

# Cycles of Innovation and Financial Propagation\*

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## Abstract

Episodes of boom-bust cycles tend to occur in sectors with recent arrivals of new technologies and are often related to excessive funding by the financial sector. In this paper, I develop a dynamic general equilibrium model consistent with a role for the financial sector in propagation during such episodes. I extend a standard Schumpeterian growth model by incorporating (a) a monopolistically competitive financial sector and (b) time-varying technological conditions in real sectors. I identify two propagation channels. The first operates through financial firms' acquisition of sector-specific knowledge (*skill channel*); financial firms chase "hot sectors" and thereby amplify fluctuations. The second channel originates in an interaction between competition in the financial sector and patent races in product markets (*competition channel*). Financial firms' temporary competitive advantages in access to new ventures imply market segmentation: financial firms maximize the surplus generated by the client firms they can currently attract, anticipating competing financial firms' future screening and funding decisions. Relative to the Pareto optimum, the competition channel generates overinvestment in sectors with temporarily improved technological conditions; excessively high growth in these sectors comes at the cost of lower growth in the economy as a whole. The model links financial propagation to time variation in the cross section of asset prices. Exposures to aggregate risk dampen amplification effects.

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# 1 Introduction

The macroeconomic role of the financial sector has been subject to a long-standing debate that, with the recent financial crises, has attracted new attention. A central topic in this debate is the financial sector's contribution toward the cyclical nature of innovation and growth in real sectors. Gompers and Lerner (2003) find, for example, that the cyclical nature of the venture capital industry tends to generate over-shooting in innovation,<sup>1</sup> that is, that "during boom periods, prevalence of overfunding of particular sectors can lead to a sharp decline in the effectiveness of venture funds" and "prolonged downturns may ... lead to good companies going unfunded." Prominent examples of innovation-related boom-bust cycles are the biotech boom in the 1980s and the internet boom in 1990s – times when sectors with path-breaking new technologies saw rapid growth, before going through spectacular busts. In these cases, not only did the real sectors go through a boom-and-bust cycle, but so did the part of the financial system backing the innovation process.

In this paper, I develop a dynamic general equilibrium model consistent with a role for the financial sector in propagation during such episodes. I extend a Schumpeterian growth model by incorporating (a) a monopolistically competitive financial sector and (b) time-varying technological conditions in real sectors. In this framework, I identify two mechanisms that amplify fluctuations in innovation. The first channel operates through financial firms' endogenous acquisition of sector-specific knowledge (*skill channel*). The second channel originates in an interaction between competition in the financial sector and patent races in product markets (*competition channel*). Relative to the Pareto optimum, the competition channel generates overinvestment in sectors with temporarily improved technological conditions; excessively high growth in these sectors comes at the cost of lower growth in the economy as a whole. The model features a tight link between financial propagation and time variation in the cross section of asset prices. Procyclical variation in technological conditions is less subject to amplification because it induces less variation in the value of financial firms'

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<sup>1</sup>For related evidence see, for example, Kortum and Lerner (2000), Gompers and Lerner (2004), Kaplan and Schoar (2005), and Gompers, Kovner, Lerner and Scharfstein (2008).

business opportunities across sectors.

My model builds on the Schumpeterian growth literature (Grossman and Helpman 1991 and Aghion and Howitt 1992), where start-up firms undertake R&D projects in order to leapfrog current incumbents and to appropriate their rents. "Creative destruction," in the Schumpeterian sense, is the engine of growth (Schumpeter 1934). In this environment, I introduce financial firms that can acquire specialized proprietary knowledge that improves their sector-specific project selection skills relative to other market participants. Financial firms enter a sector if the cost of knowledge acquisition can be amortized through profits from competitive advantages in project evaluation and funding. Thus the financial sector fulfills its classic role of resource allocation as characterized by Schumpeter (1934): It identifies and funds those entrepreneurs with the best chances of successfully developing and implementing innovative products and production processes.

**The skill channel.** Endogeneity in the provision of these financial services constitutes the model's first propagation channel, the *skill channel*. When the diffusion of a new technology favors product development in a particular sector, financial firms enter the sector because they anticipate higher funding volume and increased revenues. Since specialized financial firms have superior skills in project selection, they crowd out less skilled market participants. The increase in skill on the investor side in turn accelerates innovation in the sector. On the other hand, when product development in a sector reaches a state of technological saturation, a corresponding decline in growth may be amplified by financial firms' decisions to stop paying attention to the sector, because specialization cost can no longer be amortized. Since the remaining investors in the financial market are less skilled in evaluating projects in the sector, the drop in growth is amplified.

In contrast to many existing theories in the corporate finance literature, the skill channel is *not* based on the classic trade-off between *internal* and *external* financing,<sup>2</sup> but rather on

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<sup>2</sup>See classic theories on principal-agent problems between shareholders and managers, such as Jensen and Meckling (1976) or Jensen (1986). See Aghion, Dewatripont and Rey (1999) for a Schumpeterian growth model that features agency cost of free cash flow in the sense of Jensen (1986).

variation in the knowledge- or skill-acquisition decisions by the suppliers of *external* financing. In reality, innovative firms may not only be constrained by incentive problems between managers and investors, but also by a lack of investors who are sufficiently knowledgeable to evaluate projects that build on a new technology. In sectors with new technologies, project evaluation is typically a difficult task since, by definition, no past data exist on inventions and their future impact. In order to estimate a project's future cash flows, a financial firm has to exert effort to acquire knowledge on the industry's competitive environment, technological developments, consumer demand, and other aspects. Financial firms only acquire this type of skill if (1) it yields competitive advantages that allow the financial firm to extract rents through project funding and (2) the anticipated scale of activity is sufficiently large to yield revenues that cover the cost. The skill channel in the model covers this economic rationale. Time-varying technological conditions alter financial firms' business opportunities and skill levels across sectors, which in turn feed back into real growth.

**The competition channel.** The model's second propagation channel, the competition channel, operates through an interaction between financial market competition and patent races in product markets.<sup>3</sup> Financial firms' clients compete in patent races – they strive to develop new products that displace current industry incumbents' vintages. Financial firms in turn compete in attracting clients with good prospects in product development. The temporary nature of financial firms' competitive advantages in access to new ventures generates market segmentation: Financial firms maximize the surplus the clientele they can currently attract generates, taking competing financial firms' future entry and funding decisions as given. Relative to the Pareto optimum, the decentralized equilibrium generates booms with overinvestment in sectors with temporarily improved technological conditions.

Consider, for example, a financial firm with a current opportunity to finance new ventures with good prospects in a particular area. This financial firm will make more extensive use of its current opportunity if it expects other investors not to flood the market with rivaling

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<sup>3</sup>See, e.g., Tirole (1988), Reinganum (1989), and Fudenberg, Gilbert, Stiglitz and Tirole (1983).

ventures in the same area in the future. The fewer competitors to enter in the future, the more likely a client venture is to stay a profitable leading-edge producer for a long time. Thus, deteriorating conditions for future entrants are similar to "barriers to entry": They strengthen the competitive positions of currently funded clients. Times of overinvestment are particularly severe when financial firms are anticipated to not enter in the future, since unfavorable "financing conditions" in the future strengthen the competitive position of clients funded in current boom times. On the other hand, times of improved conditions can induce busts in other times, because they imply increased levels of entry and competition that can make it optimal for financial firms to not enter the sector in "normal" times.

New technologies, like the Internet in the 1990s, give startups opportunities to enter existing industries, because they facilitate the development of new products that can displace those that current incumbents offer. When agents anticipate that opportunities for further product improvements based on a new technology are going to be exhausted in the near future, financial firms with access to the funding of these "last opportunities" increase their investment because clients that successfully develop the latest leading-edge product at that time will have a safer incumbency position with less competition in the future. These firms are most likely to weather the remaining time of rapid product turnover and to become established incumbents once high-growth times are over. Through this mechanism, an anticipated end to improved technological and financial conditions can feed an investment boom just before the decline.

If financial firms and product developers were able to merge into one large conglomerate and eliminate competition, the described form of amplification would cease to exist. Similarly, if competitive advantages in the financial sector were not just temporary, but instead, one financial firm had a perpetual competitive advantage, this financial firm could align diverging interests. Yet temporary competitive advantages in the financial sector seem to be an appropriate description for market economies such as the United States and Great Britain, where financial firms' profitable opportunities due to informational or technological advantages tend to erode over time, as they are competed away by other market participants.

In this environment, competition has the potential to generate the described amplification effects.

**Interaction with aggregate risk.** The two propagation channels operate in the absence of aggregate risk. How would the channels interact with aggregate growth cycles? To consider this question, I further extend the model by introducing time-varying aggregate consumption growth and risk aversion on the household side.

Current technological conditions for product innovations and incumbent asset prices primarily determine financial firms' project evaluation and funding activity in the cross section. Good technological conditions and high incumbent asset prices in a sector encourage financial firms to evaluate and fund new ventures. With higher levels of risk aversion, procyclical fluctuations in technological conditions generate less variation in the value of profits financial firms can make. As divergences in financial firms' profitability across sectors fluctuate less over time, so does the activity of financial firms across sectors, implying that financial propagation effects are diminished. Keeping technological conditions for innovation fixed, sectors where incumbent asset prices are more exposed to fluctuations over the business cycle are also more exposed to procyclical financial propagation. The model thus features a tight link between financial propagation and time variation in the cross section of asset prices.

## Literature

"Financial accelerator" theories dating back to Fisher (1933) consider the role of financial frictions in the propagation of the business cycle.<sup>4</sup> A central mechanism in these theories is procyclical variation in collateral values, which implies tightening borrowing constraints in downturns. When borrowing constraints are tied to the value of tangible assets, frictions

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<sup>4</sup>See, e.g., papers by Bernanke and Gertler (1989, 1990), Bernanke, Gertler and Gilchrist (1999), and Kiyotaki and Moore (1997). Related to the financial accelerator is the so-called "credit channel," which conceptualizes how monetary policy influences real variables by affecting the quality of borrowers' or banks' balance sheets (referred to as the "balance sheet channel" and "bank-lending channel," respectively). See, e.g., Bernanke and Blinder (1988), Bernanke and Gertler (1995), Stein (1998), and Adrian and Shin (2008).

of this type may dampen booms in innovation-intensive sectors relative to other sectors, since innovative firms, in particular startups, are typically endowed with low amounts of tangible collateral. A key distinction of the mechanism in my paper is that amplification effects are induced by time-variation in financial firms' business opportunities across sectors. Financial firms' private incentives to acquire sector-specific knowledge influence the provision of external financing and may induce excessive booms in sectors with temporarily improved technological conditions for innovation.

My paper is related to a large body of literature on the relation between finance and growth.<sup>5</sup> Similar to King and Levine (1993b), my setup builds on a Schumpeterian growth model similar to Grossman and Helpman (1991), extended by a financial sector that sorts good projects from bad ones. Three central deviations from King and Levine (1993b) are (1) the specification of the financial sector, (2) time variation in technological conditions in real sectors, and (3) aggregate risk. Due to deviations (1) and (2), King and Levine (1993b) do not feature the propagation effects I address in my paper. In King and Levine (1993b), "intermediaries" are endowed with identical evaluation skills and cannot establish competitive advantages. Perfect competition implies that the financial sector does not earn any rents in equilibrium. King and Levine's model does not feature any dynamics in financial sector activity. Intermediaries evaluate all projects in the economy at all times. Aggregate growth is deterministic and unambiguously increased by intermediaries' activity. King and Levine's analysis thus essentially only addresses the growth-enhancing impact of intermediaries' screening activity.

My paper proposes a new tractable approach to model the financial sector in a Schumpeterian growth framework. Financial firms' acquisition of proprietary knowledge generates short-term competitive advantages in project selection and thereby yields rents in equilib-

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<sup>5</sup>See, e.g., Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), King and Levine (1993a), Levine (1997), Lim and Townsend (1998), Levine and Zervos (1998), Rajan and Zingales (1998, 2001a, 2001b), Beck, Levine and Loayza (2000), Allen and Gale (2001), Claessens and Laeven (2003), Gine and Townsend (2004), Jeong and Townsend (2005), Trew (2006), and Townsend and Ueda (2006).

rium. The temporary nature of competitive advantages precludes long-term relationships between financial firms and corporations. Competitive pressure thus imposes constraints on the ability to intertemporally share surplus. This idea is related to Petersen and Rajan (1995), who find creditors are more likely to finance credit-constrained firms when credit markets are concentrated, because these creditors can more easily internalize the future benefits of assisting the firms. Michalopoulos, Laeven and Levine (2009) propose a notion of "financial innovation" in a Schumpeterian growth model with two periods and linear utility. The description of the financial sector in their model shares some qualitative similarities with my approach: Financiers attempt to create better screening technologies than their competitors to maximize profits, and existing screening methodologies become obsolete as technology advances. Yet, in contrast to my paper, Michalopoulos et al. (2009) do not address the amplification of innovation cycles and do not link financing of innovation to aggregate risk and risk aversion.

The considered framework provides an analytically tractable way to analyze the decentralization of information acquisition in a dynamic general equilibrium economy. Noisy rational expectations equilibrium models in the spirit of Grossman and Stiglitz (1980)<sup>6</sup> are commonly used to study the decentralization of information acquisition in financial markets. These models are typically two-period endowment economies that do not address links between the financial sector and macro-economic growth patterns.<sup>7</sup>

The sources of propagation I discuss in this paper are clearly only a subset of potential mechanisms that may be at play in reality. In particular, overreaction by investors may account for excessive booms with overinvestment, and may coexist with the mechanisms described in this paper. For the venture capital industry, Gompers and Lerner (2003) provide a detailed discussion of behavioral and institutional frictions that can cause excessive cycli-

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<sup>6</sup>See, e.g., Verrecchia (1982), Barlevy and Veronesi (2000), Peress (2004), and Veldkamp (2006). Wang (1993) features a dynamic setup with asymmetric information but keeps information exogenous.

<sup>7</sup>Mertens (2009) analyzes a dynamic general equilibrium model that features the aggregation of dispersed information about fundamentals. Yet the model does not consider endogenous costly information acquisition.

cality. Other sources of cyclicity discussed in the literature are, for example, entrepreneurs' self-fulfilling expectations about the implementation of innovations (Shleifer 1986), rational herd behavior (Scharfstein and Stein 1990), firm-specific learning-by-doing (Stein 1997), or "financing risk" as suggested by Nanda and Rhodes-Kropf (2009).

On the asset pricing side, my paper is related to the long-run risk literature (Bansal and Yaron 2004 and Hansen, Heaton and Li 2008). The model features stochastic differential utility (Duffie and Epstein 1992) in combination with regime changes that induce variation in the local drift of aggregate consumption growth. Regime-state dynamics are specified by a continuous time Markov chain, which allows the characterization of solutions to the laissez-faire equilibrium for general utility parameterizations through a system of non-linear equations.<sup>8</sup> Related to the notion of creative destruction, Hobijn and Jovanovic (2001) argue that major technological change like the IT revolution leads to initial stock market declines since it destroys old firms and since stock-market entry of new firms and new capital takes time. Pastor and Veronesi (2009) also analyze dynamics related to innovative firms that are due to technological change. The authors develop a general equilibrium model in which stock prices of innovative firms exhibit "bubbles" during "technological revolutions" that are induced by a change in the nature of uncertainty about a new technology from idiosyncratic to systematic. A key difference of my paper relative to the existing asset pricing literature is the considered production side of the economy, which endogenizes consumption dynamics and allows the study of connections between financial sector activity and asset prices.<sup>9</sup>

He and Krishnamurthy (2008) analyze the impact of financial intermediaries on asset

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<sup>8</sup>Hung (1994) considers a discrete time endowment economy with Kreps and Porteus (1978, 1979) preferences where market fundamentals follow a bivariate Markov switching process. Chen (2009) employs related techniques in continuous time.

<sup>9</sup>Garleanu, Kogan and Panageas (2008) study the interaction between innovation and asset returns in a model based on Romer (1990), which shares common elements with Schumpeterian growth models. Yet in order to focus on matters of asset pricing, Garleanu et al. (2008) specify growth exogenously. In their model, due to a lack of inter-generational risk sharing, innovation creates a systematic risk factor ("displacement risk") that helps explain empirical patterns in asset returns like the value premium and the equity premium.

prices in an endowment economy where risk sharing is limited due to an agency friction. In their model, only some agents ("intermediaries") have direct access to risky assets; others can invest in risky assets only through intermediaries. This heterogeneity among agents is exogenous and invariant to the state of the economy. In contrast, in my model, financial firms' acquisition of proprietary knowledge responds endogenously to changes in the state of the economy and influences aggregate consumption growth. Whereas risk sharing is perfect, distortions arise through the decentralization of innovation and financing: Competitive advantages provide private firms with incentives to invest, but these incentives generally fail to induce Pareto optimal allocations.

