

Online appendices for Alvarez-Lippi 2007

Financial innovation and the transactions
demand for cash

D Alternative data sources for ATM withdrawals

In this appendix we compare data on average ATM withdrawals drawn from two sources: our households survey data (*SHIW*) and the data drawn from banks' records as reported in the ECB Blue Book (2006). Table 12.1a in the bluebook reports the total number of cash withdrawals at ATMs in a year. Table 13.1a gives the total value of cash withdrawals at ATMs in a year. The average withdrawal computed as the ratio of these two numbers for the years 2001, 2002 and 2004 is 162, 205 and 169 euros, respectively. (these years are the closest to those of the *SHIW* survey years). In the household survey we compute the analogue statistics for the years 2000, 2002 and 2004 obtaining 177, 185 and 205 euros, respectively. For each year the latter statistics were computed as the ratio between the sum across households of the amount of cash withdrawn from ATMs and the sum across households of the number of withdrawals from ATMs. For each household, the total amount of cash withdrawn from ATM was given by the average ATM withdrawal times the number of ATM withdrawals. These statistics differs from the statistics on W reported in Table 1 for three reasons. First because even for households with ATM card W includes withdrawals done at the bank desk (which are larger on average). Second W is measured in 2004 euros. Third W reports the average withdrawal per household, so the weighting is different.

E Solution for the Value Functions ODEs

ODEs of the form:

$$f(x) = a_0 + a_1x + (a_2 + a_3x) f'(x)$$

appear in this paper as Bellman equations. Their solution is

$$f(x) = A_0 + A_1x + A \left[1 + \frac{A_2}{A_3}x \right]^{A_3}$$

To see this notice that

$$f'(x) = A_1 + A A_3 \left(\frac{A_2}{A_3} \right) \left[1 + \frac{A_2}{A_3}x \right]^{(A_3-1)}$$

which requires:

$$\begin{aligned} & A_0 + A_1x + A \left[1 + \left(\frac{A_2}{A_3} \right) x \right]^{A_3} \\ = & a_0 + a_1x + (a_2 + a_3x) \left(A_1 + A A_3 \left(\frac{A_2}{A_3} \right) \left[1 + \frac{A_2}{A_3}x \right]^{(A_3-1)} \right) \end{aligned}$$

Solving the system of equations defined by the previous equality yields:

$$A_0 = a_0 + a_2 a_1 / (1 - a_3) \quad A_1 = a_1 / (1 - a_3) \quad A_2 = 1/a_2 \quad A_3 = 1/a_3$$

□

F Expressions for the model with free random withdrawals ($f = 0$) when $\pi = 0$

This appendix collects the expression that are obtained in the case of $\pi = f = 0$. In most cases they have to be obtained by using L'Hopital rule in the corresponding formulas for the general case.

For $\pi = 0$ the expression for m^* in Proposition 2 is

$$\exp\left(\frac{m^*}{c}(r+p)\right) = 1 + \frac{m^*}{c}(r+p) + (r+p)^2 \frac{b}{cR}. \quad (70)$$

and the expression for the value function in Proposition 3 is

$$V(m) = \left[\frac{pV^*(r+p) - Rc}{(r+p)^2} \right] + \left[\frac{R}{r+p} \right] m + \left(\frac{c}{r+p} \right)^2 A \exp\left(-\frac{r+p}{c}m\right).$$

The expression for the expected number of trips per unit of time n in Proposition 5 for $\pi = 0$ is

$$n(m^*; c, 0, p) = \frac{p}{1 - e^{-m^* \frac{p}{c}}} \quad (71)$$

The expression for the density of the distribution of real cash balances in Proposition 6 for $\pi = 0$ is

$$h(m) = \frac{\frac{p}{c} \exp\left(\frac{mp}{c}\right)}{\exp\left(\frac{m^* p}{c}\right) - 1} \quad (72)$$

The expression for aggregate money balances for $\pi = 0$

$$M = c \left[\frac{1}{1 - e^{-\frac{p}{c}m^*}} \frac{m^*}{c} - 1/p \right]. \quad (73)$$

