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Rybczynski's Theorem in the Heckscher–Ohlin World — Anything Goes ☆

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ABSTRACT

We demonstrate that Rybczynski's classic comparative statics can be reversed in a Heckscher–Ohlin world when preferences in each country favor the exported commodity. This taste bias has empirical support. An increase in the endowment of a factor of production can lead to an absolute curtailment in the production of the commodity using that factor intensively, and an absolute expansion of the commodity using relatively little of the same factor. This outcome – which we call "Reverse Rybczynski" – implies immiserizing factor growth. We present a simple analytical example that delivers this result with unique pre- and post-growth equilibria. In this example, production occurs within the cone of diversification, such that factor price equalization holds. We also provide general conditions that determine the sign of Rybczynski's comparative statics.

1. Introduction

Fifty-four years ago T. M. Rybczynski (1955) published a frequently referenced note in which he modeled the comparative statics associated with a change in the endowment of a factor of production. The questions that he considered are fundamental: How do the prices of final goods, and the production and consumption of these goods, depend on factor endowments? How do factor prices and the wealth of consumers vary with changes in factor endowments? What are the welfare implications of changes in factor endowments? Of similar importance to Rybczynski's contribution are the various derivatives of the Heckscher–Ohlin model in which factor endowments determine the pattern of trade. Both of these models have become cornerstones for teaching the pure theory of trade.

In this paper we reconsider Rybczynski's theoretical analysis within the framework of the Heckscher–Ohlin model. Thus, technology exhibits constant returns to scale, preferences are homothetic, and there are no factor intensity reversals. Similar to Jones (1956), and in accordance with empirical evidence (Linder, 1961; Weder, 2003), we consider a taste bias in favor of the exportable good.[3] In the context of this model we demonstrate the existence of economies in which Rybczynski's primary comparative statics' conclusions are reversed in sign. In these economies production prevails within the cone of diversification so that factor price equalization holds, and equilibrium is unique. From a theoretical perspective nothing is unusual. However, since the comparative statics of the Heckscher–Ohlin model must allow for endogenous changes in the distribution of income across

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[3] It is important to note that the original articulation of the Heckscher–Ohlin theorem by Ohlin (1933) did not rely on the assumption of identical preferences, but instead on an economic definition of factor abundance. Such a definition uses (autarky) factor prices rather than physical measures to determine relative factor abundance and renders the Heckscher–Ohlin theorem valid independently of the structure of demand. For the purpose of this article, the distinction between the economic and the physical definition of relative factor abundance does not turn out to play a role (both definitions apply). See Gandolfo (1998) for a discussion of this issue.

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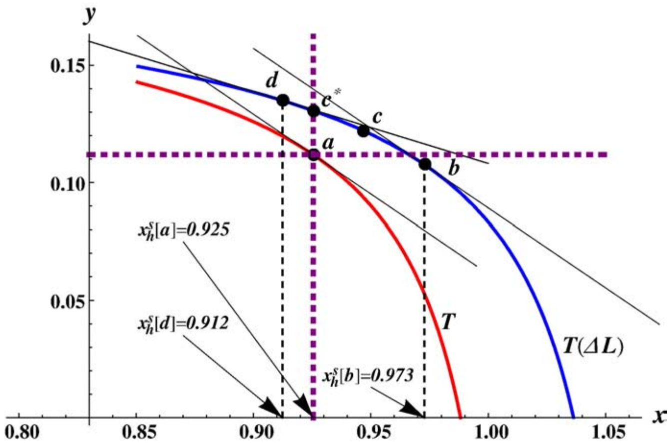

Fig. 1. Production possibilities in the home economy.

countries it is somewhat richer than the comparative statics in Rybczynski's closed economy model.

Before turning to the Heckscher–Ohlin world we ask the reader to recall that Rybczynski's analysis goes beyond the typical textbook treatment with fixed prices and includes a closed economy analysis in which prices were free to vary (see Rybczynski, 1955, p. 336). His presentation begins with the specification of an economy with two factors of production, say, capital (K) and labor (L), and two consumption goods, say, x and y, each produced according to constant returns to scale (CRS) and perfect competition. Let p denote the ratio of the price of x to the price of y. The Rybczynski theorem states that if x is labor intensive and y is capital intensive, *then for each p an increase in L leads to an increase in the equilibrium supply of x and a decrease in the equilibrium supply of y at price p*. Another way to state this conclusion is to say that if one holds the marginal rate of transformation in the production between x and y, MRT (x, y), constant then an increase in L, which allows for an increase in the production of both outputs, leads to an increase in x and a decrease in y. This is illustrated in Fig. 1 by the movement from a to b, where T is the original production possibilities frontier and $T(\Delta L)$ represents the new production possibilities when L is augmented by an increment ΔL.

