Price Transparency and Price Discriminating Noisy Rational Expectations Equilibria

Brian Weller*

December 9, 2011

Abstract

I introduce price opacity and market power into the noisy rational expectations equilibrium (NREE) of Grossman and Stiglitz (1980). I characterize the resulting price discriminating noisy rational expectations equilibria (PD-NREE) in the cases of complete price opacity and price transparency and derive analytic expressions for price levels, sensitivity to information, and informativeness. Price transparency increases price responsiveness to information and informativeness but results in increased average prices for less informed agents in the presence of price discrimination. I conclude by mapping the PD-NREE model to the corporate bond market prior to the phase-in of a transaction price dissemination system (TRACE) and discuss efficiency and distributional implications of the transition.

*University of Chicago Booth School of Business and Department of Economics. Please contact bweller@chicagobooth.edu with comments or suggestions. The most up-to-date version and web appendix are available online at http://home.uchicago.edu/~bweller/. Thanks to seminar participants at the Commodities Futures Trading Commission and the University of Chicago for their comments and suggestions. I am especially grateful to John Cochrane and Toby Moskowitz for their support and to Timothy Dore, Filipe Lacerda, Teresa Lwin, and Marina Niessner for their helpful suggestions throughout the writing process.

1 Introduction

I develop a price discriminating noisy rational expectations equilibrium (PD-NREE) by incorporating varying price opacity and a monopolistic intermediary into the noisy rational expectations equilibrium (NREE) of Grossman and Stiglitz (1980). The PD-NREE maintains the linear pricing in signal and supply innovations of the standard NREE while generating price dispersion and differences in price informativeness between uninformed and informed agents in the economy.

The central modeling innovation in the PD-NREE is the decoupling of prices between agents of different informedness on the risky asset's next-period payoff while preserving incomplete revelation of the informed agents' signal, even in a setting with transparent prices. To do this I introduce a monopolistic intermediary who identifies trader types from their demand schedules and price discriminates by offering different, privately observed prices. The monopolist's profit-maximization problem links prices among agent types, enabling uninformed agents to infer the informed agents' signal with noise. The resulting equilibrium obfuscates the signal of the informed agents from the perspective of the uninformed agents, thereby reducing the precision of the uninformed agents' estimate for the next-period payoff conditional on his price. The PD-NREE's price discrimination also delivers higher (lower) average prices and lower (higher) price sensitivity to information for uninformed (informed) agents than in the NREE. Lower average prices and uncertainty about the asset's next-period payoff typically entail a greater share of asset supply accruing to the informed agents, suggesting a novel distributional dimension associated with price transparency. Increasing price transparency by making prices public maintains the qualitative features of the price opaque PD-NREE but increases price informativeness and, counterintuitively, *ex ante* average price spreads between agent types.

The PD-NREE framework offers a useful description for understanding opaque, over-the-counter markets. The PD-NREE concept captures the price opacity, issue concentration and informational asymmetries extant in the corporate bond market prior to the 2002-2005 implementation of price dissemination in the Trade Reporting and Compliance Engine (TRACE). Likewise, concentration and price opacity in the swap market make the PD-NREE a prime candidate for analyzing informational efficiency ramifications of the new Dodd-Frank provisions for swap price reporting. Although an increase in price transparency unequivocally enhances informational efficiency of prices, uninformed agents may pay higher prices in the presence of market power under a price transparent regime.

The paper is structured as follows. Section 2 overviews the theoretical NREE and price transparency literatures as well as empirical studies pertaining to TRACE. Section 3 develops the price discriminating noisy rational expectations equilibrium concept and compares it with the standard noisy rational expectations equilibrium. Section 4 maps the model to the corporate bond market. Section 5 offers potential caveats and extensions. Section 6 concludes.

2 Background and Related Literature

2.1 Theoretical Background

The price discriminating noisy rational expectations equilibrium (PD-NREE) theory draws heavily on the noisy rational expectations equilibrium variant of Grossman and Stiglitz (1980). The PD-NREE adopts the Grossman and Stiglitz NREE as a parsimonious baseline for perturbations to price transparency and market structure among trader counterparties. Additionally, the PD-NREE's pricing functions are identical in form to those of the NREE, enabling straightforward comparison between the equilibrium concepts. Kyle (1989) derives similar linear pricing functions when linking informational efficiency and prices to counterparties' market power in an alternative REE setup. Kyle (1989) characterizes informational efficiency under imperfect competition in a non-noisy REE, but the symmetric linear equilibria he derives are difficult to modify to incorporate price opacity or take to the data. This work fills a gap in the theoretical literature by introducing a testable framework connecting price transparency with price discrimination and informational efficiency.

The price opaque and price transparent PD-NREEs differ solely on the dimension of price transparency, allowing for a variety of comparative statics for price transparency's effects on informational efficiency and the distribution of the risky asset across agent types. Madhavan (1991) builds on the model of Kyle (1989) and demonstrates that transparency only improves informational efficiency and liquidity if the market is sufficiently thick and will decrease both otherwise. Flood, Huisman, Koedijk and Mahieu (1999) shows that opaque markets facilitate faster price discovery because high search costs increase the return to information gathering. In the PD-NREE, I take as given a fixed level of information gathering and abstract away from market depth considerations; I show in the absence of these concerns that price transparency increases sensitivity to information and informational efficiency of prices regardless of market structure. Even from an informational efficiency standpoint, it is *ex ante* uncertain whether the thinly traded corporate bond market was a good candidate for the additional price transparency effected by TRACE.

In regulating the market, efficiency considerations are supplemented by the question of the incidence of costs and benefits. Pagano and Röell (1996) and Biais, Glosten and Spatt (2005) show transparency reduces average costs for uninformed traders. I refine their works by distinguishing between cases in which transparency does (NREE) and does not (price transparent PD-NREE) reduce average costs for uninformed agents.Bloomfield and O'Hara (1999, 2000) demonstrate in a lab setting and theoretically that transparency enhances informational efficiency but benefits market makers at the expense of informed traders by widening bid-ask spreads. I find an analogous result in that increasing price transparency while leaving market structure unaffected has heterogenous effects on different classes of market participants and widens price spreads between them. Without a more complete welfare analysis, the effects of transparency are ambiguous for all agent types in the economy.

I abstract from the growing corpus on competitive search in over-the-counter markets (e.g., Duffie, Gârleanu and Pedersen (2007) and Guerrieri, Shimer and Wright (2010) for recent treatments) and assume search costs are sufficiently high that retail investors contact at most one dealer. The assumption is borne out in that bond issues were highly concentrated among dealers in the 1990s and that brokers had little incentive to search for better prices for low-volume clients.[1] A richer model would link search intensity with quoted prices but likely cannot maintain the linearity and tractability of the PD-NREE. To the extent that search costs constitute a surmountable barrier to competitive pricing, the dealer market will deviate from monopoly and the PD-NREE will serve as a less useful approximation.

2.2 Empirical Facts from TRACE Implementation

This paper's application to the corporate bond market is most closely related to a series of studies evaluating TRACE's impact on lead-lag relations, transactions costs, price dispersion, and dealer market power in the corporate bond market. Chen, Wang and Wu (2006) and Downing, Underwood and Xing (2009) proxy informational efficiency in the corporate bond market by lead-lag relations between stock and bond returns and suggest lower transaction costs under TRACE do not translate into improved efficiency. In a similar spirit, I address the question of informational efficiency from the two alternative standpoints of price sensitivity to information and price informativeness and explicitly link them to TRACE's price transparency shock.

Market power plays a central role in the pricing of corporate bonds. Green, Hollifield and Schürhoff (2007) find evidence of considerable dealer market power in the comparably opaque and similarly structured municipal bond market—to the best of my knowledge, no comparable study has been conducted for corporate bonds. Goldstein, Hotchkiss and Sirri (2007) find that TRACE has little impact on liquidity or volume despite diminished transactions costs and argue that the primary benefit of increased transparency is in enhancing negotiating terms of market participants vis-à-vis dealers. Increased bargaining power brings about reduced price dispersion for retail investors. Cici, Gibson and Jr. (2011) complement their finding with data on price dispersion across mutual funds and provides evidence that price dispersion declines faster with the introduction of TRACE than the secular trend of decreased price dispersion in corporate bonds. Bessembinder and Maxwell (2008) review several of these works and conclude that "these results are consistent with the reasoning that dealers extracted rents from customers in the opaque market, and more so

[1] Green, Hollifield and Schürhoff (2007) documents similar characteristics for the municipal bond market.

from less-informed customers. In the absence of transaction reporting, customers found it difficult to know whether their trade price reflected market conditions."

These TRACE studies suggest increased price transparency does not facilitate increased trading or improve (some measures of) informational efficiency. TRACE implementation appears foremost to reduce asymmetric information on fair transaction prices to weaken dealers' bargaining positions to the benefit of smaller institutional or retail investors. Unsurprisingly, many dealers and institutional investors viewed the introduction of TRACE unfavorably, citing reduced dealer incentives to hold inventory and hence more difficulty in executing trades (e.g., Bessembinder and Maxwell (2008) and Decker (2007)). The welfare effects of TRACE on dealers and institutional investors are not well understood and also serve as a subject of this paper.

3 Model

In this section, I define and characterize the PD-NREE and compare it with the NREE on the dimensions of price levels, price sensitivity to information, price informativeness, and agent welfare.[2]

3.1 Price Discriminating Noisy Rational Expectations Equilibrium

The economy exists for two periods and consists of mass N atomistic, asymmetrically informed agents and a risk-neutral profit-maximizing dealer. Traders are risk-averse and have constant absolute risk aversion utility over consumption with common parameter α. Let the mass of informed agents be denoted by N_I and that of the remaining, uninformed agents be denoted by N_U. The agents are differentially informed between groups but are otherwise identical. All agents have labor income W to be allocated between two securities to be liquidated in the second period. The first security, the risk-free bond, has price normalized to 1 at time 1 and a gross return of $1 + r$ for sure in the next period. The second security, the corporate bond issue, has price p^i for agent i at time 1 and a gross return of d in the next period. The dealer is a profit-maximizing monopolist and is endowed with the entirety of available corporate bond issues at the beginning of the period. The presence of this intermediary and the differential prices it can charge are the key departures of the PD-NREE from the canonical NREE.

All agents know the expected next-period payoff for the bond issue $\bar{d}$. From the perspective of the uninformed agents, the liquidating payoff is distributed

$$d \sim N\left(\bar{d}, \sigma^2\right)$$

[2] To help keep track of PD-NREE concepts, I provide a guide to notation and features of the equilibrium in the first part of the Appendix.