G Average balance with precautionary motive

Proposition 15. *Assume that $\pi = 0$ and let λ denote the time elapsed between two consecutive withdrawals. Let $M(\lambda)$ be the average cash balance during this elapsed time, $W(\lambda)$ be the withdrawal at the end of a period of length λ and $\underline{M}(\lambda)$ the cash balance just prior to the withdrawal. Let M be the expected value of cash holdings under the invariant distribution and $g(\lambda)$ be the density of the distribution of the*

lengths. We then have

$$M(\lambda) = \underline{M}(\lambda) + W(\lambda)/2 = m^* - (c \lambda)/2 \quad (74)$$

$$M = \frac{\int_0^\infty M(\lambda) \lambda g(\lambda) d\lambda}{\int_0^\infty \lambda g(\lambda) d\lambda} \quad (75)$$

Proof of Proposition 15. Let $t \in [0, \lambda]$ index the time elapsed in an interval of length λ . The law of motion of cash and the optimal policy imply that cash holdings obey $m(t) = m^* - c\lambda$ for $t \in [0, \lambda)$ and $m(\lambda) = m^*$. $W(\lambda) = m^+(\lambda) - m^-(\lambda)$ and $m^* = W(\lambda) + \underline{M}(\lambda)$ imply equation (74). The ergodic theorem implies, using ω to index the sample space,

$$M = \lim_{T \rightarrow \infty} (1/T) \int_0^T m(t, \omega) dt \quad \text{in pr.} \quad (76)$$

from which equation (75) can be derived. \square

Remark 1. If the distribution of the length λ is concentrated at a single value $\bar{\lambda}$, as in a deterministic model, then $M = M(\bar{\lambda})$. Then

$$M = M(\bar{\lambda}) = \underline{M}(\bar{\lambda}) + W(\bar{\lambda})/2$$

Remark 2 When the distribution of the length λ is not degenerate then

$$M < \int_0^\infty M(\lambda) g(\lambda) d\lambda = \int_0^\infty \underline{M}(\lambda) g(\lambda) d\lambda + \frac{1}{2} \int_0^\infty W(\lambda) g(\lambda) d\lambda$$

where the inequality follows because $M(\lambda)$ is decreasing in λ . Thus M , the duration weighted expected value of $M(\lambda)$, is smaller than the unweighted expected value in the right hand side of the inequality.

H Expressions for the model with costly random withdrawals ($f > 0$) when $\pi = 0$

This appendix collects the expression that are obtained in the case of $f > 0$ and $\pi = 0$. In most cases they have to be obtained by using L'Hopital rule in the corresponding formulas for the general case.

When $\pi = 0$, for a given V^* and $0 < \underline{m} < \bar{m}$ the solution of $V(m)$ for $m \in (\underline{m}, \bar{m})$

is given by:

$$\begin{aligned} V(m) &= \varphi(m, A_\varphi) \equiv \\ &\equiv \left[\frac{-Rc}{r^2} \right] + \left[\frac{R}{r} \right] m + \left(\frac{c}{r} \right)^2 A_\varphi \exp\left(-\frac{r}{c}m\right). \end{aligned}$$

and

$$\begin{aligned} V(m) &= \eta(m, V^*, A_\eta) \equiv \\ &\equiv \left[\frac{p(V^* + f)(r + p) - Rc}{(r + p)^2} \right] + \left[\frac{R}{r + p} \right] m + \left(\frac{c}{r + p} \right)^2 A_\eta \exp\left(-\frac{r + p}{c}m\right). \end{aligned}$$

for $m \in (0, \underline{m})$ or $m \in (\bar{m}, m^{**})$.