Rybczynski understood that in a closed economy the relative price p that prevails at equilibrium depends on the demand side of the economy, and varies with factor endowments (this is a general equilibrium effect), and his presentation continues with an analysis of how outputs (which are equal to consumptions in a closed economy) and prices will change following an increase in labor.[4] In particular, Rybczynski argued that in the absence of inferior goods, and with demand generated by the smooth indifference curves of a single consumer whose income is derived from her ownership of K and L, an increase in the amount of factor L leads to an increase in the equilibrium supply (=demand) of x, but that the effect on the equilibrium value of y is ambiguous and could take the economy of Fig. 1 to any point on $T(\Delta L)$ between b and c^*, such as c. Furthermore, it is apparent that he understood that with x inferior, an increase in L can lead to an absolute decrease in x and an increase in y, as in the movement from a to d in Fig. 1. We call this outcome "Reverse Rybczynski".

Although Rybczynski's own analysis took place in the context of a closed economy, it has prominently been recast in trade theory in the context of a home economy that is small (more properly, infinitesimal) relative to the rest of the world so that p is determined by the rest of the world. In Fig. 2 the equilibrium supply in the home country is initially a (on T) and is determined by profit maximization at p. When home labor increases by ΔL the equilibrium supply moves to b on $T(\Delta L)$. If the home country acts as a single consumer with homothetic preferences, the equilibrium demand moves from $\bar{a}$ to $\bar{b}$ of Fig. 2. The increment ΔL will increase the supply of x more than demand at p (in fact, the assumption that y is not inferior is enough for this conclusion). In the Heckscher–Ohlin world that we will consider the home country is not taken to be infinitesimal. Still, at each p the increment ΔL will increase the supply of x more than demand. This powerful implication of Rybczynski's theorem is evident from Fig. 1. It will play a major role in our analysis.

We now turn explicitly to the Heckscher–Ohlin world: there are two countries, neither of which is infinitesimal, and production functions are CRS and identical across countries. Relative factor endowments are different in the two countries and demand in each country is generated by a single consumer whose income is determined by her ownership of capital and labor and who has homothetic preferences; in particular, no goods are inferior in either country. Despite this rather standard form, we show that an increase in the amount of factor L in the home country may lead to a decrease in the relative price of x that is sufficiently large so that the equilibrium supply of x in that country decreases while the production of y increases, as in the movement from a to d in Fig. 1. World production of x also declines. In other words, in general equilibrium, and without the small country assumption, the output implications of an increase in a factor endowment can be the reverse of what is established in the Rybczynski analysis; that is, "Reverse Rybczynski", even with no inferior goods in either country. Furthermore, equilibrium is unique both before and after the increase in L and both equilibria are interior.[5]

The remainder of this paper is organized as follows. A brief overview of related literature is provided in the next section, and an example of "Reverse Rybczynski" in the case of a simple Heckscher–Ohlin model appears in Section 3. We emphasize that we do not assert that "Reverse Rybczynski" is normally the case. Despite the rather innocuous form of the example that demonstrates the above possibility, we are able to provide general conditions on preferences and endowments in the Heckscher–Ohlin model under which the comparative statics in the home and world economies are more or less as they are in Rybczynski's closed economy with no inferior goods. Namely, we are able to provide conditions under which an increase in the home endowment of the factor in which x is intensive leads necessarily to an increase in the supply of x in the home country and in the world. In this case the world production of y will also increase. Finally, we show that "Reverse Rybczynski" implies immiserizing factor growth. The preceding propositions are the work of Section 4. Concluding remarks are presented in Section 5.

2. Related literature

The possibility of "Reverse Rybczynski" was of great interest to Professor Xiaokai Yang, and his interest in that possibility led to this paper. Professor Yang conjectured that "Reverse Rybczynski" could be established using the Sonnenschein–Mantel–Debreu (SMD) theorem (Sonnenschein, 1972, 1973; Mantel, 1974; Debreu, 1974). On this premise some attempts were made to prove this possibility using the idea that derivatives in an equilibrium model could be given quite arbitrary signs (Cheng et al., 2004). However, the Cobb–Douglas utility specification used by the authors means that this approach cannot succeed.[6]

"Reverse Rybczynski" was first established by Hugo Sonnenschein using an elementary version of the Sonnenschein–Mantel–Debreu

[4] Rybczynski also understood than an increase in L leads to an improvement in welfare and, with y normal, to a fall in p.

[5] Uniqueness is key here, since with multiple equilibria both before and after the increase in L, there will generally be a selection from the equilibrium set that trivially yields "Reverse Rybczynski".

[6] This is a corollary to Proposition 1 of Section 4.