The rest of the paper is organized as follows. In the following section, I present the model. Section 3 characterizes solutions to the laissez-faire equilibrium and the social planner problem. Section 4 analyzes properties of financial propagation and deviations of laissez-faire allocations from the Pareto optimum. Section 5 concludes. I collect technical results and proofs in the Appendix.

## 2 Model

The setup builds on existing Schumpeterian growth models, in particular, the framework by Grossman and Helpman (1991). I present the key extensions in subsections 2.4 and 2.5, where I describe the innovation possibilities frontier and the financial sector. In addition, my model generalizes the preference specification relative to the existing Schumpeterian growth literature by considering stochastic differential utility (Duffie and Epstein 1992) instead of power utility or linear utility. This generalization is only essential for the analysis of the relation between propagation effects and aggregate risk. For all other parts of the paper, the results are identical for the special case of power utility.

## 2.1 Preferences

The economy is in continuous time and admits a representative household that maximizes stochastic differential utility (Duffie and Epstein 1992)

$$J_t = E_t \left[ \int_t^\infty f(C_\tau, J_\tau) d\tau \right], \quad (1)$$

where  $f(C, J)$  is a normalized aggregator of current consumption and continuation utility that takes the form

$$f(C, J) = \frac{\beta}{\rho} \left( (\alpha J)^{1-\frac{\rho}{\alpha}} C^\rho - \alpha J \right), \quad (2)$$

with  $\rho = 1 - \frac{1}{\psi}$  and  $\alpha = 1 - \gamma$ , where  $\beta > 0$  is the rate of time preference,  $\gamma > 0$  is the coefficient of relative risk aversion, and  $\psi > 0$  is the elasticity of intertemporal substitution. The normalized aggregator  $f(C, J)$  takes the following form when  $\psi \rightarrow 1$ :

$$f(C, J) = \beta \alpha J \left[ \log C - \frac{1}{\alpha} \log(\alpha J) \right]. \quad (3)$$

Power utility obtains by setting  $\psi = 1/\gamma$ . The generalization to stochastic differential utility allows specifying risk aversion and intertemporal elasticity of substitution separately, which proves useful when analyzing links between aggregate uncertainty, financial propagation, and time variation in the cross section of asset prices. The existing Schumpeterian growth literature considers economies with either deterministic aggregate consumption growth with power utility or risk-neutral households; aggregate risk and its impact on allocations through risk prices are not considered.

## 2.2 Labor Supply

Two types of labor are supplied inelastically: blue-collar labor and white-collar labor with total supply  $\bar{L}_B$  and  $\bar{L}_W$ , respectively. The manufacturing of intermediate goods according to existing patents requires blue-collar labor. Financial firms and firms that undertake R&D employ white-collar labor. The allocation of labor across firms in the economy is perfectly frictionless. White-collar and blue-collar labor obtain the equilibrium wage rates  $w_W(t)$  and  $w_B(t)$ , respectively.

The separation of the work force into groups with different skills is not new to the Schumpeterian growth literature (e.g., Aghion and Howitt 1992). In my model, the preclusion of labor movements from blue-collar jobs to white-collar jobs ensures that the economy does not feature jumps in aggregate consumption upon regime changes. The assumption corresponds to the notion that skill differences in the work force tend to be persistent and prevent, for example, agents from frequently switching between manufacturing jobs and finance jobs. As a side product, the separation will generate time-varying differences between the equilibrium wages paid in manufacturing on the one hand, and R&D and finance on the other.

### 2.3 Production Technology

The production of a unique final good uses a continuum of intermediate goods. The measure of intermediate goods is normalized to 1. Intermediate good varieties are indexed by  $v \in [0, 1]$ . Process innovations lead to quality improvements for existing intermediate goods. Let  $q(v, t)$  denote the quality of the intermediate good in variety  $v$  at time  $t$ , and let  $N(v, t)$  denote the number of innovations that occurred in variety  $v$  between time 0 and time  $t$ . As is customary in the Schumpeterian growth literature, a "quality ladder" determines the evolution of quality in each intermediate good variety:

$$q(v, t) = \kappa^{N(v, t)} q(v, 0), \quad \forall v \text{ and } t, \quad (4)$$

where  $\kappa > 1$  and  $q(v, 0) \in \mathbb{R}_+$ . The quality ladder implies that each innovation leads to a proportional quality increase by an amount  $\kappa$ . The number of innovations,  $N(v, t)$ , is a random variable; quality changes in the various intermediate good varieties are thus also stochastic.

The final good is produced by competitive firms according to the Cobb-Douglas production function:

$$Y(t) = \vartheta(t) \cdot \exp \left( \int_0^1 \log [q(v, t) x(v, t|q)] dv \right), \quad (5)$$

where  $x(v, t|q)$  is the quantity of intermediate good variety  $v$  of quality  $q$  used in the production process. In equilibrium, the output flow of the final good  $Y(t)$  equals the consumption flow  $C(t)$  of the representative household. Apart from the factor  $\vartheta(t)$ , the specification of the production function is standard in the Schumpeterian growth literature.<sup>10</sup> For most of the analysis,  $\vartheta(t)$  can be ignored, that is, set equal to unity. The factor only plays a role when I analyze the impact of aggregate risk on allocations. In that case,  $\vartheta(t)$  is assumed to follow the stochastic differential equation

$$\frac{d\vartheta(t)}{\vartheta(t)} = \varpi(Z(t)) dt + \sigma_{\vartheta} dB(t), \quad \text{with } \vartheta(0) > 0, \quad (6)$$

where  $B(t)$  is a standard Brownian motion and where the local drift  $\varpi$  may depend on an observable regime state  $Z(t)$ , further specified below.

The production function (5) implicitly assumes that at any point in time, exactly one quality of any intermediate good is used in the production of the final good. This assumption is not restrictive since in equilibrium only the leading-edge quality of any intermediate good will be employed. This replacement of older vintages by new inventions represents the notion of Schumpeterian "creative destruction" in the model (Acemoglu 2009).

## 2.4 Innovation

In the following, I describe the innovation possibilities frontier of the economy. The specification is standard in the Schumpeterian growth literature except for the point where I introduce stochastic dynamics in technological conditions for innovation. R&D projects invent higher-quality vintages of intermediate goods, building on the know-how of existing vintages. The execution of an R&D project requires  $c_R$  units of white-collar labor. The costs of R&D are identical for current incumbents and new firms (also called "entrants" going forward). There is free entry into research – any firm can undertake research in any of the intermediate good varieties. A firm that makes an innovation obtains a perpetual patent

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<sup>10</sup>See, e.g., King and Levine (1993b), Francois and Lloyd-Ellis (2003), Francois and Roberts (2003), and Klette and Kortum (2004).

for the new intermediate good. Yet the patent system does not preclude other firms from undertaking research based on this know-how. A firm that owns a patent for an intermediate good of quality  $q$  needs to employ one unit of blue-collar labor to manufacture one unit of the intermediate good.

As in King and Levine (1993b), the population of R&D projects can be partitioned by success prospects, into "good" and "bad" projects. Good projects generate a strictly positive Poisson arrival rate of innovation; bad projects never lead to a successful invention. The distribution of good and bad projects in the total population of R&D projects is common public knowledge. Firms that undertake an R&D project do not have additional private information on the success prospects of their project. Let  $\phi$  denote the fraction of good projects in the population. Further, define  $g(v, t)$  as the (continuous) number of good projects undertaken in variety  $v$  at date  $t$ .  $g(v, t)$  is endogenously determined in equilibrium. The *joint* Poisson arrival rate of innovation generated by  $g(v, t)$  good R&D projects is given by

$$h(v, t) = \theta(v, Z_t) \cdot g(v, t)^{1-\eta}, \quad (7)$$

where  $0 < \eta < 1$ . The Poisson arrival rate per good project,  $\frac{h(v,t)}{g(v,t)}$ , thus declines with the total number of good projects undertaken in an intermediate good variety at a time. This specification implies there are external diminishing returns to R&D activity – R&D firms doing research at the same time and in the same field are "fishing out of the same pond" (Acemoglu 2009). The parameter  $\theta(v, Z_t)$  captures technological conditions for innovation in variety  $v$  given the economy is in state  $Z_t$ . I also refer to  $\theta(v, Z_t)$  as the "productivity" or "effectiveness" of R&D. The dynamics of  $\theta(v, Z_t)$  are governed by the state  $Z(t)$ , which follows a two-state continuous time Markov chain

$$dZ(t) = \varphi_0 [Z(t-)] d\zeta_0(t) + \varphi_1 [Z(t-)] d\zeta_1(t), \quad (8)$$

where 0 and 1 label to the two Markov states,  $Z(t-)$  denotes the left limit  $\lim_{s \uparrow t} Z(s)$ ,  $\zeta_0(t)$  and  $\zeta_1(t)$  are independent Poisson processes with intensity parameters  $\lambda(0)$  and  $\lambda(1)$ , respectively, and where  $\varphi_0[0] = -\varphi_1[1] = 1$ ;  $\varphi_0[1] = \varphi_1[0] = 0$ . Although the model can be

easily extended to any finite state setup, the basic results of the paper can be illustrated in this simple two-state case.

In reality, time variation in R&D productivity may, for example, be induced by the arrival of new general purpose technologies (GPTs), such as the internet in the 1990s. A new GPT of this kind spurs inventions in various sectors of the economy as it opens new channels for product improvements. The effect on innovation naturally varies across sectors and is transitory in nature since new potentials for innovations are exploited over time.

## 2.5 Financial Sector

The paper deviates from the existing literature in its specification of a monopolistically competitive financial sector. Among all households there is a continuum of agents, called *financiers*, that have proprietary access to "leading-edge knowledge" about new R&D projects in a variety. Leading-edge knowledge may be interpreted as a common information component of a locally defined mass of R&D projects (local in the time and variety dimension) that is inaccessible to other market participants at that time. Financial firms have to acquire this knowledge in order to obtain the ability to evaluate new R&D projects' success prospects.

Financiers can work for one financial firm at a time and cannot commit to long-term contracts with financial firms. Each financier obtains access to proprietary knowledge in a finite number of varieties at any date. Financiers obtain access in distinct varieties, and for every variety there is a financier that can provide access to leading-edge knowledge. At any point in time, each financier works for the financial firm that offers the highest compensation for current access to this knowledge. If there is no demand the financier's access remains unused.

The setup implies that at most one financial firm obtains access to proprietary knowledge in each variety at any point in time. A financial firm with access to proprietary knowledge needs to employ white-collar labor to acquire and use the knowledge. Knowledge acquisition requires  $c_P$  units of white-collar-labor flow per product variety and date. Conditional on the acquisition of leading-edge knowledge, a financial firm is capable of evaluating R&D projects

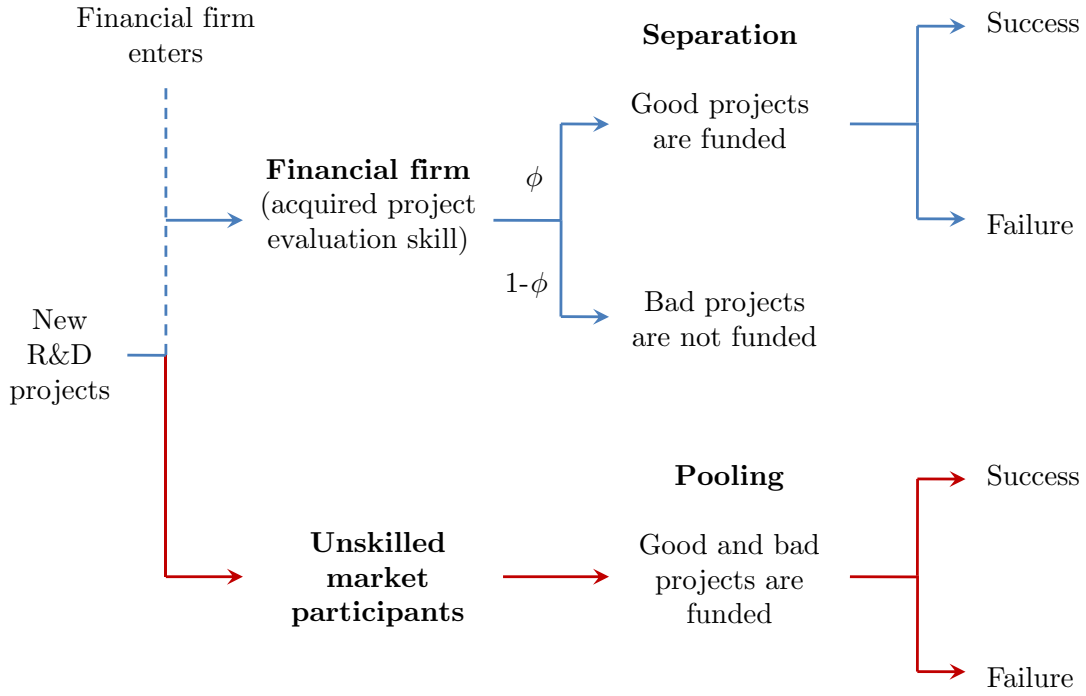


Figure 1: The figure illustrates R&D projects' evaluation and financing options.

at a constant per-project rate of  $c_E$  units of white-collar labor. The evaluation process performed by such a financial firm perfectly identifies analyzed projects' success prospects; that is, good and bad projects are separated. Because of the local nature of leading-edge knowledge in the time dimension, evaluation advantages based on the acquisition of this type of knowledge are short term in the sense that they last over an instant of time (from time  $t$  to time  $t+dt$ ). Other agents in the economy lack access to the proprietary piece of knowledge at that time, and thus cannot obtain the ability to distinguish good from bad projects – they encounter a pooled population of R&D projects. Going forward, I refer to

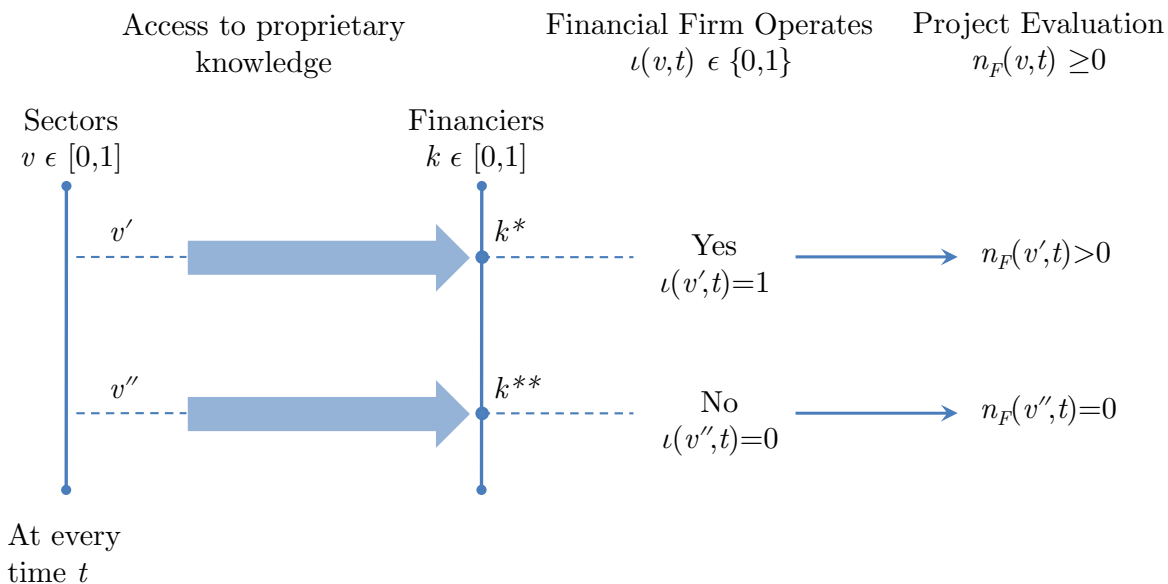


Figure 2: The figure illustrates the distribution of lead opportunities and financiers' decisions on the acquisition of proprietary knowledge and the evaluation of R&D projects.

these agents as "unskilled market participants." Although acquired knowledge is proprietary, the distribution of access to leading-edge knowledge among financiers is public information at every date. In addition, financial firms' actions are publicly observable ex post (at time  $t+dt$ ).

The local nature of leading-edge knowledge is conceptually consistent with the crowding-out effect among R&D projects discussed in the previous section. Both specifications refer to a common component among the mass of R&D projects that are undertaken in a variety at a time.

