P}_{\mathcal{K}}$, the admission rule, $r \in \{cb, q, ap\}$, and the numbers of competitors $K_{\mathcal{M}}$ and $K_{\mathcal{N}}$. Under the payoff mapping $\Pi^r(s_i, \mathbf{s}_{-i}; \theta)$ induced by mechanism r, students optimally invest based on their type, the types of potential match partners, and competition from opponents. The model defined above constitutes an asymmetric, multi-object, all-pay auction with single-unit demands. Intuitively, the centralized admissions board is analogous to an auctioneer selling off a set of heterogeneous prizes according to a rank-order mechanism, and students are like bidders with HC as their bids. The centralized auction mechanism provides a parsimonious characterization of endogenous HC investment within a complex market setting where HC plays dual roles of a productive asset and determining one's match prospects. Moreover, auction theory is a well-tailored framework for studying incentive shifts from changes in allocation mechanisms. Each admission rule $r \in \{cb, q, ap\}$ effectively defines a different pricing rule, since it (endogenously) determines the number of human capital units an individual must produce to win a given seat. Changes in bidding behavior across alternative pricing rules has been a central theme of auction theory since the seminal work of Vickrey [34], Wilson [35], and Milgrom and Weber [29].

A *group-wise symmetric equilibrium* of the game $\Gamma(K_{\mathcal{M}}, K_{\mathcal{N}}, F_{\mathcal{M}}, F_{\mathcal{N}}, \mathcal{P}_{\mathcal{K}}, r, U, C, \mathcal{S})$ is a set of functions $\sigma_j : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}_+,\ j = \mathcal{M}, \mathcal{N}$ which generate optimal HC investment $s = \sigma_j(\theta)$, given that ones' opponents behave similarly. For convenience, I denote the inverse investment functions by $\psi_j(s) \equiv \sigma_j^{-1}(s) = \theta$. Hickman [21] establishes existence, uniqueness, and strict monotonicity, $\sigma_j'(\theta) < 0\ \forall\theta$, using results from auction theory proven by Athey [5]. These conditions are based on the following assumptions.

2.3. **Model Assumptions.**

Assumption 2.1. $U_1(p,s) > 0$, $U_2(p,s) \geq 0$, *and* $U_{22}(p,s) \leq 0$ *for each* $(p,s) \in [\underline{p}, \overline{p}] \times \mathcal{S}$.

Assumption 2.2. $C'(s;\theta) > 0$, $C''(s;\theta) \geq 0$, *and* $\frac{\partial C}{\partial \theta} > 0$ *for each* $(s,\theta) \in \mathcal{S} \times [\underline{\theta}, \overline{\theta}]$.

Assumption 2.3. $\arg\max_{s \in \mathcal{S}} \left\{ U(\underline{p}, s) - C(s; \overline{\theta}) \right\} = \underline{s}$

Assumptions 2.1 and 2.2 are regularity conditions so that individuals' decision problems will have well-defined maximizers: utility is differentiable, strictly increasing in college quality, and weakly increasing and concave in HC, with analogous conditions on costs. Assumption 2.3 is similar to a market participation constraint, and provides a boundary condition $\sigma^r_{\mathcal{N}}(\overline{\theta}) = \underline{s}$, $r \in \{cb, q, ap\}$.

Assumption 2.4. $\frac{\partial^2 C}{\partial s \partial \theta} > 0$ *for each* $(s, \theta) \in \mathcal{S} \times [\underline{\theta}, \overline{\theta}]$.

Assumption 2.5. *For each* $(p, \theta) \in [\underline{p}, \overline{p}] \times [\underline{\theta}, \overline{\theta}]$, $\exists\, s_{p\theta}$ *s.t.* $C'(s; \theta) > U_2(p, s)$, $\forall s > s_{p\theta}$.

Assumption 2.6. $F_j(\theta) \in \mathcal{C}^2$, $j = \mathcal{M}, \mathcal{N}$ *and densities* $f_{\mathcal{M}}(\theta)$ *and* $f_{\mathcal{N}}(\theta)$ *are strictly positive on a common compact support* $[\underline{\theta}, \overline{\theta}]$ *with non-empty interior.*

Assumption 2.7. $F_P(p) \in \mathcal{C}^2$ *and the prize density* $f_P(p)$ *is strictly positive on a compact support* $[\underline{p}, \overline{p}]$ *with non-empty interior.*

Assumptions 2.4 - 2.6 are key for existence of a monotone pure-strategy equilibrium. Assumption 2.4 is a single-crossing condition that produces strategic complementarity—meaning that agents' best responses are increasing in the strategies of their opponents—since a smaller θ means both a lower cost of ataining s and a lower marginal cost of increasing from s to $s + \varepsilon$. Assumption 2.5 states that there is a finite choke price for every (p, θ) pair, and is needed to rationalize finite investment. Assumptions 2.6 - 2.7 are standard regularity conditions on the type and prize distributions.

Assumption 2.8. $U_{12}(p, s) \geq 0$ *for each* $(p, s) \in [\underline{p}, \overline{p}] \times \mathcal{S}$.

Assumption 2.9. $\widetilde{S}'(s) > 0 \;\; \forall s \in \mathcal{S}$.

Assumption 2.8 states that HC and school quality are complementary inputs into match value. It justifies the assumption of decentralized assortative matching, which in turn justifies the centralized rank-order mechanisms I have defined. Of course, assortative matching also requires assumptions on colleges' preferences as well. These are encapsulated in Assumption 2.9: $\widetilde{S}$ is strictly increasing, so it does not rearrange orderings within demographic groups: colleges value HC, and their preferences for HC are superseded only partially by their preferences for racial diversity. The difference $\widetilde{S}(s) - s$ is therefore a measure of how much HC they are willing to trade off for diversity, but holding race fixed, colleges always prefer to enroll a student with more HC.

2.4. **Equilibrium.** For even moderately large K, the equilibrium becomes unwieldy since expected payoffs are weighted averages of all possible match utilities, where the weight on the k^{th} utility is one's probability of being the k^{th} order statistic among competitors.

However, Hickman [21] showed that for large K the equilibrium can be approximated by considering the limiting payoff function as $K \to \infty$, effectively treating prizes and competitors as continuous, rather than discrete. The limiting allocation mechanisms under a color-blind, quota, and admission preference regime are

$$P^{cb}_{\mathcal{M}}(s) = P^{cb}_{\mathcal{N}}(s) = P^{cb}(s) = F_P^{-1}\left[G_{\mathcal{K}}(s)\right], \tag{1}$$

$$P^{q}_{j}(s) = F_P^{-1}\left[G_j(s)\right],\ j = \mathcal{M}, \mathcal{N},\ \text{and} \tag{2}$$

$$\begin{aligned} P^{ap}_{\mathcal{M}}(s) &= F_P^{-1}\left[\mu G_{\mathcal{M}}(s) + (1-\mu) G_{\mathcal{N}}\left(\widetilde{S}(s)\right)\right] \\ P^{ap}_{\mathcal{N}}(s) &= F_P^{-1}\left[\mu G_{\mathcal{M}}\left(\widetilde{S}^{-1}(s)\right) + (1-\mu) G_{\mathcal{N}}(s)\right], \end{aligned} \tag{3}$$

respectively, where $G_j(s)$ is a HC distribution for $j = \mathcal{M},\ \mathcal{N},\ \mathcal{K}$. The intuition behind each of these expressions is simple: the admissions board determines the quantile rank of a given s within the appropriate distribution, and then awards the right to match with the same quantile rank among colleges. In a color-blind system, the relevant quantile rank is with respect to the population HC distribution, whereas in a quota system the quantile rank is only relative to one's own demographic group. The limiting admission preference in equation (3) takes into account the re-ordering across race groups when computing quantile ranks relative to the distribution of transformed HC levels. Note that minority standings improve relative to non-minorities when $\widetilde{S}(s) > s$.

The limiting payoff function is $\max_{s \in \mathcal{S}} \left\{U\left[P(s), s\right] - C(s;\theta)\right\}$, with FOC

$$U_1\left[P(s), s\right] P'(s) + U_2\left[P(s), s\right] = C'(s;\theta). \tag{4}$$

The first term on the left-hand side of equation (4) depicts the competitive channel of investment incentives: a one unit increase in s produces a $P'(s)$-unit increase in the quality of one's match, which in turn renders marginal utility $U_1\left[P(s), s\right]$. The second term on the left-hand side is the standard productive channel of incentives: $U_2\left[P(s), s\right]$ is the direct marginal return to investment. This direct effect alone would be equated with marginal costs in the full information world if the planner were to perform matching directly on cost types. The presence of the competitive channel implies over-investment relative to the full information case. In other words, investment costs beyond those associated with $\sigma^*(\theta) \equiv \arg\min_{s \in \mathcal{S}} \left| U_2\left[F_P^{-1}[F_{\mathcal{K}}(\theta)], s\right] - C'(s;\theta)\right|$ are the price society pays in order to efficiently match low-cost types with high-quality schools.

Explicitly differentiating equation (4) solves for the maximizers of the limiting objective function. For the remainder of the paper I shall concentrate on the admission preference case and drop the superscript for the allocation scheme unless context requires specificity. The investment functions solve the following system of differential equations:

$$
\begin{aligned}
(5)\qquad & -U_1[P_{\mathcal{N}}(s),s]\cdot\frac{(1-\mu)f_{\mathcal{N}}\left[\psi_{\mathcal{N}}(s)\right]\psi'_{\mathcal{N}}(s)+\mu f_{\mathcal{M}}\left[\psi_{\mathcal{M}}(\widetilde{S}^{-1}(s))\right]\psi'_{\mathcal{M}}(\widetilde{S}^{-1}(s))\frac{d\widetilde{S}^{-1}(s)}{ds}}{f_P\left[P_{\mathcal{N}}(s)\right]} \\
& \qquad\qquad\qquad + U_2[P_{\mathcal{N}}(s),s] = C'\left(s;\psi_{\mathcal{N}}(s)\right) \\
& -U_1[P_{\mathcal{M}}(s),s]\cdot\frac{(1-\mu)f_{\mathcal{N}}\left[\psi_{\mathcal{N}}(\widetilde{S}(s))\right]\psi'_{\mathcal{N}}(\widetilde{S}(s))\widetilde{S}'(s)+\mu f_{\mathcal{M}}\left[\psi_{M}(s)\right]\psi'_{\mathcal{M}}(s)}{f_P\left[P_{\mathcal{N}}(s)\right]} \\
& \qquad\qquad\qquad + U_2[P_{\mathcal{M}}(s),s] = C'\left(s;\psi_{\mathcal{M}}(s)\right) \\
& \psi_{\mathcal{N}}(\underline{s}) \;=\; \overline{\theta} \text{ (boundary condition).}
\end{aligned}
$$

Hickman [21] showed that the solution to this system constitutes an *approximate equilibrium* of the finite game $\Gamma(K_{\mathcal{M}}, K_{\mathcal{N}}, F_{\mathcal{M}}, F_{\mathcal{N}}, \mathcal{P}_{\mathcal{K}}, r, U, C, \mathcal{S})$, or a set of functions that approximate equilibrium strategies and payoffs to arbitrary precision for large enough K. An alternative interpretation is that these maximizers are an approximation employed by agents themselves, rather than the researcher. If mental accounting is costly or if agents are cognitively constrained, then in large markets—*i.e.*, where the data and calculations to exactly represent the market become sufficiently complex—they may simplify computation of the optimum by viewing the market through the lens of equations (1) - (3), and then employ (5) as behavioral strategies. I now move to a discussion of the data in order to justify my large-market approximation.

3. DATA

The structural approach is helpful in overcoming two challenges to previous empirical work on AA: first, controlling for endogenous investment responses to rule change, and second, eliminating dependence upon matched student-school data, which are typically only available for elite private colleges. Given the nature of an empirical auction model, aggregate data are enough to characterize the action distributions to which agents best respond in equilibrium, which makes it possible to empirically map observed investment s into the underlying unobserved type θ. This approach was pioneered by Guerre, Perrigne, and Vuong [19, henceforth, GPV], and will be discussed at length in the next section on identification and estimation. Here I describe the set of observables used to identify the structural model.

3.1. **US College Admissions Market: 1996.** I use US college data for the academic year 1995-1996 for two main reasons. First, one can reasonably assume that, prior to that year, AA policies were stable and understood by decision-makers. In summer of 1996 the outcome of a federal lawsuit *Hopwood v. Texas* (78 F.3d 932, 5^{th} Cir. 1996) was finalized, marking the first successful legal challenge to AA in US college admissions since 1978.[7] Shortly after, other AA legal changes occurred, including state laws banning AA in Texas, California, and Michigan. The second reason for studying the 1995-1996 admissions cycle is that other unique data are available (the Baccalaureate and Beyond database) which allow for a link to salary premiums from college attendance. Thus, this specific case study investigates how AA shaped the college landscape for the parents of today's elementary and middle-school children.

3.2. **Colleges.** For a sample $\mathcal{L} = \{1, 2, \ldots, L\}$ of American four-year undergraduate colleges, I have a vector $\mathbf{X}_l$ of institutional characteristics. The first is a quality measure derived from data and methodology by US News & World Report (henceforth, USNWR) for their annual *America's Best Colleges* rankings (see Morse [30]).[8] This I interpret as college quality index p_l, and it plays the role of ranking of students' potential match partners. I argue that interpreting this index as a meaningful reflection of students' perceived value rankings is sensible for two reasons. First, USNWR solves market information frictions by providing a wealth of data on many schools, along with advice on how to interpret and summarize them. Indeed, the market response to this service has been large enough that rankings are now the primary focus of USNWR's business model and its sole remaining print publication. Second, the validity of USNWR rankings is undoubtedly reinforced in students' minds by the enthusiasm with which so many schools advertise their status in *America's Best Colleges*. A typical undergraduate admissions web page will contain at least some reference to USNWR.

The other data representing colleges are enrollment of first-time freshmen by race, provided by the National Center for Education Statistics (NCES) through their Integrated Postsecondary Education Data System (IPEDS) tool. For each school I have a tally of all first-time freshmen (including full-time and part-time) for the following 7 racial classifications: White, Black, Hispanic, Asian or Pacific Islander, American Indian or Alaskan Native, non-resident alien, and race unknown. The data representing schools are $\{p_u, M_u, N_u\}_{u=1}^{U}$, where for the u^{th} school p_u is the USNWR quality index, M_u

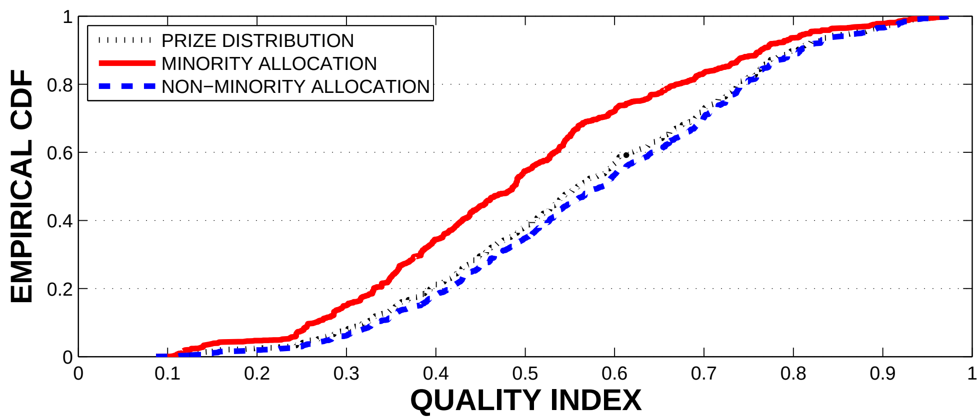
[7]On March 18, 1996 the US Fifth Circuit Court disallowed race-conscious admissions at the University of Texas law school, but appeals continued for several months afterward until the Supreme Court declined to review the Fifth Circuit's ruling. The previous successful legal challenge was in 1978, when the Supreme Court declared quotas unconstitutional in *University of California v. Bakke* (438 U.S. 265 1978).

[8] USNWR collects data on fourteen quality indicators for each institution and condenses them to a single index. These indicators include, among other things, spending per student, student-faculty ratios, and graduation rates. The measure I use for this study implements a small modification to the original USNWR methodology in order to produce a single ranking that applies to all colleges in the sample, rather than dividing them by region and Carnegie classification as in Morse [30]. Further details on the USNWR methodology and my modification are outlined in an appendix.

Table 1. Racial Representation Within Academic Quality Quintiles

Tier ↓	Black 11.19%	Hispanic 5.70%	American Indian/ Alaskan Native 0.76%	White 71.97%	Asian/ Pacific Islander 5.71%	$\mathcal{M}$ 17.65%	$\mathcal{N}$ 82.35%
I	5.45%	5.45%	0.43%	73.69%	10.17%	**11.33%**	**88.67%**
II	5.54%	4.63%	0.60%	78.06%	6.64%	**10.77%**	**89.23%**
III	10.97%	4.89%	0.76%	74.59%	4.27%	**16.62%**	**83.38%**
IV	13.53%	6.11%	0.93%	70.57%	4.53%	**20.57%**	**79.43%**
V	20.73%	7.33%	1.11%	63.70%	2.43%	**29.17%**	**70.83%**

Figure 1. College Seat Allocations



is the number of seats awarded to minorities—comprising Blacks, Hispanics, and Native Americans—and N_u is the number of seats occupied by non-minorities. These data characterize prizes and market allocations, denoted $\mathcal{P}_{\mathcal{K}} = \{p_k\}_{k=1}^{K} = \{\{p_{li}\}_{i=1}^{M_l+N_l}\}_{l=1}^{L}$, $\mathcal{P}_{\mathcal{M}} = \{p_m\}_{m=1}^{M} = \{\{p_{li}\}_{i=1}^{M_l}\}_{l=1}^{L}$, and $\mathcal{P}_{\mathcal{N}} = \{p_n\}_{n=1}^{N} = \{\{p_{li}\}_{i=1}^{N_l}\}_{l=1}^{L}$, respectively.