The uninformed agents' distribution is common knowledge. Informed agents receive a supplementary signal s distributed

$$s \sim N\left(d, \sigma_\epsilon^2\right)$$

The functional form for the next-period payoff conditional on the signal $d|s$ is known by the dealer and all traders. Informedness is thus assumed to enter as a different conditional mean combining $\bar{d}$ and s. The signal also increases the conditional precision on the next-period payoff so long as σ_ϵ^2 is finite.

Following Grossman and Stiglitz (1980), the risky asset is subject to exogenous supply noise. Supply noise can be interpreted as a fluctuation in outstanding issues by the firm, as an exogenous demand of the dealer to maintain some portion of the issue supply in his inventory, or as demand of the issue by noise traders. The risky security has time-varying supply $S + u$, where u is private information for the dealer. From the viewpoint of traders, supply noise is given by

$$u \sim N\left(0, \sigma_u^2\right)$$

By contrast, the riskless bond has perfectly elastic supply. I assume the dealer must liquidate his entire risky issue supply at the end of the period, as dealers are averse to holding inventory in practice. [3] Unlike a traditional monopolist, the dealer in this economy has variable prices but fixed quantities; profit-maximization is subject to an equality rather than to the inequality that demand be weakly less than available supply. [4]

In the first period, traders submit demand schedules $x^i\left(p^i\right)$ to the dealer, where $x^i(\cdot)$ denotes demand for the corporate bond of an agent of type i at price p^i. Unlike the price transparent noisy rational expectations equilibrium, prices are not public in the price opaque PD-NREE and the dealer may quote different prices conditional on the demand schedules received. I assume search costs are prohibitively high and disallow discovery of the other type's price p^{-i}, either due to institutional constraints, e.g., unwillingness of an intermediating broker to contact multiple dealers on behalf of retail investors, or because the bond issue is sufficiently thinly traded that other issuers do not keep the corporate bond in their inventories. In the price transparent variant of the PD-NREE, this assumption is relaxed slightly in that other agents' prices are revealed, but agents cannot negotiate to obtain the other agent's price. Because all agents of equal informedness are identical, I replace the generic x^i and p^i with $x^I(\cdot)$ and p^I for the informed agents and $x^U(\cdot)$ and p^U for the uninformed agents. I assume traders are anonymous so that agents cannot collude

[3] I provide in the web appendix an alternative set of assumptions for the dealer in which he can endogenously determine her inventory.

[4] I assume the monopolist consults with the debt-issuing firm to issue profit-maximizing level of S and, as in the corporate bond market, publicly announces the size of issue prior to trading and before knowing u. The monopolist is thereby constrained in that he may choose a non-profit maximizing level of S *ex post* upon having learned u. I also show in the web appendix fixed S profit-maximization as a function of the parameters of the economy and address another potential source of supply noise.

against the monopolistic dealer.

The PD-NREE is defined as follows:

Definition 1 (PD-NREE)**.** ***A price discriminating noisy rational expectations equilibrium*** is a set of prices $\left\{p^I, p^U\right\}$ and demand schedules $\left\{x^I\left(p^I\right), x^U\left(p^U\right)\right\}$ such that

1. (agent optimization) Demand schedules $x\left(p_t\right)$ solve

$$x\left(p_t\right)=\begin{cases}\arg\max_{x^I} -E\left\{\exp\left[-\alpha\left(\left(W-x^I p^I\right)(1+r)+x^I d\right)\right]|s^I, p^I\right\}, & x=x^I\\ \arg\max_{x^U} -E\left\{\exp\left[-\alpha\left(\left(W-x^U p^U\right)(1+r)+x^U d\right)\right]|p^U\right\}, & x=x^U\end{cases}$$

2. (market-clearing) Supply and demand must clear such that

$$N^I x^I\left(p_t^I\right)+N^U x^U\left(p^U\right)=S+u$$

3. (profit-maximization) The dealer chooses prices that maximize profits by solving

$$\max_{p_t^I, p_t^U} N^I x^I\left(p^I\right) p^I+N^U x^U\left(p^U\right) p^U \text{ s.t. } N^I x^I\left(p^I\right)+N^U x^U\left(p^U\right)=S+u$$

3.2 Characterization of the Price Discriminating Noisy Rational Expectations Equilibrium

3.2.1 Timing

I outline an action sequence, although the ordering of specifying demand functions is largely immaterial in the PD-NREE as long as traders move simultaneously. In the first step, traders of each type submit strategies in the form of demands as functions of the signal, where applicable, and hypothetical observed prices. Each submitted demand schedule may be thought of as a continuum of limit orders, with the caveat that the mapping is less applicable for uninformed agents' demands in the price transparent PD-NREE, which are conditional on both sets of prices. The monopolistic dealer then reviews the demand schedules and selects the profit-maximizing pricing pair, $\left(p^I, p^U\right)$, such that the market clears. Note that the information structure of the game is left unchanged if the ordering is reversed so long as the monopolist and traders can submit sufficiently rich strategies.

Due to the anonymity of the market and the countably infinite number of traders, collusion is infeasible and actions of other traders are taken as exogenous. For this analysis, I exclude secondary markets; however, the liquidating dividend, d, can be interpreted as the next-period price and coupon in a richer extension in which secondary markets are introduced. Likewise, orders cannot be smoothed over time. Only one trading action is allowed per trader in each period.

3.2.2 Equilibrium

Prices in the PD-NREE are characterized in the following proposition and corollary:

Proposition 1 (PD-NREE Characterization)**.** *The separating equilibrium for the PD-NREE is given by prices linear in signal and supply realizations*

$$\begin{aligned} p^U &= A + B\left[\left(s - \bar{d}\right) - Cu\right] \\ p^I &= D + F\left[\left(s - \bar{d}\right) - Gu\right] \end{aligned}$$

and demands of the constant absolute risk aversion form

$$x^U\left(p^U\right) = \frac{E\left[d|p^U\right] - (1+r)\,p^U}{\alpha\sigma^2_{d|p^U}} \tag{3.1}$$

$$x^I\left(p^I\right) = \frac{E\left[d|s\right] - (1+r)\,p^I}{\alpha\sigma^2_{d|s}} \tag{3.2}$$

for some set of constants $A, \ldots, F$. The price transparent PD-NREE is identical to the price opaque PD-NREE with the exception that

$$x^U\left(p^U\right) = \frac{E\left[d|p^U, p^I\right] - (1+r)\,p^U}{\alpha\sigma^2_{d|p^U}}$$

Proof. I compute and characterize prices and demand functions in Appendix A and A.6. □

Average prices, A and D, and price sensitivities to the signal, B and F, differ across agent types, by contrast with the standard NREE, but the PD-NREE maintains the NREE's linearity.

As the price is not informative about the next-period payoff beyond the initial signal of the informed agents, the informed agents will submit a demand function as though partaking in a Walrasian equilibrium with exogenous price p^I. The traditional channel for inference by the uninformed agents in the NREE is modified because of the private nature of prices offered to traders. Instead of inverting their offered price to obtain directly a distribution on the signal, the uninformed agents must use p^U to invert the optimization problem of the dealer to jointly back out p^I and s. Price opacity thus obscures the informed agents' signal from the perspective of the uninformed agents as this inversion is clouded by the monopolist's noisy supply.

Contrastingly, informedness and the signal are immediately observable to the dealer. The dealer knows

$\bar{d}_{t+1}$ and can invert the submitted demand schedule of a given agent to determine whether $\bar{d}$ or s is the trader's best non-price signal of the next-period payoff of the bond. Additionally, by integrating over u from the traders' perspective, the dealer can invert the informed agents' demand function to discern his signal. With full knowledge of types, signal, and supply, the monopolist's profit-maximization problem is straightforward.

The price discriminating monopolist can infer signal and agent types because the monopolist knows the payoff and supply noise distributions from the perspective of the informed and uninformed agents and the remaining one-dimensional signal is fully revealed by the informed agent's demand schedule. In practice, one of the primary concerns of market makers is in distinguishing the informedness of potential counterparties. In this respect, the parsimony of the PD-NREE in its three noise sources—one incremental source for each type of market participant—delivers full revelation for the monopolist at the expense of realism. Increasing the dimensionality of the informed agents' signal will undermine the monopolist's ability to infer s, but additional features would be required for the informed agents' demand schedule to not be sufficient to expose agent types. It may be necessary to reformulate the PD-NREE to be a perturbation of a differential information NREE to break the sufficiency of the demand curves for revealing agent types.

3.3 Pooling and Separating Equilibria

3.3.1 Pooling by Traders

The PD-NREE differs from the standard NREE in that it admits an equilibrium in which the informed agents ignore their signal and mimic the uninformed agents by submitting identical demand schedules to the dealer. In this case, the dealer can no longer differentiate among the classes of agents in the model and must quote a single price across agent types. The separating equilibrium, by contrast, entails agents differentiating themselves by their demand schedules.

The pooling equilibrium consists of all participants submitting the same demand schedules regardless of type. The pooling equilibrium obtains when the expected utility of separating is less than that of pooling for the informed agents, as only the informed agents have the ability to mimic the other trader type. The naive approach would be to assume that the price reveals nothing about the signal of the informed agents because they do not incorporate their information in emulating the uninformed agents' demand schedules. Were this to be the case, I could apply the Walrasian equilibrium concept and solve for the market-clearing price and expected utilities. However, the equilibrium selected may inform on the signal received because only some subset of admissible signal will result in the informed agents choosing to pool.

To skirt issues of equilibrium selection and the interplay between the pooling and separating cases, I introduce an infinitesimally informative signal for uninformed agents. The signal maintains agents' information levels on d but precludes the informed agents from replicating the demand functions of the uninformed agents. As the prices and demand functions are asymptotically identical to the PD-NREE detailed above, I leave further discussion of the equilibrium selection procedure to Appendix C. Throughout this work, I focus solely on the separating equilibrium.

3.3.2 Pooling by the Dealer

The strategy space for agents in the price transparent PD-NREE is richer than that of the price opaque PD-NREE because agents can specify actions over both sets of prices, $\left(p^I, p^U\right)$ regardless of type. In the price opaque PD-NREE, only the informed agents had this (unexercised) capability, as they can effectively infer u and consequently p^U. The monopolist's set of potential price signals has equal cardinality to the set of noise parameters of the model, s and u, enabling her to fully reveal s to the uninformed agents. As shown in Appendix A.6.1, the monopolist will elect to reveal sufficiently high signals $s > \bar{s}$ to the uninformed agents via linearly independent pricing functions that allow recovery of s and u. Price discrimination collapses in this instance as the monopolist seeks to exploit high demand from all agents in the economy. Additionally, in this scenario, an additional equilibrium exists in which truthful revelation by the informed agents breaks down as they choose to submit *noninvertible* demands mimicking $s < \bar{s}$ to prevent a discontinuous and undesirable jump in their prices. The effective truncation of the informed agents' signaling also complicates inference by the uninformed agents and generates nonlinearity in their demand functions.

