Price Transparency and Price Discriminating Noisy Rational Expectations Equilibria — Web Appendix

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Mathematical Appendix

A Alternative PD-REEs

A.1 PD-REE Without Noise

Absent supply noise, the monopolist's profit-maximization condition is sufficiently strong that the uninformed agents can invert their quoted price to discern the signal of the informed agents. Supply noise is therefore an essential component of the PD-NREE.

Proposition 1 (Necessity of Supply Noise)**.** *Supply noise in the PD-NREE is necessary to avoid the Grossman-Stiglitz (**?**) paradox. Further, when supply noise is removed, the profit-maximization condition allows invertibility of the uninformed agents' price* p_t^U *to identify* s_t *and thereby preclude price discrimination by the dealer.*

Proof. The price discriminating rational expectations equilibrium simplifies the PD-NREE by fixing u to be 0. I conjecture that the demand of the uninformed agents is linear in the quoted price, $x^U\left(p^U\right) = a + bp^U$. The profit-maximization condition

$$\max_{p^I, p^U} N^I x^I\left(p^I\right) p^I + N^U x^U\left(p^U\right) p^U \text{ s.t. } N^I x^I\left(p^I\right) + N^U x^U\left(p^U\right) = S$$

may be rewritten as

$$\begin{aligned} \max_{p^I, p^U} & N^I p^I \frac{\bar{d} + \beta_s\left(s - \bar{d}\right) - (1+r)\, p^I}{\alpha\sigma_{d|s}^2} + N^U p^U\left(a + bp^U\right) \\ \text{s.t. } & N^I \frac{\bar{d} + \beta_s\left(s - \bar{d}\right) - (1+r)\, p^I}{\alpha\sigma_{d|s}^2} + N^U\left(a + bp^U\right) = S \end{aligned} \tag{A.1}$$

Taking first-order conditions for the Lagrangian associated with Equation A.1 delivers

$$\begin{aligned} \mathcal{L} &= N^I p^I \frac{\bar{d} + \beta_s\left(s - \bar{d}\right) - (1+r)\, p^I}{\alpha\sigma_{d|s}^2} + N^U p^U\left(a + bp^U\right) \\ &- \lambda\left(S - N^I \frac{\bar{d} + \beta_s\left(s - \bar{d}\right) - (1+r)\, p^I}{\alpha\sigma_{d|s}^2} - N^U\left(a + bp^U\right)\right) \\ \frac{\partial \mathcal{L}}{\partial p^I} = 0 \quad &: \quad \lambda = \frac{\bar{d} + \beta_s\left(s - \bar{d}\right)}{1+r} - 2p^I \\ \frac{\partial \mathcal{L}}{\partial p^U} = 0 \quad &: \quad \lambda = -\frac{a}{b} - 2p^U \\ \frac{\partial \mathcal{L}}{\partial \lambda} = 0 \quad &: \quad S = N^I \frac{\bar{d} + \beta_s\left(s - \bar{d}\right) - (1+r)\, p^I}{\alpha\sigma_{d|s}^2} + N^U\left(a + bp^U\right) \end{aligned}$$

which in turn gives expressions relating the profit-maximizing prices, p^I and p^U

$$
\begin{aligned}
p^I &= p^U + \frac{1}{2}\left[\frac{\bar{d}+\beta_s\left(s-\bar{d}\right)}{1+r} + \frac{a}{b}\right] \\
p^U &= \frac{1}{N^U b - N^I \frac{(1+r)}{\alpha\sigma^2_{d|s}}}\left[\frac{1}{2}N^I\frac{\bar{d}+\beta_s\left(s-\bar{d}\right)-\frac{a}{b}(1+r)}{\alpha\sigma^2_{d|s}} + N^U a - S\right]
\end{aligned}
$$

as well as an equation for s in terms of variables known to all participants in the economy

$$
s = \bar{d} + \frac{1}{\beta_s}\left(\frac{2\alpha\sigma^2_{d|s}}{N^I}\left[p^U\left(N^U b - N^I\frac{(1+r)}{\alpha\sigma^2_{d|s}}\right) - N^U a + S\right] + \frac{a}{b}(1+r) - \bar{d}\right) \tag{A.2}
$$