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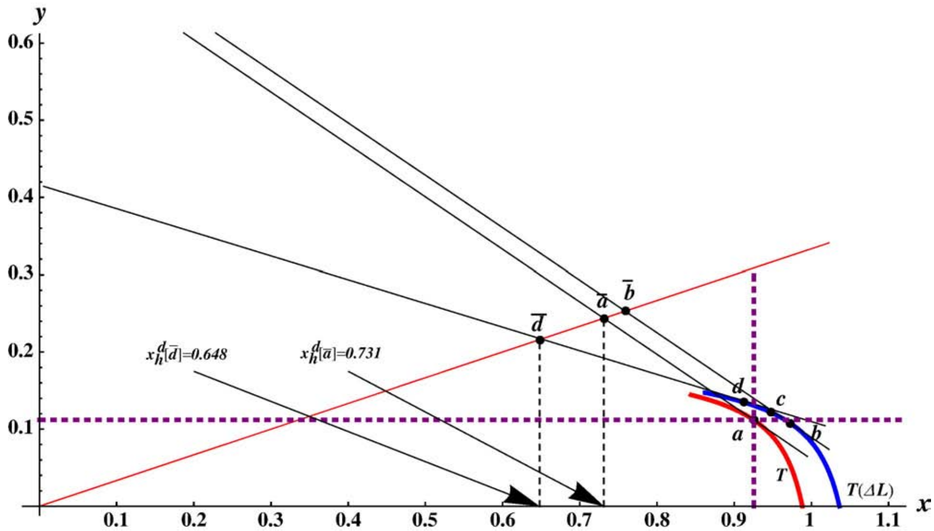

Fig. 2. Home economy's production and trade.

theorem and it was improved by Marcus Opp and Hugo Sonnenschein in the first submitted version of this paper. The couplet "Anything Goes" at the end of this paper's title captures the idea that in a Heckscher–Ohlin world an increase in L in the home country can result in a change in equilibrium supply to any one of d, c, or b in Figs. 1 and 2. The example of "Reverse Rybczynski" in Section 3, using Leontief preferences, is due to Christis Tombazos, and this led the way to an understanding by the present authors of the conditions under which "Reverse Rybczynski" is not possible, as well as to the rather complete list of comparative statics that are presented here.

Finally, the well informed reader will recall that in the Edgeworth Box framework (with two final goods) a gift of endowments can be harmful to the recipient (see Samuelson, 1952a,b), but that this requires that there be multiple equilibria. It is thus somewhat surprising that in the Heckscher–Ohlin world (again, with two final goods) factor growth can be immiserizing (Bhagwati, 1958), even when equilibrium is unique.

3. An example of "Reverse Rybczynski"

One might conjecture that "Reverse Rybczynski" requires exotic preferences and technologies, but as the following example shows this is in fact not the case. Following the conventions of the previous section, the two goods are given by x and y, the two countries are home (h) and foreign (f), and the two factors are capital (K) and labor (L). Commodity x is assumed to be the labor intensive good. As in the previous section, y is the numeraire and p is the normalized price of good x. Factor endowments are $K_h=0.2$ and $L_h=1$ in the home country, and $K_f=1$ and $L_f=0.2$, in the foreign country.

Preferences across the two countries are Leontief and are given by:

$$U_h = \min\left((1-\varepsilon)x_h^d, \varepsilon y_h^d\right), U_f = \min\left((1-\delta)x_f^d, \delta y_f^d\right), \tag{1}$$

where x_i^d, y_i^d represent the consumption of x and y in economy $i \in (h, f)$, respectively. We assume that $\varepsilon=0.750$ and $\delta=0.248$, thus there is a consumption bias in favor of the exportable. We will show later that such a bias, which is not standard in many textbook editions of the Heckscher–Ohlin model but which is consistent with the original articulation of this model (Ohlin, 1933) and which finds empirical support (Linder, 1961, and Weder, 2003), is required for the result.

Technology across the two countries is common and is given by the following Constant Elasticity of Substitution (CES) production functions:

$$x_i^s = \left(\alpha K_{i,x}^\rho + (1-\alpha)L_{i,x}^\rho\right)^{\frac{1}{\rho}}, \quad y_i^s = \left(\beta K_{i,y}^\rho + (1-\beta)L_{i,y}^\rho\right)^{\frac{1}{\rho}} \tag{2}$$

where $\alpha=0.001$, $\beta=0.999$, and $\rho=-2$, x_i^s and y_i^s represent the production of x and y in economy $i \in (h, f)$, respectively, and K_{ix} and L_{ix} (K_{iy} and L_{iy}) correspond to the quantities of capital and labor employed in industry $x(y)$ in economy $i \in (h, f)$. The common ρ across the two production functions, rules out factor intensity reversals (Arrow et al., 1961). Using Eq. (2) it can be easily shown that:

$$\kappa_y = \xi \kappa_x \tag{3}$$

where κ_y and κ_x correspond to the capital–labor ratios that are employed in the production of y and x, respectively, and $\xi = \left(\frac{(1-\alpha)\beta}{(1-\beta)\alpha}\right)^{\frac{1}{1-\rho}} = 99.93$ corresponds to the constant of proportionality.