Let  $\iota(v, t) \in \{0, 1\}$  denote an indicator variable that represents financial firm entry in product variety  $v$  at date  $t$ .  $\iota(v, t) = 1$  refers to the case where a financial firm chooses to obtain access to proprietary knowledge through a financier.  $\iota(v, t) = 0$  represents the case where no financial firm enters variety  $v$  at time  $t$ . Non-entry may be optimal for financial firms since knowledge acquisition also requires costly white-collar labor. Further, let  $n_F(v, t)$  denote the number of R&D projects that the entering financial firm chooses to evaluate, and let  $n_U(v, t)$  denote the number of R&D projects unskilled market participants finance. At each date  $t$ , the following logical order of events occurs:

1. Financial firms decide whether to enter sectors by obtaining access to leading-edge knowledge via financiers ( $\iota(v, t) \in \{0, 1\}$ ).
2. Conditional on entry ( $\iota(v, t) = 1$ ), a financial firm acquires leading-edge knowledge, evaluates  $n_F(v, t)$  projects, and provides funding to R&D projects contingent on the evaluation results.
3. Unskilled market participants offer  $n_U(v, t)$  projects funding.
4. Funded R&D projects are executed and succeed or fail.

Figures 1 and 2 illustrate the setup. At this point, I preview the basic structure of the contract financial firms offer. I present the details in section 3. Financial firms promise to evaluate  $n_F(v, t)$  projects and to fund all that are identified as good. Bad projects are not funded. In exchange for project evaluations and contingent funding, financial firms obtain a claim to the dividends of R&D projects that innovate successfully and assume incumbency (which is revealed at time  $t+dt$ ).

The financial firms in this model resemble venture capital (VC) firms or other specialized investment firms most closely. The model is consistent with the view that VC firms' profits from the funding of start-ups are founded in competitive advantages in access to proprietary information (e.g., based on network connections) and the skillful evaluation of new projects' success prospects. Information of this kind is typically time and project-type specific; the

model's local specification of common information components is an extreme representation of this notion. In addition, consistent with the contractual setup in this paper, VCs typically obtain substantial stakes in the start-ups they finance.

## 2.6 Contractual Restrictions

As typical for Schumpeterian growth models, the economy features a contractual restriction that is also customary in the patent-race literature<sup>11</sup>: The model rules out the possibility that the current and previous incumbent contract and share the higher monopoly profits that could be earned through cooperation. Similarly, the proposed industrial organization of the financial sector rules out that financial firms can contract to eliminate competition, that is, to effectively merge into one large financial firm.

Apart from these restrictions, households and firms are free to trade in a frictionless Walrasian market where Arrow Debreu securities are in zero net supply. Although households might have different endowments of white-collar labor, blue-collar labor, and financier lead opportunities, they are assumed to maintain identical wealth levels; that is, financial wealth is the corresponding residual.

## 3 Solution

In the section, I characterize the laissez-faire equilibrium and the social planner solution for the economy.

### 3.1 Laissez-faire Equilibrium

**Definition 1 (Allocation)** *An allocation in this economy is given by stochastic processes of consumption  $[C(t)]_{t=0}^{\infty}$ ; R&D efforts by incumbents and entrants  $[n_I(v, t), n_E(v, t)]_{v \in [0,1], t=0}^{\infty}$ ; stochastic processes of proprietary knowledge acquisition and evaluation decisions by financial*

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<sup>11</sup>See, e.g., Tirole (1988), Reinganum (1989), and Fudenberg et al. (1983).

firms, and funding decisions by unskilled market participants  $[\iota(v, t), n_F(v, t), n_U(v, t)]_{v \in [0,1], t=0}^\infty$ ; stochastic processes of prices and quantities of leading-edge intermediate goods and the net present discounted value of profits from those goods,  $[p^x(v, t|q), x(v, t|q), V_I(v, t|q)]_{v \in [0,1], t=0}^\infty$ ; and stochastic processes of state-prices and wage rates,  $[\xi(t), w_B(t), w_W(t)]_{v \in [0,1], t=0}^\infty$ .

**Definition 2 (Equilibrium)** *An equilibrium in this economy is given by an allocation in which*

1. *R&D decisions by entrants maximize their discounted value;*
2. *pricing, quantity, and R&D decisions by incumbents maximize their discounted value;*
3. *proprietary knowledge acquisition, evaluation, and funding decisions by financial firms maximize their discounted value;*
4. *evaluation and funding decisions by unskilled market participants maximize their discounted value;*
5. *households choose their paths of consumption optimally;*
6. *blue-collar and white-collar labor markets and capital markets clear.*

### **Final Good Producers' Maximization**

As noted previously, the final good is produced competitively. At every date, producers of final goods take the prices of intermediate goods of various available qualities,  $p^x(v, t|q)$ , as given. Prices  $p^x(v, t|q)$  are determined in the monopolistically competitive intermediate goods market. The final good producers' first-order condition yields the unit elastic intermediate good demand

$$x(v, t|q) = \frac{Y(t)}{p^x(v, t|q)}. \quad (9)$$

## Intermediate Good Producers' Maximization

Given the unit elastic demand for intermediate goods (9), the firm with the highest-quality vintage in a variety limits prices at the marginal cost of a previous incumbent who could enter using his inferior patent. The profit-maximizing monopoly price for the highest-quality product is thus given by

$$p^x(v, t|q) = \kappa \cdot w_B(t). \quad (10)$$

Combining (9) and (10) implies the intermediate good demand

$$x(v, t|q) = \frac{Y(t)}{\kappa \cdot w_B(t)}. \quad (11)$$

The intermediate good producer with the leading-edge patent in variety  $v$  at time  $t$  earns monopoly profits

$$\pi_I(v, t) = (\kappa w_B(t) - w_B(t)) x(v, t|q) = \left(1 - \frac{1}{\kappa}\right) Y(t). \quad (12)$$

Since the costs of R&D are identical for incumbents and new firms, Arrow's replacement effect implies that incumbents will not undertake R&D in their own product line. The incumbent has weaker incentives to innovate, since the innovation would replace his own intermediate good. In contrast, a new entrant does not have this replacement calculation in mind. As a result, with the same technology of innovation, the entrants are always the ones that undertake R&D investments. This finding does not imply that incumbent firms do not undertake any R&D at all: As in the model of Klette and Kortum (2004), firms that are incumbents in a finite number of intermediate varieties may optimally engage in R&D in other product lines where they are not the current incumbent; that is, firms may try to enter new varieties to "steal business" from other firms. For the results of this paper, whether entrants are also incumbents in different intermediate good varieties is irrelevant since they act in exactly the same fashion.<sup>12</sup>

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<sup>12</sup>This property is due to the separability of value maximization across intermediate good varieties.

## Financial Firm Maximization

Let  $V_I(v, t)$  denote the market value of the intermediate good producer with the leading-edge patent in variety  $v$  at time  $t$ . I will refer to  $V_I(v, t)$  also as the "incumbent value." To simplify the presentation, I define  $n(v, t)$  as the sum of the number of projects evaluated by financial firms,  $n_F(v, t)$ , and the number of projects funded by unskilled market participants,  $n_U(v, t)$ , that is,

$$n(v, t) = n_U(v, t) + n_F(v, t). \quad (13)$$

A financial firm that has obtained access to leading-edge knowledge in variety  $v$  at time  $t$  solves the problem

$$\begin{aligned} \max_{\substack{\iota(v, t) \in \{0, 1\} \\ n_F(v, t) \geq 0}} \left\{ \frac{n_F(v, t)}{n(v, t)} (\phi n(v, t))^{1-\eta} \theta(v, Z_t) V_I(v, t) \right. \\ \left. - w_W(t) (\iota(v, t) \cdot c_P + n_F(v, t) \cdot (c_E + \phi c_R)) \right\} \end{aligned} \quad (14)$$

subject to

$$(1 - \iota(v, t)) n_F(v, t) = 0 \quad (15)$$

and the complementary slackness condition resulting from potential financing by unskilled market participants:

$$\begin{aligned} \phi \cdot \theta(v, Z_t) (\phi \cdot n(v, t))^{-\eta} V_I(v, t) \leq c_R w_W(t), \quad n_U(v, t) \geq 0 \text{ and} \\ \phi \cdot \theta(v, Z_t) (\phi \cdot n(v, t))^{-\eta} V_I(v, t) = c_R w_W(t) \quad \text{if } n_U(v, t) > 0. \end{aligned} \quad (16)$$

The financial firm maximizes the expected profit flow. The expected revenue flow is given by the product of three terms: the financial firm's market share, funded projects' joint Poisson arrival rate of innovation, and the incumbent value. Costs arise through the acquisition of proprietary knowledge, project evaluation, and the funding of R&D projects. The continuous nature of  $n(v, t)$  implies that the mass of good projects undertaken is exactly a fraction  $\phi$ , that is,  $g(v, t) = \phi n(v, t)$ .

Due to Arrows replacement effect it is always a financial firm without a stake in the current incumbent which has the highest incentives to obtain access to current leading-edge

knowledge in a variety. In equilibrium the financial firm that enters thus has no incentive to protect the current incumbent. All prices are thus taken as given in the financial firm's maximization problem. The setup ensures that a financial firms' maximization is separable across time and across varieties. Financial firms enter varieties only if their expected profit flow, before compensation payments to the financier, are greater than or equal to zero. Conditional on the acquisition of proprietary knowledge, the financial firm maximizes its value by driving unskilled market participants out of the local financing market. This is feasible due to the financial firm's competitive advantage in project evaluation that allows it to offer an evaluation and funding contract that gives R&D firms marginally better financing terms. Due to free entry, R&D firms and financial firms do not obtain any rents in equilibrium. Financiers' compensation is equal to the value of profits that a financial firm generates through its access to proprietary knowledge. Financiers share rents with scarce white-collar labor that financial firms and R&D firms employ.

**Proposition 1** *For  $\frac{1-\eta}{\phi+c_E/c_R} > 1$ , constraint (16) in the financial firm's maximization problem is slack and the following relations obtain*

$$\iota(v, t) = \begin{cases} 1 & \text{for } \frac{\theta(v, Z_t) \cdot V_I(v, t)}{w_W(t)} > \left(\frac{c_P}{\eta}\right)^\eta \left(\frac{c_R + \frac{c_E}{\phi}}{1-\eta}\right)^{1-\eta} \\ 0 \text{ or } 1 & \text{for } \frac{\theta(v, Z_t) \cdot V_I(v, t)}{w_W(t)} = \left(\frac{c_P}{\eta}\right)^\eta \left(\frac{c_R + \frac{c_E}{\phi}}{1-\eta}\right)^{1-\eta} \\ 0 & \text{otherwise} \end{cases} \quad (17)$$

$$g(v, t) = \begin{cases} \phi n_F(v, t) = \left(\frac{(1-\eta)\phi}{c_E + \phi c_R} \frac{\theta(v, Z_t) V_I(v, t)}{w_W(t)}\right)^{\frac{1}{\eta}} & \text{for } \iota(v, t) = 1 \\ \phi n_U(v, t) = \left(\frac{\phi}{c_R} \frac{\theta(v, Z_t) V_I(v, t)}{w_W(t)}\right)^{\frac{1}{\eta}} & \text{for } \iota(v, t) = 0. \end{cases} \quad (18)$$

*For  $\frac{1-\eta}{\phi+c_E/c_R} < 1$ , constraint (16) is binding and the following relations obtain*

$$\iota(v, t) = \begin{cases} 1 & \text{for } \frac{\theta(v, Z_t) \cdot V_I(v, t)}{w_W(t)} > \frac{c_R}{\phi} \left(\frac{c_P \phi}{c_R(1-\phi) - c_E}\right)^\eta \\ 0 \text{ or } 1 & \text{for } \frac{\theta(v, Z_t) \cdot V_I(v, t)}{w_W(t)} = \frac{c_R}{\phi} \left(\frac{c_P \phi}{c_R(1-\phi) - c_E}\right)^\eta \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

$$g(v, t) = \left(\frac{\phi}{c_R} \frac{\theta(v, Z_t) V_I(v, t)}{w_W(t)}\right)^{\frac{1}{\eta}}. \quad (20)$$

**Proof.** See Appendix. ■

For  $\frac{1-\eta}{\phi+c_E/c_R} > 1$ , a financial firm with proprietary knowledge possesses a sufficiently strong efficiency advantage to optimally finance strictly more good projects than unskilled market participants would, implying that the financial firm is effectively not constrained by competition from unskilled market participants. Generally, we see two competing forces: Since a local crowding out effect exists among R&D projects ( $0 < \eta < 1$ ), a financial firm with a local monopoly has a tendency to ration the number of executed projects relative to the competitive outcome where each individual R&D firm takes the per project arrival rate as given (*monopoly effect*). On the other hand, more efficient investment through financial firms' superior project selection warrants a greater amount of R&D activity (*efficiency effect*). If the monopoly effect dominates, i.e. for  $\frac{1-\eta}{\phi+c_E/c_R} < 1$ , the financial firm, conditional on entry, optimally funds just as many good R&D projects as competitive unskilled market participants would, attracting all R&D projects that are undertaken in equilibrium.

The ratio of the incumbent value to the wage rate  $V_I(v, t)/w_W(t)$  is a key determinant of financial firm entry and of the amount of funding in a variety. Higher values of this ratio imply that the labor cost of R&D firms and financial firms are low relative to the private gains from innovation, that is, the value of the incumbent's profits. Similarly, ceteris paribus, a higher R&D productivity  $\theta(v, Z_t)$  encourages financial firm entry and R&D funding. Lower knowledge acquisition cost  $c_P$  lowers the bar for financial firm entry. Similarly, lower values of  $\phi$  and  $c_E/c_R$  imply the financial firm's evaluation activity is more efficient: The lower the parameter  $\phi$ , the more bad projects in the population, and the more effective the project evaluation by a financial firm. The lower the ratio  $c_E/c_R$  the lower is the labor cost of project evaluation relative to the labor cost of R&D project execution. Clearly, all such statements have to be taken with caution, as the equilibrium prices  $V_I(v, t)$  and  $w_W(t)$  also change with parameter changes. To obtain comparative statics that incorporate such effects, I solve the model numerically in section 4.2.

## External vs. Internal Financing

In equilibrium, innovation risk is completely diversifiable. Thus, due to agents' risk aversion, any reliance on internal finance is inefficient and dominated by external financing: R&D firms are funded either by unskilled market participants or a financial firm. Financial firms in turn are owned by diversified shareholders, implying perfect risk sharing.

## Equilibrium Wages

Labor markets are assumed to be competitive. The equilibrium wage rate for blue-collar labor,  $w_B(t)$ , is determined by the market-clearing condition

$$\bar{L}_B = \int_0^1 L_M(v, t) dv = \frac{Y(t)}{w_B(t)} \int_0^1 \frac{1}{\kappa} dv. \quad (21)$$

It follows immediately that the blue-collar equilibrium wage rate is

$$w_B(t) = \frac{Y(t)}{\bar{L}_B} \int_0^1 \frac{1}{\kappa} dv. \quad (22)$$

Intermediate-good producers' demand for blue-collar labor is therefore given by

$$L_M(v, t) = \frac{\bar{L}_B}{\kappa \int_0^1 \frac{1}{\kappa} dv}. \quad (23)$$

The blue-collar wage to consumption ratio,  $\tilde{w}_B \equiv \frac{w_B(t)}{C(t)}$ , and the allocations of blue-collar labor,  $L_M(v, t)$ , are thus time invariant. The market-clearing condition for white-collar labor yields the equation

$$\bar{L}_W = \int_0^1 (L_F(v, t) + L_R(v, t)) dv, \quad (24)$$

where the labor demand by financial firms and R&D firms is given by

$$L_F(v, t) = c_E \cdot n_F(v, t) + c_P, \quad (25)$$

$$L_R(v, t) = \phi \cdot c_R \cdot n_F(v, t) + c_R \cdot n_U(v, t). \quad (26)$$

From the financial firm's maximization problem, we know  $n_F(v, t)$  and  $n_U(v, t)$  are implicit functions of the white-collar wage rate  $w_W(t)$  and the incumbent value  $V_I(v, t)$ . The allocation of white-collar labor and the wage to consumption ratio  $\tilde{w}_W(t) \equiv \frac{w_W(t)}{C(t)}$  are generally varying in response to changes in the regime state  $Z_t$ .

## State Prices

Household maximization implies that a state-pricing process  $\xi_t$  may be written as follows (Duffie and Epstein 1992):

$$\xi_t \equiv \exp \left[ \int_0^t f_J (C_\tau, J_\tau) d\tau \right] f_C (C_t, J_t), \quad (27)$$

where  $\xi_\tau/\xi_t$  has the standard interpretation in terms of intertemporal substitution of income between dates  $\tau$  and  $t$ .