Figure 1 and Table 1 depict the related distributions and illustrate why the groups $\mathcal{M}$ and $\mathcal{N}$ are defined as they are. AA policies are traditionally targeted toward helping *under-represented* race groups. Note that the $\mathcal{N}$ allocation dominates the $\mathcal{M}$ allocation in the first-order sense, with two-thirds of non-minorities attending schools with a quality index of 0.5 or better, while only half of minorities enroll in the same market segment. In Table 1 schools are sorted in descending order of p and separated into five tiers, with each containing one fifth of the seats in the market. Table entries contain the mass of each race group within each tier. "Minority" race classes are all under-represented in the top two tiers and over-represented in the bottom two.

Figure 1 supports the large atomless market approximation to investment decisions with no large jumps in the empirical CDFs. The dotted line represents a total of 1,056,580 seats in the market, with any single school having only a negligible market share. The largest one in 1996 (Michigan State) filled 6,989 new freshman seats for a total market share of only 0.66%, and the next ten largest schools (in descending order of size: Purdue, UT-Austin, Texas A&M, Ohio State, U of Illinois at Urbana-Champaign, U of Wisconsin at Madison, U of Michigan at Ann Arbor, Indiana U, Virginia Tech, and Rutgers) combined for only 5.4% of total market supply. The mean, median, and standard deviation seat count within a given school are 804.09, 451, and 934.78, respectively, which translate into market shares of 0.000761, 0.000427, and 0.000885, respectively.

Three other school-level variables will be used to estimate the match utility function: median SAT score (or SAT equivalent of median ACT score) for new freshmen, denoted $S_{0.5,l}$, 6-year graduation rates for the 1996 fall adjusted cohort, denoted ρ_l (both provided by IPEDS), and the third variable is an estimate of average 10-year annual salary for students who graduated during the 1993-1994 academic year, denoted u_l.[9] The salary measure was derived from the Baccalaureate and Beyond Survey (B&B), which randomly samples colleges and then randomly samples graduates within a given college. The survey includes annual salaries for graduates, after 10 years in the workforce, and within each school I average over this number for employed graduates not seeking post-graduate education, in order to get an estimate of the expected mid-career salary within a given school. Finally, I also acquired a measure of the outside option from the Bureau of Labor Statistics, being average annual salary for high-school completers in 2002, denoted u_0. Descriptive statistics are reported for institutional data in Table 2.[10]

3.3. **Investment Data.** For 1996 graduating high-school seniors, I have individual data on composite SAT scores, race, and other characteristics for a random sample of 76,757 students (after eliminating individuals over age 25 to focus on first-time freshmen). SAT scores range between 400 and 1,600 in increments of 10, but I drop the final digit. The sample of SATs is denoted $\mathbf{S}_{\mathcal{K},T} = \{s_t, d_t\}_{t=1}^{T}$, where d_t is the demographic of the t^{th} test taker and T is the sample size. The racial investment gap is illustrated in Figure 2 where

[9] An adjusted cohort corrects the graduation rate number for transfer-outs.

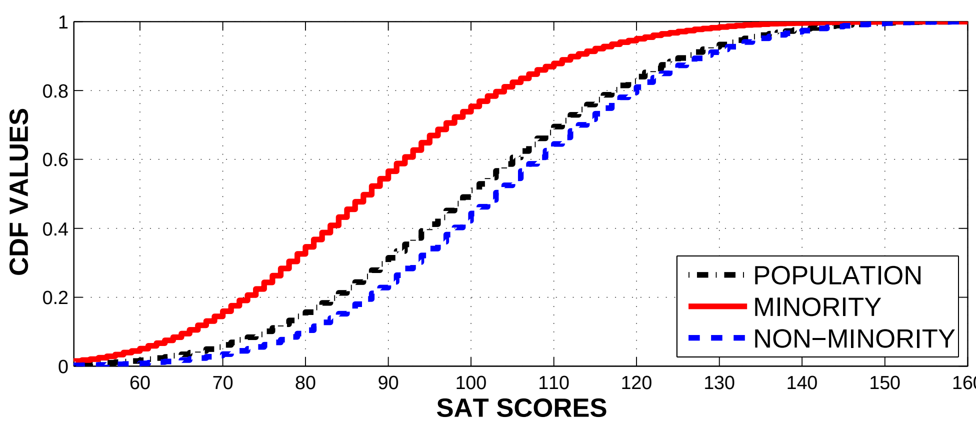
[10] In order to convert ordinal quality rankings into cardinal utilities, I project school quality and SAT score onto graduation rates and salary premiums. For the graduation rate, I use the graduation rate of the 1996 fall cohort, measured in AY2002-03, and for the salary premium, I use mid career salaries for the cohort of students graduating in AY1993-94. These students were graduating while the 1996 freshman class was in it's prime investment stage. The assumptions underlying the utility projection are thus twofold: first, that the 1996 cohort correctly anticipated the effect of school quality and HC on its own graduation prospects; and second, that they correctly anticipated mid-career income premium while they were themselves investing, and that this income premium was their motivation to compete for high-quality college seats.

TABLE 2. Descriptive Statistics for Match Partner and Utility Projection Variables

Variable	Obs	Mean	Std. Dev.	Min.	Max.
1996 Graduation Rate (IPEDS)	1281	0.524	0.180	0.033	1
1996 Adjusted Cohort Size (IPEDS)	1281	755.75	912.75	5	6547
Median SAT Score (IPEDS)	1152	999.50	121.91	567	1415
College Quality Index (USNWR)	1314	0.5129	0.189	0.087	0.973
Variable*	**Obs**	**Mean**	**Std. Dev.**	**5^{th} Percentile**	**95^{th} Percentile**
School Avg. 10-yr Ann. Salary (B&B, 2002 USD)	540 (rounded)	$54,226	$14,847	$33,000 (rounded)	$81,000 (rounded)
W/in-School Sample Size (B&B)	540 (rounded)	12.29	10.79	3	32
Avg. Salary for HS Completers (BLS, 2002 USD)	—	$26,133	—	—	—

*As per IES data security measures, the numbers of observations for B&B variables are rounded to the nearest 10. Min and max are suppressed to avoid compromising confidential data. For within-school 10-year average salaries, the 5^{th} and 95^{th} percentiles, rounded to the nearest $1,000, are reported instead.

FIGURE 2. SAT Score Distributions



the empirical distributions of SAT scores are displayed. The median for non-minorities is 103, and the median for minorities is 87, or the 19^{th} percentile for non-minorities. The structural model will illuminate how much of this gap is due to cost inequality versus asymmetry in incentives, and to what extent these drive inequality in college outcomes. Summary Statistics are given in Table 3.

TABLE 3. Summary Statistics for Normalized SAT Scores and Match Partner Quality

Sample	# of Obs	Median	Mean	StDev	Min	Max
Population SAT Scores	76,757	100	100.43	19.50	52	160
Minority SAT Scores	16,603	88	88.6	17.99	52	159
Non-Minority SAT Scores	60,154	103	103.7	18.61	52	160
Population College Quality (USNWR Index)	1,056,580	0.558	0.564	0.186	0.087	0.973
Minority College Quality (USNWR Index)	186,507	0.487	0.496	0.189	0.087	0.973
Non-Minority College Quality (USNWR Index)	870,073	0.586	0.579	0.182	0.087	0.973

4. IDENTIFICATION AND ESTIMATION

The structural objects to identify are the type distributions, $F_j(\theta)$, the cost function $C(s;\theta)$, the match utility function $U(p,s)$ and the markup $\widetilde{S}(s)$. The set of observables is the prize distribution $F_P(p)$, *ex-post* match distributions, $F_{P_j}(p)$, the minority mass μ, college-specific covariates $\mathbf{X} = \{\mathbf{X}_l\}_{l=1}^{L} = \{p_l, M_l, N_l, S_{0.5,l}, \rho_l, u_l\}_{l=1}^{L}$, outside utility u_0, and investment distributions $G_j(s)$, $j = \mathcal{M}, \mathcal{N}$. For notational ease, I denote the collective observables by $O = \left\{F_P, F_{P_{\mathcal{M}}}, F_{P_{\mathcal{N}}}, \mu, \mathbf{X}, u_0, G_{\mathcal{M}}, G_{\mathcal{N}}\right\}$.

I first show how to identify U and $\widetilde{S}$ from observables, then I discuss difficulties which arise in identification of the remaining structural primitives. In particular, $(F_{\mathcal{M}}, F_{\mathcal{N}}, C)$ are nonparametrically non-identified, even after imposing functional forms for C. These results are related to a recent literature on identification of first-price auction models (see Guerre, Perrigne, and Vuong [20] and Campo, Guerre, Perrigne, and Vuong [7]) and principal-agent models (see D'Haultfoeuille and Fevrier [15, 16]) when utility exhibits curvature. They motivate an additional assumption I employ to close the empirical model: a market clearing condition that type $\overline{\theta}$ was just indifferent between entering college and the outside option. I show that this assumption combined with a parametric restriction on C is enough to semiparametrically identify the model, whereas dropping either of these assumptions loses identification.

4.1. **Nonparametric Identification of Admission Preference Markups.** Recall that HC and race are mapped into match outcomes via the mechanisms displayed in equation (3), from which it immediately follows that

$$P_{\mathcal{M}}(s) = P_{\mathcal{N}}(\widetilde{S}(s)). \tag{6}$$

In other words, a minority student with investment s is matched to the same college as a non-minority student with investment $\widetilde{S}(s)$. This observation allows one to recover $\widetilde{S}$ from the observables by determining what rule could have produced allocations $\mathcal{P}_{\mathcal{M}}$ and $\mathcal{P}_{\mathcal{N}}$ from the investment distributions $G_{\mathcal{M}}$ and $G_{\mathcal{N}}$.

Proposition 4.1. *Let $\bar{s}_{\mathcal{M}}$ denote the upper bound of the minority investment distribution. Then under assumptions 2.6 and 2.9 $\widetilde{S}(\cdot)$ is nonparametrically identified from $(G_{\mathcal{M}}, G_{\mathcal{N}}, F_{P_{\mathcal{M}}}, F_{P_{\mathcal{N}}})$ on the interval $s \in [0, \bar{s}_{\mathcal{M}}]$.*

Proof: For $\varphi \in (0,1)$ define $s_{\mathcal{N}}(\varphi) \equiv G_{\mathcal{N}}^{-1}(\varphi)$ as the φ^{th} quantile in the non-minority HC distribution. For minorities, let $\varphi_{\mathcal{M}}(\varphi) \equiv G_{\mathcal{M}}\left(\widetilde{S}^{-1}(s_{\mathcal{N}}(\varphi))\right)$ denote the quantile rank of the *de-subsidized* version of $s_{\mathcal{N}}(\varphi)$ within the minority HC distribution. By equation (6), it follows that $F_{P_{\mathcal{M}}}^{-1}(\varphi_{\mathcal{M}}[\varphi]) = F_{P_{\mathcal{N}}}^{-1}(\varphi)$, $\forall \varphi$. By substituting in $\varphi_{\mathcal{M}}$ and rearranging, we get $G_{\mathcal{N}}^{-1}(\varphi) = \widetilde{S}\left(G_{M}^{-1}\left[F_{P_{\mathcal{M}}}\left(F_{P_{\mathcal{N}}}^{-1}[\varphi]\right)\right]\right)$, from which it follows that

$$\widetilde{S}(s) = G_{\mathcal{N}}^{-1}\left[F_{P_{\mathcal{N}}}\left(F_{P_{\mathcal{M}}}^{-1}\left[G_{\mathcal{M}}(s)\right]\right)\right],\ s \in [0, \bar{s}_{\mathcal{M}}]. \quad \blacksquare \tag{7}$$

The proof is constructive since the right-hand side of (7) is merely a composition of distribution and quantile functions which can be estimated directly from data. Aside from monotonicity, no *a priori* restrictions are imposed on the form of the markup function, not even an assumption that it aids minorities, since $\widetilde{S}(s) \geq s$ need not hold unless the data so indicate.

4.2. **Match Utility Identification.** Institutional quality p_l provides an ordinal measure of school quality, but ultimately a cardinal measure of match quality is required to identify the trade-off between investment and market payoffs. Moreover, this measure should allow for intrinsic valuing of HC, so a single measure is needed relate the units of p_l to the units of s in terms of agents' cardinal preferences. To accomplish this I project (p, s) pairs onto expected salary premiums from college attendance. Specifically, conditional on graduation, a match (p, s) produces income stream $u(p, s)$, but graduation is not guaranteed: (p, s) also produces graduation probability $\rho(p, s)$ which implies an expected salary premium of $U(p, s) = \rho(p, s)\left[u(p, s) - u_0\right]$. Given the school-level observables, ρ and u could in principle be recovered via nonparametric regression.

4.2.1. *Macro- vs. Micro-Level Phenomena.* One possible nonparametric estimator for $\widetilde{S}$ might involve constructing a maximal set of quantile ranks $\{\varphi_l\}_{l=1}^{L}$ and plugging them into (7), where the four CDFs are replaced with their Kaplan-Meier estimator. This would produce a scatterplot depicting a step function where individual school-specific markups may be fixed and tailored toward the HC level a school expects to attract.

However, high-school students would find it computationally expensive to follow an exhaustive set of individual markups (and discrete differences) when making investment decisions.

Therefore, in keeping with the behavioral interpretation of the large market approximation proposed by Hickman [21], I shall choose stimators for $\widetilde{S}$ and U smooth over a certain amount of micro-level (*i.e.*, school-specific) variation to obtain a simpler macro-level picture of broad market trends. Below I adopt estimators based on kernel-smoothed CDFs, which simplify broad market trends by providing both levels and derivatives. Similar logic motivates projection of college-specific variables and outcomes (with their idiosyncratic deviations from the conditional mean) onto a real-valued function space. Although p_l and $\bar{s}_l$ are not deterministically related—due in part to finite sampling from the student body at a given college—they are highly correlated ($CORR(p_l, \bar{s}_l) = 0.67$), so the smoothed approximation appears reasonable. In the case of estimating ρ, for which the best and most plentiful school-level data are available, the fit of the macro-level model to the micro-level data is fairly tight (see Section 5.2).

4.3. **Nonparametric Non-Identification of Types and Costs.** For ease of exposition, I shall henceforth treat $(U, \widetilde{S}, F_P, G_{\mathcal{M}}, G_{\mathcal{N}})$ as observables, in contrast to the remaining structural objects $(F_{\mathcal{M}}, F_{\mathcal{N}}, C)$ which I shall label the *data generating process* (DGP). Recall the system of differential equations (5) defining the inverse investment functions, and note that various aspects of this system can be re-written in terms of observables. By monotonicity of equilibrium investment it follows that

$$S_j \sim G_j(s) = 1 - F_j\left[\psi_j(s)\right],\ j = \mathcal{M}, \mathcal{N}. \tag{8}$$

Moreover, by monotonicity of $\widetilde{S}$, subsidized minority investment is distributed as

$$\widetilde{S}(S_{\mathcal{M}}) \sim \widetilde{G}_{\mathcal{M}}(s) = G_{\mathcal{M}}\left(\widetilde{S}^{-1}(s)\right) = 1 - F_{\mathcal{M}}\left[\psi_{\mathcal{M}}\left(\widetilde{S}^{-1}(s)\right)\right], \tag{9}$$

and de-subsidized non-minority investment is distributed as

$$\widetilde{S}^{-1}(S_{\mathcal{N}}) \sim \widetilde{G}_{\mathcal{N}}(s) = G_{\mathcal{N}}\left(\widetilde{S}(s)\right) = 1 - F_{\mathcal{N}}\left[\psi_{\mathcal{N}}\left(\widetilde{S}(s)\right)\right]. \tag{10}$$

By substituting (8)-(10) and their derivatives into (5), we get

$$\xi_{\mathcal{N}}(s) \equiv U_1[P_{\mathcal{N}}(s), s]\frac{(1-\mu)g_{\mathcal{N}}(s) + \mu\widetilde{g}_{\mathcal{M}}(s)}{f_P\left[P_{\mathcal{N}}(s)\right]} + U_2[P_{\mathcal{N}}(s), s] = C'(s;\theta)\text{, and} \tag{11}$$

$$\xi_{\mathcal{M}}(s) \equiv U_1[P_{\mathcal{M}}(s), s]\frac{\mu g_{\mathcal{M}}(s) + (1-\mu)\widetilde{g}_{\mathcal{N}}(s)}{f_P\left[P_{\mathcal{M}}(s)\right]} + U_2[P_{\mathcal{M}}(s), s] = C'(s;\theta), \tag{12}$$

where ξ_j represents the total marginal benefit of investment for group $j = \mathcal{M}, \mathcal{N}$, which is now treated as observable. I also adopt the following additional assumption:

Assumption 4.2. $C(s;\theta) = \theta c(s)$

Multiplicative separability is a standard dimension reducing assumption in structural models of Bayesian games. It allows costs to be decomposed into an idiosyncratic component θ, and a common component $c(s)$. However, even with this additional structure the model is non-parametrically non-identified.

