Economics 251 — Homework 1

Solution

1. **Currency forwards with bid-ask spreads**. Table 1 and Table 2 contain relevant data on interest rates and exchange rates on April 7, 2008. The tables contain bid and ask prices. We want to compute the implied forward 3-months exchange rate of the Euro vs. the US Dollar, to buy Euros against Dollars in 3 months. Because of bid-ask spreads, we will determine a *band* of forward rates that are compatible with no arbitrage. (Remember that for the Euro, the spot and the forward exchange rate are quoted as USD per unit of foreign currency).

   (a) What is the maximum forward exchange rate such that you would make a sure profit by taking a long position in this contract? What is the profit-making portfolio in case there's an arbitrage opportunity? Please give 4 digit accuracy.

   We look for an arbitrage opportunity, whereby we would enter the agreement to buy Eur 1 at $\$F_0$ *in 3 months, and make a profit on this. Buy* $\$F_0 e^{-r_{US}\times 0.25}$ *today (so we have the* $\$F_0$ *in 3 months), by expending Eur* $F_0 e^{-r_{US}\times 0.25}/S_0$*, which we borrow. So that in 3 months, we receive Eur 1 and owe* $e^{r_{Eur}\times 0.25}\left(F_0 e^{-r_{US}\times 0.25}/S_0\right)$.
   This is an arbitrage if this portfolio (which costs nothing by construction) returns positive payoffs in 3 months:

   $$e^{r_{Eur}\times 0.25}\left(F_0 e^{-r_{US}\times 0.25}/S_0\right) < 1$$

   that is if

   $$F_0 < S_0 e^{(r_{US}-r_{Eur})\times 0.25}$$

   To construct this portfolio, we used the saving rate in the USA (2.88%)*, the borrowing rate in Europe* (4.64%)*, and* $S_0 = 1.5713$
   Hence the lower bound on the forward is

   $$1.5713 \times e^{(0.0288-0.0464)\times 0.25} = 1.5644$$

   i.e. if the forward rate is lower than that, there's an arbitrage opportunity.

   (b) What is the minimum forward exchange rate such that you would make a sure profit by taking a short position in this contract? What is the profit-making portfolio in case there's an arbitrage opportunity? Please give 4 digit accuracy.

We look for an arbitrage opportunity, whereby we would enter the agreement to sell Eur 1 for $\$F_0$ *in 3 months, and make a profit on this. Buy Eur* $e^{-r_{Eur}\times 0.25}$ *today (so you have the Eur 1 in 3 months), and to expend nothing today, let's borrow* $\$S_0 e^{-r_{Eur}\times 0.25}$, *so that in 3 months we receive* $\$F_0$ *and have to repay* $\$e^{r_{US}\times 0.25}\left(S_0 e^{-r_{Eur}\times 0.25}\right)$. *This is an arbitrage if*

$$\begin{aligned} F_0 &> e^{r_{US}\times 0.25}\left(S_0 e^{-r_{Eur}\times 0.25}\right) \\ &> S_0 e^{(r_{US}-r_{Eur})\times 0.25} \end{aligned}$$

To construct this portfolio, we used the borrowing rate in the USA (3.03%)*, the saving rate in Europe* (4.54%)*, and* $S_0 = 1.5713$.
Hence the upper bound on the forward rate is

$$1.5713 \times e^{(0.0303-0.0454)\times 0.25} = 1.5654$$

(c) Please conclude: what is the band of forward rates?

If the forward exchange rate is out of the interval $[1.5644, 1.5654]$, *then there's an arbitrage opportunity.*

2. **Forwards on stocks with dividends**. Let's price a forward on the Dow Jones Industrial Average stock index (DJIA) for June 7, 2008, that is 2 months from now. We consider using the 3-month interest rates to discount payoffs. In addition to the bid-ask spread on the interest rate, you also have to consider transaction costs: your broker requires \$2 per unit of the index he trades on your account. On April 7, the DJIA opened at 12,612.43.