## State Variables

State variables in the economy are the regime state  $Z(t)$  and the level of aggregate consumption  $C(t) = Y(t)$ . Due to the iso-elastic properties of the setup, the economy scales by  $Y(t)$ .

**Proposition 2 (Aggregate consumption)** *Aggregate consumption follows the stochastic differential equation*

$$\frac{dY(t)}{Y(t)} = \mu_Y(Z_t) dt + \sigma_\vartheta dB(t), \quad (28)$$

where the local drift depends on the aggregate regime state  $Z_t$  and takes the form

$$\mu_Y(Z_t) = \varpi(Z_t) + \int_0^1 \log[\kappa] \cdot \bar{h}(v, Z_t) dv, \quad (29)$$

with

$$\bar{h}(v, Z_t) = \theta(v, Z_t) (M(v, Z_t) g_F(v, Z_t)^{1-\eta} + (1 - M(v, Z_t)) g_U(v, Z_t)^{1-\eta}), \quad (30)$$

$$g_F(v, Z_t) = \left( \frac{\phi \theta(v, Z_t) p_I(v, Z_t)}{c_R \tilde{w}_W(Z_t)} \right)^{\frac{1}{\eta}} \left( \frac{1 - \eta}{\phi + \frac{c_E}{c_R}} \right)^{\frac{1}{\eta}}, \quad (31)$$

$$g_U(v, Z_t) = \left( \frac{\phi \theta(v, Z_t) p_I(v, Z_t)}{c_R \tilde{w}_W(Z_t)} \right)^{\frac{1}{\eta}}. \quad (32)$$

The probability of financial firm entry  $M(v, Z_t) \equiv \Pr[\iota(v, t) = 1 | Z_t]$  satisfies

$$\begin{aligned} M(v, Z_t) &= 1 && \text{for } \frac{\theta(v, Z_t) \cdot p_I(v, Z_t)}{\tilde{w}_W(Z_t)} > \left(\frac{c_P}{\eta}\right)^\eta \left(\frac{c_R + \frac{c_E}{\phi}}{1-\eta}\right)^{1-\eta} \\ M(v, Z_t) &\in [0, 1] && \text{for } \frac{\theta(v, Z_t) \cdot p_I(v, Z_t)}{\tilde{w}_W(Z_t)} = \left(\frac{c_P}{\eta}\right)^\eta \left(\frac{c_R + \frac{c_E}{\phi}}{1-\eta}\right)^{1-\eta} \\ M(v, Z_t) &= 0 && \text{for } \frac{\theta(v, Z_t) \cdot p_I(v, Z_t)}{\tilde{w}_W(Z_t)} < \left(\frac{c_P}{\eta}\right)^\eta \left(\frac{c_R + \frac{c_E}{\phi}}{1-\eta}\right)^{1-\eta}. \end{aligned} \quad (33)$$

The white-collar wage to consumption ratio  $\tilde{w}_W(Z_t)$  solves the market clearing condition

$$\bar{L}_W = \int_0^1 \left( (g_F(v, Z_t) \left(\frac{c_E}{\phi} + c_R\right) + c_P) M(v, Z_t) + \frac{c_R}{\phi} g_U(v, Z_t) (1 - M(v, Z_t)) \right) dv. \quad (34)$$

**Proof.** See Appendix. ■

The proposition shows the local drift of aggregate consumption may be decomposed into an exogenous component  $\varpi(Z_t)$  and a component that depends on the endogenously determined Poisson arrival rates of innovation  $\bar{h}(v, Z_t)$ . Equilibria may obtain where financial firms enter a variety in state  $Z_t$  only probabilistically:  $M(v, Z_t)$  denotes the corresponding probability of financial firm entry in variety  $v$  given the economy is in state  $Z_t$ .

**Proposition 3 (Households' value function)** *Households' value function takes the form*

$$J(Y_t, Z_t) = E_t \left[ \int_t^\infty f(C_\tau, J_\tau) d\tau \right] = F(Z_t) \frac{Y_t^{1-\gamma}}{1-\gamma}, \quad (35)$$

where the function  $F(Z_t)$  solves the equations (for  $Z_t = 0, 1$ ):

$$0 = \frac{\beta\alpha}{\rho} \left( F(Z_t)^{-\frac{\rho}{\alpha}} - 1 \right) + \alpha\mu_Y(Z_t) + \frac{1}{2}\alpha(\alpha-1)\sigma_\vartheta^2 + \lambda(Z_t)\varphi_{Z_t}[Z_t] \frac{F(1) - F(0)}{F(Z_t)}. \quad (36)$$

**Proof.** See Appendix. ■

**Proposition 4 (Incumbent price)** *The price of an incumbent in variety  $v$  is given by*

$$V_I(v, Y_t, Z_t) = E_t \left[ \int_t^\infty \frac{\xi_\tau}{\xi_t} \pi_I(v, t) d\tau \right] = Y_t \cdot p_I(v, Z_t), \quad (37)$$

where  $p_I(v, Z_t)$  solves for  $Z = 0, 1$ :

$$p_I(v, Z_t) = \frac{1 - \frac{1}{\kappa}}{r_f(Z_t) + r p_I(v, Z_t) + \bar{h}(v, Z_t) - \lambda(Z_t) \frac{p_I(v, Z_t + \varphi_{Z_t}[Z_t]) - p_I(v, Z_t)}{p_I(v, Z_t)} - \mu_Y(Z_t)}, \quad (38)$$

where  $r_f(Z_t)$  denotes risk-free short rate in state  $Z$ :

$$r_f(Z_t) = \frac{\beta\alpha}{\rho} - \frac{\beta(\alpha - \rho)}{\rho} F(Z_t)^{-\frac{\rho}{\alpha}} + \gamma\mu_Y(Z_t) - \frac{1}{2}\gamma(\gamma + 1)\sigma_\vartheta^2 - \lambda(Z)\varphi_{Z_t}[Z_t] \frac{F(1)^{1-\frac{\rho}{\alpha}} - F(0)^{1-\frac{\rho}{\alpha}}}{F(Z_t)^{1-\frac{\rho}{\alpha}}}, \quad (39)$$

and where  $rp_I(v, Z_t)$  is defined as

$$rp_I(v, Z_t) = (1 - \alpha)\sigma_\vartheta^2 + \lambda(Z_t) \left( 1 - \frac{p_I(v, Z_t + \varphi_{Z_t}[Z_t])}{p_I(v, Z_t)} \right) \left( \left( \frac{F(Z_t + \varphi_{Z_t}[Z_t])}{F(Z_t)} \right)^{1-\frac{\rho}{\alpha}} - 1 \right). \quad (40)$$

**Proof.** See Appendix. ■

Propositions 2 to 4 constitute a system of non-linear equations that are satisfied in the laissez-faire equilibrium.

### 3.2 Social Planner Solution

In this section, I discuss the Pareto optimal allocation for the economy. The social planner maximizes the representative household's utility subject to the resource constraints, the final goods production technology (equations 5 and 6), the innovations possibility frontier (equations 4 and 7), and the financial sector's technology to evaluate projects. Throughout, I add an additional subscript  $S$  to indicate the social planner solution.

As noted in subsection 2.5, agents' actions and financiers' access to proprietary knowledge are public information, implying the social planner can ensure that the financial sector operates according to the Pareto optimal plan. The planner does not need to directly observe the knowledge financial firms acquire. Since actions are observable, incentives to deviate from the social planner solution can be eliminated through contingent penalties. The planner can circumvent the contractual restrictions of the competitive setup, which effectively rule out that private financial firms and corporations merge into one large corporation that maximizes surplus.

**Proposition 5 (Social planner solution)** *Under the social planner solution, aggregate consumption follows the stochastic differential equation*

$$\frac{dY_t}{Y_t} = \mu_{YS}(Z_t) dt + \sigma_{\vartheta} dB_t, \quad (41)$$

where the local drift is given by

$$\mu_{YS}(Z_t) = \varpi(Z_t) + \max_{\substack{\iota_S(v, Z_t) \in \{0, 1\}, \\ n_{US}(v, Z_t) \geq 0, \\ n_{FS}(v, Z_t) \geq 0}} \int_0^1 \log[\kappa] \cdot \bar{h}_S(v, Z_t) dv, \quad (42)$$

subject to

$$\bar{h}_S(v, Z_t) = \theta(v, Z_t) (\phi(n_{US}(v, Z_t) + n_{FS}(v, Z_t)))^{1-\eta} \quad (43)$$

$$0 = (1 - \iota_S(v, Z_t)) n_{FS}(v, Z_t) \quad (44)$$

$$\bar{L}_W = \int_0^1 (c_E n_{FS}(v, Z_t) + c_P \iota_S(v, Z_t) + \phi c_R n_{FS}(v, Z_t) + c_R n_{US}(v, Z_t)) dv, \quad (45)$$

The level of the final good consumption flow is given by

$$Y(t) = \vartheta(t) \cdot \exp\left(\int_0^1 \log[q(v, t)] dv\right) \cdot \bar{L}_B. \quad (46)$$

**Proof.** See Appendix. ■

Due to the intertemporal separability of the innovations possibility frontier, the social planner problem reduces to a static problem: The planner maximizes the local drift of consumption growth at every date. This solution provides a clear-cut benchmark: Any intertemporal dependencies that exist in the laissez-faire equilibrium will generate deviations from the social planner solution.

The laissez-faire equilibrium is generally Pareto suboptimal, which is a standard result in Schumpeterian growth models.<sup>13</sup> A special aspect of my model is the interaction between the so-called "business-stealing effect" and financial firms' temporary competitive advantages. In the literature, the business-stealing effect refers to the notion that private research firms do not internalize the loss to the previous incumbent caused by an innovation, which may cause

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<sup>13</sup>See, e.g., Aghion and Howitt (1998) for a detailed discussion.

excessive incentives for innovation. A financial firm, conditional on entry, becomes effectively marginal in setting the number of R&D projects in a variety at a point in time and may internalize gains from its clients' business-stealing. The temporary nature of financial firms' competitive advantages in access to new ventures implies that financial firms do not align diverging interests of the various R&D firms that obtain funding over time. Financial firms maximize the surplus the clientele they can currently attract generates, taking competing financial firms' future funding decisions as given. Business-stealing in the intermediate goods market thus interacts with financial firms' knowledge acquisition- and funding-decisions.

Whereas business-stealing increases incentives for innovation, two other effects tend to reduce innovation in the laissez-faire equilibrium: First, the "appropriability effect," which refers to the notion that monopolists in the intermediate good market are not able to appropriate the whole output flow since competition from the next best producer limits pricing. Second, the "intertemporal spillover effect," which is due to the fact that incumbents are replaced by new entrants in finite time with probability one, and attach no weight to the benefits that accrue beyond the succeeding innovation. In contrast, the social planner takes into account the fact that the benefit to the next innovation will be permanent, since innovations are cumulative.

## 4 Analysis

### 4.1 Symmetric Intermediate Good Varieties

In this section, I consider the special case where all structural parameters are identical across all intermediate good varieties  $v \in [0, 1]$ . In this case, the solutions to the laissez-faire equilibrium and the social planner problem simplify substantially. The results provide a useful reference point as they allow isolating effects in the model that are caused by violations of symmetry. To simplify the presentation, I restrict my attention to the part of the parameter domain where  $\frac{1-\eta}{\phi+c_E/c_R} > 1$ , that is, the case where specialized financial firms have a substantial skill advantage over unskilled market participants.

**Proposition 6 (Laissez-faire equilibrium under symmetry)** *In a symmetric setup, the probability of financial firm entry in variety  $v$  in the laissez-faire equilibrium,  $M \equiv \Pr[\iota(v, t) = 1]$ , is given by*

$$M = \begin{cases} 1, & \text{given } c_P < \underline{c}_P, \\ \frac{\bar{L}_W}{c_P} \eta - \left( \frac{c_E + \phi}{c_R} \right)^{\frac{1-\eta}{\eta}}, & \text{given } \underline{c}_P < c_P < \bar{c}_P, \\ 0, & \text{given } c_P > \bar{c}_P, \end{cases} \quad (47)$$

where

$$\underline{c}_P \equiv \eta \bar{L}_W \quad \text{and} \quad \bar{c}_P \equiv \eta \bar{L}_W \left( \frac{1-\eta}{\frac{c_E}{c_R} + \phi} \right)^{\frac{1}{\eta}-1}. \quad (48)$$

The solutions for all other variables are provided in the Appendix.

**Proof.** See Appendix. ■

**Proposition 7 (Social planner solution under symmetry)** *In a symmetric setup, the fraction of varieties where financial firms operate under the social planner solution,  $M_S$ , is given by*

$$M_S = \begin{cases} 1, & \text{given } c_P < \underline{c}_{PS}, \\ \frac{\bar{L}_W}{c_P} \eta - \frac{(1-\eta) \left( \frac{c_E + \phi}{c_R} \right)^{\frac{1}{\eta}-1}}{1 - \left( \frac{c_E + \phi}{c_R} \right)^{\frac{1}{\eta}-1}}, & \text{given } \underline{c}_{PS} < c_P < \bar{c}_{PS}, \\ 0, & \text{given } c_P > \bar{c}_{PS}, \end{cases} \quad (49)$$

where

$$\underline{c}_{PS} \equiv \eta \bar{L}_W \frac{1 - \left( \frac{c_E}{c_R} + \phi \right)^{\frac{1}{\eta}-1}}{1 - \eta \left( \frac{c_E}{c_R} + \phi \right)^{\frac{1}{\eta}-1}} \quad \text{and} \quad \bar{c}_{PS} \equiv \eta \bar{L}_W \frac{1 - \left( \frac{c_E}{c_R} + \phi \right)^{\frac{1}{\eta}-1}}{(1-\eta) \left( \frac{c_E}{c_R} + \phi \right)^{\frac{1}{\eta}-1}}. \quad (50)$$

The solutions for all other variables are provided in the Appendix.

**Proof.** See Appendix. ■

The structure of the laissez-faire equilibrium and the social planner solution are similar; there are three parameter regions: In the parameter region where entry occurs in all intermediate good varieties, financial firms evaluate all projects in the economy. An equal

number of R&D projects is evaluated and executed in each variety, and the total supply of white-collar labor determines the scale. Similarly, in the parameter region where there is no financial firm entry in any variety, white-collar labor is split evenly across R&D projects in all varieties. Further, both solutions feature a region where financial firms operate in some varieties but not all. Due to the abovementioned distortionary effects present in the laissez-faire equilibrium, the cutoff values for the regions deviate (see  $\underline{c}_P$  vs.  $\underline{c}_{PS}$  and  $\bar{c}_P$  vs.  $\bar{c}_{PS}$ ). Yet, for sufficiently low (or high) knowledge acquisition cost  $c_P$ , the laissez-faire equilibrium and the social planner solution *coincide*.

A special feature of the laissez-faire solutions under symmetry is the absence of forward-looking terms and time invariance, unless structural parameters like  $c_P$ ,  $c_E$ ,  $c_R$ , or  $\phi$  are specified as state dependent. Due to perfect symmetry, no intermediate good variety has a relative R&D productivity advantage that would favor a higher allocation of white-collar labor. Equation (17) indicates that more generally, laissez-faire financial firm entry depends on the ratio of the incumbent market value relative to the white-collar wage rate ( $V_I(v, t) / w_W(t)$ ). This ratio encodes forward-looking information. Yet in the absence of differences in the structural parameters across intermediate good varieties, incumbents' market values are identical, that is,  $V_I(v, t) = V_I(v', t)$ ,  $\forall v, v', t$ . The white-collar labor market-clearing condition then fixes the product  $\theta(v, Z_t) V_I(v, t) w_W(t)^{-1}$  to a time-invariant constant, implying that the forward-looking terms that determine financial firms' entry decision (see equation 17) can be substituted out.

This result strictly hinges on the symmetry assumption made in this section. A violation of perfect symmetry generally leads to the case where incumbent market values differ from each other in some varieties, which in turn implies that financial firms' entry decisions are influenced by forward-looking asset prices. Due to households' risk aversion, differences in incumbents' risk exposures will therefore influence allocations. In order to analyze such effects, I break the symmetry assumption in the following section.

## 4.2 Financial Propagation - A Two-Sector Example

In this section, I consider an economy with two sectors, indexed by superscripts  $A$  and  $B$ . All intermediate good varieties in a sector have identical structural parameters. For simplicity, I assume there is one large sector ( $A$ ) and a small, zero-measure sector ( $B$ ).

Due to the zero-measure assumption for sector  $B$ , the social planner solution and the laissez-faire equilibrium can be determined in two steps: First, the sector  $A$  economy is solved according to propositions 7 and 6, yielding the aggregate allocation. Second, given the corresponding (shadow) prices, the solutions for sector  $B$  can be obtained separately. I provide details for the second step in the Appendix.