To enhance tractability, I assume the monopolist cannot choose to fully reveal s to the uninformed agents. Pricing functions are thereby constrained to be linearly dependent up to the constant term and only the separating equilibrium of Appendix A.6 obtains. As both prices are revealed in the separating equilibrium, the assumption also avoids the paradox of full revelation with one traded price (p^I). An interesting extension to this work would be to numerically construct the nonlinear price function equilibrium that holds under price transparency when this assumption is relaxed.

3.4 Comparison with the NREE

Differences in the uninformed agents' learning mechanism in the NREE and the PD-NREE have implications for relative price levels, price sensitivities to the informed agents' signal, and informativeness of prices. Relations among pricing function constants are summarized in Propositions 2 and 3. Two ancillary conditions are necessary for their interpretation:

Condition 1. *The* ***relative noise condition*** *is said to hold if and only if*

$$\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2 < 0$$

The relative noise condition holds if price sensitivity to supply innovations is high enough to swamp uncertainty on the next-period payoff from the perspectives of the informed and uninformed agents. Condition 1 relates all of the noise sources in the model and controls the sign of several critical relations in Propositions 2 and 3. Failures of the relative noise condition generate counterintuitive predictions for asset holdings and price sensitivities to information. Increasing the number of informed agents in the PD-NREE decreases price sensitivity to the signal for both types of agents, suggesting that the price becomes less informative with more informed agents when the relative noise condition fails (Claim 5). Likewise, increasing the signal on the unanticipated payoff on the asset, s, only increases the demand of the informed agents for all (N_U, N_I)-pairs in the price opaque PD-NREE when the relative noise condition holds (Claim 6).

Condition 2. *The* ***bounded noise condition*** *is said to hold if and only if*

$$2\left(\sigma^2 + \sigma_\epsilon^2\right) - C^2\sigma_u^2 > 0$$

The bounded noise condition acts in conjunction with the relative noise condition to establish a region in which average prices are well-behaved across equilibrium concepts. Combining Conditions 1 and 2 ensures that supply noise is large enough to be obfuscatory but not too large to completely dominate the other sources of variation in the model. Establishing this condition empirically requires disentangling signal noise σ_ϵ^2 from supply noise $C^2\sigma_u^2$. Doing so is infeasible without making additional, restrictive assumptions on the cross-section of structural parameters. Accordingly, I instead assume without proof that supply noise is such that both conditions hold, that is

$$\sigma^2 + \sigma_\epsilon^2 < C^2\sigma_u^2 < 2\left(\sigma^2 + \sigma_\epsilon^2\right)$$

Proposition 2 (Price Levels)**.** *For risk-averse traders ($\alpha > 0$), average prices for the informed agents D, the uninformed agents A, and for all agents in the non-price discriminating NREE A_{NREE} are related by*

$$\tilde{D} < D < A_{NREE} < A < \tilde{A}$$

when $\sigma^2 + \sigma_\epsilon^2 < C^2\sigma_u^2 < 2\left(\sigma^2 + \sigma_\epsilon^2\right)$, i.e., when the relative and bounded noise conditions hold. Furthermore

$$\tilde{D} < A_{NREE} < \tilde{A}$$

and

$$D < A$$

hold more generally and do not require additional noise boundedness assumptions.

Proof. See Claim 1 in the Appendix. □

Proposition 2 offers a first step toward welfare comparison among equilibrium concepts. On average, informed agents fare better in facing lower prices in the PD-NREE. Lower prices do not come at the expense of reduced quantities, as $x^I\left(p^I\right)$ is identical across concepts with the exception of $p^I < p_{NREE}$, which in turn is associated with a larger quantity demanded. The uninformed agents pay higher prices in the PD-NREE if and only if Condition 1 holds. Given that the supply of the asset is fixed at $S + u$, the higher quantity demanded by the informed agents must come at the expense of issues sold to the uninformed agents. Formal statements cannot be made readily on the respective utility of each set of agents, but the disparities in the average price levels and quantities between equilibrium concepts suggest that the informed (uninformed) agents may be worse (better) off under the NREE.

Proposition 3 (Price Sensitivities to information)**.** *Price sensitivity to the signal s is higher for the informed agents than for the uninformed agents in the PD-NREE. The price sensitivity in the non-price discriminating NREE B_{NREE} is in the intermediate range. That is,*

$$0 \quad < \quad B < \tilde{B} < B_{NREE} < F < \tilde{F}$$

Additionally, $B < B_{NREE} < F$ holds more generally and does not require boundedness assumptions on $C^2\sigma_u^2$.

Proof. See Claim 2 in the Appendix. □

The informed agents face a steeper slope in signal innovations $s - \bar{d}$ than do the uninformed agents. The uninformed agents can only imperfectly distinguish a reduction of asset supply caused by low supply realizations from that induced by a high realization of s_t and are therefore less signal sensitive in their prices than are the informed agents. Hence the informed agents will pay more for the asset when the signal s_t is sufficiently high.

The price sensitivity to the signal in the NREE is pinned between the price sensitivities to the signal in the PD-NREE. The single price of the NREE puts a ceiling on the price sensitivity that can be enforced on the informed agents due to the uninformed agents' noisy signal and a floor on the price sensitivity that can be enforced on the uninformed agents due to the informed agents' otherwise excess (i.e., non-market-clearing) demand.

Price informativeness in the PD-NREE is lower than in the NREE in the sense that the price for the uninformed agent is less responsive to the signal. Likewise, from the perspective of a hypothetical outside observer drawing prices uniformly from among transaction prices, the "average" price sensitivity to the signal of the informed agents is less in the PD-NREE.[5] The bifurcation in prices precludes broader conclusions on price informativeness because the informed agents' price is more informative under the PD-NREE. The concept of informational efficiency as prices incorporating available information is not well-defined when prices are segmented in this fashion.

Price informativeness may also be considered in terms of the variance of the next-period payoff conditional on the observed price. Results are similar to those obtained by comparing price sensitivities, as summarized by the next proposition:

Proposition 4 (Price Informativeness)**.** *Price informativeness conditional on the uninformed agents' price is lower in the price opaque PD-NREE than in the NREE. The precision of the next-period payoff conditional on the informed agent's price is higher in the price opaque PD-NREE than in the NREE if* $\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2 < 0$. *Price informativeness in the price transparent PD-NREE is equal to that of the standard NREE. Taken together*

$$\begin{cases} \tau_{p^U} < \tilde{\tau}_{p^U} = \tau_{NREE} = \tilde{\tau}_{p^I} < \tau_{p^I}, & \sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2 < 0 \\ \tau_{p^U} < \tilde{\tau}_{p^U} = \tau_{NREE} = \tilde{\tau}_{p^I} \lesseqgtr \tau_{p^I}, & \sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2 > 0 \end{cases}$$

Proof. See Claim 3 in the Appendix. □

Supply noise inhibits the uninformed agents' inference to a greater extent in the PD-NREE and hence obscures information on the next-period payoff that would be more observable in the NREE. By contrast, conditioning on the price of the informed agent reduces the conditional variance of the next-period payoff more in the PD-NREE when Condition 1 obtains. However, no agent in the economy would evaluate price informativeness as τ_{p^I}. The uninformed agents cannot condition on the private price quoted to the informed agents and the informed agents' signal is (trivially) strictly more informative on the next-period payoff than the informed agents' price.

From the perspective of an outside observer, price informativeness need not be less in the PD-NREE. The agent mass-weighted average of price informativeness may be higher or lower then that of the NREE, with higher values associated with high ${}^{N_I}/{}_{N_U}$. The precision-based price informativeness measure is subject to the same price segmentation issue as the price sensitivity measure. Informational efficiency cannot be evaluated if informativeness of prices is higher for some agents but lower for others.

[5]The web appendix provides comparisons of signal sensitivities among a variety of related equilibrium concepts.

If the fully separating equilibrium obtains, the informed agent prefers to distinguish his demand schedule rather than to pool with uninformed agents (C). The informed agents are better off than the uninformed agents *ex ante* in the PD-NREE assuming this equilibrium is in force. It is less clear whether traders are better off under the NREE. Due to its potential analytical intractability, I leave the comparison of expected utilities across equilibrium concepts as a subject for continued work. However, comparing prices and quantities across equilibria as a function of the informed agent's signal suggests that informed agents fare worse and uninformed agents better in the NREE on average. Price dispersion no longer works to the informed agents' advantage (on average) in the NREE. Because of lower (higher) signal sensitivity to information, the uninformed (informed) agents may do better for especially high signal realizations in the PD-NREE (NREE), making the net effect of equilibrium transition ambiguous. I formalize these intuitions in the following proposition:

Proposition 5 (Distributional Consequences)**.** *Prices (quantities) for the uninformed agents are higher (lower) on average in the PD-NREE than in the non-price discriminating NREE if* $2\left(\sigma^2+\sigma_\epsilon^2\right)-C^2\sigma_u^2>0$, *but are lower (higher) for sufficiently high realizations of* s *or* u*. Prices (quantities) for the informed agents are lower (higher) on average in the PD-NREE if* $2\left(\sigma^2+\sigma_\epsilon^2\right)-C^2\sigma_u^2>0$, *but are higher (lower) for sufficiently high realizations of* s *or low realizations of* u.

Proof. See Claim 4 in the Appendix. □

3.5 Pricing Function Intuition

Pricing function constants are closely related to those of the standard noisy rational expectations equilibrium. Appendix A.3 solves for the constants of the NREE model. In parallel with the PD-NREE, A_{NREE}, B_{NREE}, and C_{NREE} denote the average price, price sensitivity to information, and price sensitivity to supply innovations, respectively.