Using the aggregate resource constraints, Eq. (3) and $MRT(x, y) = p$, it is possible to determine a closed form expression for the quantity supplied of goods x and y in each country as a function of the final goods price p:

$$x_i^s(p, L_i, K_i) = \left(\alpha\left(\frac{K_i - \xi\kappa_x^*(p)L_i}{1-\xi}\right)^\rho + (1-\alpha)\left(\frac{K_i/\kappa_x^*(p) - \xi L_i}{1-\xi}\right)^\rho\right)^{\frac{1}{\rho}} \tag{4}$$

$$y_i^s(p, L_i, K_i) = \left(\beta\left(K_i - \frac{K_i - \xi\kappa_x^*(p)L_i}{1-\xi}\right)^\rho + (1-\beta)\left(L_i - \frac{K_i/\kappa_x^*(p) - \xi L_i}{1-\xi}\right)^\rho\right)^{\frac{1}{\rho}} \tag{5}$$

where: $\kappa_x^*(p) = \left[\frac{\beta^{\frac{-\rho}{1-\rho}}(p\alpha)^{\frac{\rho}{1-\rho}}(1-\alpha) - \xi^{-\rho}(1-\beta)}{\beta - \beta^{\frac{-\rho}{1-\rho}}(p\alpha)^{\frac{\rho}{1-\rho}}\alpha}\right]^{\frac{1}{\rho}}$ denotes the optimal capital–labor ratio in sector x as a function of p. The equilibrium price ratio p can be determined by the market clearing condition for good x:

$$x_h^s(p, L_i, K_i) + x_f^s(p, L_i, K_i) = \frac{Y_h}{p + \frac{1-\varepsilon}{\varepsilon}} + \frac{Y_f}{p + \frac{1-\delta}{\delta}} \tag{6}$$

where $Y_i = px_i^s(p, L_i, K_i) + y_i^s(p, L_i, K_i)$ represents the income of country i.

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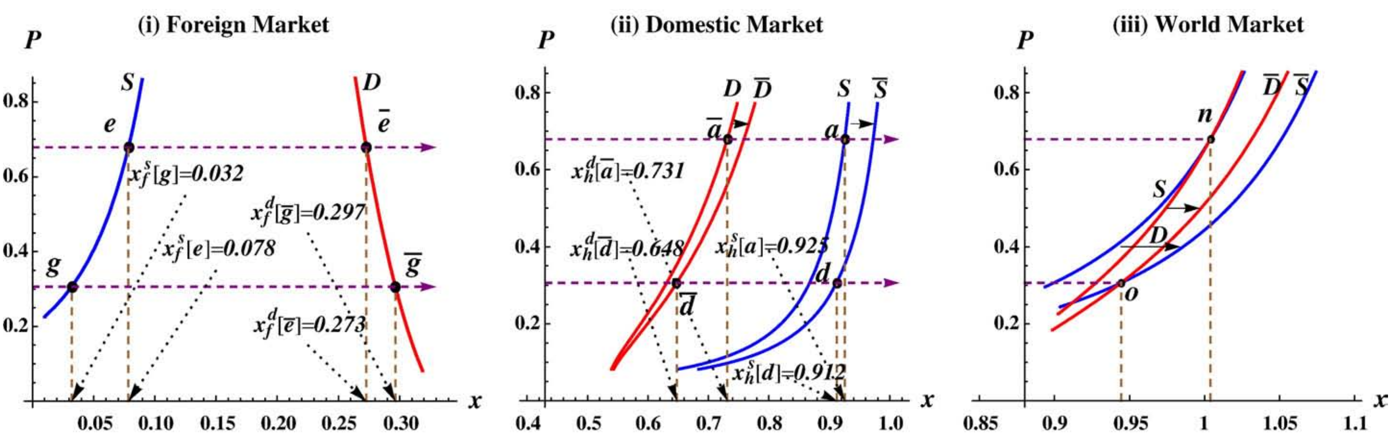

Fig. 3. Domestic, foreign, and world markets.

In the context of the assumed parameter calibrations and endowments the equilibrium price ratio is 0.679 and the capital–labor ratio in sector x is given by $\kappa_x^*(p)=0.085$. Given $\frac{k_f}{\xi}(=0.05)\leq\kappa_x^*<k_h(=0.2)$ production prevails within the cone of diversification.