Table I presents the baseline parameterization. In the baseline case, the parameters in sector  $A$  and  $B$  are identical. The parameters are chosen such that financial firms operate in all varieties under both the laissez-faire equilibrium and the social planner solution (given that a financial sector exists). As previously discussed in relation to the case of symmetric intermediate good varieties (see propositions 7 and 6), this setup also implies that all other aspects of the solutions to the laissez-faire equilibrium and the planner problem coincide in the baseline case.

### The Laissez-faire Equilibrium

In this section, I analyze how the financial sector amplifies growth fluctuations that are due to stochastic changes in technological conditions for innovation. First, I consider a setup that only features time variation in technological conditions in sector  $B$ . In the next step, I introduce stochastic aggregate growth and analyze its interaction with financial propagation.

Figure 3 illustrates how growth variability in sector  $B$ , as measured by the ratio of the Poisson arrival rate of inventions in sector  $B$  in the two states ( $\frac{\bar{h}^B(1)}{h^B(0)}$ ), depends on variability in technological conditions, as measured by the ratio of R&D productivity in states 1 and 0 ( $\frac{\theta^B(1)}{\theta^B(0)}$ ). A value of one on the horizontal axis thus represents time-invariant technological conditions, which implies time-invariant growth, that is,  $\frac{\bar{h}^B(1)}{h^B(0)} = 1$ . In the illustrations, changes in the parameter  $\theta^B(1)$  induce different values for  $\frac{\theta^B(1)}{\theta^B(0)}$ . Throughout,

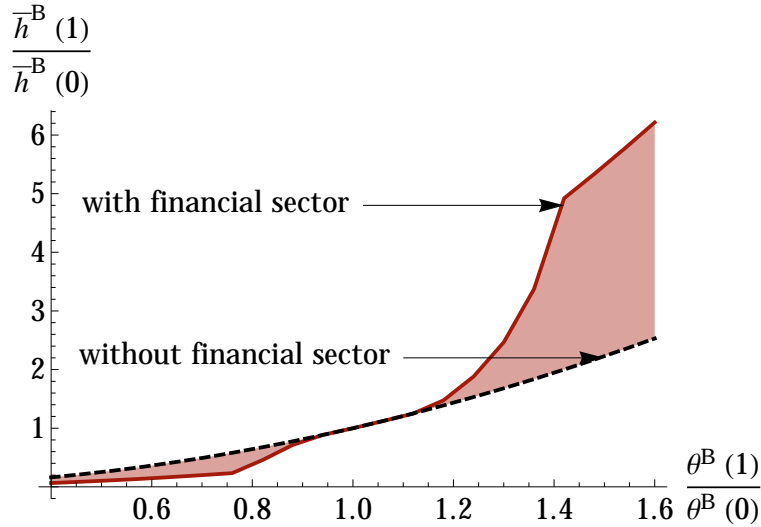


Figure 3: The figure plots growth variability in sector  $B$  in the laissez-faire equilibrium, as measured by the ratio of the Poisson arrival rate of inventions in sector  $B$  in the two states ( $\frac{\bar{h}^B(1)}{\bar{h}^B(0)}$ ), against variability in technological conditions for innovation, as measured by the ratio of R&D productivity in sector  $B$  in states 1 and 0 ( $\frac{\theta^B(1)}{\theta^B(0)}$ ). Aggregate consumption growth is constant.

the parameter  $\theta^B(0)$  is kept constant at its benchmark level. Note that for  $\frac{\theta^B(1)}{\theta^B(0)} < 1$ , lower values of  $\frac{\bar{h}^B(1)}{\bar{h}^B(0)}$  represent increased growth fluctuations and for  $\frac{\theta^B(1)}{\theta^B(0)} > 1$ , higher values of  $\frac{\bar{h}^B(1)}{\bar{h}^B(0)}$  imply increased growth fluctuations.

The positive relationship in Figure 3 is not surprising because growth fluctuations are induced by changes in technological conditions. The main purpose of the figure is the comparison between the case "without financial sector" and the case "with financial sector." Throughout, the label "without financial sector" refers to the case where financial firms do not operate in sector  $B$ . The label "with financial sector" refers to the case where financial firms potentially can enter or leave sector  $B$ . For the illustrations, parameters are chosen such that financial firms enter sector  $B$  in both regime states given that technological conditions are time invariant, that is, for  $\frac{\theta^B(1)}{\theta^B(0)} = 1$ .

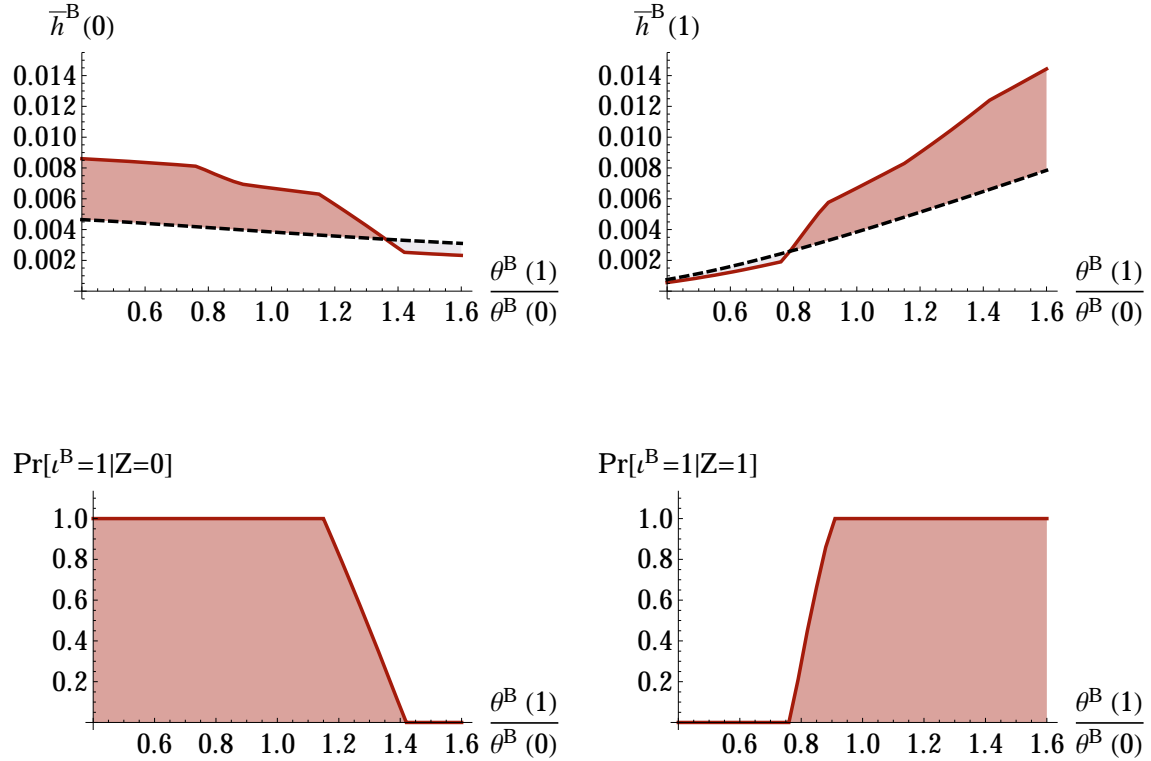


Figure 4: The plots in the upper panel illustrate growth in sector  $B$  in the laissez-faire equilibrium ( $\bar{h}^B(1)$  and  $\bar{h}^B(0)$ ), as a function of technological conditions for innovation in state 1,  $\theta^B(1)$ , scaled by the benchmark value of  $\theta^B(0)$ . The dashed line represents the case without financial sector; the solid line represents the case with financial sector. The plots in the lower panel illustrate the probability of financial firm entry in the two states.

Figure 3 indicates that growth fluctuations are systematically amplified by the existence of a financial sector. What causes this result? For  $\frac{\theta^B(1)}{\theta^B(0)} < 1$ , increased growth fluctuations are mainly due to the "skill channel." Low values of the parameter  $\theta^B(1)$  imply unfavorable conditions for product development in regime state 1. Adverse conditions in turn reduce financial firms' profits from skilled funding. If conditions are sufficiently bad, financial firms do not specialize in the sector and only unskilled market participants remain (see the lower panel in Figure 4 for financial firms' entry decisions). Due to the lack of skill in the financial market, growth is in turn further reduced in state 1, which leads to an increase in the variability of  $\bar{h}^B$  across states.

For  $\frac{\theta^B(1)}{\theta^B(0)} > 1$ , a combination of the "skill channel" and the "competition channel" induce increased growth fluctuations. Improved technological conditions in state 1 lead to a *decline* in equilibrium skill in state 0, even though technological conditions in state 0 are not altered in the illustration. The competition channel implies that positive technological conditions in state 1 make it less profitable for financial firms to operate in state 0, because ventures funded in state 0 are anticipated to face strong competition from new entrants in state 1. When conditions in state 1 are sufficiently good, financial firms choose to leave sector  $B$  in state 0. As illustrated in Figure 4, the skill channel then implies a bust in state 0 in the sense that growth is reduced markedly, below the level that obtains in a world without financial sector. On the other hand, growth in state 1 is significantly higher due the presence of skilled financial firms, which, in combination with the bust in state 0, yields an amplification of growth fluctuations.

### The Social Planner Solution

Figure 5 considers the same setup as Figure 3 but illustrates the social planner solution to the economy, not the laissez-faire equilibrium. Although an amplification of growth fluctuations is still existent for  $\frac{\theta^B(1)}{\theta^B(0)} < 1$ , no effects are present for  $\frac{\theta^B(1)}{\theta^B(0)} > 1$ , where the dashed line and the solid line lie on top of each other, indicating that growth fluctuations are identical with and without a financial sector.

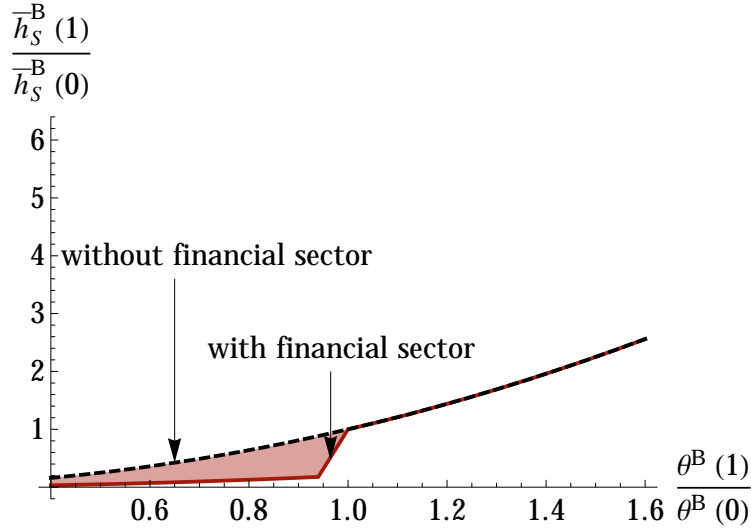


Figure 5: The figure plots growth variability in sector  $B$  under the social planner solution, as measured by the ratio of the Poisson arrival rate of inventions in sector  $B$  in the two states  $(\frac{\bar{h}_S^B(1)}{\bar{h}_S^B(0)})$ , against variability in technological conditions for innovation, as measured by the ratio of R&D productivity in sector  $B$  in states 1 and 0  $(\frac{\theta^B(1)}{\theta^B(0)})$ .

Note that the skill channel also operates under the social planner solution: The planner directs agents to acquire skill in project selection in a way that maximizes the representative household's utility. For sufficiently low values of  $\theta^B(1)$ , the planner decides against skill acquisition in sector  $B$ , which implies that fewer projects are funded in state 1.

Yet the competition channel does not operate under the social planner solution, implying that the propagation effect for  $\frac{\theta^B(1)}{\theta^B(0)} > 1$  is not present. Improved technological conditions in state 1 do not lead to a reduction in funding and skill acquisition in state 0. As noted in section 3.2, the social planner effectively maximizes growth date by date, implying that technological conditions in state 1 do not have an impact on the optimal allocation in state 0. In contrast, competition under laissez-faire generates an interdependence between states in the sense that financial firms' funding in one state has an impact on other financial firms' skill acquisition and funding in the other state. The social planner solution thus provides a

clean benchmark that allows isolating the intertemporal effects induced by the contractual restrictions present under laissez-faire.

### Financial Propagation and Aggregate Risk

Figure 6 illustrates how the laissez-faire results change when the economy features aggregate risk. The plot on the left-hand side in Figure 6 introduces state dependence in the local drift of aggregate consumption. State 1 is specified as the high-growth state and state 0 as the low-growth state. The plot on the right-hand side instead considers the case where aggregate consumption growth is exposed to local uncertainty. Table I contains details of the parameterization.

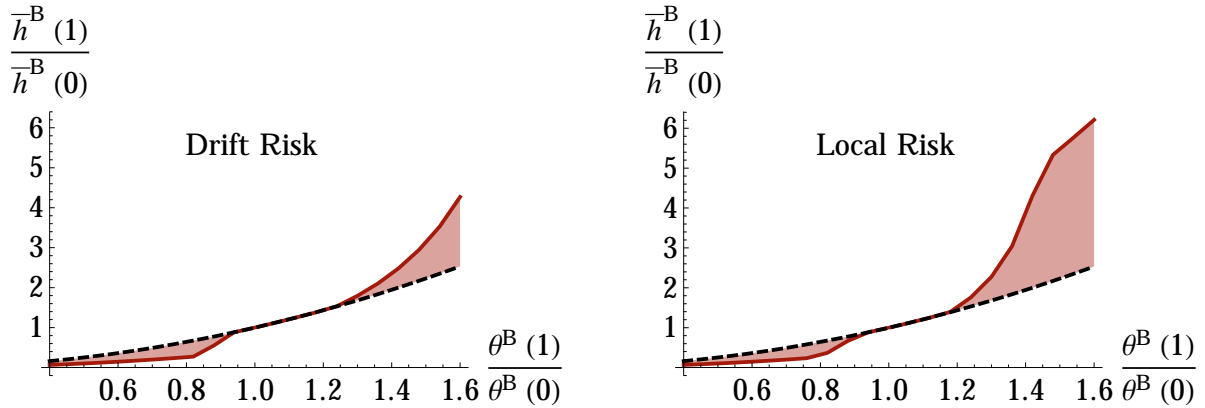


Figure 6: The figure plots growth variability in sector  $B$  in the laissez-faire equilibrium, as measured by the ratio of the Poisson arrival rate of inventions in sector  $B$  in the two states ( $\frac{\bar{h}^B(1)}{\bar{h}^B(0)}$ ), against variability in technological conditions for innovation, as measured by the ratio of R&D productivity in sector  $B$  in states 1 and 0 ( $\frac{\theta^B(1)}{\theta^B(0)}$ ). The dashed lines represent the case without financial sector; the solid lines refer to the case with financial sector. In the plot on the left-hand side, the local drift of aggregate consumption varies across states ( $\mu_Y(1) - \mu_Y(0) = 0.006$ ). On the right-hand side, aggregate consumption growth is exposed to Brownian uncertainty  $\sigma_\vartheta = 0.02$ .

First consider the plot on the left-hand side in Figure 6, that is, the setup where the local drift of aggregate consumption varies across states. For  $\frac{\theta^B(1)}{\theta^B(0)} > 1$ , technological conditions in sector  $B$  are procyclical relative to aggregate growth. The graph reveals that the amplification of growth fluctuations is muted relative to the previous case where aggregate consumption growth was constant (see Figure 3). Procyclicality in combination with agents' risk aversion implies that the value financial firms can extract through project evaluation and funding reacts less sensitively to improvements in technological conditions. Thus financial firm's funding activity is also less sensitive to improved technological conditions in state 1. Since financial firms' reaction is muted, so is financial propagation. For  $\frac{\theta^B(1)}{\theta^B(0)} < 1$ , that is, the case of countercyclical variation in technological conditions in sector  $B$ , the opposite obtains: The value of financial firms' profits in sector  $B$  reacts more sensitively to a change in the fluctuations of technological conditions. Financial propagation is thus stronger when underlying technological conditions are countercyclical.

Finally, consider the plot on the right-hand side in Figure 6. The introduction of local uncertainty hardly changes the propagation effects relative to the case without aggregate risk (see Figure 3). What causes the differential impact of the two sources of risk considered in Figure 6? Note that variation in  $\theta^B$  and variation in the local drift of aggregate consumption are both induced by the Markov state  $Z$ . In the considered two-state setup, changes in the variability of  $\theta^B$  thus directly alter the sector's exposure to aggregate risk. Since financial firms' profits in sector  $B$  are tied to technological conditions, different values of  $\frac{\theta^B(1)}{\theta^B(0)}$  also imply different discount rates for financial firms' profits. As financial firms' entry and funding decisions are tied to these discount rates, so are growth fluctuations in the sector.