As in the PD-NREE, the demand schedule of the informed agents reveals s to the dealer. Moreover, as $\sigma^2_{d|s}$ is independent of s, the signal of the informed is fully revealed by the price quoted to the uninformed agents via Equation A.2. Therefore in equilibrium, the uninformed agents become fully informed by inverting p^U. The informed and uninformed agents then possess the same information and submit identical demands. Solving for the implied demand function of the uninformed agents obtains

$$
\begin{aligned}
a + bp^U &= \frac{\bar{d}+\beta_s\left(s-\bar{d}\right)-(1+r)p^I}{\alpha\sigma^2_{d|s}} = \frac{\bar{d}+\beta_s\left(s-\bar{d}\right)-(1+r)\left[p^U+\frac{1}{2}\left[\frac{\bar{d}+\beta_s\left(s-\bar{d}\right)}{1+r}+\frac{a}{b}\right]\right]}{\alpha\sigma^2_{d|s}} \\
b &= -\frac{1+r}{\alpha\sigma^2_{d|s}} \\
a &= \frac{\frac{1}{2}\left[\bar{d}+\beta_s\left(s-\bar{d}\right)\right]-\frac{1}{2}\frac{a}{b}(1+r)}{\alpha\sigma^2_{d|s}} = \frac{\frac{1}{2}\left[\bar{d}+\beta_s\left(s-\bar{d}\right)\right]}{\alpha\sigma^2_{d|s}} + \frac{1}{2}a = \frac{\bar{d}+\beta_s\left(s-\bar{d}\right)}{\alpha\sigma^2_{d|s}}
\end{aligned}
$$

So the conjecture of linearity is verified as a solution to the PD-REE. Substitution of a and b into the expression for p^I shows that the equilibrium can only hold if $p^U = p^I = p$. Market clearing requires that $x_U(p) = x_I(p) = \frac{S}{N}$. As prices are equal across agent types, the price discriminating rational expectations equilibrium without noise reduces to the non-price discriminating rational expectations equilibrium without noise and the **?** paradox arises. □

A.2 Alternative Formulation of the PD-NREE

An alternative price discriminating noisy rational expectations equilibrium concept allows the dealer to retain inventory at the end of each period. As before, the dealer knows the value of u and inverts the value of s from the orders of the informed and is hence exposed only to variance from signal noise. This formulation necessitates abandoning risk-neutrality because the expected payoff is

$$E[d|s,u] = \bar{d} + \beta_s \left(s - \bar{d}\right)$$

which is higher than the price that can be charged either agent on account of their risk-aversion and additional exposure to u. To preclude the dealer from keeping his entire inventory, the dealer's risk-neutrality must be replaced with concave preferences. The profit maximizing condition then becomes a utility maximization whereby asset sales to the informed and uninformed agents constitute a risk-free payoff to the dealer. Using that the residual uncertainty is σ_ϵ^2 delivers a revised third equilibrium condition of

$$\max_{\{p^i\}} -\exp\left(-\alpha\left[N^I x^I\left(p^I\right)p^I + N^U x^U\left(p^U\right)p^U - DE[d|s] - \frac{1}{2}\alpha\sigma_\epsilon^2 D^2\right]\right)$$

where "excess demand" D is given by

$$D = N^I x^I\left(p^I\right) + N^U x^U\left(p^U\right) - (S+u)$$

$$\begin{aligned} p^I : 0 &= x^I\left(p^I\right)p^I - \frac{\partial x^I\left(p^I\right)}{\partial p^I}\left[E[d|s] - \alpha\sigma_\epsilon^2 D\right] \\ p^U : 0 &= x^U\left(p^U\right)p^U - \frac{\partial x^U\left(p^U\right)}{\partial p^U}\left[E[d|s] - \alpha\sigma_\epsilon^2 D\right] \end{aligned}$$

These first-order conditions are significantly more complicated than those obtained in the perturbation I consider in the paper. The constants that would arise from this (more) nonlinear system will be less comparable with those obtained from the non-price discriminating NREE and are less likely to yield similar testable implications. This variant warrants further consideration for its additional consequences for volume (unretained assets) and transactions costs (the premium required by the dealer over the current price to take on or sell additional securities).