T of Figs. 1 and 2 is an exact representation of the production possibilities of the home country with the above technology and resource endowments. Point a on this frontier is the equilibrium of the production vector $(x_h^s[a], y_h^s[a])$ in the home country with values given by (0.925, 0.112).[7] The corresponding equilibrium values of the consumption vector, given in Fig. 2 by $\bar{a}=(x_h^d[\bar{a}], y_h^d[\bar{a}])$, correspond to (0.731, 0.244).

Now assume that the endowment of labor in the home country, given by L_h, increases by $\Delta L=0.05$ to 1.05. The new equilibrium values of κ_x^* and p are given by 0.061 and 0.306, respectively. Since $\frac{k_f}{\xi}(=0.05)\leq\kappa_x^*<k_h(=0.19)$ continues to hold equilibrium remains within the cone of diversification. $T(\Delta L)$ of Fig. 1 is an exact representation of the new set of production possibilities in the home country when the supply of labor is augmented. Given the new relative endowments, the equilibrium production choice in the home country is given by $d=(x_h^s[d], y_h^s[d])$ which corresponds to (0.912, 0.135).

The movement from a to b $\{=(x_h^s[b]=0.973, y_h^s[b]=0.108)\}$ is the Rybczynski (fixed price) effect associated with the increase in the labor endowment of the home country. The movement from b to d is the price effect that results from the increase in labor endowment. The "net" movement from a to d represents a decrease in the home production of the labor intensive good (denoted by x_h^s) and an increase in the production of the capital intensive commodity (denoted by y_h^s); thus "Reverse Rybczynski".

One should observe that despite the unexceptional aspects of this example, the general equilibrium demand function for x in the home country is increasing in its relative price p. To be precise, if we define $x_h^d(\cdot)$ to represent the demand for x in the home country when prices for the final goods are given by p for x and 1 for y, and where income in the home country is derived from the value of its labor and capital, then $x_h^d(\cdot)$ slopes upward in a neighborhood of the initial equilibrium value of p (the prices of labor and capital are determined by the zero profit requirement). Exact representations of this upward sloping home demand can be seen in Panel (ii) of Fig. 3 where its pre- and post-growth manifestations are denoted by D and $\bar{D}$, respectively. Similar exact representation of the foreign and world markets are given in Panels (i) and (iii) of Fig. 3, respectively.

Before explaining the intuition behind the previous comparative statics, it is useful to understand why the general equilibrium demand function $x_h^d(\cdot)$ that we generated slopes upward despite the fact that preferences are homothetic. This is an easy consequence of the fact that as price falls the home country must sacrifice more of the x commodity that it has produced in order to purchase y, and when it has a low elasticity of substitution in consumption it is forced to reduce both its consumption of x and y in order to maintain equilibrium. In the example under consideration the elasticity of substitution in consumption is zero; however, a more moderate assumption would suffice in order to generate a positive slope for $x_h^d(\cdot)$. If there is a low elasticity of substitution in consumption in the two countries, and if in addition both countries favor the commodity that they export, then in a neighborhood of equilibrium a small decrease in price reduces the home country's consumption of x more than it increases the foreign country's consumption of that commodity. This is the case under consideration and it results in an upward slope for the world's general equilibrium demand for x, given by $x_{\text{world}}^d(\cdot)=x_h^d(\cdot)+x_f^d(\cdot)$, in a neighborhood of the equilibrium price.

We are now able to explain the intuition behind the possibility of "Reverse Rybczynski" comparative statics, and we trace this through in Fig. 3. Begin with the equilibrium that is associated with the original global demand and supply curves for commodity x, given by D and S of Panel (iii), respectively. Observe that the global demand function is upward sloping at the equilibrium price p, but that the aggregate supply function for x is flatter than demand function at this price. (For the aggregate supply function to be flatter than the demand function it is required that there be sufficient output substitution between x and y in production.) When the home country's initial labor resource is augmented by ΔL, the demand function $x_h^d(\cdot)$ will shift right at each price less than the supply function shifts right. This is the implication of Rybczynski's theorem that was discussed earlier, and it is displayed in Panel (ii) of Fig. 3. Observe that the demand and supply functions in the foreign country, given in Panel (i), are not affected by the change in endowment in the home country. Thus, world demand changes from D to $\bar{D}$ in Panel (iii) of Fig. 3. As a consequence of the increase in labor in the home country there is now an excess supply of x at the initial equilibrium p and so the price of x must fall relative to the price of y. The reduction in price that is necessary in order to restore equilibrium is sufficiently large that it leads to a reduction in the equilibrium production of x in the home country, despite the fact that the supply of x is now larger at every price as a result of the increase in labor.[8] This is "Reverse Rybczynski".