The analysis reveals direct links between financial propagation in the laissez-faire equilibrium and asset pricing. Note that the social planner solution is *not* altered by the introduction of aggregate risk. This result is again due to the fact that the social planner effectively maximizes aggregate consumption growth date by date. This maximization is independent of the two exogenous sources of aggregate risk considered in the illustrations. In other words, these sources of aggregate risk influence financial propagation through the

competition channel; their impact on allocations is tied to the contractual frictions implied by monopolistic competition in the financial sector and intermediate good markets.

## 5 Conclusion

In this paper, I develop a dynamic general equilibrium model consistent with a role for the financial sector in propagation of cycles of innovation. The model features two propagation channels: the "skill channel" and the "competition channel." The skill channel operates through financial firms' acquisition of sector-specific knowledge. Financial firms specialize in sectors with good technological conditions and thereby accelerate fluctuations. The competition channel originates in an interaction between competition in the financial sector and patent races in product markets. Financial firms' temporary competitive advantages in access to new ventures imply market segmentation. Financial firms maximize the surplus generated by the client firms they can currently attract, taking competing financial firms' future funding decisions as given. Relative to the Pareto optimum, the competition channel generates overinvestment in sectors with temporarily improved technological conditions; excessively high growth in these sectors comes at the cost of lower growth in the economy as a whole. Excessive booms are shown to amplify busts, and vice versa. The model links financial propagation to time variation in the cross section of asset prices. High incumbent asset prices encourage financial firms' investment in proprietary information, and increase the scale of project evaluation and funding. Propagation effects are muted in sectors that are more exposed to aggregate growth cycles, because risk exposures reduce the sensitivity of financial firms' profits to improved technological conditions.

With regard to future work, the model might be useful for two lines of research. First, it might provide a stepping stone for explorations of the impact of changes to the industrial organization of the financial sector, in particular, how adjustments to the incentives financial firms face influence the cyclicity of growth. Second, the model might prove useful for studies of cross-sectional properties of asset prices in production economies with Schumpeterian

competition and long-run risk.

**Table I**  
**Two-Sector Analysis: Baseline Parameters Used**  
**in the Numerical Analysis**

Parameter Descriptions		Notation	Parameter Values
1.	Rates of transition between states	$\lambda(0), \lambda(1)$	0.200
2.	Fraction of good entrepreneurs	$\phi$	0.100
3.	Productivity of R&D	$\theta$	0.013
4.	Size of innovations	$\kappa$	2.000
5.	Diminishing returns to R&D	$\eta$	0.500
6.	Labor-flow cost of R&D	$c_R$	1.000
7.	Labor-flow cost of project evaluation(*)	$c_E$	0.100
8.	Labor-flow cost of proprietary knowledge acqu. (*)	$c_P$	0.470
9.	White-collar labor supply	$\bar{L}_W$	1.000
10.	Blue-collar labor supply	$\bar{L}_B$	5.000
11.	Local drift of $\vartheta(t)$	$\omega$	0.010
12.	Local risk exposure of $\vartheta(t)$	$\sigma_\vartheta$	0.000
13.	Rate of time preference	$\beta$	0.010
14.	Elasticity of intertemporal substitution	$\psi$	2.000
15.	Coefficient of relative risk aversion	$\gamma$	28.000

Parameters marked with (\*) only apply to the case "with financial sector."

Extensions  
Aggregate Risk

Parameter Descriptions		Notation	Parameter Values	
			Z=0	Z=1
a.	Local drift of $\vartheta(t)$	$\omega$	0.007	0.013
b.	Local risk exposure of $\vartheta(t)$	$\sigma_\vartheta$	0.020	

**Table II**  
**Two-Sector Analysis: Results for the Laissez-faire**  
**Equilibrium**

Baseline Parameterization			
	Variable Descriptions	Notation	Values
1.	Local drift of aggregate consumption growth	$\mu_Y$	0.015
2.	Risk-free rate	$r_f$	0.017
3.	Financial firms' profits (scaled)	$\frac{\pi_F}{Y}$	0.020
4.	White-collar wage rate (scaled)	$\frac{w_W}{Y}$	0.337
5.	Blue-collar wage rate (scaled)	$\frac{w_B}{Y}$	0.100

Extensions

a. Drift Risk

	Variable Descriptions	Notation	Values	
			Z=0	Z=1
1.	Local drift of aggregate consumption growth	$\mu_Y$	0.012	0.018
2.	Risk-free rate	$r_f$	0.016	0.018
3.	Financial firms' profits (scaled)	$\frac{\pi_F^A}{Y}$	0.020	0.020
4.	White-collar wage rate (scaled)	$\frac{w_W}{Y}$	0.326	0.328
5.	Blue-collar wage rate (scaled)	$\frac{w_B}{Y}$	0.100	0.100
6.	Incumbent risk adjustment	$rp_I$	0.001	0.001

b. Local Risk

	Variable Descriptions	Notation	Values
1.	Local drift of aggregate consumption growth	$\mu_Y$	0.015
2.	Risk-free rate	$r_f$	0.009
3.	Financial firms' profits (scaled)	$\frac{\pi_F^A}{Y}$	0.016
4.	White-collar wage rate (scaled)	$\frac{w_W}{Y}$	0.259
5.	Blue-collar wage rate (scaled)	$\frac{w_B}{Y}$	0.100
6.	Incumbent risk adjustment	$rp_I$	0.011

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# A Appendix

## Aggregate Consumption Dynamics

Given that manufacturing one unit of the intermediate good requires one unit of blue-collar labor input, one may substitute  $x(v, t|q) = L_M(v, t)$  in the final good's production technology to obtain

$$Y(t) = \vartheta(t) \exp\left(\int_0^1 \log[q(v, t)] dv\right) \exp\left(\int_0^1 \log[L_M(v, t|q)] dv\right). \quad (51)$$

In addition, we obtain from intermediate good producers' profit maximization

$$L_M(v, t) = \frac{\bar{L}_B}{\kappa \int_0^1 \frac{1}{\kappa(s)} ds}, \quad (52)$$

which yields

$$Y(t) = \vartheta(t) Q(t) \Psi, \quad (53)$$

where I define

$$Q(t) \equiv \exp\left(\int_0^1 \log[q(v, t)] dv\right), \quad (54)$$

$$\Psi \equiv \frac{\bar{L}_B}{\int_0^1 \frac{1}{\kappa(s)} ds} \exp\left(-\int_0^1 \log[\kappa] dv\right). \quad (55)$$

Innovation risk in various varieties is idiosyncratic. The aggregate quality level  $Q(t)$  thus grows at the regime dependent rate

$$\frac{dQ(t)}{Q(t)} = \mu_Q(Z_t) = \int_0^1 \log[\kappa] \theta(v, Z_t) (\phi n(v, t))^{1-\eta} dv. \quad (56)$$

Thus the final goods output flow follows the stochastic differential equation

$$\frac{dY_t}{Y_t} = \mu_Y(Z_t) dt + \sigma_\vartheta dB_t, \quad (57)$$

where

$$\mu_Y(Z_t) \equiv \varpi(Z_t) + \mu_Q(Z_t). \quad (58)$$

## Household Value Function

Conjecture that the value function takes the form

$$J(C_t, Z_t) = F(Z_t) \frac{C_t^\alpha}{\alpha}, \quad (59)$$

Since the representative household's consumption flow  $C_t$  is equal to the final good output flow  $Y_t$ , Itô's lemma yields

$$\frac{dJ_t}{J_t} = \left( \alpha \mu_Y(Z_t) + \frac{1}{2} \alpha (\alpha - 1) \sigma_\vartheta^2 \right) dt + \alpha \sigma_\vartheta dB_t + \frac{F(Z_t) - F(Z_{t-})}{F(Z_t)}. \quad (60)$$

Moreover,  $dJ_t = \mu_J(t) dt + dM_t$ , where  $M$  is a local martingale and

$$\frac{\mu_J(t)}{J_t} = \alpha \mu_Y(Z_t) + \frac{1}{2} \alpha (\alpha - 1) \sigma_\vartheta^2 + \lambda(Z_{t-}) \varphi_{Z(t-)}[Z_{t-}] \frac{F(1) - F(0)}{F(Z_t)}. \quad (61)$$

Note that the normalized aggregator under the conjecture  $J(Y, Z) = F(Z) \frac{Y^\alpha}{\alpha}$  takes the form

$$f(Y, J) = \frac{\beta \alpha}{\rho} J \left( F(Z)^{-\frac{\rho}{\alpha}} - 1 \right). \quad (62)$$

Thus  $F(Z_t)$  solves the equation

$$0 = \frac{\beta \alpha}{\rho} J_t \left( F(Z)^{-\frac{\rho}{\alpha}} - 1 \right) + \alpha \mu_Y(Z_t) + \frac{1}{2} \alpha (\alpha - 1) \sigma_\vartheta^2 + \lambda(Z_{t-}) \varphi_{Z(t-)}[Z_{t-}] \frac{F(1) - F(0)}{F(Z_t)} \Big) J_t \quad (63)$$

Dividing by  $J$  yields

$$0 = \frac{\beta \alpha}{\rho} \left( F(Z)^{-\frac{\rho}{\alpha}} - 1 \right) + \alpha \mu_Y(Z_t) + \frac{1}{2} \alpha (\alpha - 1) \sigma_\vartheta^2 + \lambda(Z_{t-}) \varphi_{Z(t-)}[Z_{t-}] \frac{F(1) - F(0)}{F(Z_t)}. \quad (64)$$

### State Prices Process $\xi_t$ and Risk-free Rate $r_f$

Household maximization implies that a state-pricing process  $\xi_t$  may be written as follows (Duffie and Epstein 1992):

$$\xi_t \equiv \exp \left[ \int_0^t f_J(C_\tau, J_\tau) d\tau \right] f_C(C_t, J_t). \quad (65)$$

Under the conjecture for the value function we obtain

$$\xi_t = Y_t^{\alpha-1} F_2(Z_t) \exp \left\{ \int_0^t F_1(Z_t) d\tau \right\}, \quad (66)$$

where I define

$$F_1(Z_t) = \frac{\beta(\alpha - \rho)}{\rho} F(Z_t)^{-\frac{\rho}{\alpha}} - \frac{\beta \alpha}{\rho}, \quad (67)$$

$$F_2(Z_t) = \beta F(Z_t)^{1-\frac{\rho}{\alpha}}, \quad (68)$$

and where I use the aggregate market-clearing relation  $C_t = Y_t$ . By Itô's lemma, we may write

$$\begin{aligned} \frac{d\xi_t}{\xi_t} &= \left( F_1(Z_t) + (\alpha - 1)\mu_Y(Z_t) + \frac{1}{2}(\alpha - 1)(\alpha - 2)\sigma_\vartheta^2 \right) dt \\ &\quad + (\alpha - 1)\sigma_\vartheta dB_t + \frac{F_2(Z_t) - F_2(Z_{t-})}{F_2(Z_t)}. \end{aligned} \quad (69)$$

Moreover,  $d\xi_t = \mu_\xi(t)dt + dM_{\xi t}$ , where  $M_{\xi t}$  is a local martingale and

$$\begin{aligned} \frac{\mu_\xi(t)}{\xi_t} &= F_1(Z_t) + (\alpha - 1)\mu_Y(Z_t) + \frac{1}{2}(\alpha - 1)(\alpha - 2)\sigma_\vartheta^2 \\ &\quad + \lambda(Z_{t-})\varphi_{Z(t-)}[Z_{t-}] \frac{F_2(1) - F_2(0)}{F_2(Z_t)}. \end{aligned} \quad (70)$$

Thus the short rate  $r_f(Z_t) = -\frac{\mu_\xi(t)}{\xi_t}$  is given by

$$\begin{aligned} r_f(Z_t) &= \frac{\beta\alpha}{\rho} - \frac{\beta(\alpha - \rho)}{\rho} F(Z_t)^{-\frac{\rho}{\alpha}} - (\alpha - 1)\mu_Y(Z_t) - \frac{1}{2}(\alpha - 1)(\alpha - 2)\sigma_\vartheta^2 \\ &\quad - \lambda(Z_{t-})\varphi_{Z(t-)}[Z_{t-}] \frac{F(1)^{1-\frac{\rho}{\alpha}} - F(0)^{1-\frac{\rho}{\alpha}}}{F(Z_t)^{1-\frac{\rho}{\alpha}}}. \end{aligned} \quad (71)$$

## Incumbent Firm Value

The net present value of an intermediate good producer that owns a patent for a blueprint of quality  $q_I$  in variety  $v$  at time  $t$  is given by

$$\begin{aligned} V_I(v, Y_t, Z_t | q_I) &= E_t \left[ \int_t^\infty \frac{\xi_\tau}{\xi_t} \pi(v, t | q_I) d\tau \right], \\ &= E_t \left[ \int_t^\infty \frac{\xi_\tau}{\xi_t} Y(t) \left(1 - \frac{1}{\kappa}\right) \mathbf{1}_{\{q_I = q(v, \tau)\}} d\tau \right] \end{aligned} \quad (72)$$

where  $q(v, \tau)$  denotes the highest available quality level in variety  $v$  at time  $\tau$ , and  $\mathbf{1}_{\{q_I = q(v, \tau)\}}$  is an indicator variable that is one when  $q_I = q(v, \tau)$  and zero otherwise. We obtain the Hamilton Jacobi Bellman equation

$$\begin{aligned} 0 &= \xi_t Y(t) \left(1 - \frac{1}{\kappa}\right) \mathbf{1}_{\{q_I = q(v, t)\}} + \xi_t V_I(v, Y_t, Z_t) \frac{E_t [d(\mathbf{1}_{\{q_I = q(v, t)\}})]}{dt} \\ &\quad + \frac{E_t [d(\xi_t V_I(v, Y_t, Z_t))]}{dt}, \end{aligned} \quad (73)$$

where I use the fact that conditional on R&D effort by entrants, the Poisson rate of replacement arrivals are independent of the states  $Z_t$  and  $Y_t$ , that is,

$$\begin{aligned} &\frac{E_t [d(\xi_t V_I(v, Y_t, Z_t) \mathbf{1}_{\{q_I = q(v, t)\}})]}{dt} \\ &= \xi_t V_I(v, Y_t, Z_t) \frac{E_t [d(\mathbf{1}_{\{q_I = q(v, t)\}})]}{dt} + \frac{E_t [d(\xi_t V_I(v, Y_t, Z_t))]}{dt}. \end{aligned} \quad (74)$$

In addition, conditional on  $\mathbf{1}_{\{q_I=q(v,t)\}} = 1$ , we have  $\frac{E_t[d(\mathbf{1}_{\{q_I=q(v,t)\}})]}{dt} = -\bar{h}(v, Z_t)$ . Further, define

$$U_t = \xi_t Y_t p_I(v, Z_t) = Y_t^\alpha F_2(Z_t) p_I(v, Z_t) \exp\left\{\int_0^t F_1(Z_\tau) d\tau\right\}. \quad (75)$$

By Itô's lemma, we obtain

$$\begin{aligned} \frac{dU_t}{U_t} &= \left(F_1(Z_t) + \alpha\mu_Y(Z_t) + \frac{1}{2}\alpha(\alpha-1)\sigma_\vartheta^2\right) dt \\ &\quad + \alpha\sigma_\vartheta dB_t + \frac{F_2(Z_t) p_I(v, Z_t) - F_2(Z_{t-}) p_I(v, Z_{t-})}{F_2(Z_t) p_I(v, Z_t)} \end{aligned} \quad (76)$$

and  $dU_t = \mu_U(t) dt + dM_{U_t}$ , where  $M_U$  is a local martingale and

$$\begin{aligned} \frac{\mu_U(t)}{U_t} &= F_1(Z_t) + \alpha\mu_Y(Z_t) + \frac{1}{2}\alpha(\alpha-1)\sigma_\vartheta^2 \\ &\quad + \lambda(Z_{t-}) \varphi_{Z(t-)}[Z_{t-}] \frac{F(1)^{1-\frac{\rho}{\alpha}} p_I(v, 1) - F(0)^{1-\frac{\rho}{\alpha}} p_I(v, 0)}{F(Z_t)^{1-\frac{\rho}{\alpha}} p_I(v, Z_t)}. \end{aligned} \quad (77)$$