   (a) Abstracting from bid-ask spreads and transaction costs, suppose the price of the June forward of the DJIA opened at exactly 12,650.0 on April 7. Please infer the dividend yield q (continuously compounded) on the DJIA. For this question *only*, assume that $r_{US} = 2.95\%$.

   Since $F_0 = S_0 \exp\left[(r-q)\,T\right]$, *it has to be that*

$$\begin{aligned} q &= r - \frac{1}{T}\log\left(\frac{F_0}{S_0}\right) \\ &= 0.0295 - 6\log\left(\frac{12650}{12612.43}\right) \\ &= 0.0117 \end{aligned}$$

(b) Now we reintroduce bid-ask spreads and transaction costs, and use the dividend yield q we found in (a). Given that the interest rates for saving and borrowing are those given in next pages' tables (the same as for question 1), and given that your broker charges \$2 per unit of the index you trade (be it to give it or take it from the broker), please find the lower and upper bound on the price of the 2-months DJIA forward such that you can't exploit any arbitrage opportunity (use the same method as for question 1). You don't need to give any decimal for this question.

Suppose first that the price of the forward is too low, so that you want to take a long position in this contract: you short $e^{-q\frac{2}{12}}$ units of the DJIA (which generates transaction costs $e^{-q\frac{2}{12}} \times \$2$), save the proceeds (at the saving interest rate of 2.88%), and give back one unit of the DJIA to your broker (this includes the dividends which would have been continuously reinvested: your broker agreed to exchange $e^{-q\frac{2}{12}}$ now against 1 in 2 months) in June. That is, you save $\$e^{-q\frac{2}{12}}(12,612.43-2)$ at 2.88% for 2 months, then you have to expend $\$(\underline{F}_0+2)$ in two months, to comply with the forward contract, and give back the stock index. This is an arbitrage opportunity if

$$e^{-q\frac{2}{12}}(12,612.43-2)\,e^{0.0288\times\frac{2}{12}} \geq 2+\underline{F}_0$$

$$\Longrightarrow \underline{F}_0 = 12,644$$

Second, suppose that the price of the forward is too high, so that you want to take a short position in this contract: you buy $e^{-q\frac{2}{12}}$ units of the DJIA and pay an additional TC $\$\left(e^{-q\frac{2}{12}}\times 2\right)$ to your broker for executing the transaction, borrowing the total cost (at the borrowing interest rate of 3.03%). In two months, to comply with the forward contract, you sell the unit of the DJIA for $\overline{F}_0$ and reimburse the loan. This is an arbitrage opportunity if:

$$e^{-q\frac{2}{12}}(12,612.43+2)\,e^{0.0303\times\frac{2}{12}} \leq \overline{F}_0-2$$

$$\Longrightarrow \overline{F}_0 = 12,656$$

We find $\left[\underline{F}_0,\overline{F}_0\right] = [12644, 12656]$.

3. **Commodity forwards with storage costs**. Suppose you are an American commodity arbitageur on the NYMEX: you can buy oil barrels and store them at some cost, however you cannot short them. On April 7, the NYMEX crude oil barrel traded at \$109.09. For what values of the 3-months forward would you find an arbitrage? The interest rates are the

same as for question 1 and 2 and should be read from next pages' tables, and the storage costs are 1% of the value of the asset.

If the forward price is high enough, you take a short position in this contract, which you hedge by buying oil now, storing it and delivering it in 3 months.

In practice, the payoff from this portfolio is

$$F_0 - S_0 \exp\left[(r+u) \times 0.25\right]$$

which is positive if

$$F_0 > 109.09 \exp\left[(3.03+1)\,\% \times 0.25\right]$$

$$\Longrightarrow F_0 > 110.19$$

You could not construct an arbitrage portfolio with a long position in the forward contract because you would have to short the underlying asset which you cannot always do for commodities, (and in particular in this question). The relevant interest rate is the borrowing rate (3.03%) *since your 0-cost portfolio requires to borrow to buy oil now.*