[7] All reported figures are rounded at the three digit level.

[8] The rate of reduction in the production of x in the home country, which is given by the elasticity of supply in that country, is determined by the specified CES technologies. These technologies generate sufficient substitution in production between x and y to make "Reverse Rybczynski" possible. "Reverse Rybczynski" depends on the availability of such substitution.

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4. General analysis

While the previous section established the existence of non-pathological economies exhibiting "Reverse Rybczynski" it did not investigate the particular circumstances under which such an outcome might prevail. In this section, we extend the analysis by listing conditions on preferences and factor endowments that imply downward sloping world demands, and consequently render "Reverse Rybczynski" impossible. We then proceed to relate these general conditions with the characteristics of the world economy of the example of Section 3. Finally, we present a general welfare analysis of factor endowment changes which establishes the link between immiserizing growth and "Reverse Rybczynski".

The first order of business is to provide conditions on the Heckscher–Ohlin model under which an increase in labor endowment in the home country leads to adjustments in the home and world economies that are similar to what is found in Rybczynski's closed economy. In particular, we provide conditions under which an increase in labor endowment in the home country leads to an increase in the production of the labor intensive good x in the home country and in the world.

As in previous sections, we rely on the assumption that production is constant returns to scale and common across the two countries, that the home country is labor abundant, and that good x is labor intensive. In addition, for reasons that we have previously discussed, we assume that there is a unique equilibrium. Alternatively, we could make the weaker assumption that all changes take place in a neighborhood of a Walrasian stable equilibrium. In either case, the slope of world demand for x does not exceed the slope of world supply in a neighborhood of the equilibrium under consideration. As before, the home country exports x in a neighborhood of equilibrium. The fact that in a neighborhood of equilibrium the slope of world demand for x does not exceed the slope of world supply, coupled with the fact that an increase in home labor endowment increases supply more than demand at each price, means that the relative price of x must decrease with an increase in home labor endowment. As a consequence, an increase in labor endowment in the home country must decrease the price of labor and increase the price of capital (Stolper and Samuelson, 1941).[9]

In the Heckscher–Ohlin model the occurrence of "Reverse Rybczynski" requires that world demand for x, shown in Panel (iii) of Fig. 3, has a positive slope in a neighborhood of equilibrium. This follows from the fact that if world demand for x has a negative slope at equilibrium, then an increase in labor endowment in the home country means that both $x_h^d(\cdot)$ and $x_h^s(\cdot)$ shift right, and so the equilibrium value of world supply of x must increase. However, since p falls the foreign production of x cannot increase. Thus, the home production of x must increase, and so "Reverse Rybczynski" is not possible. Since downward sloping home demand rules out upward sloping world demand, it also rules out the occurrence of "Reverse Rybczynski". This is Lemma 1.

Lemma 1. *If demand for the labor intensive good x in the home country slopes downward in a neighborhood of equilibrium then world demand must also slope downward. Hence, growth in the home country's endowment of labor cannot generate "Reverse Rybczynski": Growth of labor in the home country leads this country to increase the quantity of good x that it supplies.*

The following proposition gives sufficient conditions under which world demand for x has a negative slope in a neighborhood of equilibrium. All of these conditions rely on the assumption of homothetic preferences and allow for the possibility that the production of y in the home economy may either increase or decrease.

Proposition 1. *World demand for good x slopes downward in a neighborhood of equilibrium, and as a result "Reverse Rybczynski" is impossible, if any of the following conditions are satisfied:*

(i) Preferences in the two countries are identical;
(ii) Endowments in the two countries are proportional;
(iii) The home country prefers the importable commodity when compared to the foreign country in the sense that at each p, $y_h^d(p)/x_h^d(p) > y_f^d(p)/x_f^d(p)$;
(iv) The elasticity of substitution in consumption in the home country is greater than, or equal to, one.

Before providing the proof of Proposition 1 we comment on the four conditions. The first condition, identical preferences, is commonly made in implementations of the Heckscher–Ohlin model; however, as pointed out in footnote 3 it is not a requirement for the result of the theory. There is, in fact, empirical support for differences in taste that are opposite to condition (iii). Thus, neither (i) nor (iii) are particularly attractive ways to obtain downward sloping demand at equilibrium, and by Lemma 1 to rule out "Reverse Rybczynski". The second condition is similarly not very attractive, since when endowments are proportional, we rule out the primary reason for trade in the Heckscher–Ohlin theory.[10] Finally, the attractiveness of (iv) depends on the manner in which we aggregate commodities; that is, whether or not what we arbitrarily call the aggregate commodity x is highly substitutable for other commodities which we call y.