For $\pi = 0$ the range of inaction ($m^* - \underline{m}$) is given by:

$$\frac{f c}{R} = [m^* - \underline{m}]^2 \left(\frac{1}{2} + \sum_{j=3}^{\infty} \frac{1}{j!} \left[(m^* - \underline{m}) \frac{r}{c} \right]^{j-2} \right) \quad (77)$$

Calculations for $m^* - \underline{m}$ for the case of $\pi = 0$. To see how we obtain the result for $\pi = 0$, start with the expression for $z^* = m^* - \underline{m}$:

$$z^* = \frac{1}{r/c} \left(\exp \left[z^* \frac{r}{c} \right] - 1 \right) - f \frac{r}{R}.$$

Write this expression as:

$$\exp \left[z^* \frac{r}{c} \right] = 1 + \left[z^* \frac{r}{c} \right] + \left[z^* \frac{r}{c} \right]^2 \left(\frac{1}{2} + \sum_{j=3}^{\infty} \frac{1}{j!} \left[z^* \frac{r}{c} \right]^{j-2} \right)$$

hence

$$\frac{f c}{R} = [m^* - \underline{m}]^2 \left(\frac{1}{2} + \sum_{j=3}^{\infty} \frac{1}{j!} \left[z^* \frac{r}{c} \right]^{j-2} \right)$$

The CDF for $\pi = 0$.

For $m \in (0, \underline{m})$ we have

$$H(m) = \frac{A_0}{p/c} \exp(pm/c) - B_0 \quad (78)$$

$$\begin{aligned} H(\underline{m}) &= \frac{1 - \exp(-p(\underline{m}/c))}{p(m^* - \underline{m})/c + 1 - \exp(-p(\underline{m}/c))} \\ A_0 &= \frac{H(\underline{m})(p/c)}{[\exp(p\underline{m}/c) - 1]} \end{aligned} \quad (79)$$

$$B_0 = \frac{A_0}{p/c} \quad (80)$$

For $m \in (\underline{m}, m^*)$ we have

$$H(m) = \frac{A_1}{\pi} \log \left(1 + \pi \frac{m}{c} \right) - B_1 \quad (81)$$

$$[1 - H(\underline{m})] = \frac{p(m^* - \underline{m})/c}{p(m^* - \underline{m})/c + 1 - \exp(-p\underline{m}/c)}$$

$$A_1 = \frac{1 - H(\underline{m})}{(m^* - \underline{m})/c} \quad (82)$$

$$B_1 = A_1 m^*/c - 1 \quad (83)$$

The average money holdings and withdrawals for $\pi = 0$

$$M = m^* - \frac{A_0}{(p/c)} \left\{ \frac{[\exp(p\underline{m}/c) - 1]}{(p/c)} - \underline{m} \right\} - \frac{A_1}{c} ((m^*)^2 - (\underline{m})^2) + [A_1 m^*/c - 1] (m^* - \underline{m}) \quad (84)$$

where A_0 , A_1 and B_1 are given in (79),(82) and (83).

If $\pi = 0$ the average withdrawal W is given by:

$$W = m^* \left[1 - \frac{p}{n} H(\underline{m}) \right] + \left[\frac{p}{n} H(\underline{m}) \right] \frac{\int_0^{\underline{m}} (m^* - m) h(m) dm}{H(\underline{m})} \quad (85)$$

where

$$\frac{\int_0^{\underline{m}} (m^* - m) h(m) dm}{H(\underline{m})} = m^* - \underline{m} - \frac{\frac{[\exp(p\underline{m}/c) - 1]}{(p/c)} - \underline{m}}{\exp(p\underline{m}/c) - 1}$$

I Expression (with derivation) for M when $f > 0$

$$\int_0^{\underline{m}} mh(m) dm = \left[H(\underline{m}) \underline{m} - H(0) 0 - \int_0^{\underline{m}} H(m) dm \right]$$

where

$$\int_0^{\underline{m}} H(m) dm = \int_0^{\underline{m}} \frac{A_0}{p/c} \left(1 + \pi \frac{m}{c} \right)^{\frac{p}{\pi}} dm - B_0(\underline{m}) = \frac{A_0}{p/c} \left[\frac{\left(1 + \frac{\pi}{c} \underline{m} \right)^{\frac{p}{\pi} + 1} - 1}{(p + \pi)/c} - \underline{m} \right]$$

and

$$\int_{\underline{m}}^{m^*} mh(m) dm = \left[m^* - H(\underline{m}) \underline{m} - \int_{\underline{m}}^{m^*} H(m) dm \right]$$

$$\int_{\underline{m}}^{m^*} H(m) dm = \int_{\underline{m}}^{m^*} \frac{c}{\pi} A_1 \log \left(1 + \pi \frac{m}{c} \right) dm - B_1 (m^* - \underline{m})$$

where

$$\int_{\underline{m}}^{m^*} \log \left(1 + \pi \frac{m}{c} \right) dm = \frac{c}{\pi} \left(1 + \pi \frac{m}{c} \right) \left[\log \left(1 + \frac{\pi}{c} m \right) - 1 \right] \Big|_{\underline{m}}^{m^*}$$