If the dealer were forced to charge the same price to both classes of agents in the price opaque PD-NREE, the profit-maximization condition becomes superfluous and the PD-NREE reduces to the NREE. The NREE framework requires only optimization and market-clearing to uniquely pin down prices and demand schedules for both types of agents so the support for the third condition reduces to a single point. This alternative equilibrium allows equal prices across agent types to no longer make trivial the monopolistic dealer's optimization problem. Comparing between this equilibrium and the PD-NREE thus offers the ability to distinguish the effects of pure market power from those of market power interacted with price discrimination.

B Revenue-Maximization and Issue Supply

The monopolist knows the parameters of the economy, N_I, N_U, etc., but does not yet observe u. Supply noise is generated by the monopolist being unaware of $\bar{d}$, but seeing a signal $\hat{d} \sim N\left(\bar{d}, \sigma_\zeta^2\right)$.

Claim 1. *The revenue-maximizing choice of issue supply, S^*, is in $(0, \infty)$ and is increasing in average asset payoff $\bar{d}$. Furthermore, supply is normally distributed around mean $S^*\left(\bar{d}\right)$.*

Proof. Denote $\pi^*|\hat{d}$ as the monopolist's expected revenue conditional on the signal on $\bar{d}$. □

$$\begin{aligned}
\pi^*|\hat{d} &= \max_S E\left[N_I p^I(S)\, x^I(S) + N_U p^U(S)\, x^U(S)\right] \\
&= \max_S E\left[N_I p^I(S) \frac{\bar{d} + \left(s - \bar{d}\right)\beta_s - (1+r)\, p^I(S)}{\alpha\sigma^2_{d|s}}\right. \\
&\quad \left. + N_U p^U(S) \frac{\frac{\sigma^2}{B}\left(p^U(S) - A(S)\right) + \left[\bar{d} - (1+r)\, p^U(S)\right]\left(\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right)}{\alpha\sigma^2\left(\sigma_\epsilon^2 + C^2\sigma_u^2\right)} | \hat{d}\right] \\
&= \max_S E\left[N_I \frac{\left[\bar{d} + \left(s - \bar{d}\right)\beta_s\right] p^I(S) - (1+r)\left(p^I(S)\right)^2}{\alpha\sigma^2_{d|s}}\right. \\
&\quad \left. + N_U \frac{\left[\frac{\sigma^2}{B}\left(p^U(S) - A(S)\right) + \bar{d}\left(\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right)\right] p^U(S) - (1+r)\left(p^U(S)\right)^2\left(\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right)}{\alpha\sigma^2\left(\sigma_\epsilon^2 + C^2\sigma_u^2\right)} | \hat{d}\right] \\
&= \max_S N_I \frac{\hat{d}D(S) - (1+r)\, D^2(S)}{\alpha\sigma^2_{d|s}} + N_U \frac{\hat{d}A(S) - (1+r)\, A^2(S)}{\alpha\sigma^2_{d|p^U}} + E\left[K|\hat{d}\right]
\end{aligned}$$

where $\frac{dE\left[K|\hat{d}\right]}{dS} = 0$. Taking first-order conditions obtains

$$\begin{aligned}
0 &= \frac{N_I}{\sigma^2_{d|s}}\left[\hat{d} - 2(1+r)\, D(S)\right] D'(S) + \frac{N_U}{\sigma^2_{d|p^U}}\left[\hat{d} - 2(1+r)\, A(S)\right] A'(S) \\
&= N_I \frac{1}{\sigma^2_{d|s}}\left[1 + \frac{2S\alpha\sigma^2_{d|s}}{\left[\hat{d} - 2(1+r)\, A(S)\right] N_I \frac{C^2\sigma_u^2}{\sigma^2 + \sigma_\epsilon^2}}\right] D'(S) + N_U \frac{1}{\sigma^2_{d|p^U}} A'(S)
\end{aligned}$$