In the PD-NREE and NREE, average prices for the uninformed and informed agents may be recast as

$$\begin{aligned}
A &= \frac{\bar{d}}{1+r}-\frac{S\alpha\sigma_{d|s}^2}{(1+r)\left[N_I+N_U\frac{\sigma_{d|s}^2}{\sigma_{d|p^U}^2}\right]\left(\frac{-C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2}\right)}\\
D &= A-\frac{S\alpha\sigma_{d|s}^2}{(1+r)\left[N_I\frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2}\right]}\\
A_{NREE} &= \frac{\bar{d}}{1+r}-\frac{S\alpha\sigma_{d|s}^2}{(1+r)\left[N_I+N_U\frac{\sigma_{d|s}^2}{\sigma_{d|p}^2}\right]}
\end{aligned}$$

In the NREE, the denominator is the effective number of informed agents in the model. The weight placed on uninformed agents is equal to their ability to infer the next-period payoff relative to that of the informed agents. Higher relative precisions on the next-period payoff, $\frac{1/\sigma^2_{d|p^U}}{1/\sigma^2_{d|s}}$, are assigned a weight closer to unity. In the PD-NREE, the effective number of informed agents is modified from traders' perspectives. Heuristically, when the relative noise condition holds, that is, when $\sigma^2 + \sigma^2_\epsilon - C^2\sigma^2_u < 0$, the multiplying ratio is greater than one, the denominator is larger, and uninformed agents act as though their prices are more informative vis-à-vis those of the NREE.[6] Because of this perceived increase in their informedness, the average price uninformed agents are willing to pay is higher in the PD-NREE ($A_{NREE} < A$). The reverse situation holds for informed agents. Although less clear notationally, subtracting the additional term from A will increase the denominator under the bounded noise condition, making trading in the asset less attractive for the informed agents. Decreased willingness to pay for the bond issue in the PD-NREE translates into $A_{NREE} > D$.

Average prices are lower across agents and equilibrium concepts for higher numbers of effective informed agents, larger issue supplies, higher risk aversion, and higher interest rates. Larger issue supplies decrease average prices through the traditional supply and demand channel. Increasing risk aversion or the interest rate boosts the attractiveness of the outside option of investing in the risk-free bond with guaranteed gross return of $1 + r$, necessitating a decreased price for the corporate bond issue for markets to clear.

Price sensitivities to information may also be rewritten to build intuition

$$\begin{aligned}
B &= \frac{\left(\frac{\sigma^2}{\sigma^2+\sigma^2_\epsilon}\right)\left[\frac{1}{2}N_I + N_U\frac{\sigma^2_\epsilon}{\sigma^2_\epsilon+C^2\sigma^2_u}\right]}{(1+r)\left[N_I + N_U\frac{\sigma^2_{d|s}}{\sigma^2_{d|p^U}}\right]} \\
F &= \frac{\left(\frac{\sigma^2}{\sigma^2+\sigma^2_\epsilon}\right)\left[N_I + N_U\frac{\sigma^2_\epsilon}{\sigma^2_\epsilon+C^2\sigma^2_u} + \frac{1}{2}N_U\frac{\sigma^2_{d|s}}{\sigma^2_{d|p^U}}\right]}{(1+r)\left[N_I + N_U\frac{\sigma^2_{d|s}}{\sigma^2_{d|p^U}}\right]} \\
B_{NREE} &= \frac{\left(\frac{\sigma^2}{\sigma^2+\sigma^2_\epsilon}\right)\left[N_I + N_U\frac{\sigma^2_\epsilon}{\sigma^2_\epsilon+C^2\sigma^2_u}\right]}{(1+r)\left[N_I + N_U\frac{\sigma^2_{d|s}}{\sigma^2_{d|p}}\right]}
\end{aligned}$$

Price sensitivities to information, like price levels, have the effective number of informed agents in the denominator. The numerator offers an alternative means of evaluating the number of effective informed agents in the model. Rather than weighting by the ratio of conditional precisions, the uninformed agents instead take weights equal to their ability to extract the informed agents' signal from the mix of signal and

[6] Strictly speaking, the conditions required for relating A to A_{NREE} are more complex because $\sigma^2_{d|p^U} \neq \sigma^2_{d|p}$. Throughout this section I take license with comparisons of $\sigma^2_{d|p^U}$ and $\sigma^2_{d|p}$ to offer the intuition behind the sign relations, as their formal proofs are at best opaque. Claims 1 and 2 in the Appendix offer formal sufficient conditions for $D < A_{NREE} < A$ and $B < B_{NREE} < F$.

supply noise captured in p^U

$$\frac{var\left(B\left(s-\bar{d}\right)\right)}{var\left(B\left(s-\bar{d}\right)-BCu\right)}=\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2+C^2\sigma_u^2}$$

The $1/2$ coefficients applied to N_I and $N_U\frac{\sigma^2_{d|s}}{\sigma^2_{d|p^U}}$ are artifacts of the double slope of the marginal revenue curve relative to the demand curve under monopoly. In all equilibrium concepts, a higher number of effective informed agents (in the numerator) influencing the price will increase the price's sensitivity to information.[7] The PD-NREE decreases (increases) the number of effective informed agents for the uninformed (informed) segment relative to that in the NREE, yielding the $B < B_{NREE} < F$ relation. Increasing the interest rate reduces agents' desire to incorporate their information into prices by making the outside option of not investing in the information-sensitive issue relatively more appealing.

Supply noise sensitivity in each equilibrium concept may be written as:

$$C_{NREE}=\frac{\alpha\sigma_\epsilon^2}{N_I}=\frac{\alpha}{N_I\frac{1}{\sigma_\epsilon^2}}=\frac{1}{2}C$$

Stronger effects of supply noise in the PD-NREE ($BC > B_{NREE}C_{NREE}$, proved in the web appendix) are simply a function of a steeper MR curve under monopoly than under perfect competition. C is twice C_{NREE} because the price discriminating monopolist rides horizontally on the MR curve with supply shocks rather than vertically as in the price sensitivity to information coefficients. The $BC = FG$ relation, which defines G, essentially ensures that a profit-maximizing monopolist allocates units of the bond issue so that the marginal unit for both agent types has an equal contribution toward his revenue.

As the information content of the price with respect to the signal increases, the monopolist's ability to profitably disguise supply fluctuations increases. The monopolist need not reduce prices much in the face of a positive supply shock because uninformed agents confuse it with strong demand by the informed agents and are willing to pay a premium when that signal is particularly precise. Hence C is inversely proportional to the number of informed agents and to the precision of the informed agents' signal $\frac{1}{\sigma_\epsilon^2}$. Increasing traders' risk aversion will make them more wary of conflating signal innovations with issue supply shocks, driving them toward the risk-free asset and increasing C.

[7] Claim 5 in the Appendix describes further how price sensitivity parameters vary with the number of informed and uninformed agents in each model. Comparative statics are counterintuitive in the PD-NREE when the relative noise condition fails in that adding more informed (uninformed) agents to the model decreases (increases) price sensitivity to information.

4 Mapping the PD-NREE to the Corporate Bond Market

Although nearly half the size of the U.S. stock market, the corporate bond market historically has been far more opaque.[8] Until the early 2000s, traders in the corporate bond market rarely had access to dealer price quotes or transaction price data. The launch of a centralized transaction price dissemination database, the Trade Reporting and Compliance Engine (TRACE), from 2002 to 2005 provides a natural laboratory for gauging the consequences of price transparency for price dynamics and informational efficiency.

TRACE dramatically enhances price transparency of affected issues and may have precipitated a decline in dealer market power.[9] I model TRACE implementation as replacing the price opaque PD-NREE with either the standard, competitive NREE or the (still monopolistic) price transparent PD-NREE to embed these changes in a simple analytical framework. The PD-NREE captures the idea that dealers can extract rents by offering different and private prices across the informedness spectrum before transactions prices become available through TRACE.

4.1 Background

In 1998, the Securities and Exchange Commission (SEC) requested that the National Association of Securities Dealers (NASD) develop a system for universal corporate bond transaction price reporting and immediate dissemination. On January 23, 2001, SEC Rule 6200 Series codified the structure and rollout of the Trade Reporting and Compliance Engine (TRACE). Billed by the Financial Industry Regulatory Authority (FINRA) as "what may be the most significant innovation benefiting retail bond investors in decades," TRACE dramatically enhanced price transparency within the corporate bond market by providing a timely, public database for transaction price reporting. Although roughly 99% of corporate bond transactions occur in the over-the-counter (OTC) market, prior to TRACE it was difficult for traders to obtain access to OTC price data. The absence of public transaction price data enabled dealers to execute trades at a range of prices depending on the demands of their OTC counterparties. Small institutional or retail investors, who generated roughly two-thirds of all trades, were especially ill-apprised with historical transaction price information with which to compare their prices. Bloomberg and comparable systems offered dealer quotes for a few issues, but the price at which sizable trades would clear could diverge markedly from those values.

TRACE was deployed in four waves. NASD implemented TRACE Phase I on July 1, 2002, with 500 bond issues with initial issue sizes of greater than \$1 billion.[10] Phase II brought the number of TRACE-eligible

[8] The U.S. corporate bond and equity markets had aggregate valuations of approximately \$7.6 trillion and \$17.3 trillion, respectively, as of March 31, 2011. Market size data sourced from the Securities Industry and Financial Markets Association (www.sifma.org/uploadedfiles/research/statistics/statisticsfiles/cm-us-bond-market-outstanding-sifma.xls) and the World Federation of Exchanges (http://www.world-exchanges.org/statistics/ytd-monthly).

[9] As suggested by Goldstein, Hotchkiss and Sirri (2007) and Bessembinder and Maxwell (2008).

[10] Details of TRACE scope and implementation are provided in the TRACE Announcements Archive and summarized in the

securities to 4,600 on April 14, 2003, where the system then comprised bonds with issue sizes of greater than \$100 million and A or higher ratings. Phase IIIA, initiated on October 1, 2004, increased the number of eligible bond issues to 17,000 by including (almost) all other investment-grade securities. Implementation of Phase IIIB on February 7, 2005, brought TRACE coverage to 99% of all corporate bond trades. The 120 securities in the Phase IIIA pilot and the 177 securities rotated through the Phase IIIB pilot are excluded from this analysis.

The time lag between transaction and price dissemination decreased coincidently with the TRACE roll-outs. The initial maximal reporting lag time was 75 minutes, but the lag was progressively reduced to the current maximal reporting lag of 15 minutes by July 1, 2005. As of late 2004, more than 82% of transactions already were reported with a delay of less than five minutes, effectively instantaneously on the timescale of the thinly traded corporate bond market.[11] With the additional transparency provided by TRACE, transaction prices immediately entered the public domain and enabled less informed agents, e.g., retail investors, to draw inference from recent transactions and build more informed demand schedules.

4.2 Mapping to the Corporate Bond Market

I model the pre-TRACE period as a price discriminating noisy rational expectations equilibrium (PD-NREE) to capture high price dispersion across the agent informedness spectrum (e.g., as in Goldstein et al. (2007) and Cici et al. (2011)) as well as the considerable dealer market power described by Goldstein et al. (2007) and Bessembinder and Maxwell (2008). With the introduction of TRACE and its associated price transparency, I hypothesize that this equilibrium collapses to the standard noisy rational expectations equilibrium (NREE), in which a single price obtains and dealers operate competitively, a stylization of the reduced dealer bargaining power result of Goldstein et al. (2007). Alternatively, if the market power of dealers persists under TRACE, the market may resemble a price transparent PD-NREE with its associated unintuitive implications for average prices for uninformed agents.

