

Economics 251 — Homework 2

Due Thursday, May 15th, 2008

Please follow these instructions:

- Write your answers in the space provided on this sheet.
- Please write your name and student ID number here: $\left\{ \begin{array}{l} \textit{name} : \text{-----} \\ \textit{student ID} : \text{-----} \end{array} \right.$
- You can work in groups of up to 3 students and hand a common answer sheet.
- Use continuous compounding throughout.
- As a rule, when you answer that there's an arbitrage opportunity, you should (1) explicitly construct a portfolio that exploits it, and (2) give a quantitative measure of how much you can earn, e.g. per dollar you take in each position, or per unit of the underlying asset you buy/short. If how much you earn depends on the price of the underlying at expiration, explain how.
- Please put your problem set before 4pm Thursday, May 15th, in my folder in front of R0 203.

1. **Bound on the slope.** Consider two European put options on the same underlying asset, with same expiration date $T = 1$ year from now, and with strike prices K_1 and $K_2 = K_1 + 2$. The risk-free rate is $r = 8\%$ and it is compounded continuously. The prices of the put options are $P_1 = \$4$ and $P_2 = \$6$. Is there an arbitrage opportunity? If so construct the portfolio that exploits this opportunity. Also if there's an arbitrage opportunity, please compute how much you would earn if you limit yourself to holding or shorting at most 1 of each option.

2. **Arbitrage bounds.** Consider a European call option on a stock worth $S_0 = \$10$, with expiration date exactly one year from now and with strike price $\$9.5$. The risk-free rate is $r = 15\%$ and it is compounded continuously.

(a) The call is worth $\$1.80$. Is there an arbitrage opportunity? If so construct the portfolio that exploits this opportunity. Also, if there's an arbitrage opportunity, please compute how much you would earn if you limit yourself to holding or shorting at most 1 of each option and underlying asset.

(b) Now suppose that the call is worth $\$12$. Is there an arbitrage opportunity? If so construct the portfolio that exploits this opportunity. Also, if there's an arbitrage opportunity, please compute how much you would earn if you limit yourself to holding or shorting at most 1 of each option and underlying asset.

3. **Varying the strike price.**

(a) Consider three European call options on the same stock, with same expiration date T and strike prices $K_1 = 22$, $K_2 = 24$, $K_3 = 28$. Suppose that their price is, respectively, $C_1 = 1$, $C_2 = 2$ and $C_3 = 4$. Is there an arbitrage opportunity? If so construct the portfolio that exploits this opportunity.

- (b) Now consider three European put options on the same stock, with same expiration date T and strike prices $K_1 = 22$, $K_2 = 24$, $K_3 = 26$. Suppose that their price is, respectively, $C_1 = 3$, $C_2 = 3.45$ and $C_3 = 4$. Is there an arbitrage opportunity? If so construct the portfolio that exploits this opportunity.

4. **American Options.** Suppose that $r = 10\%$. Consider an American call option which expires one year from now, with strike price $K = 90$. The underlying stock pays no dividend ever.

- (a) If the current stock price is $S_0 = \$100$, what are the range(s) of prices C for the call such that you could find some arbitrage? What portfolio(s) would you construct to make money.

- (b) When will you exercise the option? Please justify your answer.

5. **Put-Call Parity.** Consider a European put and a European call on the same non-dividend paying stock with the same strike price, both maturing in T years. Let S_0 be the current price of the underlying asset, K the strike

price, P the price of the put option, C the price of the call option and r the annual, continuously compounded, risk free interest rate.

(a) Show that if the call option is *in the money* (that is, $K < S_0$) the price of the put is lower than the price of the call.

(b) Now assume the call option is *out of the money* (that is, $K > S_0$). Is the price of the put higher than the price of the call? Justify your answer.

(c) **Arbitrage opportunity.** Suppose that the stock price is $S_0 = \$31$, the strike price is $K = \$30$, the annual risk free interest rate is $r = 10\%$, the price of a three month European call option is $C = \$3$, and the price of a three-month European put option is $P = \$2.25$. Is there an arbitrage opportunity? If so show (i) what trades you would do and (ii) their payoffs today and at maturity.

- (d) **Put-call parity on a dividend paying stock.** Now assume that over the period $[0, T]$, the stock pays a dividend whose present value at interest r is D (using continuous compounding). Consider the following portfolios:

Portfolio A : one European call option plus an amount of cash $D + Ke^{-rT}$.

Portfolio B : one European put option plus one share.

What are the payoffs of portfolios A and B at expiration of the options? Derive from there a put-call parity relation for European options expiring at T with dividend paying stocks.