Rearranging delivers

$$S = \frac{\hat{d}}{2\alpha}\left(\frac{\sigma^2_{d|s}\frac{\sigma^2 + \sigma_\epsilon^2}{C^2\sigma_u^2}}{N_I + N_U\frac{\sigma^2_{d|s}}{\sigma^2_{d|p^U}}\left[N_I - \frac{\sigma^2 + \sigma_\epsilon^2}{\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2}\left[N_I + N_U\frac{\sigma^2_{d|s}}{\sigma^2_{d|p^U}}\right]\right]^{-1}} + \frac{\sigma^2_{d|s}\frac{\sigma^2 + \sigma_\epsilon^2}{C^2\sigma_u^2}}{\frac{\sigma^2 + \sigma_\epsilon^2}{-\left(\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2\right)}\left[N_I + N_U\frac{\sigma^2_{d|s}}{\sigma^2_{d|p^U}}\right]}\right)^{-1}$$

If the relative noise condition holds, $\frac{-C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2} > 0$, $A'(S) < 0$ and $D'(S) < 0$. Then all quantities are positive, as is S. Because $\sigma_{d|s}^2 \frac{\sigma^2+\sigma_\epsilon^2}{C^2\sigma_u^2} > 0$ and the denominators are finite, S will be finite. That ${}^{dS^*}/_{d\bar{d}} > 0$ is immediate upon inspection.

If the assumption on the source of supply noise is too restrictive, the profit-maximization above can be coupled with alternative putative noise sources, e.g., the activities of noise traders. Imposing $\hat{d} \sim N\left(\bar{d}, \sigma_\zeta^2\right)$ obtains that unconditionally, S^* is normally distributed with mean $S|\bar{d}$. In the notation of the rest of the model

$$\begin{aligned} S &= \frac{\bar{d}}{2\alpha}\left(\frac{\sigma_{d|s}^2 \frac{\sigma^2+\sigma_\epsilon^2}{C^2\sigma_u^2}}{N_I + N_U \frac{\sigma_{d|s}^2}{\sigma_{d|p^U}^2}\left[N_I - \frac{\sigma^2+\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2}\left[N_I + N_U \frac{\sigma_{d|s}^2}{\sigma_{d|p^U}^2}\right]\right]^{-1}} + \frac{\sigma_{d|s}^2 \frac{\sigma^2+\sigma_\epsilon^2}{C^2\sigma_u^2}}{\frac{\sigma^2+\sigma_\epsilon^2}{-(\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2)}\left[N_I + N_U \frac{\sigma_{d|s}^2}{\sigma_{d|p^U}^2}\right]}\right)^{-1} \\ u &\sim N\left(0, \sigma_u^2\right) \end{aligned}$$

Note that the relation between σ_u^2 and σ_ζ^2 is *ex ante* ambiguous and may impose bounds on σ_u^2 as a function of model primitives.

C Additional Pricing Relations

C.1 Average Prices

Claim 2. *The average price for the informed agent D is less than the unconditional expected discounted next-period payoff, $\frac{\bar{d}}{1+r}$. The average price for the uninformed agent A is weakly greater than this value if and only if $\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2 < 0$.*

Proof. Results follow immediately upon inspection for the price transparent PD-NREE. For the price opaque PD-NREE, subtracting the unconditional expected discounted next-period payoff from the average price for the uninformed agent obtains

$$A - \frac{\bar{d}}{1+r} = \frac{S \frac{\alpha}{1+r} \frac{\sigma^2\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}}{\frac{\frac{1}{2}\sigma^2 N_I}{\sigma^2-(1+r)B[\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2]} - N_I - N_U \frac{\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2)}{(\sigma^2+\sigma_\epsilon^2)(\sigma_\epsilon^2+C^2\sigma_u^2)}}$$

which is positive if and only if

$$\begin{aligned} \sigma^2 - (1+r)B\left[\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right] &> 0 \\ \Leftrightarrow \sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2 &< 0 \end{aligned}$$