For concreteness, I classify institutional and retail investors as informed and uninformed agents. Breaks into the categories need not coincide with business structure or assets under management, as the assumption of equally endowed agents is immaterial with CARA utility. However, it is important that no institution is large enough to act strategically in the corporate bond market; large players change the nature of the game played by traders with the dealer by eliminating price exogeneity conditional on agent type.

Prior to TRACE, issue concentration was such that it was difficult to find other dealers with an issue,

TRACE Fact Books, available at www.finra.org/Industry/Compliance/MarketTransparency/TRACE/Announcements/P117685 and http://www.finra.org/Industry/Compliance/MarketTransparency/TRACE/FactBook/.

[11] If the reporting delay constraint were binding, variation in maximal reporting lag would make for a good source of cross-sectional variation in price transparency beyond the binary pre-TRACE/TRACE split. However, the prevalence of near-immediate reporting makes exploiting such variation unlikely.

particularly for retail investors. The PD-NREE stylizes this feature of the corporate bond market by taking market concentration to the monopolistic extreme.[12]The PD-NREE also captures dealers' desire to not keep inventory to avoid taking directional positions. To abstract from more nuanced utility functions for the dealer, I again stylize by exogenously forcing the dealer to liquidate holdings at the end of the period. As discussed in Section 3, issue supply noise is interpretable as exogenous desire of the monopolist to maintain inventory or as activity of noise traders. This setup contrasts with more plausible fully dynamic model in which assets are traded repeatedly in a secondary market.

The information structure of the pre-TRACE corporate bond market also resembles that of the PD-NREE. Before TRACE, Bloomberg and comparable systems offered dealer quotes for only a few issues and the absence of public transaction price data enabled dealers to execute trades at a range of prices. Perfect price opacity and price discrimination reflects the difficulty in finding and negotiating prices. I extend an asymmetric information NREE rather than a differential information NREE to focus on the price and welfare effects of equilibrium type on heterogeneously well-informed agents in the economy.

The mapping between the corporate bond market in the pre-TRACE period and the PD-NREE achieves correspondences between agent types, market structure, informational environments, and (differentiated) prices. However, the realism of the PD-NREE fails on three fronts. First, no secondary market exists for trading the asset. Insofar as a non-trivial share of corporate bond issues are held to maturity by insurance companies and pension funds on purchase, this shortcoming may be qualitatively unimportant. Second, all risky securities are transacted in a one-shot, simultaneous mechanism. Especially when prices are transparent and rapidly disseminated, market participants will learn from transactions of other agents. Sequential learning is absent from the PD-NREE mechanism and is instead replaced by a weaker simultaneous form of learning encapsulated in demand functions. Finally, I assume traders submit their entire demand functions to dealers, thereby enabling the monopolist to infer agent types with certainty. In some markets, as in equities, it may be sensible to submit a range of limit orders to effectively trace out a demand curve, yet accurate inference by market makers remains elusive. However, Bessembinder and Maxwell (2008) offer substantial anecdotal evidence that dealers in the corporate bond market nonetheless sensed the sophistication of their counterparties and charged them accordingly.

4.3 TRACE Impact

If market power declines with TRACE, as argued by Goldstein et al. (2007), TRACE will be associated with decreased average prices for uninformed agents, who were not able to bargain effectively previously without

[12]A slightly more realistic treatment might followKyle (1989)'s framework of monopolistic competition, but the PD-NREE concept offers a reasonable approximation while maintaining a high degree of tractability.

price information, and higher average prices for informed agents. Excess returns will therefore increase (decrease) for the uninformed (informed) traders when the increase in price transparency mitigates the information asymmetry among agent types. The model also predicts higher sensitivity to informed agents' information in the uninformed agents' price under the TRACE regime. On average, retail investors will also obtain a greater share of corporate bond issue supply. However, if issue supply remains concentrated and prices become more transparent without substantially reducing dealer market power, TRACE will *increase* the average price spread between agent types and have a muted effect on price sensitivity to informed agents' information.

Neither changes in price levels nor in price responsiveness to information have been studied empirically for the TRACE transition. Documenting how price transparency changed these measures will inform on (1) whether the PD-NREE is an accurate depiction of the pre-TRACE period and (2) whether the price opaque PD-NREE was replaced by a price transparent PD-NREE or by an NREE. The answer to these questions carries important efficiency and distributional implications for the *ex post* desirability of the TRACE system.

5 Caveats and Extensions

In this section, I consider primarily the counterfactual ability of the dealer to identify trader types and the absence of dynamics in the PD-NREE. I also suggest several directions for future work to address these and other concerns.

5.1 Trader Identification

The conditional precision of the next-period payoff $\sigma^2_{d|s}$ is larger for informed agents even when signals are identical across agent classes. The slope of the demand equations (3.1) with respect to own-price is therefore higher for the uninformed agents. By observing the entirety of the demand schedule for both classes of agents, the monopolist can thus infer without error traders' levels of informedness. By contrast with market microstructure models in which spreads between bids and asks on the same security are in part justified by an information risk premium for the market-maker, the PD-NREE has spreads only between trader types and no information risk because trader types are immediately observed. Summary identification causes the model to suffer in realism, as does Grossman and Stiglitz (1980) on which it is built.

The PD-NREE fully reveals agent types to the monopolist because traders counterfactually submit the entirety of their demand schedules rather than a single (or stream of) order(s). Submitting a continuous demand correspondence is required to ensure an equilibrium exists in this setting, although only two points from a single account are necessary to identify agent types[13] Although not attempted here, breaking the dependence on a continuous demand correspondence would simultaneously give rise to implications for bid-ask spreads and restore plausibility to the order submission mechanism of the PD-NREE.

5.2 Dynamics

The PD-NREE models presented are fundamentally static in nature. The PD-NREE provides a snapshot in which parameters of the economy are counterfactually fixed over time or across equilibrium concepts. Even under the assumption of stable parameters, normally distributed prices, and hence, returns, do not tractably compound across periods, precluding empirical work applying the model to corporate bond or swap market data.

When comparing across equilibrium concepts, transition dynamics take on an additional degree of importance. In implementing TRACE, the Securities and Exchange Commission wanted to increase price transparency in the market to benefit small investors. If the SEC were successful in this objective, more small traders should enter the market *ceteris paribus*. Likewise informed investors' expected prices and quantities decrease with price transparency and competitiveness, thereby reducing the benefits to becoming informed. Reduced profitability of informed trading may decrease *ex ante* information gathering and convert otherwise informed agents into uninformed traders, further tilting the N_U/N_I ratio. The volatility of signals may also change in response to regime changes as informed agents reduce their investment in learning about the security. Empirically, TRACE implementation may have affected other structural parameters of the model, as well: Decker (2007) suggests that dealers are more reluctant to hold inventory under TRACE, which may result in differences in supply or supply noise.

The PD-NREE additionally excludes the possibility of secondary markets. In the price transparent form of the PD-NREE, in particular, uninformed agents may prefer to wait to purchase the assets at an intermediate price, $p^* \in \left(p^U, p^I\right)$, rather than purchase directly from the monopolistic dealer. The static nature of the PD-NREE described here cannot readily extend to incorporate these behaviors. Specifying monopoly and price opacity in a dynamic setting is beyond the scope of this work, but represents an essential extension for accommodating estimation and strategic behavior in addition to the changes in parameters discussed above.

[13] Only one point would be sufficient for grouping an agent with his type, but two points are necessary for labeling groups as "informed" and "uninformed" and price discriminating accordingly.

5.3 Other Extensions

Transactions costs are a barrier to incorporation of information into prices and can serve as another measure of price informativeness besides the two evaluated here. Unlike many microstructure models applied to corporate bonds, the PD-NREE makes no statements for trading volume or transactions costs. As such implications would be readily testable, this absence is especially lamentable. Trading costs act only through reducing $\bar{d}_{t+1}$ and are taken as exogenous or excluded.

In a similar vein, although the model does not speak to bid-ask spreads, reduced price dispersion implies reduced bid-bid spreads over short intervals. A richer pre-TRACE dataset would enable direct testing of the reduced price dispersion of the PD-NREE to NREE concept, although the posited equilibrium transition is not unique in having this property.

6 Conclusion

I introduce a price discriminating noisy rational expectations equilibrium (PD-NREE) that incorporates elements of price transparency and concentrated market power into the standard NREE framework. This model is built to match mechanically the increased price transparency and decreased price dispersion associated with TRACE (Goldstein et al. (2007) and Cici et al. (2011)) while enabling evaluation of those changes on informational efficiency. The PD-NREE inherits several of the convenient analytical properties of the NREE such as linear pricing and intuitive relations governing pricing coefficients. Like the NREE, the PD-NREE makes predictions for price informativeness and asset allocation to informed and uninformed classes of investors that are difficult to assess directly from the data. The PD-NREE's primary distinction is in its segmented and private prices whereby each class of investors partakes in linked, but differentially informationally efficient markets. The PD-NREE concept offers a useful guide for gauging transparency's effects outside of the traditional channels of transactions costs and lead-lag relations. I demonstrate also that the PD-NREE can be modified slightly to disentangle the effects of price transparency, market power, and supply noise in rational expectations equilibria.

I compare the price opaque PD-NREE to the NREE and show that average prices are lower (higher) for informed (uninformed) investors than under the canonical NREE and that price sensitivities to information and informational content are higher (lower). These differences have natural testable implications for expected returns and (*ex post*) price informativeness when transitioning from one equilibrium concept to the other. Price transparency thus can have a first-order effect on observed moments of prices and on the distribution of inelastically supplied securities among market participants in addition to the more abstract concepts of price informativeness and informational efficiency.

The PD-NREE to NREE model accords with intuition in that, on average, price transparency and competition improve informational efficiency of prices. However, I also show that if changes in market structure do not accompany transparency improvements, welfare effects for uninformed agents may be mixed. Perversely, increasing transparency in a PD-NREE setting leads to higher average prices for less well-informed market participants. Evaluation of the efficacy of TRACE implementation and of price transparency as a whole clearly entails more than contrasting transactions costs and lead-lag relations between regimes. Comparison across PD-NREE and NREE variations with alternative market or information structures may identify avenues by which trader welfare can be enhanced less ambiguously.

References

Bessembinder, Hendrik and William Maxwell, "Markets: Transparency and the Corporate Bond Market," *Journal of Economic Perspectives*, 2008, *22* (2), 217–234.

Biais, Bruno, Larry Glosten, and Chester Spatt, "Market microstructure: A survey of microfoundations, empirical results, and policy implications," *Journal of Financial Markets*, 2005, *8* (2), 217 – 264.