Hence

$$\begin{aligned} \int_{\underline{m}}^{m^*} H(m) dm &= A_1 \left(\frac{c}{\pi} \right)^2 \left\{ \left(1 + \frac{\pi}{c} m^* \right) \left[\log \left(1 + \pi \frac{m^*}{c} \right) - 1 \right] \right. \\ &\quad \left. - \left(1 + \frac{\pi}{c} \underline{m} \right) \left[\log \left(1 + \frac{\pi}{c} \underline{m} \right) - 1 \right] \right\} - B_1 (m^* - \underline{m}) \end{aligned}$$

Thus

$$\begin{aligned} M &= m^* - \int_0^{\underline{m}} H(m) dm - \int_{\underline{m}}^{m^*} H(m) dm \\ &= m^* - \frac{c}{p} A_0 \left[\frac{\left(1 + \frac{\pi}{c} \underline{m} \right)^{\frac{p}{\pi} + 1} - 1}{(p + \pi)/c} - \underline{m} \right] \\ &\quad - A_1 \left(\frac{c}{\pi} \right)^2 \left\{ \left(1 + \frac{\pi}{c} m^* \right) \left[\log \left(1 + \pi \frac{m^*}{c} \right) - 1 \right] - \left(1 + \frac{\pi}{c} \underline{m} \right) \left[\log \left(1 + \frac{\pi}{c} \underline{m} \right) - 1 \right] \right\} \\ &\quad + (m^* - \underline{m}) \left(\frac{c}{\pi} A_1 \log \left(1 + \pi \frac{m^*}{c} \right) - 1 \right) \end{aligned}$$

where

$$\begin{aligned} A_0 &= \frac{p}{c} \frac{1}{\left[\left(1 + \frac{\pi}{c} \underline{m} \right)^{\frac{p}{\pi}} - 1 \right]} H(\underline{m}) \\ A_1 &= \frac{(1 - H(\underline{m})) (\pi/c)}{\log \left(1 + \pi \frac{m^*}{c} \right) - \log \left(1 + \frac{\pi}{c} \underline{m} \right)} \end{aligned}$$

J Expression (with derivation) for W when $f > 0$

The expression

$$\frac{\int_0^{\underline{m}} (m^* - m) h(m) dm}{H(\underline{m})}$$

is the expected withdrawal conditional on being done by an agent with $m > 0$, or conditional on being a withdrawal that happens due to a chance meeting with the

intermediary.

$$\int_0^{\underline{m}} (m^* - m) h(m) dm = m^* H(\underline{m}) - \int_0^{\underline{m}} mh(m) dm$$

$$\int_0^{\underline{m}} mh(m) dm = \underline{m}H(\underline{m}) - \int_0^{\underline{m}} H(m) dm$$

with

$$\int_0^{\underline{m}} H(m) dm = \frac{A_0}{p/c} \left[\frac{(1 + \frac{\pi}{c}\underline{m})^{\frac{p}{\pi}+1} - 1}{(p + \pi)/c} - \underline{m} \right]$$

Thus

$$\int_0^{\underline{m}} (m^* - m) h(m) dm = (m^* - \underline{m}) H(\underline{m}) + \frac{A_0}{p/c} \left[\frac{(1 + \frac{\pi}{c}\underline{m})^{\frac{p}{\pi}+1} - 1}{(p + \pi)/c} - \underline{m} \right]$$

$$\left(\frac{A_0}{p/c} \right) / H(\underline{m}) = \frac{1}{(1 + \frac{\pi}{c}\underline{m})^{\frac{p}{\pi}} - 1}$$