Analogously, subtracting the unconditional expected discounted next-period payoff from the average price for the informed agent gives

$$D - \frac{\bar{d}}{1+r} = -\left(A - \frac{\bar{d}}{1+r}\right)\left[\frac{\frac{\sigma^2}{\sigma_\epsilon^2} N_I \left(C^2\sigma_u^2\right) + N_U\left(\frac{\sigma^2\left(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right)}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)}{N_I \frac{\sigma^2}{\sigma_\epsilon^2}\left[\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2\right]}\right]$$

The numerator of the second component is strictly positive. The denominator has the same sign as $\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2$, as does $A - \frac{\bar{d}}{1+r}$. The expression $D - \frac{\bar{d}}{1+r}$ is therefore strictly negative abstracting from the knife-edge case of $\sigma^2 + \sigma_\epsilon^2 = C^2\sigma_u^2$. □

C.2 Price Sensitivity to Information

Claim 3. Price sensitivities among equilibrium concepts are related by

$$B_{Wal} \lesseqgtr B < \tilde{B} \lesseqgtr B_{AVE} < B_{NREE} < F < \tilde{F} = B_{NN-REE}$$

Proof. I compare price sensitivities to the informed agents' signal among several additional equilibrium concepts. I show $B < B_{NREE} < F$ in Claim 2 of the Appendix. Denote the transaction-weighted average price sensitivity as

$$B_{AVE} \equiv \frac{N_I F + N_U B}{N}$$

Then, the average price sensitivity in the PD-NREE relative to the NREE is

$$\begin{aligned}
& B_{AVE} - B_{NREE} \\
= \quad & \frac{N_I\left(B + \frac{1}{2}\frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\right) + N_U B}{N} - B_{NREE} \\
= \quad & (B - B_{NREE}) + \frac{1}{2}\frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\frac{N_I}{N} \\
= \quad & \frac{\left[\frac{1}{2}\frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right] - \left[\frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]}{(1+r)\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]} \\
+ \quad & \frac{\frac{1}{2}\frac{N_I}{N}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]}{(1+r)\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]} \\
= \quad & \frac{-\frac{1}{2}\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]\left[\frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} - \frac{N_I}{N}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]\right] - \frac{N}{N_U}\left[\frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2} - \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]}{(1+r)\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]}
\end{aligned}$$

All terms are negative and $B_{AVE} < B_{REE}$ if the second bracketed expression is positive. This is easily verified, as

$$\frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2}-\frac{N_I}{N}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\left[\frac{N}{N_U}+\frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2}+\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]=\frac{N_I}{N}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\left[\frac{\sigma^2\left(C^2\sigma_u^2\right)}{\sigma_\epsilon^2\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)}\right]>0$$

Comparing average price sensitivities in each equilibrium concept obtains

$$\begin{aligned} B_{AVE}-\tilde{B} &= \\ \frac{1}{1+r}\left[\frac{\left(\frac{\sigma^2}{2\sigma_\epsilon^2}\right)+\frac{N_U}{N_I}\left(\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)}{\left(\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}\right)+\frac{N_U}{N_I}\left(\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)}+\frac{1}{2}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\left(1+\frac{N_U}{N_I}\right)^{-1}-\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2+\tilde{C}^2\sigma_u^2}\right]\end{aligned}$$

which is bounded in

$$\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}-\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}<0<\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}-\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}$$

so the relationship is ambiguous. Increasing N_U/N_I makes the difference more negative. The average price sensitivity to information in the price transparent PD-NREE is

$$\begin{aligned}\tilde{B}_{AVE} &= \frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2+\tilde{C}^2\sigma_u^2}\left[1-\left(1+\frac{N_U}{N_I}\right)^{-1}\right]+\frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\left(1+\frac{N_U}{N_I}\right)^{-1} \\ &= \frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\left[1+\frac{1}{4}\frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2}\left(1+\frac{N_U}{N_I}\right)^{-1}\right]\end{aligned}$$