Proposition 4.3. *Let $\left(U, \widetilde{S}, F_P, G_{\mathcal{M}}, G_{\mathcal{N}}\right)$ satisfy Assumptions 2.1 and 2.7 – 2.9, and assume $G_j(s) \in \mathcal{C}^2,\ j = \mathcal{M}, \mathcal{N}$, and that costs are separable. Then the model is non-parametrically non-identified in the sense that, if there exists at least one DGP $(F_{\mathcal{M}}, F_{\mathcal{N}}, c)$ satisfying Assumptions 2.2 – 2.6 that rationalizes $(G_{\mathcal{M}}, G_{\mathcal{N}})$ as an approximate equilibrium of the college admissions game with large K, then one can construct another distinct DGP $(\widetilde{F}_{\mathcal{M}}, \widetilde{F}_{\mathcal{N}}, \widetilde{c}) \neq (F_{\mathcal{M}}, F_{\mathcal{N}}, c)$ with separable costs that also rationalizes the observables as well.*

Proof: Note that the observables uniquely determine a total marginal benefit function ξ_j, and from Equations (11) and (12), we have $\theta_j(s) = \frac{\xi_j(s)}{c'(s)},\ j = \mathcal{M}, \mathcal{N}$. In order for the observables to be rationalizable under the model, it must be that costs c satisfy $\theta_j'(s) < 0$ for each $s \in [\underline{s}, \overline{s}],\ j = \mathcal{M}, \mathcal{N}$. Assuming this is so, then for $\theta_j = \xi_j(s)/c'(s) \sim F_j$, it follows that the triple $(F_{\mathcal{M}}, F_{\mathcal{N}}, c)$ rationalizes the observables. Now consider an alternative cost function $\widetilde{c}(s) \equiv c(s)\phi(s)$, where $\phi : \mathcal{S} \to [1, \infty)$ is differentiable, strictly increasing, and strictly convex. Note that since $\widetilde{c}'(s) = c'(s)\phi(s) + c(s)\phi'(s) > 0$ and $\widetilde{c}''(s) = c''(s)\phi(s) + 2c'(s)\phi'(s) + c(s)\phi''(s) > 0$, it follows that $\widetilde{C}(s;\theta) = \theta\widetilde{c}(s)$ also satisfies all regularity conditions required by the model. Moreover, since $\widetilde{c}'(s) = c'(s)\phi(s) + c(s)\phi'(s) > c'(s)$ it follows that $\widetilde{\theta}_j(s) = \frac{\xi_j(s)}{\widetilde{c}'(s)} = \frac{\xi_j(s)}{c'(s)\phi(s)+c(s)\phi'(s)}$ is also a strictly decreasing function, and therefore the alternative DGP $(\widetilde{F}_{\mathcal{M}}, \widetilde{F}_{\mathcal{N}}, \widetilde{c}) \neq (F_{\mathcal{M}}, F_{\mathcal{N}}, C)$, where $\widetilde{\theta}_j = \xi_j(s)/\widetilde{c}'(s) \sim \widetilde{F}_j$, also rationalizes the observables. ■

The above argument shows how separability alone is not enough to identify a unique DGP. Indeed, for any convex and separable cost function that rationalizes the data, if itn is further "convexified" by multiplication with ϕ then another rationalizing DGP will result. In fact, by induction one can always find an infinite number of DGPs to rationalize the data by $\widetilde{c}_k(s) \equiv c(s)\left(\phi(s)\right)^k$, $k = 1, 2, 3, \ldots$ Note that further parametric restrictions on costs would not suffice either, since the logic of the proof assumes nothing about the form of c above regularity conditions and separability.

A question related to identification is testability, or whether the model is falsifiable through restrictions it places on the set of observables that could be consistent with some DGP $(F_{\mathcal{M}}, F_{\mathcal{N}}, C)$. The theory model does place some restrictions on the set of observables under some parametric classes of cost functions. For example, if costs are linear, *i.e.*, $C(s;\theta) = \theta \nu s,\ \nu > 0$, then $\theta_j(s) = \xi_j(s)/\nu$, and only configurations of observables such that $\xi_j'(s) < 0,\ j = \mathcal{M},\mathcal{N}$ can be rationalized, which in general need not be true. However, given only the additional restrictions imposed by cost separability, any ξ_j can be rationalized by some DGP.

Proposition 4.4. *Let* $\left(U, \widetilde{S}, F_P, G_{\mathcal{M}}, G_{\mathcal{N}}\right)$ *satisfy Assumptions 2.1 and 2.7 – 2.9, and assume investment distributions* $G_j(s) \in \mathcal{C}^2,\ j = \mathcal{M},\mathcal{N}$. *Then there exists some DGP* $(F_{\mathcal{M}}, F_{\mathcal{N}}, c)$ *satisfying Assumptions 2.2 – 2.6 and 4.2 that rationalizes the investment distributions as an approximate equilibrium of the college admissions game with large K.*

Proof: Once again, rationalizability requires monotonicity of inverse investment $\theta_j(s) = \xi_j(s)\left(c'(s)\right)^{-1}$. This condition can be expressed as

$$\theta_j'(s) = \xi_j'(s)\left(c'(s)\right)^{-1} - \xi_j(s)\left(c'(s)\right)^{-2} c''(s) < 0\ \forall s \in [\underline{s}, \bar{s}]. \tag{13}$$

Note that the second term on the right-hand side of the equality is always negative for any $(c, \xi_{\mathcal{M}}, \xi_N)$ triple satisfying Assumptions 2.2 – 2.9. Therefore, monotonicity hinges on whether it is negative enough to preserve negativity of the sum of the first term (which may be positive) with the second term. If $\xi_j'(s) < 0$ for each $s \in [\underline{s}, \bar{s}]$, then this is true for any increasing, differentiable, and weakly convex cost function c. However, if ξ_j is non-monotone for some $j = \mathcal{M},\mathcal{N}$, then $c''(s)$ must be large enough at each point to keep the sum negative everywhere. More specifically, suppose ξ_j is non-decreasing only on a single interval, I_j and let $s_j^* \equiv \inf\left\{s \in [\underline{s}, \bar{s}] : \xi_j'(s) \le 0\right\}$.[11] Let $\underline{c}_j''$ denote the *lower convexity bound function* for group j, and without loss of generality, normalize $\underline{c}_j$ so that $\underline{c}_j(s_j^*) = \underline{c}_j'(s_j^*) = 1$. Then (13) implies that on I_j the lower convexity bound for j is the solution to the initial value problem $\underline{c}_j''(s) = \frac{d\log\left(\xi_j(s)\right)}{ds}\underline{c}_j'(s)$, such that $\underline{c}_j'(s_j^*) = 1$, with $\underline{c}_j''(s)$ being defined as zero on the set $[\underline{s}, \bar{s}] \setminus I_j$. Now I can define the *lower convexity envelope*, $\zeta : [\underline{s}, \bar{s}] \to \mathbb{R}_+$, which characterizes the minimal degree of cost convexity required for the model to be consistent with observables, as $\zeta(s) \equiv \max\left\{\underline{c}_{\mathcal{M}}''(s), \underline{c}_{\mathcal{N}}''(s)\right\}$. Since it is always possible to find a cost function c satisfying the regularity conditions, and also $c''(s) > \zeta(s),\ \forall s$, the result follows. ■

In short, only the set of cost functions c satisfying $c''(s) > \zeta(s),\ \forall s$ can be consistent with the observables within the confines of a Bayes-Nash (approximate) equilibrium. This argument illustrates how the shape of the total marginal benefit function translates into restrictions that the theory places on the DGP. In particular, non-decreasing

[11] The arguments here can be adapted simply but tediously in piecewise fashion to the case where ξ_j is non-decreasing on a set of non-connected intervals.

segments of ξ_j help to limit the set of cost functions which could have produced the observed investment distributions via Bayes-Nash equilibrium play.

4.4. **Semiparametric Identification of Types and Costs.** In this section I will show that, fortunately, only an additional restriction on a single quantile of the type distributions will suffice in order to obtain semiparametric identification. I first adopt a parametric class for the common component of the separable cost function.

Assumption 4.5. $C(s;\theta) = \theta e^{\nu(s-\underline{s})}$, $\nu > 0$

Assumption 4.5 is motivated by a two basic factors. First, it satisfies regularity conditions required by the game theoretic model of investment. Second, the exponential family exhibits a high degree of curvature, helping it to rationalize the data. This property of the exponential family is formalized in the following result.

Proposition 4.6. *Any* $\left(U, \widetilde{S}, F_P, G_{\mathcal{M}}, G_{\mathcal{N}}\right)$ *satisfying Assumptions 2.1, 2.8 – 2.7, and* $G_j(s) \in \mathcal{C}^2$, $j = \mathcal{M}, \mathcal{N}$, *can be rationalized by (approximate) Bayes-Nash equilibrium for some DGP* $(F_{\mathcal{M}}, F_{\mathcal{N}}, C)$ *with costs* $C(s;\theta) = \theta e^{\nu(s-\underline{s})}$, $\nu > 0$, *where* $\theta_j = \xi_j(s) / \left(\nu e^{\nu}(s-\underline{s})\right) \sim F_j$.

Proof: Once again, the question of rationalizeability hinges on a monotone inverse investment function. Assume for simplicity that there is a single connected interval $I_j \subseteq [\underline{s}, \bar{s}]$ on which $\xi_j'(s) \geq 0$. Fix $\varepsilon > 0$ and let

$$\nu_\varepsilon^* \equiv \max_{j \in \{\mathcal{M}, \mathcal{N}\}} \left\{ \max_{s \in I_j} \left\{ \frac{d \log\left(\xi_j(s)\right)}{ds} \right\} \right\} + \varepsilon. \tag{14}$$

Note that by continuity and compactness, ν_ε^* exists. I claim that an exponential cost function with curvature parameter ν_ε^* rationalizes the observables. To see why, note that monotonicity implies $\theta_j'(s) = \frac{\xi_j'(s)}{c'(s)} - \frac{\xi_j(s)}{(c'(s))^2} c''(s) < 0$, $\forall s$. By explicitly taking derivatives for the power family of costs and simplifying, we get $\frac{d\log\left(\xi_j(s)\right)}{ds} < \nu$, $\forall s$. Since ν_ε^* satisfies this condition for $j = \mathcal{M}, \mathcal{N}$, it follows that $(F_{\mathcal{M}}^*, F_{\mathcal{N}}^*, C^*)$ rationalizes the observables, where $C^*(s;\theta) = \theta e^{\nu_\varepsilon^*(s-\underline{s})}$ and $\theta_j^* = \xi_j / \left(\nu_\varepsilon^* e^{\nu_\varepsilon^*(s-\underline{s})}\right) \sim F_j^*(\theta)$, $j = \mathcal{M}, \mathcal{N}$. The extension of this logic to multiple disconnected intervals is similar and straightforward. ■

The above argument demonstrates that the exponential family of cost functions is able to rationalize any set of well-behaved observables as being consistent with Bayes-Nash behavior in a large market setting. Note also that there are infinitely many exponential cost functions that can rationalize well-behaved observables since ε need only be strictly positive (which follows from the non-identification argument in the previous section). Therefore, additional structure is needed to close the empirical model:

Assumption 4.7. ***Zero Surplus Condition (ZSC):*** $C(\underline{s};\overline{\theta}) = U(\underline{p},\underline{s})$

The ZSC is a natural participation constraint condition stating that the marginal individual (*i.e.*, highest cost) in the market was just indifferent between college and the outside option. It can be thought of as a market clearing condition in the broader post-secondary labor market, where students make decisions on the extensive margin of college attendance. For type $\overline{\theta}$ the expected salary premium from a college degree is just enough to offset the entry cost of participation in the college market.

In order to interpret condition 4.7 mathematically, I first impose an innocuous cost normalization $c(\underline{s}) = 1$.[12] Then under separability the ZSC is the same as parameterizing the upper extremal quantile of the type distribution since 4.7 is equivalent to $\overline{\theta} = U(\underline{p},\underline{s}) = \rho(\underline{p},\underline{s})u(\underline{p},\underline{s})$. This identification strategy is similar to that proposed by Campo, Guerre, Perrigne, and Vuong [7, henceforth, CGPV] within the context of first-price auction models with utility curvature. CGPV [7] showed that modeling one quantile of the private value distribution as a function of auction-specific covariates allows for a data-driven approach to utility curvature within a single-parameter family for utility. Knowing utility curvature means the standard GPV[19] method applies. Similar logic holds in this context as well, the main difference being that instead of observing many auctions with a single (observably) heterogeneous good for sale, the US college market is like a single auction with many heterogeneous objects for sale and a large number of bidders. In both cases, variation in items sold identifies utility curvature.

Proposition 4.8. *Let* $\left(U, \widetilde{S}, F_P, G_{\mathcal{M}}, G_{\mathcal{N}}\right)$ *satisfy Assumptions 2.1 and 2.7 – 2.9, and* $G_j(s) \in \mathcal{C}^2$, $j = \mathcal{M}, \mathcal{N}$. *Moreover, assume exponential costs* $C(s;\theta) = \theta e^{\nu(s-\underline{s})}$, $\nu > 0$ *and that the zero surplus condition 4.7 holds. Then if there exists a DGP* $(F_{\mathcal{M}}, F_{\mathcal{N}}, \nu)$ *that rationalizes the observables as consistent with (approximate) Bayes-Nash equilibrium, it is unique, though one need not exist. Thus, the model is both testable and semiparametrically identified from observables.*

Proof: Assumption 4.5 implies that equations (11) and (12) can now be re-written as $\theta_j(s) = \frac{\xi_j(s)}{\nu e^{\nu(s-\underline{s})}}$, $j = \mathcal{M}, \mathcal{N}$. This in combination with the ZSC implies $\overline{\theta} = \frac{\xi_{\mathcal{N}}(\underline{s})}{\nu} =$

[12] As D'Haultfoeuille and Fevrier [15, 16] point out in a related context, a model of costs $C(s;\theta) = \theta c(s)$ is equivalent to any other where the idiosyncratic component is scaled by a constant ι and the common component is divided by the same, since $(\iota \cdot \theta)\,(c(s)/\iota) = \theta c(s)$. More generally, all models with the same total costs for each (θ, s) pair are equivalent. For any parametric model of costs $(\widetilde{\theta}, \widetilde{c}(s))$ one can eliminate the ambiguity by normalizing $(\theta, c(s)) \equiv \left(\widetilde{c}(\underline{s}) \cdot \widetilde{\theta}, \widetilde{c}(s)/\widetilde{c}(\underline{s})\right)$, which implies $c(\underline{s}) = 1$. Assumption 4.5 has this normalization built in.

$U(\underline{p}, \underline{s})$,[13] which is equivalent to

$$\nu = \frac{U_1(\underline{p}, \underline{s})P'_{\mathcal{N}}(\underline{s}) + U_2(\underline{p}, \underline{s})}{U(\underline{p}, \underline{s})}. \tag{15}$$

Equation (15) uniquely determines the cost curvature parameter, ν, and also F_j since it is the distribution of $\theta_j(s) = \xi_j(s)/\left(\nu e^{\nu(s-\underline{s})}\right)$, as long as $\theta_j(s)$ is monotone decreasing. Note, however, that (15) need not be consistent with $\nu > \nu_0^*$, where ν_ε^* is defined in equation 14. Therefore, following similar logic as in the proof of Proposition 4.6, monotonicity need not follow and the model is both testable and semiparametrically identified whenever it is not rejected by the data. ■

An interesting implication of the proof is that the level of cost curvature which equilibrates the market on the extensive margin is equal to the percent change in total utility from a one-unit increase in investment for the marginal market participant. As it turns out, semiparametric identification and testability can be established for another model as well, the power family: $C(s;\theta) = \theta(s/\underline{s})^{\ddot{\nu}}$ $\ddot{\nu} \geq 1$. Note that the power family nests linear costs as a special case. Later on the following result will be helpful in seeing whether alternative cost specifications are also compatible with the US college market data.