so

$$\frac{\int_0^{\underline{m}} (m^* - m) h(m) dm}{H(\underline{m})} = (m^* - \underline{m}) + \frac{A_0}{p/c} \left[\frac{(1 + \frac{\pi}{c}\underline{m})^{\frac{p}{\pi}+1} - 1}{(p + \pi)/c} - \underline{m} \right]$$

$$= (m^* - \underline{m}) + \frac{(1 + \frac{\pi}{c}\underline{m})^{\frac{p}{\pi}+1} - 1}{(p + \pi)/c} - \underline{m}$$

$$= (m^* - \underline{m}) + \frac{(1 + \frac{\pi}{c}\underline{m})^{\frac{p}{\pi}} - 1}{(1 + \frac{\pi}{c}\underline{m})^{\frac{p}{\pi}} - 1}$$

K Cash-Flows Identity

We derive the following relationship (equation 49 in the paper)

$$c = -\pi M + nW$$

between the average (real) cash balances M , average (real) withdrawal amount, W , average (real) consumption flow c , average number of withdrawals n per unit of time, and the inflation rate π for a (large) class of cash management policies.

In what follows we fixed a particular path and denote the real cash balances at time t by $m(t)$, and let τ_i be the times at which there are withdrawals for this sample path and w_i the corresponding withdrawals amounts. In between withdrawals cash balances satisfy

$$\frac{dm(t)}{dt} = -c - m(t)\pi$$

At times $t = \tau_i$, a withdrawal of size w_i occurs, defined as an upward jump on m :

$$w_i \equiv \lim_{t \downarrow \tau_i} m(t) - \lim_{t \uparrow \tau_i} m(t) > 0.$$

Thus we have that

$$m(t) = m(0) - \int_0^T (c + \pi m(s)) ds + \sum_{i=1}^{N(T)} w_i$$

where $N(T)$ denotes the number of (upward) jumps up to time T in the path:

$$N(T) \equiv \{N : \tau_N \leq T \leq \tau_{N+1}\}.$$

Dividing by T and rearranging:

$$\frac{m(t) - m(0)}{T} = -c - \pi \frac{1}{T} \int_0^T m(s) ds + \left[\frac{N(T)}{T} \right] \left[\frac{1}{N(T)} \sum_{i=1}^{N(T)} w_i \right]$$

Defining :

$$M \equiv \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T m(s) ds, \quad n \equiv \lim_{T \rightarrow \infty} \frac{N(T)}{T}, \quad \text{and } W \equiv \lim_{T \rightarrow \infty} \frac{1}{N(T)} \sum_{i=1}^{N(T)} w_i$$

where M , n , W are the average money balances, average number of withdrawals per unit of time, and the average amount of withdrawal. Assuming that, for almost all paths, the limits M , n and W are well defined, and that the process is ergodic, so that these time averages converge to the unconditional expectations for almost all paths, we obtain (49). In all the models we analyze, these limits exist and coincide for all paths as a consequence of basic results on renewal theory, but of course their validity is much more general.

L Solving for b and f

Here we describe how to find b and f given $(m^*, \underline{m}, r, \pi, R)$ For convenience we rewrite equation (66) for $m_2^*(\cdot)$:

$$m^* = \left[\frac{r}{r + \pi} \right] \underline{m} + \frac{c}{(r + \pi)} \left(\frac{[1 + \pi \frac{m^*}{c}]^{1 + \frac{r}{\pi}}}{[1 + \frac{\pi}{c} \underline{m}]^{\frac{r}{\pi}}} - 1 \right) - f \frac{r}{R}$$

to find f . It is given by:

$$f = \frac{\left[\frac{r}{r+\pi} \right] \underline{m} + \frac{c}{(r+\pi)} \left(\frac{\left[1 + \pi \frac{m^*}{c} \right]^{1 + \frac{r}{\pi}}}{\left[1 + \frac{\pi}{c} \underline{m} \right]^{\frac{r}{\pi}}} - 1 \right) - m^*}{r/R}$$