Taking differences with B_{NREE} obtains

$$\begin{aligned}\tilde{B}_{AVE}-B_{NREE} &= \\ \frac{1}{1+r}\left[\frac{\sigma^2\left(1+\frac{1}{4}\frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2}\left(1+\frac{N_U}{N_I}\right)^{-1}\right)}{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}-\frac{\frac{\sigma^2}{\sigma_\epsilon^2}+\frac{N_U}{N_I}\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}}{\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}+\frac{N_U}{N_I}\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}}\right]\end{aligned}$$

And taking first-order conditions with respect to N_U/N_I gives

$$\frac{N_U}{N_I}=-\frac{2\sqrt{\frac{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}}-\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}}{2\sqrt{\frac{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}}-\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}}$$

which is negative for $\sigma^2<1.5\sigma_\epsilon^2$. If this condition does not hold, comparing $\tilde{B}_{AVE}-B_{NREE}$ is infeasible. If the condition holds, only corner solutions for $\frac{N_U}{N_I}$ exist and $\tilde{B}_{AVE}=B_{NREE}$.

Inspection shows that $\tilde{B}_{AVE}>B_{AVE}$, as $\tilde{B}>B$ and $\tilde{F}>F$ and $\tilde{B}_{AVE}\lesseqgtr F$ depending on N_U/N_I.

Comparing the price sensitivity for the informed agents in the PD-NREE with that in the non-noisy REE

$$
\begin{aligned}
& B_{NN-REE} - F \\
= \quad & \frac{\beta_s}{1+r} - B - \frac{1}{2}\frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} \\
= \quad & \frac{1}{1+r}\left(\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} - \frac{N_I\frac{\sigma^2}{2\sigma_\epsilon^2} + N_U\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}}{N_I\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2} + N_U\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}} - \frac{1}{2}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\right) \\
= \quad & \frac{1}{1+r}\left(\frac{N_U\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}}{N_I\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2} + N_U\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}}\right)\left[\frac{1}{2}\frac{C^2\sigma_u^2-\sigma^2-\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}\right]
\end{aligned}
\tag{C.1}
$$

which is positive if and only if the relative noise condition holds, that is

$$\sigma^2+\sigma_\epsilon^2 - C^2\sigma_u^2 < 0$$

The price sensitivity to information in the non-noisy REE should serve as an upper bound of price sensitivities among equilibria as the uninformed agents become fully informed via prices. That F can be larger than B_{NN-REE} is a symptom of the inefficiency imposed by the monopolistic dealer.

I also compare price sensitivity to information with that in the Walrasian equilibrium concept:

$$B_{Wal} - B = \frac{N_I\beta_s}{(1+r)\left(N_I + N_U\frac{\sigma_{d|s}^2}{\sigma^2}\right)} - \frac{N_I\frac{\sigma^2}{2\sigma_\epsilon^2} + N_U\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}}{(1+r)\left[N_I\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2} + N_U\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]}$$

which is negative if and only if

$$\frac{N_I\frac{\sigma^2}{\sigma_\epsilon^2}}{N_I\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}+N_U} - \frac{N_I\frac{\sigma^2}{\sigma_\epsilon^2} + N_U\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}}{N_I\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}+N_U+N_U\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}} + \frac{N_I\frac{\sigma^2}{2\sigma_\epsilon^2}}{N_I\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}+N_U+N_U\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}} < 0$$

Consider the first two terms. Let $K = N_U\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2} > 0$. Then the first two terms equals

$$\frac{N_I\frac{\sigma^2}{\sigma_\epsilon^2}}{N_I\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}+N_U} - \frac{N_I\frac{\sigma^2}{\sigma_\epsilon^2}+K}{N_I\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}+N_U+K} = \frac{-K\left(N_I+N_U\right)}{\left(N_I\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}+N_U\right)\left(N_I\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}+N_U+K\right)}$$