Under the conjecture for the value function

$$V_I(v, Y_t, Z_t) = Y(t) p_I(v, Z_t) \quad (78)$$

and assuming the firm is the leading-edge patent owner at time  $t$  ( $\mathbf{1}_{\{q_I=q(v,t)\}} = 1$ ), we thus may write the HJB as follows:

$$\begin{aligned} 0 &= \xi_t Y(t) \left(1 - \frac{1}{\kappa}\right) - \bar{h}(v, Z_t) \xi_t Y(t) p_I(v, Z_t) \\ &\quad + \left(\frac{\beta(\alpha-\rho)}{\rho} F(Z_t)^{-\frac{\rho}{\alpha}} - \frac{\beta\alpha}{\rho} + \alpha\mu_Y(Z_t) + \frac{1}{2}\alpha(\alpha-1)\sigma_\vartheta^2\right. \\ &\quad \left.+ \lambda(Z_{t-}) \varphi_{Z(t-)}[Z_{t-}] \frac{F(1)^{1-\frac{\rho}{\alpha}} p_I(v, 1) - F(0)^{1-\frac{\rho}{\alpha}} p_I(v, 0)}{F(Z_t)^{1-\frac{\rho}{\alpha}} p_I(v, Z_t)}\right) \cdot \xi_t Y_t p_I(v, Z_t). \end{aligned} \quad (79)$$

Dividing by  $\xi_t Y_t$  yields

$$\begin{aligned} 0 &= 1 - \frac{1}{\kappa} + \left(\frac{\beta(\alpha-\rho)}{\rho} F(Z_t)^{-\frac{\rho}{\alpha}} - \frac{\beta\alpha}{\rho} + \alpha\mu_Y(Z_t) + \frac{1}{2}\alpha(\alpha-1)\sigma_\vartheta^2 - \bar{h}(v, Z_t)\right. \\ &\quad \left.+ \lambda(Z_{t-}) \varphi_{Z(t-)}[Z_{t-}] \frac{F(1)^{1-\frac{\rho}{\alpha}} p_I(v, 1) - F(0)^{1-\frac{\rho}{\alpha}} p_I(v, 0)}{F(Z_t)^{1-\frac{\rho}{\alpha}} p_I(v, Z_t)}\right) \cdot p_I(v, Z_t). \end{aligned} \quad (80)$$

Using the definition for  $r_f$  and rearranging terms, we finally obtain

$$p_I(v, Z_t) = \frac{1 - \frac{1}{\kappa}}{r_f(Z_t) + rp_I(v, Z_t) + \bar{h}(v, Z_t) - \lambda(Z_t) \left( \frac{p_I(v, Z_t + \varphi_{Z_t}[Z_t])}{p_I(v, Z_t)} - 1 \right) - \mu_Y(Z_t)}, \quad (81)$$

where  $rp_I(v, Z_t)$  is defined as

$$rp_I(v, Z_t) = (1 - \alpha) \sigma_{\vartheta}^2 + \left( \left( \frac{F(Z_t + \varphi_{Z_t}[Z_t])}{F(Z_t)} \right)^{1 - \frac{\rho}{\alpha}} - 1 \right) \lambda(Z_t) \left( 1 - \frac{p_I(v, Z_t + \varphi_{Z_t}[Z_t])}{p_I(v, Z_t)} \right). \quad (82)$$

## Social Planner Solution

The social planner maximizes

$$J_t = E_t \left[ \int_t^{\infty} f(C_{\tau}, J_{\tau}) d\tau \right], \quad (83)$$

subject to the resource constraints

$$\int_0^1 L_{MS}(v, Z_t) dv \leq \bar{L}_B \quad (84)$$

$$\int_0^1 (L_{FS}(v, Z_t) + L_{RS}(v, Z_t)) dv \leq \bar{L}_W \quad (85)$$

and the final goods production technology (equations (5) and (6)), the innovations possibility frontier (equations (4) and (7)) and the financial sector's technology to evaluate projects.

Since the resource constraints (84) and (85) are static, and since innovation-related activity on the one hand (R&D and project evaluation) and intermediate goods production on the other, draw on separate resources (white-collar labor and blue-collar labor, respectively), the social planner's decisions are separable across time and separable between innovation and intermediate goods production. By monotonicity of utility in final good consumption, the planner optimally maximizes the level of consumption by allocating blue-collar labor to produce various intermediate goods, and, separately, maximizes the growth rate of the aggregate quality index  $Q(t)$  date by date by allocating white-collar labor among R&D projects and financial firms. The dynamic properties of the factor  $\vartheta(t)$  are by assumption exogenous. It follows that under the social planner solution, aggregate consumption follows the stochastic differential equation

$$\frac{dY_t}{Y_t} = \mu_{YS}(Z_t) dt + \sigma_{\vartheta} dB_t, \quad (86)$$

where the local drift is given by

$$\mu_{Y_S}(Z_t) = \varpi(Z_t) + \max_{\substack{\iota_S(v,t) \in \{0,1\}, \\ n_{US}(v,t) \geq 0, \\ n_{FS}(v,t) \geq 0}} \int_0^1 \log[\kappa] \cdot \theta(v, Z_t) \cdot (\phi n_S(v, Z_t))^{1-\eta} dv, \quad (87)$$

subject to

$$n_S(v, t) = n_{US}(v, t) + n_{FS}(v, t) \quad (88)$$

$$0 = (1 - \iota_S(v, t)) n_{FS}(v, t) \quad (89)$$

$$L_{FS}(v, Z_t) = c_E \cdot n_{FS}(v, Z_t) + c_P \cdot \iota_S(v, Z_t) \quad (90)$$

$$L_{RS}(v, Z_t) = \phi \cdot c_R \cdot n_{FS}(v, Z_t) + c_R \cdot n_{US}(v, Z_t) \quad (91)$$

$$\bar{L}_W = \int_0^1 (L_{FS}(v, Z_t) + L_{RS}(v, Z_t)) dv. \quad (92)$$

The level of the final good consumption flow is given by

$$Y(t) = \vartheta(t) \cdot \exp\left(\int_0^1 \log[q(v, t)] dv\right) \cdot \bar{L}_B \quad (93)$$

### Social Planner Solution: Symmetric Intermediate Good Varieties

To maximize the local drift of the aggregate quality index  $Q(t)$ , the planner solves the problem

$$\max_{\substack{\iota_S(v,t) \in \{0,1\}, \\ n_{US}(v,t) \geq 0, \\ n_{FS}(v,t) \geq 0}} \int_0^1 \log[\kappa] \cdot \theta(v, Z_t) \cdot (\phi n_S(v, Z_t))^{1-\eta} dv, \quad (94)$$

which, given symmetric parameters  $\kappa$  and  $\theta(v, Z_t)$ , simplifies to

$$\max_{\substack{\iota_S(v,t) \in \{0,1\}, \\ n_{US}(v,t) \geq 0, \\ n_{FS}(v,t) \geq 0}} \int_0^1 n_S(v, Z_t)^{1-\eta} dv \quad (95)$$

subject to

$$n_S(v) = n_{FS}(v) + n_{US}(v) \quad (96)$$

$$0 = n_{FS}(v) (1 - \iota_S(v)) \quad (97)$$

$$\bar{L}_W = \int_0^1 (n_{US}(v) \cdot c_R + n_{FS}(v) (c_E + \phi c_R) + \iota_S(v) c_P) dv \quad (98)$$

Consider a solution where in a mass  $0 \leq M_S \leq 1$  of intermediate good varieties, financial firms evaluate projects and do not operate in the remaining varieties  $(1 - M_S)$ . An amount

$L_{FS}$  of white-collar labor is allocated to all varieties with financial firm entry and an amount  $L_{US}$  to each of the remaining varieties. The fact that all varieties are optimally used for R&D follows from the R&D technology's INADA type properties ( $0 < \eta < 1$ ), and from the symmetry in the structural parameters of all intermediate good varieties. The problem may be rewritten as follows:

$$\max_{\substack{0 \leq M_S \leq 1, \\ L_{US} \geq 0, \\ L_{FS} \geq 0}} \left\{ M_S \left( \frac{L_{FS} - c_P}{c_E + \phi c_R} \right)^{1-\eta} + (1 - M_S) \left( \frac{L_{US}}{c_R} \right)^{1-\eta} \right\} \quad (99)$$

subject to

$$M_S L_{FS} + (1 - M_S) L_{US} = \bar{L}_W.$$

At the optimum of an interior solution ( $0 < M_S^* < 1$ ,  $L_{US}^* > 0$ ,  $L_{FS}^* > 0$ ), the following marginal conditions hold

$$M_S^* \frac{1-\eta}{c_E + \phi c_R} \left( \frac{L_{FS}^* - c_P}{c_E + \phi c_R} \right)^{-\eta} - l^* M_S^* = 0 \quad (100)$$

$$(1 - M_S^*) \frac{1-\eta}{c_R} \left( \frac{L_{US}^*}{c_R} \right)^{-\eta} - l^* (1 - M_S^*) = 0 \quad (101)$$

$$\left( \frac{L_{FS}^* - c_P}{c_E + \phi c_R} \right)^{1-\eta} - \left( \frac{L_{US}^*}{c_R} \right)^{1-\eta} - l^* (L_{FS}^* - L_{US}^*) = 0. \quad (102)$$

This yields

$$L_{FS}^* = c_P \frac{\frac{1}{\eta} - \left( \frac{c_E}{c_R} + \phi \right)^{\frac{1}{\eta}-1}}{1 - \left( \frac{c_E}{c_R} + \phi \right)^{\frac{1}{\eta}-1}} \quad (103)$$

$$L_{US}^* = c_P \frac{\left( \frac{1}{\eta} - 1 \right) \left( \frac{c_E}{c_R} + \phi \right)^{\frac{1}{\eta}-1}}{1 - \left( \frac{c_E}{c_R} + \phi \right)^{\frac{1}{\eta}-1}}. \quad (104)$$

Then the constraint

$$M_S^* L_{FS}^* + (1 - M_S^*) L_{US}^* = \bar{L}_W \quad (105)$$

yields

$$M_S^* = \frac{\bar{L}_W}{c_P} \eta - \frac{\left( \frac{c_E}{c_R} + \phi \right)^{\frac{1}{\eta}-1}}{1 - \left( \frac{c_E}{c_R} + \phi \right)^{\frac{1}{\eta}-1}} (1 - \eta). \quad (106)$$

The corner solution, where  $M_S^* = 0$ , obtains when

$$\frac{\bar{L}_W}{c_P} \leq \frac{1-\eta}{\eta} \frac{\left( \frac{c_E}{c_R} + \phi \right)^{\frac{1}{\eta}-1}}{1 - \left( \frac{c_E}{c_R} + \phi \right)^{\frac{1}{\eta}-1}}. \quad (107)$$

The corner solution, where  $M_S^* = 1$ , obtains when

$$\frac{\bar{L}_W}{c_P} \geq \frac{\frac{1}{\eta} - \left(\frac{c_E}{c_R} + \phi\right)^{\frac{1}{\eta}-1}}{1 - \left(\frac{c_E}{c_R} + \phi\right)^{\frac{1}{\eta}-1}}. \quad (108)$$

Define the corresponding thresholds for  $c_P$ ,

$$\bar{c}_{PS} \equiv \bar{L}_W \frac{\eta}{1-\eta} \left( \left( \frac{c_E}{c_R} + \phi \right)^{1-\frac{1}{\eta}} - 1 \right) \quad (109)$$

$$\underline{c}_{PS} \equiv \bar{L}_W \frac{1 - \left(\frac{c_E}{c_R} + \phi\right)^{\frac{1}{\eta}-1}}{\frac{1}{\eta} - \left(\frac{c_E}{c_R} + \phi\right)^{\frac{1}{\eta}-1}}. \quad (110)$$

The solution may be characterized as follows:

$$M_S^* = \begin{cases} 0 & \text{for } c_P > \bar{c}_{PS} \\ \frac{\bar{L}_W}{c_P} \eta - \frac{\left(\frac{c_E}{c_R} + \phi\right)^{\frac{1}{\eta}-1}}{1 - \left(\frac{c_E}{c_R} + \phi\right)^{\frac{1}{\eta}-1}} (1 - \eta) & \text{for } \underline{c}_{PS} \leq c_P \leq \bar{c}_{PS} \\ 1 & \text{for } c_P < \underline{c}_{PS} \end{cases} \quad (111)$$

$$L_{US}^* = \begin{cases} \bar{L}_W & \text{for } c_P > \bar{c}_{PS} \\ c_P \frac{\left(\frac{1}{\eta}-1\right)\left(\frac{c_E}{c_R} + \phi\right)^{\frac{1}{\eta}-1}}{1 - \left(\frac{c_E}{c_R} + \phi\right)^{\frac{1}{\eta}-1}} & \text{for } \underline{c}_{PS} \leq c_P \leq \bar{c}_{PS} \\ \text{n.d.} & \text{for } c_P < \underline{c}_{PS} \end{cases} \quad (112)$$

$$L_{FS}^* = \begin{cases} \text{n.d.} & \text{for } c_P > \bar{c}_{PS} \\ c_P \frac{\frac{1}{\eta} - \left(\frac{c_E}{c_R} + \phi\right)^{\frac{1}{\eta}-1}}{1 - \left(\frac{c_E}{c_R} + \phi\right)^{\frac{1}{\eta}-1}} & \text{for } \underline{c}_{PS} \leq c_P \leq \bar{c}_{PS} \\ \bar{L}_W & \text{for } c_P < \underline{c}_{PS}. \end{cases} \quad (113)$$

$$l^* = \begin{cases} \frac{1-\eta}{c_R} \left(\frac{\bar{L}_W}{c_R}\right)^{-\eta} & \text{for } c_P > \bar{c}_{PS} \\ \frac{1-\eta}{c_R} \left( \frac{c_P \left(\frac{1}{\eta}-1\right)\left(\frac{c_E}{c_R} + \phi\right)^{\frac{1}{\eta}-1}}{1 - \left(\frac{c_E}{c_R} + \phi\right)^{\frac{1}{\eta}-1}} \right)^{-\eta} & \text{for } \underline{c}_{PS} \leq c_P \leq \bar{c}_{PS} \\ \frac{1-\eta}{c_E + \phi c_R} \left(\frac{\bar{L}_W - c_P}{c_E + \phi c_R}\right)^{-\eta} & \text{for } c_P < \underline{c}_{PS} \end{cases} \quad (114)$$

In addition, we obtain

$$n_S(v, t) = \begin{cases} n_{US}^* & \text{given } \iota_S(v, t) = 1 \\ n_{FS}^* & \text{given } \iota_S(v, t) = 0, \end{cases} \quad (115)$$

where I define

$$n_{US}^* \equiv \begin{cases} \frac{\bar{L}_W}{c_R} & \text{given } c_P > \bar{c}_{PS} \\ \frac{c_P}{c_R} \frac{(\frac{1}{\eta}-1)(\frac{c_E}{c_R}+\phi)^{\frac{1}{\eta}-1}}{1-(\frac{c_E}{c_R}+\phi)^{\frac{1}{\eta}-1}} & \text{given } \underline{c}_{PS} \leq c_P \leq \bar{c}_{PS} \\ \text{n.d.} & \text{given } c_P < \underline{c}_{PS} \end{cases} \quad (116)$$

$$n_{FS}^* \equiv \begin{cases} \text{n.d.} & \text{given } c_P > \bar{c}_{PS} \\ \frac{c_P \frac{\frac{1}{\eta} - (\frac{c_E}{c_R} + \phi)^{\frac{1}{\eta} - 1}}{1 - (\frac{c_E}{c_R} + \phi)^{\frac{1}{\eta} - 1}} - c_P}{c_E + \phi c_R} & \text{given } \underline{c}_{PS} \leq c_P \leq \bar{c}_{PS} \\ \frac{\bar{L}_W - c_P}{c_E + \phi c_R} & \text{given } c_P < \underline{c}_{PS}. \end{cases} \quad (117)$$

Finally, we have

$$\begin{aligned} \bar{h}_S(v, Z_t) &= \theta(v, Z_t) ((\phi(n_{US}^* + n_{FS}^*))^{1-\eta}) \\ M_S &\left( \frac{L_{FS} - c_P}{c_E + \phi c_R} \right)^{1-\eta} + (1 - M_S) \left( \frac{L_{US}}{c_R} \right)^{1-\eta} \end{aligned}$$

### Social Planner Solution: A Two-Sector Example

Since the small sector has zero measure, the solution for the large sector is identical to the symmetric case. Define the marginal increase in the growth rate of the aggregate quality index  $Q(t)$  for an additional unit of white-collar labor

$$l(Z_t) = l^* \cdot \log[\kappa^A] \cdot \theta^A(Z_t) \cdot \phi^{1-\eta}. \quad (118)$$