Bloomfield, R and M O'Hara, "Market transparency: who wins and who loses?," *Review of Financial Studies*, 1999, *12* (1), 5–35.

Bloomfield, Robert and Maureen O'Hara, "Can transparent markets survive?," *Journal of Financial Economics*, 2000, *55* (3), 425 – 459.

Chen, Peter, Junbo Wang, and Chunchi Wu, "The Informativeness of Corporate Bond Trades," Working Paper 2006.

Cici, Gjergji, Scott Gibson, and John J. Merrick Jr., "Missing the marks? Dispersion in corporate bond valuations across mutual funds," *Journal of Financial Economics*, 2011, *101* (1), 206–226.

Decker, Michael, "FINRA's TRACE and the U.S. Corporate Bond Market," *Securities Industry and Financial Markets Association*, 2007.

Downing, Chris, Shane Underwood, and Yuhang Xing, "The Relative Informational Efficiency of Stocks and Bonds: An Intraday Analysis," *Journal of Financial and Quantitative Analysis*, 2009, *44*, 1081–1102.

Duffie, Darrell, Nicolae Gârleanu, and Lasse Heje Pedersen, "Valuation in Over-the-Counter Markets," *Review of Financial Studies*, 2007, *20* (6), 1865–1900.

Flood, MD, R Huisman, KG Koedijk, and RJ Mahieu, "Quote disclosure and price discovery in multiple-dealer financial markets," *Review of Financial Studies*, 1999, *12* (1), 37–59.

Goldstein, Michael A., Edith S. Hotchkiss, and Erik R. Sirri, "Transparency and Liquidity: A Controlled Experiment on Corporate Bonds," *Review of Financial Studies*, 2007, *20* (2), 235–273.

Green, Richard C., Burton Hollifield, and Norman Schürhoff, "Financial Intermediation and the Costs of Trading in an Opaque Market," *Review of Financial Studies*, 2007, *20* (2), 275–314.

Grossman, Sanford J. and Joseph E. Stiglitz, "On the Impossibility of Informationally Efficient Markets," *The American Economic Review*, 1980, *70* (3), 393–408.

Guerrieri, Veronica, Robert Shimer, and Randall Wright, "Adverse Selection in Competitive Search Equilibrium," *Econometrica*, 2010, *78* (6), 1823–1862.

Kyle, Albert S, "Informed Speculation with Imperfect Competition," *Review of Economic Studies*, July 1989, *56* (3), 317–55.

Madhavan, A., "Security Prices and Market Transparency," Weiss Center Working Papers 1-92, Wharton School - Weiss Center for International Financial Research 1991.

Pagano, Marco and Ailsa Röell, "Transparency and Liquidity: A Comparison of Auction and Dealer Markets with Informed Trading," *The Journal of Finance*, 1996, *51* (2), pp. 579–611.

Notation and Model Synopsis

Guide to Notation

Basic Parameters of the Economy

N_I, N_U, N The number of informed agents, uninformed agents, and agents of both types in the economy.

α The coefficient of absolute risk aversion.

W The initial agent endowment.

r The risk-free rate.

S The risky asset supply.

Sources of Risk and Uncertainty

$d, \bar{d}, \sigma^2$ The realized terminal payoff on the risky asset and its *ex ante* mean and variance.

s, σ_ϵ^2 The informed agents' signal on the terminal payoff and its variance.

u, σ_u^2 The supply noise for the risky security and its variance.

Equilibrium Quantities and Prices

x^I, x^U The quantity demanded of the risky asset for informed and uninformed agents, respectively.

p^I, p^U The price of the risky asset for informed and uninformed agents, respectively.

Coefficients of the Pricing Functions

$A, D, \tilde{A}, \tilde{D}$ *Ex ante* average prices for uninformed agents and informed agents in the price opaque and price transparent PD-NREEs, respectively.

$B, F, \tilde{B}, \tilde{F}$ Price sensitivities to the informed agents' signal for uninformed agents and informed agents in the price opaque and price transparent PD-NREEs, respectively.

$C, G, \tilde{C}, \tilde{G}$ Price sensitivities to supply noise (relative to price sensitivities to the signal) for uninformed agents and informed agents in the price opaque and price transparent PD-NREEs, respectively.

$A_{NREE}, B_{NREE}, C_{NREE}$ *Ex ante* average prices, price sensitivities to the informed agents' signal, and price sensitivities to supply noise in the NREE.

Price Discriminating Noisy Rational Expectations Equilibrium Synopsis

I summarize below the features distinguishing the price discriminating noisy rational expectations equilibrium (PD-NREE) from the canonical noisy rational expectations equilibrium of Grossman and Stiglitz (1980). The PD-NREE model is detailed in Section 3.

Agents NREE + a monopolistic "dealer," massless traders

Preferences NREE + dealer is profit-maximizing rather than risk-averse

Endowments NREE + dealer holds the entirety of risky issue supply at the beginning of the period

Technology NREE + dealer must liquidate holdings by the end of the period

Information NREE + dealer observes u_t directly

Prices Rather than a single prevailing price p for the risky asset, agents pay p^I and p^U per unit and are

- Unable to observe prices for the other type (price opaque PD-NREE)
- Able to observe prices for the other type (price transparent PD-NREE)

Mathematically, the PD-NREE is given by Definition 1, repeated here for completeness:

Definition. ***A price discriminating noisy rational expectations equilibrium*** is a set of prices $\left\{p^I, p^U\right\}$ and demand schedules $\left\{x^I\left(p^I\right), x^U\left(p^U\right)\right\}$ such that

1. (agent optimization) Demand schedules $x\left(p_t\right)$ solve

$$x\left(p_t\right)=\begin{cases}\arg\max_{x^I} -E\left\{\exp\left[-\alpha\left(\left(W-x^Ip^I\right)(1+r)+x^Id\right)\right]|s^I,p^I\right\}, & x=x^I\\ \arg\max_{x^U} -E\left\{\exp\left[-\alpha\left(\left(W-x^Up^U\right)(1+r)+x^Ud\right)\right]|p^U\right\}, & x=x^U\end{cases}$$

2. (market-clearing) Supply and demand must clear such that

$$N^I x^I\left(p_t^I\right)+N^U x^U\left(p^U\right)=S+u$$

3. (profit-maximization) The dealer chooses prices that maximize profits by solving

$$\max_{p_t^I,p_t^U} N^I x^I\left(p^I\right)p^I+N^U x^U\left(p^U\right)p^U \text{ s.t. } N^I x^I\left(p^I\right)+N^U x^U\left(p^U\right)=S+u$$

The price transparent variant of the PD-NREE differs only in that the agents condition also on the other agent type's price in the agent optimization criterion.

Mathematical Appendix

A Solving the Price Opaque PD-NREE

I provide conditional expectations and variances of the next-period payoff, as I use these frequently in solving the price discriminating noisy rational expectations equilibrium. The informed agents condition on their signal and see

$$\begin{aligned} E[d|s] &= \bar{d} + \left(s - \bar{d}\right)\beta_s \\ \sigma^2_{d|s} &= var(d) - \beta_s^2 var(s) = \frac{\sigma^2\sigma_\epsilon^2}{\sigma^2 + \sigma_\epsilon^2} \\ \beta_s &= \frac{cov(d,s)}{var(s)} = \frac{\sigma^2}{\sigma^2 + \sigma_\epsilon^2} \end{aligned}$$

whereas the uninformed agents condition on their price p^U and perceive

$$\begin{aligned} E\left[d|p^U\right] &= \bar{d} + \left(p^U - E\left[p^U\right]\right)\beta_{p^U} \\ \sigma^2_{d|p^U} &= var(d) - \beta_{p^U}^2 var\left(p^U\right) \\ \beta_{p^U} &= \frac{cov\left(d,p^U\right)}{var\left(p^U\right)} \end{aligned}$$

Throughout, I omit time and bond issue superscripts to enhance clarity.

A.1 General Demand Functions

I conjecture that the price for the uninformed agents p^U is linear in signal and supply innovations as in the standard NREE

$$p^U = A + B\left[\left(s - \bar{d}\right) - Cu\right]$$

Substituting p^U into the expression for $E\left[d|p^U\right]$ gives

$$E\left[d|p^U\right] = E[d] + \left(p^U - E\left[p^U\right]\right)'\beta_p = \bar{d} + \frac{\sigma^2\left(p^U - A\right)}{B\left[\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right]}$$

As the unconditional expectation of the signal is $\bar{d}$ and of the supply innovation is 0, $E\left[p^U\right] = A$. Plugging in the expression for p^U gives the next-period payoff sensitivity to price β_{p^U} as

$$\beta_{p^U} = \frac{cov\left(d,p^U\right)}{var\left(p^U\right)} = \frac{Bcov(d,s)}{B^2\left[var(s) + C^2 var(u)\right]} = \frac{\sigma^2}{B\left[\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right]}$$

Finally, the conditional variance $\sigma^2_{d|p^U}$ of the next-period payoff given the uninformed agents' price is

$$\begin{aligned}\sigma^2_{d|p^U} &= \sigma^2 - \beta^2_{p^U} var\left(p^U\right) \\ &= \frac{\sigma^2\left(\sigma^2_\epsilon + C^2\sigma^2_u\right)}{\sigma^2 + \sigma^2_\epsilon + C^2\sigma^2_u}\end{aligned}$$

CARA utility delivers immediately the optimal demand of the uninformed $x^U\left(p^U\right)$ in terms of expected payoffs, payoff variance, and risk aversion

$$\begin{aligned}x^U\left(p^U\right) &= \frac{E\left[d|p\right] - (1+r)\,p^U}{\alpha\sigma^2_{d|p^U}} \\ &= \frac{\bar{d} + \beta_{p^U}\left(p^U - A\right) - (1+r)\,p^U}{\alpha\sigma^2_{d|p^U}} \\ &= \frac{\bar{d} + \frac{\sigma^2\left(p^U - A\right)}{B\left[\sigma^2+\sigma^2_\epsilon+C^2\sigma^2_u\right]} - (1+r)\,p^U}{\alpha\frac{\sigma^2\left(\sigma^2_\epsilon+C^2\sigma^2_u\right)}{\sigma^2+\sigma^2_\epsilon+C^2\sigma^2_u}} \\ &= \frac{\frac{\sigma^2}{B}\left(p^U - A\right) + \left[\bar{d} - (1+r)\,p^U\right]\left(\sigma^2 + \sigma^2_\epsilon + C^2\sigma^2_u\right)}{\alpha\sigma^2\left(\sigma^2_\epsilon + C^2\sigma^2_u\right)}\end{aligned}$$

Note that so long as A, B, and C are constants, x^U will be linear in p^U.