Proposition 4.9. *Under the same assumptions as Proposition 4.8, except that costs belong to the power family $C(s;\theta) = \theta(s/\underline{s})^{\ddot{\nu}}$, $\ddot{\nu} \geq 1$, the model is both testable and semiparametrically identified from observables.*

Proof: For the power family the FOC is now $\theta_j(s) = \frac{\xi_j(s)}{(\ddot{\nu}/\underline{s})\cdot(s/\underline{s})^{\ddot{\nu}-1}}$, $j = \mathcal{M}, \mathcal{N}$. This condition and the ZSC imply that $\ddot{\nu} = \underline{s} \cdot \frac{U_1(\underline{p},\underline{s})P'_{\mathcal{N}}(\underline{s}) + U_2(\underline{p},\underline{s})}{U(\underline{p},\underline{s})}$, so the cost curvature parameter $\ddot{\nu}$ and the type distributions are semiparametrically identified as long as they can rationalize the observables. By similar derivations as above I get the following definition

$$\ddot{\nu}_\varepsilon^* \equiv \max_{j\in\{\mathcal{M},\mathcal{N}\}} \left\{ \max_{s\in I_j} \left\{ s \cdot \frac{d\log\left(\xi_j(s)\right)}{ds} \right\} \right\} + 1 + \varepsilon, \tag{16}$$

and for the power family of costs, monotonicity is equivalent to $\ddot{\nu} > \ddot{\nu}_0^*$. Since this condition need not be satisfied, the model is both testable and identified whenever it is not rejected by the data. ■

Before moving on, a caveat is worth discussing, related to the fact that the extensive margin is used to identify the relevant segment of cost types. Problems arise when

[13] Note that this expression is based on the allocation function for the non-minority group. This is because, when AA is geared toward aiding under-represented minorities, it may be the case that $\widetilde{S}(\underline{s}) > \underline{s}$, in which case additional complications arise because the marginal minority participant may have positive surplus, or additional entry into the market may be induced.

$\widetilde{S}(\underline{s}) > \underline{s}$, which could induce additional entry by minority students (outside the identified set), or result in the marginal minority student having positive surplus. If $\widehat{\widetilde{S}}(\underline{s}) = \underline{s}$—*i.e.*, both races are represented within a neighborhood of the lower bound, which is easily verified—the common support assumption is not problematic. Moreover, only counterfactuals where $\widetilde{S}(\underline{s}) = \underline{s}$ are identified because matching at the extensive margin does not change. In the case of the counterfacutal quota I consider, it is important to note that the targeted mass of earmarked seats is μ, the mass of minority freshmen, *not* the population mass of minorities overall, which is larger. Specifying a quota in this way leaves the extensive margin unchanged, whereas targeting the overall mass of minorities would induce entry by minorities (and exit by non-minorities) from the college market, and is therefore not identified. This is an important distinction between the quota considered here and some that are implemented abroad, notably in South Africa (see *The Economist* [1]). Though this sort of counterfactual is non-identified, Coate and Loury [13] develop theory which shows moving abruptly to this extreme version of a quota fully representative of population masses can perform poorly. On the other hand, the more conservative quota explored below does quite well on incentive provision.

4.5. **A Semiparametric Estimator.**

4.5.1. *Investment Lower Bound.* I begin with a preliminary technical concern: the interpretation of minimal investment level $\underline{s}$. A cursory look at the data suggests a positive SAT lower bound. Another complication is that the SAT is an aptitude test (*i.e.*, it is designed to measure ability), as opposed to the ACT, an achievement test (*i.e.*, it is designed to measure acquired knowledge). Both are used in college admissions. In order to resolve this I turn to the ACT, which near the lower bound measures something closer to investment (see appendix for further discussion).

It is theoretically possible for a student to score zero on the ACT, but such a feat is difficult unless one knows enough to achieve a perfect score: with probability near one, a student will get a positive ACT score by random responding, due to the multiple-choice format of the test. I interpret zero investment above the minimal entry level as a student who randomly responds to all ACT test questions, and I simulated such behavior on an actual ACT exam 100,000 times in order to compute the distribution of resulting scores. The resulting mean, median, and modal scores are 12.12, 12, and 12, respectively (see appendix for further details). The equivalent SAT score range (using concordance tables provided by the test designers) is between 520 and 590, having quantile ranks of 0.0043 and 0.0134, respectively. Data from the supply side of the market lend further support to this as an appropriate range for the minimal investment level: the lowest within-school median SAT score for all colleges in the sample is 567. For the remainder of the paper, $\underline{s}$

will be set to a value of 52 (or 520 in the original SAT units). Note also that counterfactual results are not sensitive to perturbations of $\underline{s}$ within the range $[52, 59]$.

4.5.2. *Match Utilities.* I use some parametric forms to estimate match utility projection $U(p,s) = \rho(p,s)\,[u(p,s) - u_0]$, where ρ is graduation probability and $[u(p,s) - u_0]$ is the college salary premium. I specify the former as a flexible polynomial and the latter as a Cobb-Douglass production function so that

$$\rho = \beta_0 + \beta_1 p + \beta_2 p^2 + \beta_3 s + \beta_4 s^2 + \beta_5 ps + \varepsilon_\rho, \text{ and} \tag{17}$$

$$\log(u - u_0) = \alpha_0 + \alpha_1 \log(p) + \alpha_2 \log(s) + \varepsilon_u. \tag{18}$$

The interpretations of ε_ρ and ε_u are as measurement error in the outcome variable due to finite sampling of students within a given school. Given the better data available for measuring ρ—for ρ we have 1152 observations based on an average within-school sample size of 756, while for u we have 530 observations with an average within-school sample size of 12—the predictive power of the conditional graduation probability model is greater (see below). Equations 17 and 18 are both estimated via weighted least squares (*i.e.*, placing more weight on observations with lower variances), using the college-level covariates $\{\rho_l, \omega_{\rho l}, u_l, \omega_{ul}, p_l, S_{0.5,l},\}_{l=1}^{L}$, where weighting variables $\omega_{\rho l}$ and ω_{ul} represent the adjusted cohort size and the within-school 10-year salary sample size, respectively.

In addition, I estimate $\rho(p,s)$ by constrained least squares to ensure that probabilities don't exceed 1.[14] Specifically, I impose $\beta_0 + \beta_1 \overline{p} + \beta_2 \overline{p}^2 + \beta_3 \overline{s} + \beta_4 \overline{s}^2 + \beta_5 \overline{p} \cdot \overline{s} \leq 1$, where $\overline{p} = 0.973$ and $\overline{s} = 160$ are the maximal quality and HC. This transforms 17 into

$$\rho_l - 1 = \beta_1(p_l - \overline{p}) + \beta_2(p_l^2 - \overline{p}^2) + \beta_3(s_l - \overline{s}) + \beta_4(s_l^2 - \overline{s}^2) + \beta_5(p_l s_l - \overline{ps}) + \varepsilon_{\rho l}. \tag{19}$$

4.5.3. *Two-Stage Estimation Procedure.* The remaining estimation uses kernel density estimators (KDEs). Reliable density estimates at the lower boundaries of the supports are important, but ordinary KDEs are inconsistent at the boundary, due to a phenomenon known as the *boundary effect*.[15] To fix this problem, I instead use a boundary-corrected KDE developed by Zhang, Karunamuni, and Jones [36] and Karunamuni and Zhang [24], which is uniformly consistent on the closure of the support. Hickman and Hubbard [22] conducted a Monte-Carlo study which documented substantial small-sample performance gains from boundary correction in nonparametric estimation of first-price auctions; the interested reader is directed there for further discussion.

[14] The lower constraint $\rho(p,s) \geq 0$ was never binding in the data, but in principal, this constraint could be imposed in similar fashion as well.

[15] Since estimates are based on existing data within a neighborhood of a point, and data cannot exist within a half-neighborhood of the boundary, standard kernel estimators naturally penalize estimates downward near the support extrema.

My proposed college market estimator consists of the following three-step process:

Preliminary Stage:

(i) Estimate the minority mass $\widehat{\mu} = \frac{\sum_{u=1}^{U} M_u}{\sum_{u=1}^{U}(M_u+N_u)}$ from prize data;

(ii) Estimate boundary-corrected kernel densities and distributions for match allocations $\widehat{f}_{P_j}(p)$ and $\widehat{F}_{P_j}(p)$, $j = \mathcal{M}, \mathcal{N}$ from prize data; and

(iii) Estimate boundary-corrected kernel densities and distributions for investment $\widehat{g}_j(s)$ and $\widehat{G}_j$, $j = \mathcal{M}, \mathcal{N}$ from SAT data.

Stage 1: (i) Estimate $\widetilde{S}$ as outlined in Section 4.1: $\widehat{\widetilde{S}}(s) = \widehat{G}_{\mathcal{N}}^{-1}\left[\widehat{F}_{P_{\mathcal{N}}}\left(\widehat{F}_{P_{\mathcal{M}}}^{-1}\left[\widehat{G}_{\mathcal{M}}(s)\right]\right)\right]$.

(ii) Estimate regressions (17) and (18) to obtain $\widehat{U}(\cdot,\cdot)$, $\widehat{U}_1(\cdot,\cdot)$, and $\widehat{U}_2(\cdot,\cdot)$.

(iii) Given the previous parts, estimate the cost curvature parameter from equation $\widehat{\nu} = \frac{\widehat{U}_1(\underline{p},\underline{s})\widehat{P}'_{\mathcal{N}}(\underline{s}) + \widehat{U}_2(\underline{p},\underline{s})}{\widehat{U}(\underline{p},\underline{s})}$.

(iv) Estimate samples of pseudo-private costs $\widehat{\mathbf{\Theta}}_{\mathcal{N},T_{\mathcal{N}}}$ and $\widehat{\mathbf{\Theta}}_{\mathcal{M},T_{\mathcal{M}}}$ from the FOCs by $\theta_{jt} = \frac{\widehat{\xi}_j(s_{jt})}{\widehat{\nu} e^{\widehat{\nu}(s_{jt}-\underline{s})}}$, $j = \mathcal{M}, \mathcal{N}$, where $\widehat{\xi}_j$ is the empirical analog of the total marginal benefit functions defined in (11) and (12).

Stage 2: (i) Estimate boundary-corrected kernel densities $\widehat{f}_j$ for cost types using the samples of pseudo-private costs from Stage 1.

4.5.4. *Asymptotics.* In a related setting CGPV [7] developed a semiparametric estimator of a first-price auction model with utility curvature, and proved asymptotic properties of their estimator. They parameterized bidder utility and the upper quantile of the private value distribution in order to identify the model. They then used observed variation in auction characteristics to nonparametrically estimate the conditional upper quantile of the bid distribution. This upper bid quantile was mapped into an upper private value quantile and utility curvature estimate via non-linear least squares. Conditional on the utility curvature estimate, they recovered type distributions according to the two-stage GPV method. CGPV prove asymptotic normality and show that their utility curvature estimator converges at the optimal rate of $K^{(R+1)/(2R+3)}$, where R is the number of continuous derivatives of the type distributions. Type distribution estimators were shown to be asymptotically normal and to converge also at the optimal (two-stage) rate.

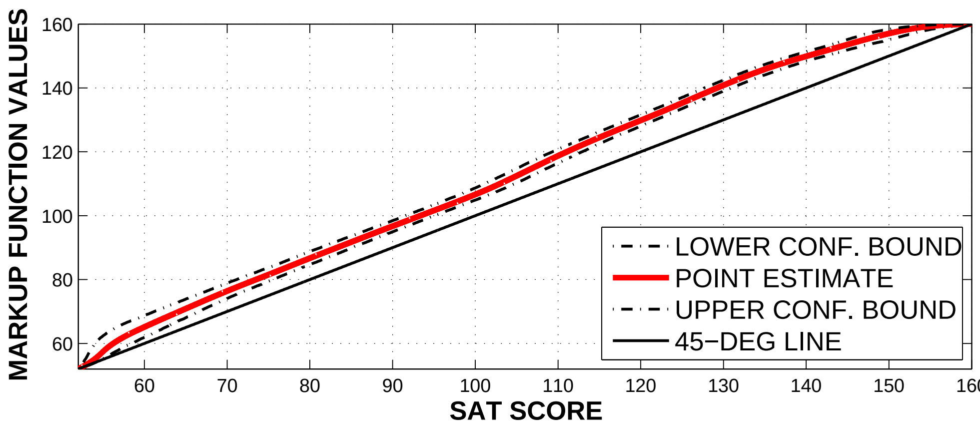
Development of asymptotic theory within the current model is beyond the scope of this paper, but it is worth pointing out parallels between the above estimator and CGPV for which asymptotic theory is known. The main difference in the two is that CGPV analyze data from many single-object auctions, whereas the college market consists of a single multi-object auction with many bidders. Otherwise, the estimators I have proposed for ν, $F_{\mathcal{M}}$, and $F_{\mathcal{N}}$ are similar in several ways. I exploit variation in objects being auctioned (college outcomes) to identify utility curvature, which I estimate using a combination of least squares and nonparametric techniques. Specifically, my cost curvature estimator $\widehat{\nu} = \frac{\widehat{U}_1[\underline{p},\underline{s}]\widehat{P}'_{\mathcal{N}}(\underline{s})+\widehat{U}_2[\underline{p},\underline{s}]}{\widehat{U}[\underline{p},\underline{s}]} = \left[\widehat{U}_1(\underline{p},\underline{s})\frac{(1-\widehat{\mu})\widehat{g}_{\mathcal{N}}(\underline{s})+\widehat{\widehat{g}}_{\mathcal{M}}(\underline{s})}{\widehat{f}_P[\underline{p}]} + \widehat{U}_2(\underline{p},\underline{s})\right] \Big/ \widehat{U}(\underline{p},\underline{s})$ combines least squares—to project (p,s) pairs onto cardinal utility measure $\widehat{U}$—and KDEs—to estimate $\widehat{P}'_{\mathcal{N}}(\underline{s})$. Moreover, type distribution estimates are conditioned on the curvature parameter, and a parametric restriction of a single quantile of the type distribution, with $\widehat{F}_{\mathcal{M}}$ and $\widehat{F}_{\mathcal{N}}$ being otherwise nonparametric as in CGPV.

5. ESTIMATION RESULTS

In this section I present the estimation results and counterfactual policy experiments. Except for match utility projections, all confidence bounds were estimated via the nonparametric bootstrap. For institutional data $\{p_l, M_l, N_l, S_{0.5,l}, \rho_l, \omega_{\rho l}, u_l, \omega_{ul}\}_{l=1}^{L}$ resampling was done at the college level, and for the SAT data $\{s_t, d_t\}_{t=1}^{T}$ it was done at the student level. The model was estimated on $1,100$ simulated bootstrap samples, with confidence bounds on estimates computed in the usual way. One thing, however, should be emphasized: since I have not developed the requisite asymptotic theory which justifies use of the nonparametric bootstrap, I must *assume* bootstrap validity in order to draw inference by this method. As a full treatment of asymptotic theory is beyond the scope of this exercise, I proceed with recognition of the limitation that statements on sampling variability presented here are contingent upon the assumption of bootstrap validity.

One challenge to implementing the estimator with the US college data is sparseness of observations near the upper bound of the SAT support. The maximal observed within-school median SAT score $\max\{S_{0.5,l}\}$ was 1415, which is just slightly above the 98^{th} percentile of the individual SAT data. The 99^{th} percentile for non-minority (minority) students was a score of 1460 (1330), meaning that there is a score range near the upper bound of 1600 with very few observations, for which estimated cardinal utility projections $\widehat{U}$ must be extrapolated outside the domain of the institutional data. This problem is particularly salient for minorities, for which there is only a single observation (of 1590) above 1540, which was the 99.994^{th} percentile. The combination of extrapolation and data sparseness caused estimates of the total marginal benefit function near the upper bound to be negative for minorities, whereas everywhere else it was positive. I dealt

FIGURE 3. Grade Markup Estimate w/95% Confidence Bands



with this problem by replacing $\widehat{\xi}_{\mathcal{M}}(159)$ with $\widehat{\xi}_{\mathcal{M}}(154)/10$ in order to get a small but positive number.[16]

5.1. **Minority Mass and Markups.** For the 1996 freshmen enrollment data, there were a total of 1,056,580 seats, with 186,507 going to minority students. This results in a demographic parameter estimate of $\widehat{\mu} = 0.17652$, with a standard error of 0.00678. The nonparametric markup estimate is depicted as the solid thick line in Figure 3, with bootstrapped confidence bands depicted by the dashed lines. Note that the lower confidence bound is above the 45°-line, which is evidence that minority students receive special consideration on the basis of race for admission to college. Moreover, the estimated racial admission preference is substantial throughout all segments of the quality spectrum for the market. The estimate implies an average markup of 68.57 SAT points on average for minority students or 35% of a standard deviation. When averaging over the top 5% of minority students, the markup rises to 101.89 SAT points, or 52% of a standard deviation. This is because, for SAT scores below 1400, the quantity $\widetilde{S}(s) - s$ is slightly increasing in s, or in other words, we see evidence of an increasing marginal markup. As Hickman [21] shows, this is an important property for investment incentive provision through the competitive channel.

These findings are consistent with previous work on AA by Chung, Espenshade and Walling [12] who estimate the average SAT-equivalent grade boost received by minority students at elite universities in admissions decisions. They find that minority students receive a substantial SAT-equivalent boost in admission decisions—230 points for African

[16] For 9 of the 1,100 bootstrap samples this same problem occurred at or above a score of 1580 for non-minorities, or the 99.957$^{\text{th}}$ percentile. For those simulations I threw out the corresponding estimates before computing confidence bounds, but nothing changes qualitatively if I instead employ the same coping technique as for extreme minority SAT scores.

Americans and 185 points for Hispanics. While these figures are not directly comparable to my measure of the admission preference—Chung, *et al.* measure AA in terms of acceptance probabilities, whereas $\widetilde{S}$ measures AA in terms of enrollment outcomes—both find that race plays a significant role in how matching rights are awarded, and the admission preference is generally highest for applicants with top SAT scores.