Putting everything together obtains

$$\frac{1}{2}\left(1+\frac{N_U}{N_I}\right)^{-1}\left(\frac{\sigma^2}{\sigma_\epsilon^2}\right)^2 \quad < \quad \left(\frac{N_U}{N_I}\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2} - \frac{\sigma^2}{2\sigma_\epsilon^2}\right)$$

which is ambiguous, as the left-hand-side is weakly positive and decreasing in $\frac{N_U}{N_I}$ and the right-hand-side positive or negative but increasing in $\frac{N_U}{N_I}$. The price responsiveness to information for the uninformed agents in the noisy price discriminating REE is larger than in the Walrasian equilibrium (WE) for high $\frac{N_U}{N_I}$. This is an unusual result in that the uninformed agents in the WE do not learn from price, but, when they are few in the noisy price discriminating REE, the profit-maximization of the dealer induces them to be even less responsive to the informed agents' signal ($B_{Wal} > B$)!

It is well-established that $B_{Wal} < B_{NREE}$ so I do not repeat the calculation here. For completeness, I compare B_{NREE} to B_{NN-REE}

$$\begin{aligned}
B_{NN-REE} - B_{NREE} &= \frac{\beta_s}{1+r} - \frac{N_I \frac{\sigma^2}{\sigma_\epsilon^2} + N_U \frac{\sigma^2}{\sigma_\epsilon^2 + \frac{1}{4}C^2\sigma_u^2}}{(1+r)\left[N_I \frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2} + N_U \frac{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]} \\
&= \frac{N_U\sigma^2}{(1+r)\left(\sigma_\epsilon^2 + \frac{1}{4}C^2\sigma_u^2\right)\left(\sigma^2+\sigma_\epsilon^2\right)}\left(\frac{\frac{1}{4}C^2\sigma_u^2}{N_I \frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2} + N_U \frac{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}}\right) > 0
\end{aligned}$$

□

Claim 4. *Price responsiveness to supply noise is higher in the price opaque PD-NREE than in the NREE. Price responsiveness to supply noise is higher (lower) in the price transparent PD-NREE for informed (uninformed) agents. In sum*

$$\tilde{B}\tilde{C} < B_{NREE}C_{NREE} < BC < \tilde{F}\tilde{C}$$

Proof. Although agents cannot observe supply noise, we can still evaluate how prices respond to innovations in u in the price discriminating and non-price discriminating noisy REEs. As established in solving for the price opaque PD-NREE, $BC = FG$ and both agent types' prices respond equally to supply innovations. It remains only to be be shown how BC relates to $B_{NREE}C_{NREE}$. Differencing obtains

$$\begin{aligned}
BC - B_{NREE}C_{NREE} &= \left(B - \frac{1}{2}B_{NREE}\right)C \\
&= \left(\frac{N_I \frac{\sigma^2}{2\sigma_\epsilon^2} + N_U \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}}{(1+r)\left[N_I \frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2} + N_U \frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]} - \frac{1}{2}\frac{N_I \frac{\sigma^2}{\sigma_\epsilon^2} + N_U \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}}{(1+r)\left[N_I \frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2} + N_U \frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]}\right)C \\
&= \frac{\frac{1}{2}CN_U \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}}{(1+r)\left[N_I \frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2} + N_U \frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]} > 0
\end{aligned}$$

Comparison with the price transparent PD-NREEs is immediate

$$\begin{aligned}
\tilde{B}C_{NREE} - B_{NREE}C_{NREE} &= \left(\tilde{B} - B_{NREE}\right)C_{NREE} < 0 \\
\tilde{F}C_{NREE} - B_{NREE}C_{NREE} &= \left(\tilde{F} - B_{NREE}\right)C_{NREE} > 0
\end{aligned}$$