Likewise, I conjecture that for the informed agents, the price p^I is linear in signal and supply innovations

$$p^I = D + F\left[\left(s - \bar{d}\right) - Gu\right]$$

For the informed agents, conditioning on the signal gives a demand function $x^I\left(p^I\right)$ only of model primitives except for where the price explicitly enters

$$x^I\left(p^I\right) \equiv x^I\left(p^I; s\right) = \frac{E\left[d|s\right] - (1+r)\,p^I}{\alpha\sigma^2_{d|s}} = \frac{\bar{d} + \left(s - \bar{d}\right)\beta_s - (1+r)\,p^I}{\alpha\sigma^2_{d|s}}$$

I prove that equilibria exist in the PD-NREE and NREE under linear pricing rules. However, I do not prove that non-linear pricing rules do not give rise to alternative equilibria. I leave consideration of uniqueness in pricing as a subject for additional research.

Lemma 1 (Truth-telling of the Informed Agents)**.** *Informed agents will submit demand schedules as though prices are exogenous with signal s if they are restricted to demand schedules monotone in the signal.*[14]

Proof. Consider any alternative demand schedule $x'\left(p^I(s), s\right)$ such that $x'(\cdot)$ is monotone in s. As the monopolistic dealer understands the demand correspondence mapping s to $x'(\cdot)$, the informed agents' demand

[14] Monotonicity in s is not unreasonable. In some sense, monotonicity is in force whenever the asset becomes more desirable at least as fast as its price increases.

schedules are fully revealing of their signal. Furthermore, because traders are massless, they cannot individually affect prices outside of the channel of revealing their types to the monopolist, so the optimal demand schedule conditional on p^I being exogenous is also the unconditional optimal demand schedule. Therefore, the demand schedule $x^I\left(p^I\left(s\right),s\right)$ of Appendix A.1 is weakly preferred to all monotone $x'\left(\cdot\right)$.[15] $\square$

A.2 k-size Economies

N_I and N_U denote the number of informed and uninformed agents, respectively. These quantities are stand-ins for agent type proportions in more general k-size economies in which agents of each type are of mass kN_I and kN_U. The normalizations below render quantities in the economies independent of k. Letting $k \to \infty$ (k_∞) carries the advantage of populating the economy with massless agents, which has the desirable feature of avoiding hold-out problems by agents aware of the dealer's necessity to liquidate asset supply. Throughout, I assume a k_∞ economy but retain the standard (N_I, N_U) notation.

CARA utility creates the primary hurdle to adjusting k: regardless of wealth, agents of equal risk aversion will select the same level of holdings of the risky asset. Hence increasing the number of agents in the economy k-fold will multiply aggregate demands by k. To prevent prices from exploding, supply must scale accordingly, as must supply noise to prevent it from becoming trivial. Other adjustments entail keeping aggregate wealth finite or, equivalently, keeping the interest rate bounded if the risk-free asset's supply is not infinitely elastic. Formally, I let hat variables denote $k = 1$ supply, supply noise, and wealth, respectively, and unaccented characters denote analogies in the k-size economies in the text:

$$\begin{aligned} k\hat{S} &= S \\ k\hat{\sigma}_u &= \sigma_u \\ \frac{\hat{W}}{k} &= W \end{aligned}$$

These transforms make all price and demand functions independent of k; in the equations that follow, using hat variables and replacing N_I with kN_I and N_U with kN_U leaves all relations unaffected.

[15] Kyle (1989) develops a non-competitive NREE in which agents are strategic in submitting their demand schedules. Market power on the part of the traders is too involved to explore here but would make for an interesting extension of the PD-NREE concept.

A.3 Non-Price Discriminating Noisy Rational Expectations Equilibrium[16]

I specialize the demand equations above to solve the NREE for comparison with the PD-NREE and to illustrate the solution method for the PD-NREE in a simpler setting. The linear pricing of the NREE is preserved in the PD-NREE, allowing for straightforward comparison of pricing constants across equilibrium concepts. The NREE is primarily differentiated from the PD-NREE in that

1. The intermediating dealer and associated market clearing condition are removed.

2. All agents pay the same price for the corporate bond issue, $p^I = p^U = p$.

As in the PD-NREE, I conjecture a linear price function in s and u

$$p = A + B\left[\left(s - \bar{d}\right) - Cu\right] \tag{A.1}$$

The market clearing condition gives

$$N_I x^I(p) + N_U x^U(p) = S + u$$

Plugging in the demands of the informed and uninformed agents delivers

$$N_I \frac{\bar{d} + \left(s - \bar{d}\right)\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} - (1+r)\,p}{\alpha\frac{\sigma^2\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}} + N_U \frac{\frac{\sigma^2}{B}(p-A) + \left[\bar{d} - (1+r)\,p\right]\left(\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right)}{\alpha\sigma^2\left(\sigma_\epsilon^2 + C^2\sigma_u^2\right)} = S + u \tag{A.2}$$

Equation A.2 can be broken down into three equations in terms of the constant, $s - \bar{d}$, and u, respectively, as it must hold for arbitrary s and u. Plugging in the conjectured linear price function of Equation A.1, these equations are

$$\begin{aligned}
N_I \frac{\bar{d} - (1+r)\,A}{\alpha\frac{\sigma^2\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}} + N_U \frac{\left[\bar{d} - (1+r)\,A\right]\left(\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right)}{\alpha\sigma^2\left(\sigma_\epsilon^2 + C^2\sigma_u^2\right)} &= S \\
N_I \frac{\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} - (1+r)\,B}{\alpha\frac{\sigma^2\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}} + N_U \frac{\sigma^2 - (1+r)\left(\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right)B}{\alpha\sigma^2\left(\sigma_\epsilon^2 + C^2\sigma_u^2\right)} &= 0 \\
BC\left[N_I \frac{(1+r)}{\alpha\frac{\sigma^2\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}} + N_U \frac{-\frac{\sigma^2}{B} + (1+r)\left(\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right)}{\alpha\sigma^2\left(\sigma_\epsilon^2 + C^2\sigma_u^2\right)}\right] &= 1
\end{aligned}$$

[16] The notation and technique that follows in this section is largely due to Leonid Kogan and Stavros Panageas.

Solving the system obtains the NREE pricing constants

$$\begin{aligned}
A_{NREE} &= \frac{\bar{d}}{1+r} - \frac{S\alpha\frac{\sigma^2\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}}{(1+r)\left[N_I + N_U\frac{\sigma_\epsilon^2\left(\sigma^2+\sigma_\epsilon^2+C_{NREE}^2\sigma_u^2\right)}{\left(\sigma^2+\sigma_\epsilon^2\right)\left(\sigma_\epsilon^2+C_{NREE}^2\sigma_u^2\right)}\right]} = \frac{\bar{d}}{1+r} - \frac{S\alpha\sigma_{d|s}^2}{(1+r)\left(N_I + N_U\frac{\sigma_{d|s}^2}{\sigma_{d|p}^2}\right)} \\
B_{NREE} &= \frac{N_I\frac{\sigma^2}{\sigma_\epsilon^2} + N_U\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}}{(1+r)\left[N_I\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2} + N_U\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]} \\
C_{NREE} &= \frac{\alpha\sigma_\epsilon^2}{N_I}
\end{aligned}$$

Constants are independent of s and u, verifying the conjecture of linear pricing. Demand correspondences follow by plugging in $p = A + B\left[\left(s - \bar{d}\right) - Cu\right]$ into the expressions for $x^I(p)$ and $x^U(p)$.

A.4 Deriving Equations for Unknowns in the Price Opaque PD-NREE

By contrast with the traditional NREE concept, the PD-NREE requires a condition to supplement market clearing to solve for the unknown constants. The market clearing condition must be taken in conjunction with the dealer's profit-maximization to obtain A, B, C, D, F, and G. The two equations are respectively

$$\begin{aligned}
N_I x^I\left(p^I\right) + N_U x^U\left(p^U\right) &= S + u \\
\left(p^I, p^U\right) = \arg\max_{\{p^I,p^U\}} N_I x^I\left(p^I\right)p^I + N_U x^U\left(p^U\right)p^U \quad &\text{s.t.} \quad N_I x^I\left(p^I\right) + N_U x^U\left(p^U\right) = S + u
\end{aligned}$$

As in the standard NREE, the demand equation is only satisfied if and only if the constant term, functions of s, and functions of u are identically equal to 0 in the excess demand form of the market clearing condition. This restriction delivers three equations in the six unknowns. Using the conjectured functional forms for prices, these equations are

$$\begin{aligned}
\frac{\bar{d}}{1+r}\left(\frac{N_I}{\sigma_{d|s}^2} + \frac{N_U}{\sigma_{d|p^U}^2}\right) - S\frac{\alpha}{1+r} &= \frac{N_I}{\sigma_{d|s}^2}D + \frac{N_U}{\sigma_{d|p^U}^2}A \\
N_I\frac{(1+r)}{\sigma_{d|s}^2}F - N_U\frac{\beta_{p^U} - (1+r)}{\sigma_{d|p^U}^2}B &= N_I\frac{\beta_s}{\sigma_{d|s}^2} \\
C\left(\beta_s - (1+r)F\right) + (1+r)FG &= \frac{\alpha\sigma_{d|s}^2}{N_I}
\end{aligned} \tag{A.3}$$

Now, imposing the profit-maximization condition obtains the Lagrangian

$$\begin{aligned}
\mathcal{L} &= N_I x^I\left(p^I\right)\left(p^I-\lambda\right)+N_U x^U\left(p^U\right)\left(p^U-\lambda\right)+\lambda\left(S+u\right) \\
&= N_I \frac{\bar{d}+\beta_s\left(s-\bar{d}\right)-\left(1+r\right)p^I}{\alpha\sigma^2_{d|s}}\left(p^I-\lambda\right)+N_U\frac{\bar{d}+\beta_{p^U}\left(p^U-A\right)-\left(1+r\right)p^U}{\alpha\sigma^2_{d|p^U}}\left(p^U-\lambda\right)+\lambda\left(S+u\right)
\end{aligned}$$

Taking first-order conditions for the Lagrangian gives two non-redundant equations

$$\begin{aligned}
\bar{d}+\beta_s\left(s-\bar{d}\right)-\left(1+r\right)\left(2p^I-\lambda\right) &= 0 \\
\bar{d}-A\beta_{p^U}+\left(\beta_{p^U}-\left(1+r\right)\right)\left(2p^U-\lambda\right) &= 0
\end{aligned}$$

which combined eliminate the multiplier as follows

$$\begin{aligned}
p^U-p^I &= -\frac{1}{2}\left[\frac{\bar{d}-A\beta_{p^U}}{\beta_{p^U}-\left(1+r\right)}+\frac{\bar{d}+\beta_s\left(s-\bar{d}\right)}{1+r}\right] \\
&= -\frac{1}{2}\left[\frac{\bar{d}B\left[\sigma^2+\sigma^2_\epsilon+C^2\sigma^2_u\right]-A\sigma^2}{\sigma^2-\left(1+r\right)B\left[\sigma^2+\sigma^2_\epsilon+C^2\sigma^2_u\right]}+\frac{\bar{d}+\beta_s\left(s-\bar{d}\right)}{1+r}\right] \\
&= \left(A-D\right)+\left(B-F\right)\left(s-\bar{d}\right)+\left(FG-BC\right)u
\end{aligned}$$