Proof. We prove parts (i) and (ii) of this proposition by reinterpreting T in Fig. 1 as the production possibilities frontier of the world economy. Recall that if either preferences are homothetic and identical across countries, or if preferences are homothetic and endowments proportional across countries, then the demand side of the world economy is generated by the homothetic preferences of a single consumer who owns the aggregate endowments of the world. This is Eisenberg's theorem (Shafer and Sonnenschein, 1982) and Chipman's (1974, 2006) corollary of this theorem. Thus, in the case in which either (i) or (ii) is satisfied, equilibrium world production and consumption are at point a of Fig. 1. At this point the boundary of the production possibilities frontier T is tangent to the highest world indifference curve, and world prices are given by the negative of the slope of the line tangent at a.

For prices lower (higher) than the equilibrium price, consumption must occur below (above) the ray emerging from the origin and passing through a. By revealed preference, it must also be above the price line tangent to a. Thus, the world demand for x must increase (decrease). Hence, if world demand is differentiable at equilibrium, which we assume, it must slope down. In fact, as long as demand is generated by smooth social indifference curves that do not exhibit inferiority it is clear that demand must slope down.

Next assume (iii) and consider the Edgeworth Box pure exchange economy defined by endowing the representative consumer in each country with the equilibrium supply of x and y that is chosen in his country at the equilibrium price p (and associated equilibrium factor prices). Since the home country has relatively more labor, the endowment in the Edgeworth Box is below the diagonal connecting the origins of the home and foreign countries. The linear income expansion paths associated with the equilibrium price ratio p intersect at the equilibrium consumption allocation, and by (iii) this is above the diagonal. At lower p, then, by the envelope theorem, the rate of change in demand in each country is "as if" production, from which income is derived, does not readjust as prices change. At this lower price, demand must be on the new (flatter) budget line, below the home country's original income consumption path, and above the foreign country's original consumption path. This means that world

[9] It can also be shown that the decrease in the price of labor is proportionally larger than the decrease in the price of x (Mas-Colell et al., 1995, p. 543).

[10] Of course, with taste biases, trade can take place even when factor endowments are proportional.

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demand must now be positive. In other words, world demand has a negative slope in a neighborhood of p.

Now assume (iv). Since foreign demand is generated by homothetic preferences its demand for the importable has a negative slope in the neighborhood of the equilibrium price p. Thus, it is sufficient to prove that home demand has a negative slope at the equilibrium price p. From the envelope theorem it follows that $dx_h^d(p)/dp$ is the same whether one holds the home country to its profit maximizing supply at p, from which its income is derived, or allows the firms to readjust their production so that it remains profit maximal at p. Let $[x_s^h(p), y_s^h(p)] > \mathbf{0}$ be the profit maximizing supply of outputs at the equilibrium price p and hold this vector constant. It is elementary that with income derived from this vector and elasticity of substitution unitary $dx_h^d(p)/dp<0$. The assumption that the elasticity of substitution is greater than one magnifies this result. Q.E.D.

The reader will observe that the example of Section 3 relies on the assumptions that preferences across the two countries are different and that the home country is labor abundant which violate parts (i) and (ii) of Proposition 1, respectively. Jointly these violations allow world demand in our example to not be effectively generated by the preferences of a single consumer who owns world endowments and in this sense they ensure that home demand and foreign demand are meaningfully disentangled. This sets the scene for the possibility that world demand for one of the two goods (our example concentrates on x) is upward sloping. For this to happen however, there are two additional requirements. First, we require that home demand for x is upward sloping which, consistent with a violation of part (iv) of Proposition 1, can be introduced by assuming a sufficiently small elasticity of substitution in consumption (our example eliminates substitution altogether by assuming Leontief preferences). Second, in accordance with violation of part (iii) of Proposition 1, each country has a relative preference for its exportable. Consequently, as p decreases the home consumption of x decreases by a greater amount than foreign consumption of x increases. Hence, world demand for this good in our example must be upward sloping.

Finally, we discuss the welfare implications associated with a change in resource endowments. Since the welfare effects of an endowment change can be completely classified according to whether or not the changing country is relatively abundant in the endowment that is being changed, this issue is completely addressed by considering an increase in each of the endowments in the home country. Because welfare in the foreign country varies inversely with p, welfare in the foreign country increases with an increase in L in the home country and falls with an increase in K in the home country. This result requires only that equilibrium is unique (or Walrasian stable) and that preferences are "normal". If the endowment of K increases in the home country, then the price of x must increase, and as a result welfare must increase in the home country. The ambiguous case arises when the home endowment of L increases. Then, it is possible that utility will fall and this will be a case of immiserizing factor growth. If the home country is infinitesimal (or sufficiently small) relative to the market, then p will not change (or change very little) as home endowment increases, and home welfare increases. However, as we have argued, p may decrease at a substantial rate when home endowment of L increases, and this can cause factor growth to be immiserizing. The next proposition demonstrates that factor growth of L in the home country is always immiserizing in the presence of "Reverse Rybczynski".[11]