5.2. **Match Utility Projection.** Table 4 presents estimates for match utility projections with heteroskedasticity-robust standard errors for two alternative specifications of the conditional probability equation. Specification 1 provides a simpler picture of how school and student quality help students to make it through college. There, HC plays the primary role, and institutional quality affects graduation probability only through increasing the effectiveness of the HC input. Since both specifications have nearly the same explanatory power, I adopt Specification 1 going forward. Table 5 displays estimates for equation 18, which specifies the salary premium as a Cobb-Douglas production function. The table depicts two different specifications; since α_2 is statistically insignificant in Specification 2, and since 12% more observations are available for Specification 1, I adopt the latter, simpler model of salary premium for use in Stage 2 of estimation.[17][18] Counterfactual predictions that will follow later are robust to choice of specifications for the graduation probability and salary premiums.

5.3. **Cost Curvature and Model Evaluation.** Table **??** summarizes cost curvature estimates under the two specifications considered. It is now possible investigate whether they are consistent with the data. Recall that Propositions 4.8 and 4.9 suggest a simple specification test based on the monotonicity conditions $\nu > \nu_0^*$ and $\ddot{\nu} > \ddot{\nu}_0^*$. Computing the curvature lower bounds requires derivatives for $\xi_j(s),\ j = \mathcal{M},\mathcal{N}$, but the discrete nature of the SAT data renders the method of finite differences problematic. Instead, I compute $d\xi_j(s)/ds$ as the derivative of a cubic spline interpolant of $\widehat{\xi}_j(s)$ at the grid of observed SAT scores $s = 52,\ldots,160$, where the boundaries of the interpolant are determined by the standard not-a-knot condition. So $\widehat{\nu}_0^*$ and $\widehat{\ddot{\nu}}_0^*$ are computed from equations 14 and 16 with the numerical derivatives, with the maximum taken over the

[17]Rounding sample sizes to the nearest 10 for data security as required by IES, for roughly 60 observations where u_l was available, the on-campus score midpoint $S_{0.5,l}$ was not. This is why the sample sizes differ in Table 5. This also accounts for why the point estimate of α_1 actually drops in Specification 1, even though p and s are positively correlated. When Specification 1 is re-estimated using the same sample as that used for Specification 2, $\widehat{\alpha}_1 = 0.6751$ is the result.

[18]Rounding sample sizes to the nearest 10 for data security as required by IES, Specification 1 uses 530 observations, rather than the 540 total reported in Table 2, because for roughly 10 observations (or 2% of the data) the mean salary estimate was less than u_0. In all such cases the within-school sample size was weakly less than 5, making it plausible that sampling variability caused the estimated means to be below the value of the outside option.

TABLE 4. Conditional Graduation Probability Estimates $\widehat{\rho}(p,s)$

	Specification 1		**Specification 2**	
N	1131		1131	
R^2	0.7525		0.7525	
Parameter	**Point Est.**	**Std. Err.**	**Point Est.**	**Std. Err.**
p (β_1)	–	–	0.3094578	(0.2999203)
p^2 (β_2)	–	–	0.0424256	(0.2004996)
s (β_3)	0.0077443^{***}	(0.0013667)	0.0055225^{**}	(0.0025169)
s^2 (β_4)	-0.0000487^{***}	(0.00000471)	-0.0000281	(0.0000244)
$p \cdot s$ (β_5)	0.0076061^{***}	(0.0002437)	0.0041589	(0.0047692)
const. (β_0)	-0.175845^{**}	(0.0839349)	0.1889114	(0.4763975)

NOTE: Significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

TABLE 5. Cobb-Douglass Salary Premium Results: $u(\widehat{p,s)-u}_0$

	Specification 1		**Specification 2**	
N	530		470	
R^2	0.1668		0.1818	
Parameter	**Point Est.**	**Std. Err.**	**Point Est.**	**Std. Err.**
p-share (α_1)	0.5363185^{***}	(0.05971)	0.6218095^{***}	(0.108838)
s-share (α_2)	–	–	0.2085263	(0.3405296)
const. ($\log(A)$)	10.60251^{***}	(0.0391162)	9.675291^{***}	(1.622419)

As per IES data security procedures, school sample sizes in the table have been rounded to the nearest 10.
NOTE: Significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively.

set $I_j = \left\{ j \in \{52, 53, \ldots, 157\} : \xi_j'(j) > 0 \right\}$.[19] Estimates are reported in Table **??**. Note that $\widehat{\nu}$ exceeds $\widehat{\nu}_0^*$ for the exponential specification but not for the power specification; *i.e.*, the exponential point estimate is consistent with monotone investment under the model, whereas the power family is not. For this reason I shall adopt the exponential specification of costs moving forward. Figures 14 and 13 in the appendix display the lower convexity envelopes for the two specifications.

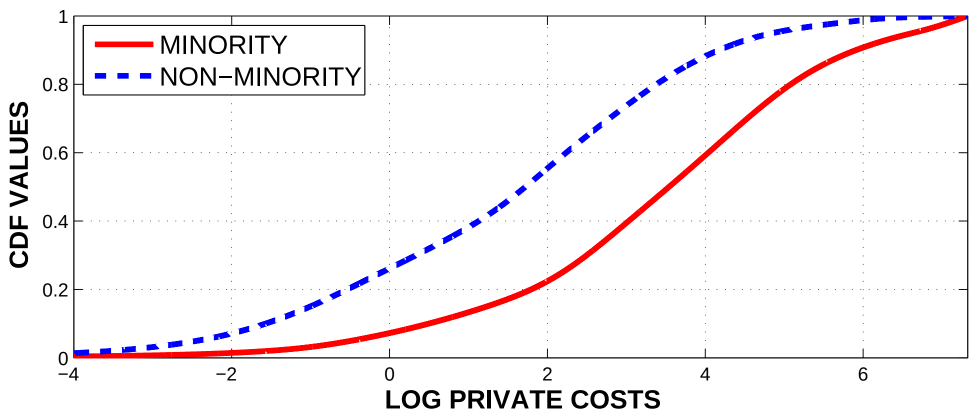
Type distributions are displayed in Figure 4. Note the strong first-order stochastic dominance relationship between minority costs and non-minority costs; in fact, the two are nearly (but not quite) ordered by likelihood ratio dominance. These pictures by

[19]Note that I ignore the sparse upper tail $\{158, 159, 160\}$ in estimating the curvature upper bound, for the reasons discussed at the beginning of this section.

TABLE 6. Cost Curvature Estimation

Cost Specification:	**Exponential**	**Power**
Cost Curvature Point Estimate	$\widehat{\nu}$: 0.1285	$\widehat{\nu}$: 6.6818
Bootstrapped 95% Conf. Int.	$[0.0567, 0.2165]$	$[2.9488, 11.2554]$
Monotonicity LB Point Estimate	$\widehat{\nu}_0^*$: 0.1076	$\widehat{\nu}_0^*$: 7.3457

FIGURE 4. Type Distributions and Densities



themselves do not give a clear idea of the implications of cost inequality, but this is explored in the counterfactual analysis in Section 6. Before moving on though, I explore the relative roles of the productive and competitive channels of investment incentives in HC investment.

5.4. **Productive Versus Competitive Channels of Investment Incentives.** Recall from the first-order condition $U_1[P_j(s), s]P_j'(s) + U_2[P_j(s), s] = C'(s;\theta)$ that the neatly decomposes incentives into the productive channel—the direct marginal benefit of an additional unit of HC—and the competitive channel—the indirect benefit of investment from improved match quality. Each student equates the sum of these two marginal benefits with the marginal cost of investment (the right-hand side) in order to optimize. However, if types were observable the planner could match students to colleges based on their production costs and allow them to invest *ex post*, in which case the FOCs would be $U_2[P_j(\theta), s] = C'(s;\theta)$. Here, $P_j(\theta)$, $j = \mathcal{M}, \mathcal{N}$ would be chosen to implement a desired allocation; specifically, the distribution of college seats awarded to group j under a given mechanism. Under certain conditions, this scenario could be termed the first-best implementation, since it would lead to an increase in student surplus.

Table **??** displays numbers relating to the full information comparison. The first column displays the average change in HC investment resulting from a full-information

TABLE 7. Private Information vs. Full Information Comparison

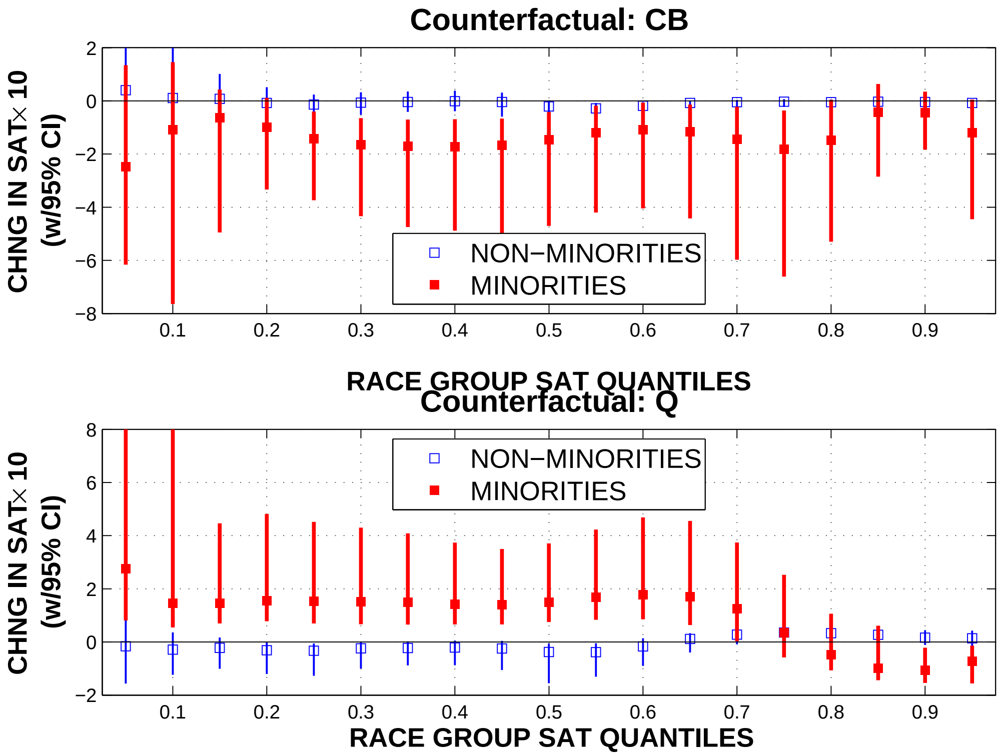
Full Info. Implementation	**Group**	**Avg. ΔS (SAT Units)**	**% Investment Due to Competition**	**% Surplus Increase**
AP	$\mathcal{M}$:	-106.6	12.0%	4.1%
	$\mathcal{N}$:	-119.2	11.5%	4.4%
	All:	-117.0	11.6%	4.3%
CB	$\mathcal{M}$:	-107.7	12.3%	3.8%
	$\mathcal{N}$:	-116.0	11.2%	4.2%
	All:	-114.5	11.4%	4.1%
Q	$\mathcal{M}$:	-102.7	11.5%	4.0%
	$\mathcal{N}$:	-121.4	11.7%	4.5%
	All:	-118.1	11.7%	4.4%

implementation, the second column displays this same number as a percentage of actual investment in the private information world, and the third column displays the percent increase in average surplus under full information. Observed average investment can be decomposed into match-specific incentives, accounting for about 89%, and competitive forces, accounting for 11%. Surplus rises under full information, with similar patterns under all three mechanisms. For minorities, the mechanism leading to the largest (smallest) role for competitive forces in investment is the color-blind (quota); *i.e.*, AA has the effect of shielding minorities from competitive forces. However, because of student-school complementarity and the productive channel of incentives—note that if $U_{12}(p,s) > 0$ then $U_2(p,s)$ is increasing in p—this *does not* imply the conventional wisdom that therefore minority investment must decrease. As shown in the next section, the opposite turns out to be true of the US college market: average minority investment under admission preferences or a quotas is actually *higher*.

6. COUNTERFACTUAL POLICY EXPERIMENTS

In this section I compute equilibria under two counterfactual policies which differ from allocations in the 1996 US college market. The first is a color-blind rule, or $\widetilde{S}(s) = s$, and simulates an AA ban. The second is a representative race quota, a form of AA more stringent than the US one which has been widely used abroad. For each alternative mechanism I computed equilibrium, holding fixed the supply of college seats, costs, the match utility function, and the number and types of competitors. Various measures are evaluated, including expected investment, college placement, graduation rates, surplus,

FIGURE 5. Investment Change, Relative to US Admission Preference



and racial inequality.[20] These calculations were done for each of the bootstrapped samples as well, in order to compute confidence intervals on differences between alternative policies. I present results in various tables and figures below.

Some comments on interpretation are worth discussion. Reported investment is measured in the original SAT units, $s \in [520, 1600]$. College salary premium numbers are measured in units of 2002 US dollars, as they are given by the Baccalaureate and Beyond 10-year salary data for graduates of the class of 1993. Within the model, investment costs play the role of a compensating differential relating economic outcomes to SAT units. Therefore, investment costs are also measured in units of *annual salary premium dollars*, since $C(s;\theta)$ gives the value of time and resources sunk into HC, as an equivalent amount of annual salary premium which justifies s as a best response to market forces.

6.1. **Investment.** The first question posed in this paper concerns the role of AA in provision of investment incentives. Figure 5 depicts counterfactual changes that would occur in HC investment as quantified by SAT scores, broken down by half deciles. Solid boxes

[20]Numerical integration for expectations was done via Simpson's rule quadrature. Step size for integrals was chosen to be small enough so that numerical error was at least an order of magnitude smaller than figures reported in Table 7 below.

TABLE 8. Enrollment %-Change in Academic Tiers, Relative to US AP

	Tier:	I	II	III	IV	V
$\mathcal{M}$	**CB:**	**-52.4%**	**-17.3%**	**-22.6%**	-1.6%	**+40.8%**
	95% CI:	[-64.3,-45.2]	[-34.1,-1.4]	[-49.0,-6.9]	[-23.1,14.6]	[27.6,65.3]
	Q:	**+59.3%**	**+52.5%**	+6.4%	**-14.5%**	**-37.4%**
	95% CI:	[41.5,79.3]	[32.3,67.1]	[-7.7,28.4]	[-23.0,-6.1]	[-42.7,-29.6]
$\mathcal{N}$	**CB:**	**+6.5%**	+2.3%	**+4.5%**	+0.4%	**-16.0%**
	95% CI:	[5.5,8.3]	[-0.1,4.9]	[1.4,9.1]	[-3.6,6.3]	[-23.6,-12.4]
	Q:	**-7.4%**	**-6.9%**	-1.3%	**+3.8%**	**+14.7%**
	95% CI:	[-9.1,-5.5]	[-8.3,-4.8]	[-4.4,1.9]	[1.4,6.9]	[9.7,19.8]
$\mathcal{M}$	**Naive CB:**	-48.0%	-13.9%	-14.8%	+3.5%	+29.8%
$\mathcal{N}$	**Naive CB:**	+6.8%	+4.6%	+2.1%	-0.6%	-15.5%

NOTE: Point estimates in bold font indicate that the 95% confidence interval does not contain zero.

are point estimates for the change in minority investment, and the empty boxes are for non-minorities. Changes from moving from the estimated US admission preference to a color-blind (quota) alternative are depicted in the upper (lower) pane.

Most minority students decrease investment by 10-20 SAT points in absence of AA, so enen though it partially shields them from competition, it still increases investment. This is because AA strengthens the productive channel by increasing access to colleges of higher quality. These effects become more vivid in the lower pane, which depicts further increases in investment that would occur under a quota, a more extreme form of AA. The comparison between the color-blind case and a quota can be inferred by adding the vertical distances between point estimates and the zero line in the upper and lower panes, implying a positive investment change near or exceeding 20 SAT points for most minority types. As required by theory, the best and brightest minorities are disincentivized from investment under either form of AA, but empirically this negative effect is small and limited to the lowest 2% of the minority cost distribution. Predominantly, minority incentives induced by either form of AA are positive.

Interestingly, AA mechanisms can be used by a policy-maker to improve the total stock of human capital in the economy. As it turns out, average investment under a quota mechanism is 3.2 SAT points higher than under a color-blind mechanism. In fact, there is nearly a stochastic dominance relationship between the model-generated HC distributions: except between the 54$^{\text{th}}$ and 67$^{\text{th}}$ percentiles, $G^{q}_{\mathcal{K}}(s)$ lay strictly to the right of $G^{cb}_{\mathcal{K}}(s)$, which means a quota motivates 87% of the population to invest more than they would have under a color-blind mechanism. As Hickman [21, Theorem 5.4] showed, when cost distributions are ordered by likelihood ratio dominance (a strong form of first-order dominance), two groups increase investment—middle- to high-cost minorities and low-cost non-minorities—and the rest of the population decreases investment. Empirically, since these two groups are most plentiful within their demographic categories, the positive effect dominates for most quantiles of the general population. This highlights a surprising result: if the social planner values HC above the private returns accruing to students, *e.g.*, due to positive externalities, then it is possible that an extreme form of AA such as a quota could be strictly preferred to a color-blind mechanism for it's superior investment incentive provision.