It only remains to be shown that $\tilde{F}\tilde{C} > BC$. Taking differences obtains

$$\begin{aligned}
\tilde{F}C_{NREE} - 2BC_{NREE} &= \left(\tilde{F} - 2B\right) C_{NREE} \\
&\geq \frac{1}{1+r} C_{NREE} \left[\frac{\sigma^2}{\sigma^2 + \sigma_\epsilon^2} - \max_{N_U/N_I} \frac{\frac{\sigma^2}{\sigma_\epsilon^2} + 2\frac{N_U}{N_I}\left(\frac{\sigma^2}{\sigma_\epsilon^2 + C^2\sigma_u^2}\right)}{\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2} + \frac{N_U}{N_I}\left(\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)} \right] \\
&= \frac{1}{1+r} C_{NREE} \left[\frac{\sigma^2}{\sigma^2 + \sigma_\epsilon^2} - \max_{N_U/N_I} \left\{ 2\left(\frac{\sigma^2}{\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2}\right), \frac{\sigma^2}{\sigma^2 + \sigma_\epsilon^2} \right\} \right] \\
&= 0
\end{aligned}$$

where the last line follows when the relative noise condition holds. Furthermore, the inequality is strict when the number of agents of both types is positive. □

C.3 General Price Relations

Claim 5. For s large enough, $p^I > p^U$.

Proof. The difference in prices for the informed and uninformed agents in the price opaque price discriminating noisy rational expectations equilibrium is given by

$$\begin{aligned}
p^U - p^I &= (A - D) + (B - F)\left(s - \bar{d}\right) + (FG - BC)\, u \\
&= \frac{1}{2}\frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} \frac{S\alpha\sigma^2\sigma_\epsilon^2}{\frac{1}{2}\sigma^2 N_I - \left(\sigma^2 - (1+r)\,B\left[\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right]\right)\left[N_I + N_U\frac{\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2)}{(\sigma^2+\sigma_\epsilon^2)(\sigma_\epsilon^2+C^2\sigma_u^2)}\right]} \\
&- \frac{1}{2}\frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\left(s - \bar{d}\right)
\end{aligned}$$

In expectation, $E[s] = E[d+\epsilon] = \bar{d}$ and $E\left[p^U - p^I\right] > 0$, as I established that $A > D$ for $\alpha > 0$ in Claim 2. However, the second term reveals that as the signaled deviation of the next-period payoff from its average value increases, the price discrepancy falls. The uninformed agents cannot back out the signal sufficiently to be as responsive to the deviation as the informed agents are, so the informed agents will bid up their own price to take units from the uninformed. The negative average price difference between p^I and p^U will reverse if

$$\begin{aligned}
\frac{1}{2}\frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\left[\frac{S\alpha\sigma^2\sigma_\epsilon^2}{\frac{1}{2}\sigma^2 N_I - \left(\sigma^2 - (1+r)\,B\left[\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right]\right)\left[N_I + N_U\frac{\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2)}{(\sigma^2+\sigma_\epsilon^2)(\sigma_\epsilon^2+C^2\sigma_u^2)}\right]} - \left(s - \bar{d}\right)\right] &< 0 \\
\Leftrightarrow \frac{S\alpha\sigma^2\sigma_\epsilon^2}{\frac{1}{2}\sigma^2 N_I - \left(\sigma^2 - (1+r)\,B\left[\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right]\right)\left[N_I + N_U\frac{\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2)}{(\sigma^2+\sigma_\epsilon^2)(\sigma_\epsilon^2+C^2\sigma_u^2)}\right]} &< \left(s - \bar{d}\right)
\end{aligned}$$

The difference in constants $A - D$ on the left-hand-side is positive and not a function of s. Therefore, for large enough values of the signal, the price $p^I > p^U$, as accords with the intuition that the informed know the asset has a high next-period payoff, whereas the uninformed attribute part of their quoted price to a negative supply shock rather than to a high signal. Analogous logic holds for the price transparent PD-NREE. $\square$