Proposition 2. *"Reverse Rybczynski" implies that factor growth is necessarily immiserizing.*

Proof. Recall that if the endowment of L increases in the home country, then p will fall. Since x is not inferior in the foreign country, the quantity demanded of good x must increase in the foreign country and quantity supplied must fall by the law of supply. If welfare does not decrease in the home country, then the value of consumption at price p for x and unity for y must not decrease. But since p is reduced and since home preferences are homothetic, the ratio of x to y demanded must not decrease. The previous two sentences guarantee that the demand for x in the home country cannot fall. But the hypothesis of "Reverse Rybczynski" means that the home supply of x must fall, and so the (negative) excess demand for x must increase in the home country. But in this case world demand for x must exceed supply at the new equilibrium. This contradiction means that welfare must have been reduced in the home country. Q.E.D.

5. Notes and conclusions

Within the context of the Heckscher–Ohlin model of international trade we have given a rather complete account of the manner in which outputs, consumption, prices, and welfare change as a result of changes in the factors of production. Thus, we take a significant step in furthering the analysis that was begun by Rybczynski 54 years ago. The "surprise" in our results is the fact that Rybczynski's single definite finding concerning output can be completely reversed as a result of very strong price effects, even when preferences are homothetic. If "Reverse Rybczynski" occurs factor growth is necessarily immiserizing.

The key observation regarding the difference between Rybczynski's closed economy analysis and the Heckscher–Ohlin analysis is that in the latter the world demand function for a final good can slope upward in a neighborhood of equilibrium, even when preferences are homothetic, production is constant returns to scale, and equilibrium is unique. This is what makes "Reverse Rybczynski" possible.

References

Arrow, K.J., Chenery, H.B., Minhas, B.S., Solow, R.M., 1961. Capital–labour substitution and economic efficiency. Review of Economics and Statistics 43, 225–250.

Bhagwati, J., 1958. Immiserizing growth: a geometrical note. Review of Economic Studies 25, 201–205.

Cheng, W.L., Sachs, J., Yang, X., 2004. An extended Heckscher–Ohlin model with transaction costs and technological comparative advantage. Economic Theory 23, 671–688.

Chipman, J.S., 1974. Homothetic preferences and aggregation. Journal of Economic Theory 8, 26–38.

Chipman, J.S., 2006. Aggregation and estimation in the theory of demand. History of Political Economy 38, 106–129.

Debreu, G., 1974. Excess demand functions. Journal of Mathematical Economics 1, 15–21.

Gandolfo, G., 1998. International Trade Theory and Policy. Springer, Berlin.

Jones, R.W., 1956. Factor proportions and the Heckscher–Ohlin theorem. The Review of Economic Studies 24, 1–10.

Linder, S.B., 1961. An Essay on Trade and Transformation. John Wiley and Sons, New York.

Mantel, R.R., 1974. On the characterization of aggregate excess demand. Journal of Economic Theory 7, 348–353.

Mas-Colell, A., Whinston, M.D., Green, J.R., 1995. Microeconomic Theory. Oxford University Press, New York.

Ohlin, B., 1933. International and Interregional Trade. Harvard University Press, Cambridge, MA.

Rybczynski, T.M., 1955. Factor endowments and relative commodity prices. Economica 22, 336–341.

Samuelson, P.A., 1952a. Foundations of Economic Analysis. Harvard University Press, Massachusetts.

Samuelson, P.A., 1952b. The transfer problem and transport costs: the terms of trade when impediments are absent. Economic Journal 62, 278–304.

Stolper, W.F., Samuelson, P.A., 1941. Protection and real wages. Review of Economic Studies 9, 58–73.

Shafer, W., Sonnenschein, H., 1982. Market demand and excess demand functions. In: Arrow, K.J., Intriligator, M.D. (Eds.), Handbook of Mathematical Economics, vol. 2. North-Holland, Amsterdam, pp. 671–693.

Sonnenschein, H., 1972. Market excess demand functions. Econometrica 40, 549–563.

Sonnenschein, H., 1973. Do Walras' identity and continuity characterize the class of community excess demand functions? Journal of Economic Theory 6, 345–354.

Weder, R., 2003. Comparative home-market advantage: an empirical analysis of British and American exports. Review of World Economics 139, 220–247.

[11] Bhagwati (1958) in his pioneering work on immiserizing growth is clearly interested in the relationship between factor growth and welfare. However, he does not consider the possibility of "Reverse Rybczynski" nor does he connect "Reverse Rybczynski" with immiserization.

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