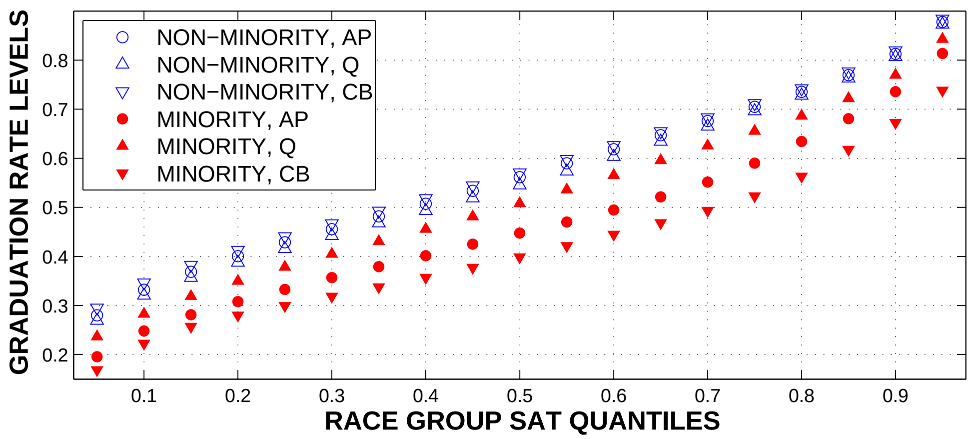
6.2. **Enrollment.** Table 6 presents counterfactual enrollment changes within the five quintiles of the college quality spectrum, relative to the estimated US admission preference. Tier I represents the highest quality quintile, and so on. The top two sections of the table display percent changes and for comparison, the lower section displays predictions under a naive color-blind counterfactual holding SATs fixed. The naive estimate tends to under-predict the impact of an AA ban because it does not take into account endogenous changes to investment.

The full counterfactual estimates paint a striking picture. Under an AA ban, minority enrollment in Tiers I–III would fall by 52.4%, 17.3%, and 22.6%, respectively. Note also the resulting shift toward minority enrollment in the lowest tier, which would increase by 40.8%. These figures illustrate the large role played by AA in shaping student bodies at American colleges. Of course, US AA does not achieve racially representative enrollment, as under a quota (see Q row), which would require further substantial positive shifts in enrollment for minority college candidates. Enrollment shifts for non-minorities on a percentage basis are much smaller, but this is merely an artifact of the small minority mass of 17.7%, since each additional top seat occupied by an $\mathcal{M}$ candidate must be vacated by an $\mathcal{N}$ candidate. However, market welfare more broadly is not a zero-sum game since costs incurred and gross match utilities depend on investment.

6.3. **Gross Utility: Graduation Rates and Salary Premia.** Table 7 displays model generated group averages under each mechanism for various measures including investment, investment costs, graduation rates, 10-year salary premia, and surplus. For comparisons, the right-most panel displays differences across alternative policies. The first two columns show average differences when moving from the US admission preference to a color-blind or quota system, respectively. For completeness, the third column shows differences when moving from a color-blind system to a quota.

In terms of projected counterfactual graduation rates $\rho(p,s)$, the model predicts minority rates of 47.1%, 42.4%, and 52% under the admission preference, color-blind, and quota mechanisms, respectively. To put this in perspective, an AA ban would result

FIGURE 6. Estimated Graduation Rate Levels by Mechanism



in a projected 11% decrease in the number of Black, Hispanic, and Native American college graduates, and a more stringent quota regimen would produce a similar additional increase in the number of minority graduates. These effects are achieved through a combination of increased HC and improved college placement.

Figures 6 and 7 break these average effects down by the half deciles. The former displays projected graduation rate levels under different mechanisms, and the latter displays changes relative to the estimated US admission preference. Note that some of the largest gains from AA in terms of graduation rates is had by candidates roughly in the middle two quartiles. This suggests arguments put forth by AA opponents (see Sowell [33] for an extensive exposition) that such policies benefit primarily the most fortunate minority competitors, may not apply to the US college market. On the other hand, that the average change in graduation tendency for Whites and Asians occurs in opposite directions so that overall graduation rates are left virtually unaffected.

Similar patterns arise for projected salary premiums, $u(p,s) - u_0$. Note, however, that salary premium considered in isolation is effectively a zero-sum game, since investment does not enter into Specification 1 reported in Table 5 above.[21] Relative to a color-blind mechanism, AA in the 1996 American form amounted to a transfer of \$1,975 in annual salary to the average minority college student (a 7.9% increase), and of \$423 away from the average non-minority student (a 1.4% decrease). A switch from the status quo admission preference to a quota system is projected to produce an additional transfer of roughly the same size. Figure 8 breaks down these average effects by the half deciles. Once again, the figure suggests that medium- and high-cost minority students gain from

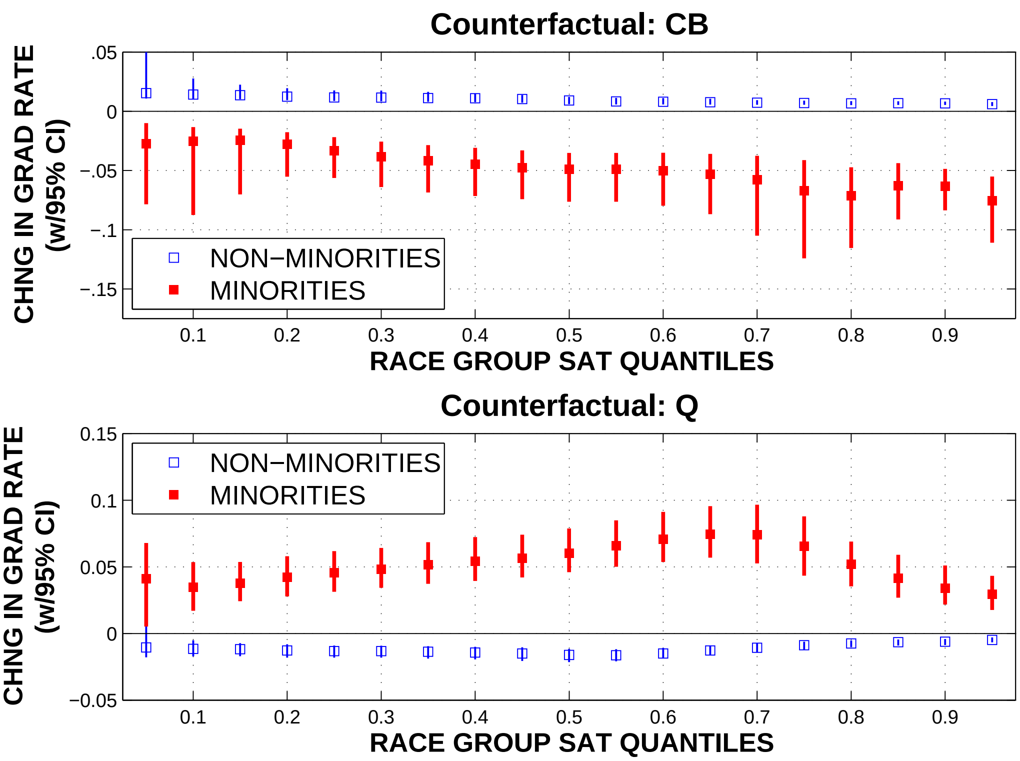
[21]Even under Specification 2 in Table 5, the role of investment in this component of utility is small.

TABLE 9. Counterfactual Means by Group

		Group	Mechanism AP	Mechanism CB	Mechanism Q	Differences* Δ_{CB-AP}	Differences* Δ_{Q-AP}	Differences* Δ_{Q-CB}
INVESTMENT	$\mathbb{E}[S]$	$\mathcal{M}$:	886	873	895	**-12.5**	**+9.7**	**+22.2**
		95% CI:	[879, 889]	[850, 880]	[890, 912]	[−32.7, −6.7]	[4.8, 32.7]	[12.5, 62.6]
		$\mathcal{N}$:	1037	1037	1036	-0.2	-1.0	**-0.9**
		95% CI:	[1020, 1038]	[1035, 1038]	[1030, 1038]	[−0.7, 15.4]	[−2.3, 10]	[−6.2, −0.1]
		All:	1010	1008	1011	-2.4	**+0.8**	**+3.2**
		95% CI:	[995, 1012]	[1002, 1011]	[1008, 1013]	[−3.6, 7.7]	[0.4, 13.2]	[1.9, 6.2]
COSTS	$\mathbb{E}[C(S;\theta)]$	$\mathcal{M}$:	$3,277	$2,800	$3,635	**-$476**	**+$358**	**+$835**
		95% CI:	[1984, 6944]	[1709, 5824]	[2185, 8434]	[−1237, −247]	[170, 1308]	[455, 2537]
		$\mathcal{N}$:	$3,410	$3,386	$3,379	-$24	-$31	-$7
		95% CI:	[2076, 6912]	[2071, 7266]	[2062, 7094]	[−54, 188]	[−107, 52]	[−170, 12]
		All:	$3,387	$3,283	$3,424	**-$104**	**+$38**	**+$141**
		95% CI:	[2068, 6914]	[2009, 6981]	[2085, 7597]	[−184, −37]	[17, 361]	[80, 308]
GRAD RATE	$\mathbb{E}[\rho(P(S), S)] \times 100$	$\mathcal{M}$:	47.1	42.4	52.0	**-4.7**	**+4.9**	**+9.7**
		95% CI:	[45.4, 49.3]	[40.4, 44]	[50.4, 54]	[−7.1, −3.7]	[3.9, 6]	[8.7, 12]
		$\mathcal{N}$:	56.8	57.8	55.7	**+1**	**-1.1**	**-2.1**
		95% CI:	[55.2, 58]	[56.5, 59.2]	[54.3, 57]	[0.8, 1.8]	[−1.4, −0.7]	[−2.6, −1.9]
		All:	55.1	55	55	-0.04	-0.1	+0.03
		95% CI:	[53.5, 56.5]	[53.7, 56.5]	[53.7, 56.4]	[−0.1, 0.3]	[−0.1, 0.3]	[−0.1, 0.04]
SAL PREMIUM	$\mathbb{E}[u(P(S), S) - u_0]$	$\mathcal{M}$:	$27,016	$25,041	$29,109	**-$1,975**	**+$2,094**	**+$4,068**
		95% CI:	[25827, 28530]	[23664, 26303]	[27867, 30333]	[−3221, −1508]	[1332, 2606]	[3337, 5022]
		$\mathcal{N}$:	$29,558	$29,981	$29,109	**+$423**	**-$449**	**-$872**
		95% CI:	[28251, 30821]	[28712, 31287]	[27867, 30333]	[328, 662]	[−581, −276]	[−1084, −715]
		All:	$29,109	$29,109	$29,109	$0	$0	$0
		95% CI:	[27867, 30333]	[27867, 30333]	[27867, 30333]	N/A	N/A	N/A
SURPLUS	$\mathbb{E}[U(P(S), S) - C(S;\theta)] + u_0$	$\mathcal{M}$:	$36,657	$34,931	$38,659	**-$1,726**	**+$2,001**	**+$3,728**
		95% CI:	[33294, 38277]	[31412, 36241]	[33907, 40332]	[−2295, −1337]	[836, 2320]	[2736, 4197]
		$\mathcal{N}$:	$40,491	$40,981	$39,981	**+$490**	**-$510**	**-$1000**
		95% CI:	[36540, 42139]	[37031, 42647]	[36100, 41620]	[378, 625]	[−633, −365]	[−1116, −870]
		All:	$39,814	$39,913	$39,747	**+$98**	**-$66**	**-$165**
		95% CI:	[35925, 41459]	[36070, 41517]	[35327, 41395]	[46, 181]	[−269, −46]	[−309, −113]
INEQUALITY*			**$3,833**	**$6,050**	**$1,322**	**+$2,217**	**-$2,511**	**-$4,727**
		95% CI:	[3022, 4364]	[5310, 6631]	[1030, 1933]	[1734, 2849]	[−2930, −1304]	[−5276, −3688]

For reported differences, bold font indicates the 95% CI does not contain zero. Computation of expectations in the table involved numerical integration of equilibrium distributions via Simpson's rule quadrature. Step size for integrals was chosen so that numerical error was at least an order of magnitude smaller than figures reported above.

FIGURE 7. Change in Graduation Rates, Relative to US Admission Preference



AA in terms of gross income just as much as the lowest-cost segment of their group, and even more so under a quota.

6.4. **Surplus.** Figure 9 and the lower panel of Table 7 depict counterfactuals on surplus, or match utilities $U(p,s) = \rho(p,s)[u(p,s) - u_0]$ net of monetized investment disutility $C(s;\theta)$. The model predicts that the 1996 US admission preference amounted to an increase of \$1,726 in annual salary dollars for minorities, or a 4.9% increase in surplus, relative to the color-blind alternative. This comparison also entailed a drop of 1.2% in average non-minority surplus. Switching to a quota system from the status quo mechanism would induce an additional surplus gain for minorities of \$2,001 annual salary dollars or 5.5%, with an additional shift of 1.3% away from non-minorities.

In terms of market surplus, the argument that advantaged minorities benefit most is somewhat more fitting. In Figure 9 the difference between minority color-blind surplus and admission preference surplus is increasing in SAT quantile. This is because low-cost minorities derive benefits in graduation rates and income from AA, while increasing investment by less than their medium- and high-cost counterparts. On the other hand, surplus gains moving from an admission preference to a quota trail off somewhat toward

FIGURE 8. Change in Salary Premium

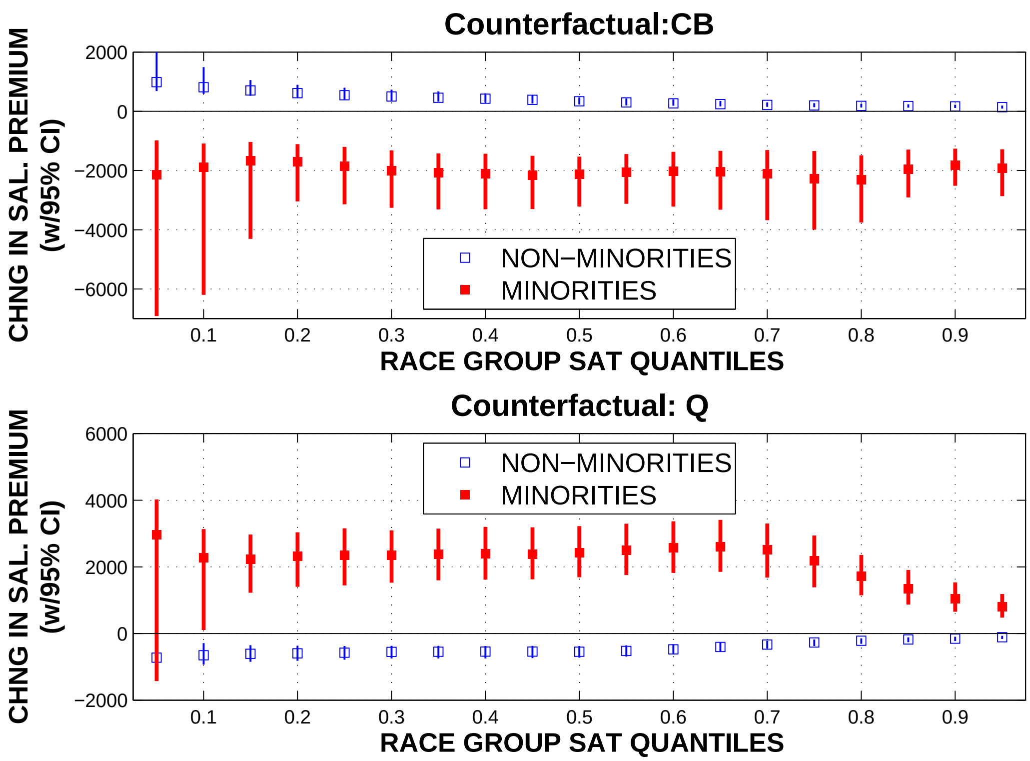
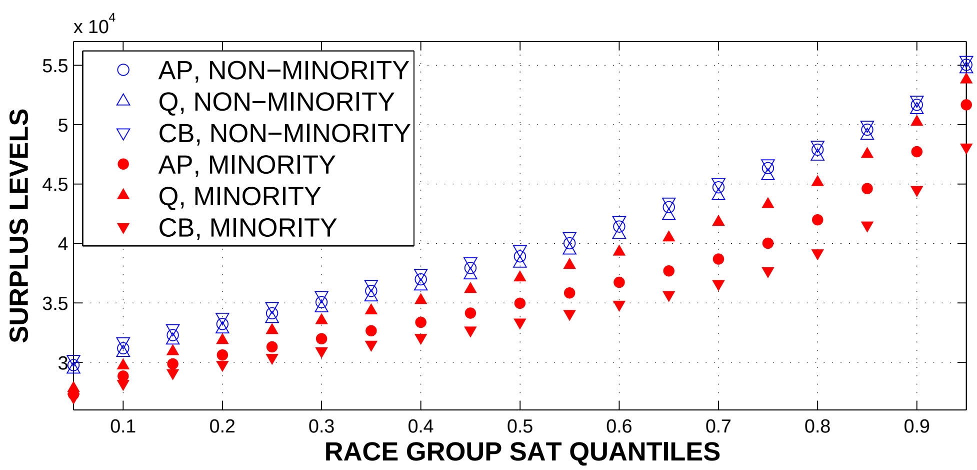


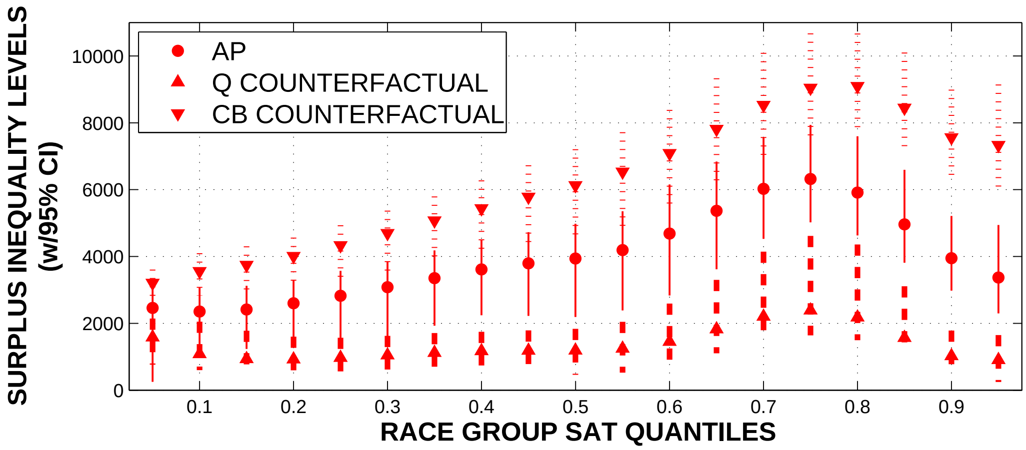
FIGURE 9. Counterfactual Surplus Levels by Race Group



the upper end of the distribution, and at the end of the day, either form of AA delivers substantial surplus increases to a large segment of the minority group.