Given this shadow price, the planner solves the following maximization problem for the zero measure sector  $B$ :

$$\begin{aligned} \max_{\substack{\iota_S^B(Z) \in \{0,1\}, \\ n_{US}^B(Z) \geq 0, \\ n_{FS}^B(Z) \geq 0}} \{ &\log[\kappa^B] \cdot \theta^B(Z_t) \cdot (\phi n_S^B(Z_t))^{1-\eta} \\ &- (c_R \cdot n_{US}^B(Z_t) + n_{FS}^B(Z_t) (c_E + \phi c_R) + \iota_S^B(Z_t) c_P) l(Z_t) \} \end{aligned} \quad (119)$$

subject to

$$(1 - \iota_S^B(Z_t)) \cdot n_{FS}^B(Z_t) = 0. \quad (120)$$

Then the marginal conditions yield

$$n_S^B(Z_t) = \begin{cases} n_{FS}^B(Z_t) = \frac{1}{\phi} \left( \frac{(1-\eta) \cdot \log[\kappa^B] \cdot \theta^B(Z_t) \cdot \phi}{(c_E + \phi c_R) \cdot l(Z_t)} \right)^{\frac{1}{\eta}} & \text{for } \iota_S^B(Z_t) = 1 \\ n_{US}^B(Z_t) = \frac{1}{\phi} \left( \frac{(1-\eta) \cdot \log[\kappa^B] \cdot \theta^B(Z_t) \cdot \phi}{c_R \cdot l(Z_t)} \right)^{\frac{1}{\eta}} & \text{for } \iota_S^B(Z_t) = 0. \end{cases} \quad (121)$$

The entry condition for the small sector is given by

$$\begin{aligned} & \log [\kappa^B] \cdot \theta^B (Z_t) \cdot (\phi n_{US}^B (Z_t))^{1-\eta} - c_R \cdot n_{US}^B (Z_t) l (Z_t) \\ & > \log [\kappa^B] \cdot \theta^B (Z_t) \cdot (\phi n_{FS}^B (Z_t))^{1-\eta} - (n_{FS}^B (Z_t) (c_E + \phi c_R) + c_P) l (Z_t), \end{aligned} \quad (122)$$

which may be rewritten as follows:

$$c_P < \left( \frac{(1-\eta) \cdot \log [\kappa^B] \cdot \theta^B (Z_t) \cdot \phi}{c_R \cdot l (Z_t)} \right)^{\frac{1}{\eta}} \frac{c_R}{\phi} \frac{\eta}{1-\eta} \left( \left( \frac{c_E}{c_R} + \phi \right)^{1-\frac{1}{\eta}} - 1 \right). \quad (123)$$

### Laissez-faire Equilibrium: Symmetric Intermediate Good Varieties

Due to the assumption of perfect symmetry, incumbent prices across varieties are identical at any point in time. For notational simplicity, I drop variety indices in the following. Non-negative expected profits condition for financial firm entry is given by

$$(n_F^* (t) \phi)^{1-\eta} \theta (Z_t) V_I (t) - w_W (n_F^* (t) (c_E + \phi c_R) + c_P) \geq 0, \quad (124)$$

where  $n_F^* (t)$  denotes the optimal number of evaluated projects *conditional* on financial firm entry, that is,

$$n_F^* (t) = \frac{1}{\phi} \left( \frac{(1-\eta) \theta (Z_t) V_I (t)}{\left( \frac{c_E}{\phi} + c_R \right) w_W (t)} \right)^{\frac{1}{\eta}}. \quad (125)$$

For  $1-\eta > \phi + \frac{c_E}{c_R}$ , we obtain

$$n_U^* (t) = \frac{1}{\phi} \left( \frac{\theta (Z_t) V_I (t)}{c_R w_W (t)} \right)^{\frac{1}{\eta}} < n_F^* (t) \quad (126)$$

Given optimal entry, a financial firm maximizes its expect profit flow by attracting all projects in its variety at that time. The non-negative expected profits condition may then be rewritten as follows:

$$\frac{V_I (t)}{w_W (t)} \geq \frac{1}{\theta (Z_t)} \left( \frac{c_P}{\eta} \right)^\eta \left( \frac{\frac{c_E}{\phi} + c_R}{1-\eta} \right)^{1-\eta}. \quad (127)$$

Let  $M (t)$  denote the measure of product varieties where a financial firm enters. The labor market-clearing condition is given by

$$M (t) \cdot (n_F^* (t) \cdot (c_E + \phi \cdot c_R) + c_P) + (1 - M (t)) \cdot (n_U^* (t) \cdot c_R) = \bar{L}_W \quad (128)$$

with

$$0 \leq M (t) \leq 1.$$

The assumption of perfect symmetry implies that in the case of  $0 < M(t) < 1$ , financial firm entry in each variety  $v$  is equally likely at any date. Independent random draws imply that financial firm entry does not generate any deviations from perfect symmetry. Substituting  $n_F^*(t)$  and  $n_U^*(t)$  into the labor market clearing and solving for the incumbent value to wage ratio yields:

$$\frac{V_I(t)}{w_W(t)} = \frac{1}{\theta(Z_t)} \left( \frac{\bar{L}_W - M(t)c_P}{M(t)(1-\eta)^{\frac{1}{\eta}} \left(\frac{c_E}{\phi} + c_R\right)^{1-\frac{1}{\eta}} + (1-M(t)) \left(\frac{c_R}{\phi}\right)^{1-\frac{1}{\eta}}} \right)^\eta. \quad (129)$$

Dividing the parameter space into three distinct regions is helpful: a region where financial firms enter in all varieties ("region A"), a region where entry occurs in some varieties ("region B"), and a region where entry does not occur in any variety ("region C"). I discuss these regions below.

**Region A: Financial Firm Entry in All Varieties** ( $c_P < \underline{c}_P$ ) In region A, financial firms enter in all varieties, that is,  $M = 1$ . Combining the labor market-clearing condition with the financial firms' non-negative profits condition yields

$$\frac{V_I(t)}{w_W(t)} = \frac{1}{\theta(Z_t)} \left( \frac{\bar{L}_W - c_P}{(1-\eta)^{\frac{1}{\eta}} \left(\frac{c_E}{\phi} + c_R\right)^{1-\frac{1}{\eta}}} \right)^\eta \quad (130)$$

$$> \frac{1}{\theta(Z_t)} \left(\frac{c_P}{\eta}\right)^\eta \left(\frac{c_E}{\phi} + c_R\right)^{1-\eta}, \quad (131)$$

or, solving for  $c_P$ ,

$$c_P < \underline{c}_P \equiv \eta \bar{L}_W, \quad (132)$$

implying

$$n(t) = n_F(t) = \frac{\bar{L}_W - c_P}{c_E + \phi c_R}. \quad (133)$$

**Region C: No Financial Firm Entry** ( $c_P > \bar{c}_P$ ) In region C, financial firms do not enter any variety, that is,  $M(t) = 0$ . Analogously to the above, we obtain the condition

$$\frac{V_I(t)}{w_W(t)} = \frac{1}{\theta(Z_t)} \left( \frac{\bar{L}_W}{\left(\frac{c_R}{\phi}\right)^{1-\frac{1}{\eta}}} \right)^\eta < \frac{1}{\theta(Z_t)} \left(\frac{c_P}{\eta}\right)^\eta \left(\frac{c_E}{\phi} + c_R\right)^{1-\eta}, \quad (134)$$

or, solving for  $c_P$ ,

$$c_P > \bar{c}_P \equiv \eta \bar{L}_W \left(\frac{1-\eta}{\frac{c_E}{\phi} + \phi}\right)^{\frac{1-\eta}{\eta}}. \quad (135)$$

Note that the assumption  $1 - \eta > \phi + \frac{c_E}{c_R}$  ensures that  $c_P > \underline{c}_P$ . We obtain

$$n(t) = n_U(t) = \frac{\bar{L}_W}{c_R}. \quad (136)$$

**Region B: Financial Firm Entry in Some Varieties** ( $\underline{c}_P < c_P < \bar{c}_P$ ) In region  $B$ , financial firms enter in some varieties, but not in all, that is,  $0 < M < 1$ . In every variety, financial firms with access to proprietary knowledge are just indifferent between operation and non-operation. Financiers obtain no compensation for providing access to proprietary knowledge since a financial firm's ex ante value from entry is zero. Similar to the other regions, one obtains the relation

$$\begin{aligned} \frac{V_I(t)}{w_W(t)} &= \frac{1}{\theta(Z_t)} \left( \frac{\bar{L}_W - M(t) c_P}{M(t) (1 - \eta)^{\frac{1}{\eta}} \left( \frac{c_E}{\phi} + c_R \right)^{1 - \frac{1}{\eta}} + (1 - M(t)) \left( \frac{c_R}{\phi} \right)^{1 - \frac{1}{\eta}}} \right)^\eta \\ &= \frac{1}{\theta(Z_t)} \left( \frac{c_P}{\eta} \right)^\eta \left( \frac{\frac{c_E}{\phi} + c_R}{(1 - \eta)} \right)^{1 - \eta}. \end{aligned} \quad (137)$$

Solving for  $M$  yields

$$M(t) = \frac{\eta \frac{\bar{L}_W}{c_P} - \left( \frac{\frac{c_E}{\phi} + c_R}{1 - \eta} \right)^{\frac{1 - \eta}{\eta}}}{1 - \left( \frac{\frac{c_E}{\phi} + c_R}{1 - \eta} \right)^{\frac{1 - \eta}{\eta}}}. \quad (138)$$

Thus, the following solutions obtain:

$$\iota(v, t) = \begin{cases} 1 & \text{for all } v, & \text{given } c_P < \underline{c}_P \\ 1 & \text{for a mass } M(t) \text{ of varieties } v, & \text{given } \underline{c}_P \leq c_P \leq \bar{c}_P \\ 0 & \text{for all } v, & \text{given } c_P > \bar{c}_P \end{cases} \quad (139)$$

$$M(t) = \begin{cases} 1 & \text{given } c_P < \underline{c}_P \\ \frac{\eta \frac{\bar{L}_W}{c_P} - \left( \frac{\frac{c_E}{\phi} + c_R}{1 - \eta} \right)^{\frac{1 - \eta}{\eta}}}{1 - \left( \frac{\frac{c_E}{\phi} + c_R}{1 - \eta} \right)^{\frac{1 - \eta}{\eta}}} & \text{given } \underline{c}_P \leq c_P \leq \bar{c}_P \\ 0 & \text{given } c_P > \bar{c}_P \end{cases} \quad (140)$$

$$n(v, t) = \begin{cases} n_F^*(t) & \text{given } \iota(v, t) = 1 \\ n_U^*(t) & \text{given } \iota(v, t) = 0 \end{cases} \quad (141)$$

and

$$n_F^*(t) = \begin{cases} \frac{\bar{L}_W - c_P}{c_E + \phi c_R} & \text{given } c_P < \underline{c}_P \\ \frac{1-\eta}{\eta} \frac{c_P}{c_E + \phi c_R} & \text{given } \underline{c}_P \leq c_P \leq \bar{c}_P \\ n.d. & \text{given } c_P > \bar{c}_P \end{cases} \quad (142)$$

$$n_U^*(t) = \begin{cases} n.d. & \text{given } c_P < \underline{c}_P \\ \frac{c_P}{\eta c_R} \left( \frac{c_E + \phi}{c_R} \right)^{\frac{1-\eta}{\eta}} & \text{given } \underline{c}_P \leq c_P \leq \bar{c}_P \\ \frac{\bar{L}_W}{c_R} & \text{given } c_P > \bar{c}_P. \end{cases} \quad (143)$$

The incumbent market value is determined given the (mean) hazard rate of replacement

$$\bar{h}(t) = \begin{cases} \theta(Z_t) \left( \phi \frac{\bar{L}_W - c_P}{c_E + \phi c_R} \right)^{1-\eta} & \text{given } c_P < \underline{c}_P \\ M(t) (\phi n_F^*(t))^{1-\eta} + (1 - M(t)) (\phi n_U^*(t))^{1-\eta} & \text{given } \underline{c}_P \leq c_P \leq \bar{c}_P \\ \theta(Z_t) \left( \phi \frac{\bar{L}_W}{c_R} \right)^{1-\eta} & \text{given } c_P > \bar{c}_P. \end{cases} \quad (144)$$

The white-collar wage rate is given by

$$w_W(t) = \begin{cases} V_I(t) \theta(Z_t) \left( \frac{\bar{L}_W - c_P}{(1-\eta)^{\frac{1}{\eta}} (\frac{c_E}{\phi} + c_R)^{1-\frac{1}{\eta}}} \right)^{-\eta} & \text{for } c_P < \underline{c}_P \\ V_I(t) \theta(Z_t) \left( \frac{c_P}{\eta} \right)^{-\eta} \left( \frac{\frac{c_E}{\phi} + c_R}{(1-\eta)} \right)^{\eta-1} & \text{for } \underline{c}_P \leq c_P \leq \bar{c}_P \\ V_I(t) \theta(Z_t) \left( \frac{\bar{L}_W}{(\frac{c_R}{\phi})^{1-\frac{1}{\eta}}} \right)^{-\eta} & \text{for } c_P > \bar{c}_P. \end{cases} \quad (145)$$

### Laissez-faire Equilibrium: A Two Sector Example

Define the profit flow of a financial firm in the zero measure sector  $B$ :

$$\pi_F^B(Y_t, Z_t, M^B(0), M^B(1)) \equiv \max_{n_F^*} \{ (\phi n_F^*(v, t))^{1-\eta} \theta^B(Z_t) V_I^B(Y_t, Z_t, M^B(0), M^B(1)) - w_W(t) (c_P + n_F^*(v, t) \cdot (c_E + \phi c_R)) \}, \quad (146)$$

where

$$n_F^B(t) = \frac{1}{\phi} \left( \frac{\phi \cdot \theta(v, Z_t) \cdot V_I(v, t)}{c_R \cdot w_W(t)} \right)^{\frac{1}{\eta}} \left( \frac{1-\eta}{\phi + \frac{c_E}{c_R}} \right)^{\frac{1}{\eta}} \quad (147)$$

$$n_U^B(t) = \frac{1}{\phi} \left( \frac{\phi \cdot \theta(v, Z_t) \cdot V_I(v, t)}{c_R \cdot w_W(t)} \right)^{\frac{1}{\eta}}. \quad (148)$$

Define the mean hazard rate of replacement

$$\bar{h}^B(Z_t) = M^B(Z_t)\theta^B(Z_t)(\phi \cdot n_F^B(t))^{1-\eta} + (1 - M^B(Z_t))\theta^B(Z_t)(\phi \cdot n_U^B(t))^{1-\eta} \quad (149)$$

Privately optimal financial firm entry yields the relations

$$\begin{aligned} M^B(Z_t) &= 1 \text{ for } \pi_F^B(Y_t, Z_t, M^B(Z_t) = 1, M^B(Z_t + \varphi_{Z_t}[Z_t])) > 0 \\ M^B(Z_t) &= 0 \text{ for } \pi_F^B(Y_t, Z_t, M^B(Z_t) = 0, M^B(Z_t + \varphi_{Z_t}[Z_t])) < 0 \\ \pi_F^B(Y_t, Z_t, M^B(Z_t), M^B(Z_t + \varphi_{Z_t}[Z_t])) &= 0 \text{ for } M^B(Z_t) \in (0, 1). \end{aligned} \quad (150)$$

Finally the market price of an incumbent in sector  $B$  is given by

$$V_I^B(Y_t, Z_t) = Y_t \cdot p_I^B(Z_t), \quad (151)$$

where  $p_I^B(Z_t)$  solves for  $Z_t = 0, 1$ :

$$p_I^B(Z_t) = \frac{1 - \frac{1}{\kappa}}{r_f(Z_t) + rp_I^B(Z_t) + \bar{h}^B(Z_t) - \lambda(Z_t) \left( \frac{p_I^B(Z_t + \varphi_{Z_t}[Z_t])}{p_I^B(Z_t)} - 1 \right) - \mu_Y(Z_t)}, \quad (152)$$

and where  $rp_I^B(Z_t)$  is defined as

$$\begin{aligned} rp_I^B(Z_t) &= (1 - \alpha) \sigma_{\vartheta}^2 \\ &+ \left( \left( \frac{F(Z_t + \varphi_{Z_t}[Z_t])}{F(Z_t)} \right)^{1-\frac{\rho}{\alpha}} - 1 \right) \lambda(Z_t) \left( 1 - \frac{p_I^B(Z_t + \varphi_{Z_t}[Z_t])}{p_I^B(Z_t)} \right). \end{aligned} \quad (153)$$