Given f and $(m^*, \underline{m}, r, \pi, R, p)$ use equation (65) for $m_1^*(\cdot)$:

$$m^* = \frac{\left(\frac{c}{r+p} \right) \left[\frac{p f}{c} - \frac{R}{(r+p+\pi)} \right]}{\left(\frac{R}{r+p} \right)} + \frac{\left[\frac{R}{r+p+\pi} \right] \underline{m} + b \left[1 + \frac{\pi}{c} \underline{m} \right]^{-\frac{r+p}{\pi}} - f}{\left(\frac{R}{r+p} \right) \left[1 - \left[1 + \frac{\pi}{c} \underline{m} \right]^{-\frac{r+p}{\pi}} \right]}$$

to find b . It is given by

$$b = \frac{\left(m^* - \frac{\left(\frac{c}{r+p} \right) \left[\frac{p f}{c} - \frac{R}{(r+p+\pi)} \right]}{\left(\frac{R}{r+p} \right)} \right) \left(\frac{R}{r+p} \right) \left[1 - \left[1 + \frac{\pi}{c} \underline{m} \right]^{-\frac{r+p}{\pi}} \right] - \left[\frac{R}{r+p+\pi} \right] \underline{m} + f}{\left[1 + \frac{\pi}{c} \underline{m} \right]^{-\frac{r+p}{\pi}}}$$

M Decomposition of the cost of financing c

Let $v(R, \pi, p, b/c)/c$ be the per unit cost of financing cash purchases given the vector $(R, \pi, p, b/c)$, which is then expressed in number of days of cash purchases. To measure the *Reduction in cost in # cash days* in Table 11 we define

$$\Delta v_{t,i} \equiv v(R_{0,i}, \pi_{0,i}, p_{0,i}, (b/c)_{0,i})/c_{0,i} - v(R_{t,i}, \pi_{t,i}, p_{t,i}, (b/c)_{t,i})/c_{t,i} \quad (86)$$

$$\Delta v_{t,i,(p,b)} \equiv v(R_{0,i}, \pi_{0,i}, p_{0,i}, (b/c)_{0,i})/c_{0,i} - v(R_{0,i}, \pi_{0,i}, p_{t,i}, (b/c)_{t,i})/c_{t,i} \quad (87)$$

$$\Delta v_{t,i,(R,\pi)} \equiv v(R_{0,i}, \pi_{0,i}, p_{0,i}, (b/c)_{0,i})/c_{0,i} - v(R_{t,i}, \pi_{t,i}, p_{0,i}, (b/c)_{0,i})/c_{t,i} \quad (88)$$

for each year t and province type i , where $(R_{t,i}, \pi_{t,i}, p_{t,i}, (b/c)_{t,i})$ are the estimated values for year-province-type t, i and where we use 0 to denote the value in the first year of the sample, 1993. The first row of the table reports the mean of $\Delta v_{t,i}$ across provinces, i.e. the total reduction in cost. The second row reports the mean of $\Delta v_{t,i,(p,b)}$ across provinces, i.e. the reduction in cost due to the change in technology. The third row reports the mean of $\Delta v_{t,i,(R,\pi)}$ across provinces, i.e. the reduction in cost due to the disinflation. The fourth row computes the percentage of the total cost due to the changes in technology, by taking the ratio of the entries reported in the second and third rows. Notice that the sum of the second and third rows does not add up to the first row due to the interactions of (p, b) with (R, π) .

Table 11: Total and counterfactual cumulative reductions in the cost of financing c

<i>Reduction in cost in # cash days for HH w/o ATM</i>						
	1993	1995	1998	2000	2002	2004
Total, due to p, b, R, π	0	0.268	1.28	1.46	1.6	1.55
due to p, b	0	0.102	0.769	0.986	1.07	0.938
due to R, π	0	0.184	0.747	0.846	1.01	1.09
due to p, b , % of total	-	35.6	50.7	53.8	51.4	46.2
<i>Reduction in cost in # cash days for HH w. ATM</i>						
	1993	1995	1998	2000	2002	2004
Total, due to p, b, R, π	0	0.187	0.785	0.87	0.93	0.882
due to p, b	0	0.0842	0.464	0.544	0.56	0.427
due to R, π	0	0.066	0.432	0.499	0.629	0.678
due to p, b , % of total	-	56	51.8	52.1	47.1	38.6

Figure 6: Cost of financing cash purchases (per year)