For total market surplus, Table 7 indicates that a ban on AA would produce a small but statistically significant improvement of \$98 annual salary dollars or 0.24%. A quota mechanism also produces a decrease in surplus of \$66 annual salary dollars or 0.16%,

FIGURE 10. Inequality Counterfactuals by Group Quantiles



relative to the US admission preference. These numbers show that AA induces market inefficiency as one might expect, with the loss being driven mainly by increased investment costs through weakened assortativity of the match. Though they are not reported, average changes in gross match utilities are very small across different mechanisms.

6.5. **Inequality.** Inequality figures in Table 7 were computed as the difference in average surplus between the two groups, under a given allocation mechanism. A ban on AA would produce a 57.8% increase in surplus inequality. On the other hand, replacing the US admission preference with a quota would further reduce inequality by 65.5% relative to the current market state.[22] Figure 10 once again breaks down this picture more finely; it displays the difference between surplus for the φ-ranked non-minority student and the φ-ranked minority student, for $\varphi \in \{0.05, 0.1, \ldots, 0.95\}$.

Given that the three mechanisms can be ranked by their magnitude of allocative distortions—color-blind, US admission preference, and quota, in that (increasing) order—one might anticipate that inequality and market surplus are both decreasing in the distortion ordering. Many have argued that equality and racial diversity on college campuses is a social good in itself, so a question remains about the equity-efficiency trade-off: can a given decrease in racial inequality justify the price of diminished market surplus? This is a difficult question to answer without knowing the social planner's objective function, but the counterfactual exercise may shed some light on it.

Assume there exists a well-defined social welfare function $\mathcal{U}$ and consider three simple specifications involving a trade-off between surplus, denoted $\mathcal{E}_j(\theta) = U\left(P(\sigma_j(\theta)), \sigma_j(\theta)\right) -$

[22]Note that a quota mechanically eliminates inequality of allocations, but it does not eliminate inequality of investment, which is tied to exogenous cost types. This is why a quota does not decrease surplus inequality by 100% even though it does so for allocations of college seats.

$C(\sigma_j(\theta);\theta) + u_0$, $j = \mathcal{M}, \mathcal{N}$, and race inequality, denoted $\mathcal{I} = \overline{\mathcal{E}}_{\mathcal{N}} - \overline{\mathcal{E}}_{\mathcal{M}}$, with $\overline{\mathcal{E}}_j = \mathbb{E}\left[\mathcal{E}_j(\theta)\right]$, $j = \mathcal{M}, \mathcal{N}$ denoting group surplus mean, and let $\overline{\mathcal{E}}_{\mathcal{K}} = \mu\overline{\mathcal{E}}_{\mathcal{M}} + (1-\mu)\overline{\mathcal{E}}_{\mathcal{M}}$ denote the total mean. For each policy $r \in \{ap, cb, q\}$, define the following social utility specifications: $\mathcal{U}_1\left(\overline{\mathcal{E}}^r_{\mathcal{K}}\mathcal{I}^r\right) \equiv \overline{\mathcal{E}}^r_{\mathcal{K}} - \omega_1\mathcal{I}^r$, $\mathcal{U}_2\left(\overline{\mathcal{E}}^r_{\mathcal{M}}, \overline{\mathcal{E}}^r_{\mathcal{N}}\right) \equiv (1-\mu)\overline{\mathcal{E}}^r_{\mathcal{N}} + (\mu\omega_2)\,\overline{\mathcal{E}}^r_{\mathcal{M}}$, and $\mathcal{U}_3\left(\mathcal{E}^r_{\mathcal{M}}, \mathcal{E}^r_{\mathcal{N}}\right) \equiv \left[(1-\mu)\int_{-\infty}^{\infty}\left[\mathcal{E}^r_{\mathcal{N}}(\theta)\right]^{(1-\eta)} f_{\mathcal{N}}(\theta)d\theta + (\mu\omega_3)\int_{-\infty}^{\infty}\left[\mathcal{E}^r_{\mathcal{M}}(\theta)\right]^{(1-\eta)} f_{\mathcal{M}}(\theta)d\theta\right]^{\left(\frac{1}{1-\eta}\right)}$.

The first specification assumes simply that society derives (linear) utility from surplus and disutility from racial inequality, with ω_1 quantifying the trade-off. The second specification has a different interpretation: in it both non-minority and minority surplus are valued, but the per-capita weight on non-minority and minority surplus may differ. Finally, the third specification is a generalization of the second: it assumes a standard constant elasticity welfare function while allowing the per-capita weight on minority surplus to differ according to ω_3. The elasticity parameter η controls the trade-off between socioeconomic inequality and total surplus: as $\eta \to 0$ social preferences become utilitarian ($\mathcal{U}_2$ is this special case), and as $\eta \to \infty$ social preferences become strongly egalitarian until the planner derives utility only from the surplus level of its poorest member.[23]

These social welfare function specifications set up a simple thought experiment to explore the efficiency versus racial equity trade-off. Since the US admission preference was implemented over a color-blind mechanism which maximizes total surplus, suppose that society is just indifferent between market outcomes under the two mechanisms. This supposition allows one to infer the value of the equity-efficiency parameters ω_l, $l = 1,2,3$ by solving $\omega_1^* \equiv \left\{\omega_1 \in \mathbb{R} : \mathcal{U}_1\left(\overline{\mathcal{E}}^{ap}_{\mathcal{K}}, \mathcal{I}^{ap}\right) = \mathcal{U}_1\left(\overline{\mathcal{E}}^{cb}_{\mathcal{K}}, \mathcal{I}^{cb}\right)\right\}$ for Specification 1, $\omega_2^* \equiv \left\{\omega_2 \in \mathbb{R} : \mathcal{U}_2\left(\overline{\mathcal{E}}^{ap}_{\mathcal{M}}, \overline{\mathcal{E}}^{ap}_{\mathcal{N}}\right) = \mathcal{U}_2\left(\overline{\mathcal{E}}^{cb}_{\mathcal{M}}, \overline{\mathcal{E}}^{cb}_{\mathcal{N}}\right)\right\}$ for Specification 2, and $\omega_3^* \equiv \left\{\omega_3 \in \mathbb{R} : \mathcal{U}_3\left(\mathcal{E}^{ap}_{\mathcal{M}}, \mathcal{E}^{ap}_{\mathcal{N}}\right) = \mathcal{U}_3\left(\mathcal{E}^{cb}_{\mathcal{M}}, \mathcal{E}^{cb}_{\mathcal{N}}\right)\right\}$ for Specification 3. For the first two, the equity-efficiency parameter is pinned down by the ap versus cb comparison. For the third there are two unknown parameters, so I shall consider various elasticity values η and solve for ω_3^* under each. With these numbers in hand, one can then draw comparisons between a quota and the other two mechanisms by asking whether social utility would rise under the more extreme form of AA. Table 8 displays the results.

In Specification 1 we see that in order for society to be indifferent between the admission preference and color-blind mechanisms, the planner would have to be willing to trade \$0.0446 of total surplus for \$1 of less racial inequality. This suggests that only a small preference against racial inequality is needed in order to justify current AA practices. Specification 2 requires a somewhat larger preference for equity: the planner

[23]See Mas-Colell, Whinston, and Green [28, Chapter 22] for more discussion on properties of social welfare functions.

TABLE 10. Efficiency vs. Racial Equity Comparison

$\mathcal{U}_l$ **Specification** $l = 1, 2, 3$	η	ω_l^*	$\left(\mathcal{U}^q\|_{\omega_l=\omega_l^*} - \mathcal{U}^{ap}\|_{\omega_l=\omega_l^*}\right)$
1	–	0.0446	44.6
2	0	1.325	48.2
3	0.1	1.325	43.0
3	0.2	1.325	45.8
3	0.4	1.325	48.9
3	0.8	1.324	63.6
3	1*	1.324	0.00136
3	2	1.320	54.0
3	6	1.293	67.1
3	10	1.260	65.6

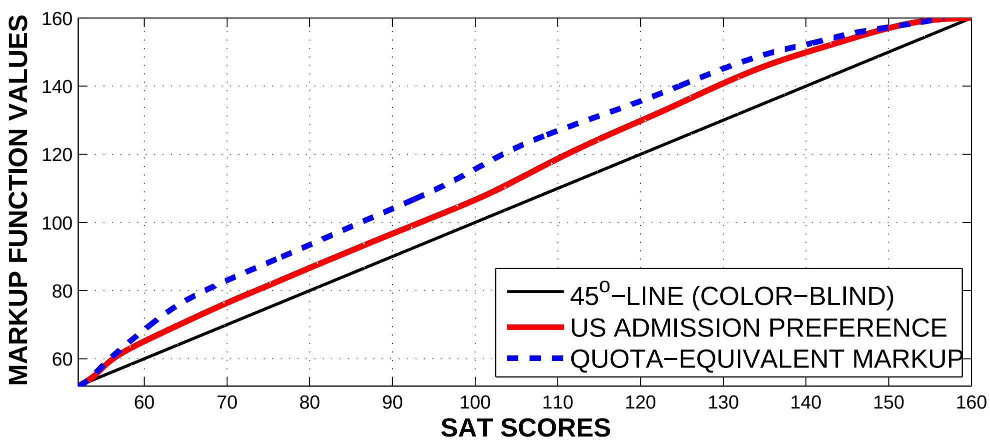
*NOTE: For $\eta = 1$ we have
$\mathcal{U}_3 \equiv (1-\mu)\int \log[\mathcal{E}_{\mathcal{N}}(\theta)] f_{\mathcal{N}}(\theta)d\theta + \mu\omega_3 \int \log[\mathcal{E}_{\mathcal{M}}(\theta)] f_{\mathcal{M}}(\theta)d\theta$

would have to value \$1.33 of non-minority surplus the same as \$1 for minorities in order for society to be indifferent between color-blind admissions and the US admission preference. In Specification 3 this race preference (ω_3^*) weakens only slightly as preferences for socioeconomic inequality (η) move from utilitarian to egalitarian.

For all three welfare functions the difference reported in the final column is positive. This suggests that racial equality preferences which justify the US admission preference over color-blind mechanism also imply that a quota mechanism is preferred to both for producing a more favorable trade-off between efficiency and racial inequality. Of course, this simple exercise omits important factors which may also play a role. If society has an aversion to redistributing wealth away from some of its members on the basis of race—most legal challenges to AA are based on this premise—then it would tip the scales in favor of less extreme forms of AA. If on the other hand the social planner desires a level of investment above that seen in the market—say, due to positive externalities from HC, or because high-school students are too near-sighted when they are investing (see Laibson [25])—then she may prefer a quota mechanism. Despite these omissions, Table 8 is still enlightening as to the nature of the equity-efficiency trade-off: if admission preferences are at least indifferent to color-blind admissions on the basis of this trade-off alone, then a quota is strictly preferred.

6.6. **Policy Function Comparisons.** As a final illustration of the differences between the three mechanisms considered in this study, Figure 11 depicts the corresponding markup functions. The thin solid line is a color-blind mechanism, or $\widetilde{S}(s) = s$, the thick solid

FIGURE 11. Comparison of Markup Functions: AP vs. Q vs. CB



line is the US admission preference $\widehat{\widetilde{S}}$, and the thick dashed line represents a quota-equivalent markup, or $\widetilde{S}(s) = G_{\mathcal{N}}^{-1}\left[G_{\mathcal{M}}(s)\right]$. Under a quota markup, admissions officers would rank students by their group-specific quantile instead of their raw SAT score. Rather than earmarking representative sets of college seats from a common set as in an actual quota, such a markup would instead merge two sets of students in a similar way so as to achieve identical outcomes and incentives.

Whether bans against quotas extend to equivalent markup systems is an interesting legal question, but beyond the scope of this study. Regardless of its legality though, the comparison is still interesting from the perspective of incentive provision. Toward the extremes, a quota markup resembles the US admission preference, but within a central region it actually provides greater investment incentives by increasing the *marginal markup* for higher investment. A linear approximation to the quota markup within the inter-quartile range of minority scores (between 770 and 1010) gives roughly an 8% SAT inflation factor with a fixed markup of 62 SAT points. Contrast this to a linear approximation of the US admission preference on that same interval, which offers close to a 0% inflation with a fixed markup of 68 SAT points. By inflating scores on a percentage basis rather than just relying on a level shift, the quota markup rewards students for increasing investment with a larger markup. This is why it is able to provide stronger investment incentives than the admission preference: it subsidizes *marginal costs* of human capital production rather than total costs, and in so doing it induces many students to invest as if their marginal cost type θ was actually lower.

7. CONCLUSION

This paper has developed identification and estimation results for endogenous human capital within large two-sided matching markets, based on a model of investment with incentives provided by a centralized ank-order mechanism for allocation of matching rights. The auction-theoretic framework allowed for an empirical investigation of the impacts of affirmative action on the entire 4-year-degree market in the United States, rather than focusing only on elite private colleges for which matched school-student applications data are available.

AA is a prominent feature of the entire college market, and has significant consequences for investment, redistribution, and welfare. Minority students invest more in human capital in response to improved opportunities afforded them by AA, and in turn, their graduation rates and economic outcomes improve. Moreover, AA policies can be used as a tool to increase the overall stock of human capital in the market, relative to a color-blind mechanism. From a policy standpoint, if AA is a viable tool to improve the stock of minority human capital, then the long-run impacts of such policies on inequality for subsequent generations may be important considerations for future research.

Despite improvements in racial inequality, AA does induce a loss in total market surplus. However, under various specifications of social welfare functions that include a racial equity-efficiency trade-off, an extreme form of affirmative action—a strict racial quota—is preferred to both the color-blind and US admission preference. Relative to these two alternatives, a quota also provides the highest HC investment incentives by subsidizing marginal academic production costs for minority students.

References

[1] Fool's gold. *The Economist Magazine*, April 27, 2013.

[2] Peter Arcidiacono. Affirmative action in higher education: How do admission and financial aid rules affect future earnings? *Econometrica*, 73(5):1477–1524, 2005.

[3] Peter Arcidiacono, Esteban Aucejo, Hanming Fang, and Ken Spenner. Does affirmative action lead to mismatch? a new test and evidence. *Quantitative Economics*, 2(3):303–333, 2011.

[4] Peter Arcidiacono, Esteban Aucejo, and Ken Spenner. What happens after enrollment? an analysis of the time path of racial differences in gpa and major choice. *IZA Journal of Labor Economics*, 1:Article 5, 2012.

[5] Susan Athey. Single crossing properties and the existence of pure strategy equilibria in games of incomplete information. *Econometrica*, 69(4):861–869, 2001.

[6] William G. Bowen and Derek Bok. *The Shape of the River: Long-Term Consequences of Considering Race in College and University Admissions*. Princeton, NJ: Princeton University Press, 1998.

[7] Sandra Campo, Immanuel Guerre, Isabelle Perrigne, and Quang Vuong. Semiparametric estimation of first-price auctions with risk averse bidders. *The Review of Economic Studies*, 78:112–147, 2011.

[8] Dario Cestau, Dennis Epple, and Holger Sieg. Admitting students to selective education programs: Merit, profiling, and affirmative action. *typescript, Carnegie Mellon University, Tepper School of Business*, 2012.

[9] Hector Chade, Gregory Lewis, and Lones Smith. Student portfolios and the college admissions problem. *Typescript, Arizona State University, W. P. Carey School of Business*, 2013.

