Cash Flow of Future Contracts

$F_{t,T}$ forward price at time t with maturity at T

Dates	Underlying	Future Price	Cash Flow Margin Account
0	S_0	$F_{0,T}$	0
1	S_1	$F_{1,T}$	$F_{1,T} - F_{0,T}$
2	S_2	$F_{2,T}$	$F_{2,T} - F_{1,T}$
...	...	...	...
$T-1$	S_{T-1}	$F_{T-1,T}$	$F_{T-1,T} - F_{T-2,T}$
T	S_T	$F_{T,T}$	$S_T - F_{T-1,T}$

Relationship between Future and Forward Prices

- Assume that interest rates r are constant (also a constant dividend yield, convenience yield and storage cost, if applicable)

- Consider strategy of starting at period $t = 0$ with $(1 + r)^{-T}$ contracts, and increasing them by a factor $(1 + r)$ between days, so that at date T you will have 1 contract (r is the "daily" interest rate).

- Increasing # contracts at date t has no cash flow effect at date t.

- Roll over at rate r the balance in the margin account, so that $1 at date t becomes $(1 + r)^{T-t}$ at date T.

- This strategy has NO cash flow at dates $t = 0, 1, 2, ..., T - 1$

- We show below that at maturity (date T) the cash flow is equal to:

$$S_T - F_{0,T}$$

which is identical to a Forward.

- Hence the future prices are equivalent to forwards as long as r is constant.

t	$a)$ # contracts	$b)$ Margin per contract	$c) = a) \times b)$
0	$(1+r)^{-T}$	0	0
1	$(1+r)^{-T+1}$	$F_{1,T} - F_{0,T}$	$\frac{F_{1,T} - F_{0,T}}{(1+r)^{T-1}}$
2	$(1+r)^{-T+2}$	$F_{2,T} - F_{1,T}$	$\frac{F_{2,T} - F_{1,T}}{(1+r)^{T-2}}$
...	...	...	...
$T-1$	$(1+r)^{-1}$	$F_{T-1,T} - F_{T-2,T}$	$\frac{F_{T-1,T} - F_{T-2,T}}{(1+r)}$
T	1	$S_T - F_{T-1,T}$	$S_T - F_{T-1,T}$

Rolling over values on column $c)$ gives the following at date T :

For example for date 1: $\frac{F_{1,T} - F_{0,T}}{(1+r)^{T-1}} \times (1+r)^{T-1} = F_{1,T} - F_{0,T}$,

$$F_{1,T} - F_{0,T} + F_{2,T} - F_{1,T} + ... + F_{T-1,T} - F_{T-2,T} + S_T - F_{T-1,T}$$
$$= S_T - F_{0,T}$$

- Interest rates do not need to be constant for the previous arguments.
- It suffices that they are known in advance.
- If interest rate are NOT known in advance, then strategy fails (we do not know how many contracts to enter) and in general

$$Forwards \neq Futures$$

- Same problem if dividend yields, convenience yields or storage cost are NOT known in advance.

- In practice this is not a problem when variations on interest rate are small relative to variations on the price of the underlying.

- In such case, the deviations between forwards and futures are small. Consider, for example, the case where the underlying is oil (compare variability of S_0 with variability of e^{rT})

- One case where forwards can be substantially different from futures is if the underlying is a bond, since its price changes with interest rates.

- Another case where forwards can be substantially different from futures is if the convenience yield varies a lot. This happens for some consumption assets (some commodities).