[10] David L. Chambers, Timothy T. Clydesdale, William C. Kidder, and Richard O. Lempert. The real impact of eliminating affirmative action in american law schools: An empirical critique of richard sander's study. *Stanford Law Review*, 57:1855–1897, 2005.

[11] Chang Y. Chung and Thomas J. Espenshade. The opportunity cost of admission preferences at elite universities. *Social Science Quarterly*, 86(2):293–305, 2005.

[12] Chang Y. Chung, Thomas J. Espenshade, and Joan L. Walling. Admission preferences for minority students, athletes, and legacies at elite universities. *Social Science Quarterly*, 85(5):1422–1446, 2004.

[13] Stephen Coate and Glenn Loury. Anti-discrimination enforcement and the problem of patronization. *The American Economic Review*, 83(2):92–98, 1993.

[14] Christopher Cotton, Brent R. Hickman, and Joseph Price. Affirmative action and human capital investment: Evidence from a randomized field experiment. *typescript, University of Chicago Economics Dept.*, 2012.

[15] Xavier D'Haultfoeuille and Philippe Fevrier. Identification of a class of adverse selection models with contracts variation. *typescript, Centre de Resherche en Economie et Statistique*, 2010.

[16] Xavier D'Haultfoeuille and Philippe Fevrier. The provision of wage incentives: A structural estimation using contracts variation. *typescript, Centre de Resherche en Economie et Statistique*, 2011.

[17] Dennis Epple, Richard Romano, and Holger Sieg. On the demographic composition of colleges and universities in market equilibrium. *The American Economic Review*, 92(2):310–314, 2002.

[18] Bruno Ferman and Juliano Assunção. Does affirmative action enhance or undercut investment incentives? evidence from quotas in brazilian public universities. *Typescript, Massachusetts Institute of Technology Department of Economics*, 2011.

[19] Emmanuel Guerre, Isabelle Perrigne, and Quang Vuong. Optimal nonparametric estimation of first-price auctions. *Econometrica*, 68(3):525–574, 2000.

[20] Emmanuel Guerre, Isabelle Perrigne, and Quang Vuong. Nonparametric identification of risk aversion in first-price auctions under exclusion restrictions. *Econometrica*, 77(4):1193–1227, 2009.

[21] Brent R. Hickman. Using auction theory to study human capital investment in frictionless matching markets: A look at affirmative action in college admissions. *Typescript, University of Chicago Department of Economics*, 2012.

[22] Brent R. Hickman and Timothy P. Hubbard. Replacing sample trimming with boundary correction in nonparametric estimation of first-price auctions. *Typescript, University of Chicago Department of Economics*, 2012.

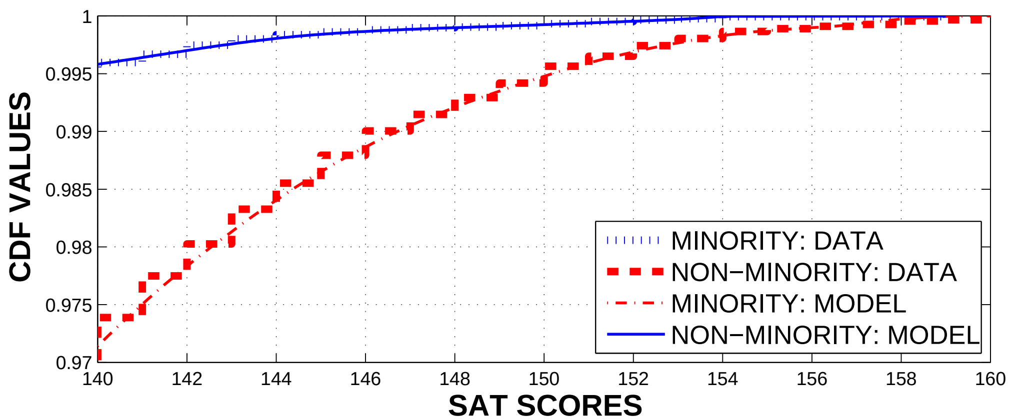
[23] Jessica S. Howell. Assessing the impact of eliminating affirmative action in higher education. *Journal of Labor Economics*, 28(1):113–66, 2010.

[24] Rhoana J. Karunamuni and Shunpu Zhang. Some improvements on a boundary corrected kernel density estimator. *Statistics & Probability Letters*, 78:499–507, 2008.

[25] David Laibson. Golden eggs and hyperbolic discounting. *The Quarterly Journal of Economics*, 112(2):443–478, 1997.

[26] Mark C. Long. College quality and early adult outcomes. *Economics of Education Review*, 27:588–602, 2008.

FIGURE 12. Model Fit: Investment in the Upper Tails



[27] Linda Datcher Loury and David Garman. Selectivity and earnings. *Journal of Labor Economics*, 13(2):289–308, 1995.
[28] Andreu Mas-Colell, Michael D. Whinston, and Jerry R. Green. *Microeconomic Theory*. Oxford University Press, 1995.
[29] Paul Milgrom and Robert Weber. A theory of auctions and competitive bidding. *Econometrica*, 50(5):1089–1122, 1982.
[30] Robert J. Morse, editor. *America's Best Colleges*. Washington D.C.: US News & World Report, 1996.
[31] Jesse Rothstein and Albert H. Yoon. Affirmative action in law school admissions: What do racial preferences do? *University of Chicago Law Review*, 75(2):649–714, 2008.
[32] Richard H. Sander. A systemic analysis of affirmative action in american law schools. *Stanford Law Review*, 57:367–483, 2004.
[33] Thomas Sowell. *Affirmative Action Around the World*. Yale University Press, 2004.
[34] William Vickrey. Counterspeculation, auctions, and competitive sealed tenders. *Journal of Finance*, 16(1):8–37, 1961.
[35] Robert Wilson. A Bidding Model of Perfect Competition. *The Review of Economic Studies*, 44(3):511–518.
[36] S. Zhang, Rhoana J. Karunamuni, and M. C. Jones. An Improved Estimator of the Density Function at the Boundary. *Journal of the American Statistical Association*, 94:1231–1241, 1999.

APPENDIX

7.1. **Additional Tables and Figures.** Figure 12 depicts the fit of the model-generated investment distribution within the upper tail of the distribution. The region depicted is the one above the 1415 cutoff (*i.e.*, the 98$^{\text{th}}$ percentile), where cardinal utility projections $\widehat{U}$ are extrapolated. The model appears to fit well, despite the sparseness of data and modifications to the estimator in the upper tail.

For the exponential cost and power cost models, Figures 13 and 14 plot the non-parametric lower bound $\widehat{\zeta}(s)$ (*i.e.*, the lower bound on the level of ν needed to ensure monotonicity) and the point estimate $\widehat{\nu}$. This gives an idea of which segments of the market place the strongest restrictions on the model.

FIGURE 13. Curvature Lower Bound: Exponential Costs

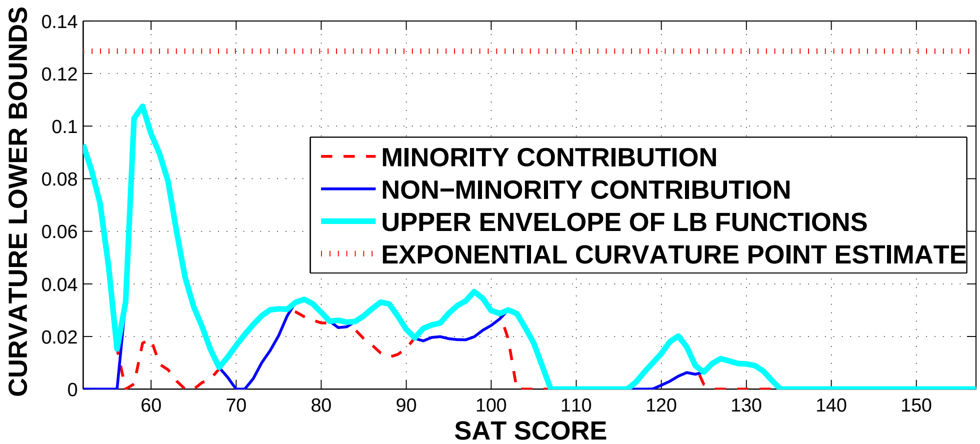
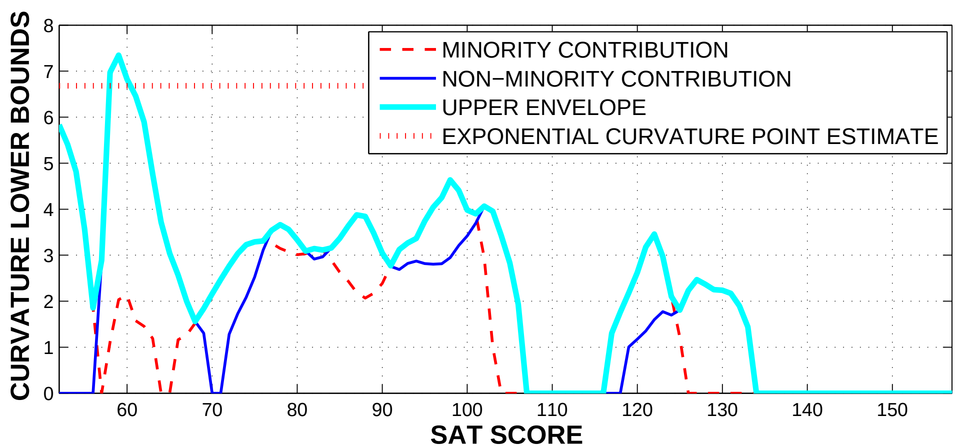


FIGURE 14. Curvature Lower Bound: Power Costs



7.2. **USNWR Data and Methodology.** As mentioned in Section 3, USNWR collects a set of 14 objective quality indicators which they use to compute a single index of quality for their *America's Best Colleges* rankings. They classify the 14 indicators into 6 categories: *selectivity*, comprised of application acceptance rate, yield, average entrance test scores, and % of first-time freshmen in the top quartile of their high school class; *faculty resources*, comprised of % of full-time instructional faculty with a PhD or terminal degree, % of instructional faculty who are full-time, average faculty compensation, and student/faculty ratio; *financial resources*, comprised of education spending per student and non-education spending per student; *retention*, comprised of graduation rate and freshman retention rate; *alumni satisfaction*, comprised of % of living alumni contributing to annual fund drives; and *academic reputation*, comprised of a ranking measure taken from a survey of college administrators. A single index of quality is determined by computing empirical distributions for each indicator, and taking a weighted average of the 14 empirical CDF values for a given school.

The USNWR method separates schools by Carnegie classification (*i.e.*, national/regional universities and national/regional liberal arts colleges) and geographic region (see Morse [30] for more details) when computing empirical rankings. In order to obtain one common ranking for all schools, I alter the original method by combining all schools into the same set. This does not pose a problem for most of the quality indicators, except one: the academic reputation score. This score is determined by asking college administrators to rank the schools in their Carnegie class and region. Since the reputation score loses its meaning when taken outside of these smaller subsets of schools (and was not provided anyway), I drop it from the list and generate the quality index with the remaining 13 indicators, uniformly spreading the reputation weight among the remaining 5 categories. This is of little consequence for the overall rankings, due to the high degree of correlation among the quality indicators.

Table 9 summarizes weights and descriptive statistics for each the individual quality indicators. In cases where USNWR lacks a certain datum for some school, their methodology replaces it with the lowest value observed for schools within the same region and Carnegie classification, so I do the same. Finally, in computing their quality index, USNWR maps average SAT and ACT scores into the corresponding cumulative distribution values within the SAT and ACT score distributions. This allows for comparisons of scores on different tests. The mean and standard deviation for average test scores in the table reflect this transformation.

A common criticism of the USNWR methodology is that the rankings are dependent upon an arbitrary weighting scheme. Critics sometimes accuse USNWR of manipulating the weights assigned to the different quality indicators, in order to alter the relative standings of elite schools. However, this objection is inconsequential if one takes the larger picture into account. Because of the high degree of correlation among the 13 quality indicators, the overall prize distribution is fairly robust to substantial changes in the weighting scheme. While it is possible that the relative rankings among the top 50 schools are affected somewhat by such changes, this set makes up a very small segment of the market—less than 2%—with induced changes in quantile ranks for the overall market being small.

7.3. **Zero Achievement Cutoff.** Recall that the working interpretation of a student with zero investment is one who simply engages in random responding to test questions he or she does not understand. I simulated 100,000 random responses to a published practice test for the ACT. The results are plotted in Figure 15. The upper pane is the

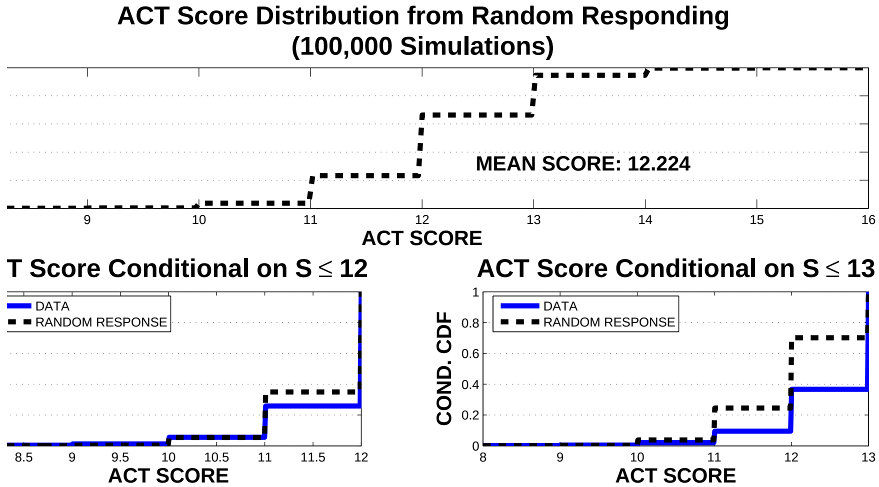
Table 11. USNWR Quality Indicators (Total of 1,314 Schools):

Category/Variable Description	Weight	Mean	StDev.	Observations
SELECTIVITY	**15%**			
Acceptance Rate		.7597	.1553	1,226
Yield (% accepted students who enroll)		.4428	.1518	1,226
Avg. SAT/ACT Percentile of Matriculants		.5515	.2101	1,152
% First-Time Freshmen in Top HS Quartile		.5227	.2038	1,008
FACULTY RESOURCES	**20%**			
% Full-Time Instructional Faculty w/Terminal Degree		.7622	.1665	1,221
% Full-Time Instructional Faculty		.6505	.1891	1,231
Avg. Faculty Compensation		$52,409.23	$12,982.11	1,291
Student/Faculty Ratio		14.99	4.2	1,245
FINANCIAL RESOURCES	**10%**			
Education Spending/Student		$9,494.56	$5,283.01	1,193
Non-Education Spending/Student		$5,951.12	$8,321	1,292
RETENTION	**25%**			
Avg. Graduation Rate		.5353	.6581	1,154
Freshman Retention Rate		.7396	.1146	1,224
ALUMNI SATISFACTION	**5%**			
Alumni Giving Rate		.2105	.1237	1,165
ACADEMIC REPUTATION	**25%**			
College Administrator Ranking Poll		N/A	N/A	N/A

unconditional distribution of simulated random responses. The mean of the distribution is 12.1224, with a standard deviation of .9224.

The question of whether 12 or 13 is the appropriate zero-achievement cutoff is addressed in the lower two panes. On the left is a comparison of the simulated distribution and the distribution from the data, conditional on a score of 12 or less; the right pane is the same for a cutoff of 13. On the right side the two distributions are closer, so I interpret an ACT score of 12 as the zero investment level $\underline{s}$. I used ACT–SAT score concordance tables to determine the equivalent zero cutoff on the SAT test. Score concordances are jointly computed by the designers of the ACT and SAT using data on students who took both tests. The result is an interval of SAT scores being mapped into each outcome-comparable ACT score (since SAT scores occur on a finer grid). These indicate typical

FIGURE 15. Defining Zero Effort



outcomes one can expect on the SAT for a student with a given score on the ACT, and vice versa. The SAT-equivalent range for an ACT score of 12 is 520-580

The reader may wonder why the random responding exercise was not performed using an SAT practice test instead. As it turns out, the mean score from random responding on the SAT is 450, significantly lower than the 520-580 range predicted by the concordance study. Moreover, conditional score distributions for the SAT do not render a similar fit as in the lower left pane of Figure 15: random responding and actual data distributions conditional on low scores differ significantly in shape. This is not surprising since the SAT is designed to test one's aptitude (*i.e.*, ability for abstract reasoning), whereas the ACT is designed to test one's achievement (*i.e.*, acquisition of knowledge). Although HC investment undoubtedly plays a role in determining both, the distinction between achievement versus aptitude becomes more pronounced near the lower extreme. As the concordance study suggests, individuals withe low levels of knowledge—*i.e.*, with ACT scores statistically indistinguishable from random responding—typically have aptitudes that allow them to beat random responding on the SAT. For more information on the distinction between the ACT and SAT tests, see http://www.act.org/aap/concordance/understand.html.

DEPARTMENT OF ECONOMICS, UNIVERSITY OF CHICAGO, 1126 E. 59th ST., CHICAGO, IL, 60637, USA
E-mail address: `HickmanBR@UChicago.Edu`