Using the same coefficient-matching approach as in Equation A.3, I obtain the additional three equations needed to close the non-linear system

$$\begin{aligned}
A-D &= -\frac{1}{2}\left[\frac{\bar{d}-A\beta_{p^U}}{\beta_{p^U}-\left(1+r\right)}+\frac{\bar{d}}{1+r}\right] \\
B-F &= -\frac{1}{2}\frac{\beta_s}{1+r} \\
BC-FG &= 0
\end{aligned} \tag{A.4}$$

A.5 Solving the System

I use the second set of equations Equation A.4 to eliminate D, F, and G. It remains to be shown that the resulting system of three non-linear equations in three unknowns has a (unique) solution. The equations are

$$
\begin{aligned}
\frac{N_I}{\sigma^2_{d|s}}\left(A+\frac{1}{2}\left[\frac{\bar{d}-A\beta_{p^U}}{\beta_{p^U}-(1+r)}+\frac{\bar{d}}{1+r}\right]\right)+\frac{N_U}{\sigma^2_{d|p^U}}A &= \frac{\bar{d}}{1+r}\left(\frac{N_I}{\sigma^2_{d|s}}+\frac{N_U}{\sigma^2_{d|p^U}}\right)-S\frac{\alpha}{1+r} \\
\frac{N_I}{\sigma^2_{d|s}}(1+r)\left(B+\frac{1}{2}\frac{\beta_s}{1+r}\right)-\frac{N_U}{\sigma^2_{d|p^U}}\left(\beta_{p^U}-(1+r)\right)B &= \frac{N_I}{\sigma^2_{d|s}}\beta_s \\
C\left(\beta_s-(1+r)\left(B+\frac{1}{2}\frac{\beta_s}{1+r}\right)\right)+(1+r)BC &= \frac{\sigma^2_{d|s}}{N_I}\alpha
\end{aligned}
$$

Substituting in for σ's and β's gives

$$
\begin{aligned}
\left(\frac{\bar{d}}{1+r}-A\right)\left(\frac{\frac{1}{2}\sigma^2 N_I}{\sigma^2-(1+r)B\left[\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right]}-N_I-N_U\frac{\sigma_\epsilon^2\left(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right)}{\left(\sigma^2+\sigma_\epsilon^2\right)\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)}\right)+S\frac{\alpha}{1+r}\frac{\sigma^2\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2} &= 0 \\
N_I\left(\frac{\frac{1}{2}\sigma^2-(1+r)B\left(\sigma^2+\sigma_\epsilon^2\right)}{\sigma_\epsilon^2}\right)+N_U\left(\frac{\sigma^2-\left(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right)(1+r)B}{\sigma_\epsilon^2+C^2\sigma_u^2}\right) &= 0 \\
C\left(\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}-(1+r)\left(B+\frac{1}{2}\frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\right)\right)+(1+r)BC-\frac{\alpha}{N_I}\frac{\sigma^2\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2} &= 0
\end{aligned}
$$

The third equation reduces neatly to

$$
C = 2\frac{\alpha\sigma_\epsilon^2}{N_I} > 0
$$

The second equation is a function of only B and C, so I can solve directly for B

$$
B=\frac{N_I\left(\frac{\sigma^2}{2\sigma_\epsilon^2}\right)+N_U\left(\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)}{(1+r)\left[N_I\left(\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}\right)+N_U\left(\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)\right]} > 0
$$

Finally, the first equation relates A, B, and C. Solving obtains

$$
A = \frac{\bar{d}}{1+r}+\frac{S\frac{\alpha}{1+r}\frac{\sigma^2\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}}{\frac{\frac{1}{2}\sigma^2 N_I}{\sigma^2-(1+r)B\left[\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right]}-N_I-N_U\frac{\sigma_\epsilon^2\left(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right)}{\left(\sigma^2+\sigma_\epsilon^2\right)\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)}}
$$

D, F, and G follow immediately from Equation A.4

$$
\begin{aligned}
D &= A + \frac{1}{2}\left[\frac{\bar{d}B\left[\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right]-A\sigma^2}{\sigma^2-(1+r)\,B\left[\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right]}+\frac{\bar{d}}{1+r}\right] \\
&= A + \frac{\sigma^2}{2}\left[\frac{-A+\frac{\bar{d}}{1+r}}{\sigma^2-(1+r)\,B\left[\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right]}\right] \\
F &= B + \frac{1}{2}\frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} \\
G &= C\frac{B}{B+\frac{1}{2}\frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}}
\end{aligned}
$$

As desired, none of the parameters are functions of s or u. p^I and p^U are therefore indeed linear equilibrium price functions, verifying the conjectured form of the price functions.

A.6 Price Transparency in the PD-NREE

The PD-NREE to NREE model assumes an increase in competitiveness in the dealer side of the market in conjunction with the increase in price transparency. In this section, I show that price transparency and market power can be disentangled in an alternative PD-NREE that does not trivialize price formation under the price transparent regime. When prices are constrained to be linearly dependent (up to a constant) to avoid an informedness paradox, the resulting price transparent PD-NREE offers a potential alternative to the price opaque PD-NREE.

As in the price opaque PD-NREE defined in Appendix A, the informed agents submit their demand schedules as though operating under a Walrasian equilibrium,[17] that is, their signals are sufficient statistics for the information contained in prices so inference through the price channel is ignored.

Uninformed agents in the standard PD-NREE submit demand functions $x^U\left(p^U\right)$. Implementing transparency without altering market power complicates the uninformed agents' demand functions by allowing the uninformed agents to condition on p^U and p^I; hence, demand for the uninformed agents will be $x^U\left(p^U, p^I\right)$. Assume as in the PD-NREE and the NREE that prices are linear in signal and noise innovations. Define the price function constants as

$$
\begin{aligned}
p^U &= \tilde{A} + \tilde{B}\left[\left(s-\bar{d}\right)-\tilde{C}u\right] \\
p^I &= \tilde{D} + \tilde{F}\left[\left(s-\bar{d}\right)-\tilde{G}u\right]
\end{aligned}
$$

Consider the two cases in which price functions are linearly dependent or linearly independent up to the

[17] By Lemma 1, neither class of agent will act strategically.

constant terms. If prices are linearly dependent, that is, $\tilde{C} = \tilde{G}$, informed agents will submit differentiated demand schedules and price discrimination will obtain. The demand for the uninformed agents becomes

$$\begin{aligned} x^U\left(p^U, p^I\right) &= \frac{E\left[d|p\right] - (1+r)\, p^U}{\alpha\sigma^2_{d|p^U}} \\ &= \frac{\bar{d} + \beta_{p^U}\left(p^U - \tilde{A}\right) - (1+r)\, p^U}{\alpha\sigma^2_{d|p^U}} \end{aligned}$$

as p^I offers no additional information due to the linear dependence of prices. The solution method in the price transparent PD-NREE is the same as in Appendix A with the exception that $\tilde{C}$ is imposed to be the same as $\tilde{G}$. Equality of $\tilde{C}$ and $\tilde{G}$ is equivalent to $p^U = a + bp^I$ for some constants a and b. The analogous market-clearing condition in the price transparent PD-NREE is

$$N_I \frac{\bar{d} + \beta_s\left(s - \bar{d}\right) - (1+r)\, p^I}{\alpha\sigma^2_{d|s}} + N_U \frac{\bar{d} + b\beta_{p^U}\left(p^I - \tilde{D}\right) - (1+r)\left(a + bp^I\right)}{\alpha\sigma^2_{d|p^U}} = S + u$$

which delivers three equations in three unknowns by coefficient matching

$$\begin{aligned} N_I \frac{\bar{d} - (1+r)\,\tilde{D}}{\alpha\sigma^2_{d|s}} + N_U \frac{\bar{d} - (1+r)\left(a + b\tilde{D}\right)}{\alpha\sigma^2_{d|p^U}} &= S \\ N_I \frac{\beta_s - (1+r)\,\tilde{F}}{\alpha\sigma^2_{d|s}} + N_U b\tilde{F} \frac{\beta_{p^U} - (1+r)}{\alpha\sigma^2_{d|p^U}} &= 0 \\ \tilde{F}\tilde{G}\left[N_I \frac{1+r}{\alpha\sigma^2_{d|s}} + N_U b \frac{-\beta_{p^U} + (1+r)}{\alpha\sigma^2_{d|p^U}}\right] &= 1 \end{aligned} \tag{A.5}$$

The Lagrangian and non-redundant first-order-conditions for the monopolist are

$$\begin{aligned} \mathcal{L} &= N_I x^I\left(p^I\right)\left(p^I - \lambda\right) + N_U x^U\left(a + bp^I\right)\left(a + bp^I - \lambda\right) + \lambda\left(S + u\right) \\ 0 &= \bar{d} + b\beta_{p^U}\left(p^I - \tilde{D}\right) - (1+r)\left(2a + 2bp^I - \lambda\right) \\ 0 &= \bar{d}p^I - a\beta_{p^U}\left(p^I - \tilde{D}\right) + \left[\beta_{p^U}\left(p^I - \tilde{D}\right) - (1+r)\, p^I\right]\left(2a + 2bp^I - \lambda\right) \\ 0 &= -N_I \frac{(1+r)\left(2p^I - \lambda\right)}{\alpha\sigma^2_{d|s}} + N_U b\left[\frac{\bar{d} + \beta_{p^U}\left(p^I - \tilde{D}\right) - (1+r)\left(a + bp^I\right)}{\alpha\sigma^2_{d|p^U}} b + \frac{\beta_{p^U} - (1+r)}{\alpha\sigma^2_{d|p^U}}\left(a + bp^I - \lambda\right)\right] \end{aligned}$$

The six equation system uniquely identifies the six unknowns a, b, $\tilde{D}$, $\tilde{F}$, $\tilde{G}$, and λ. The first and second first-order-conditions combine to give

$$\left[(1+r)\left(a + b\tilde{D}\right) - \bar{d}\right] + b\left(p^I - \tilde{D}\right)\left[(1+r) - \beta_{p^U}\right] = 0$$

which yields two equations by coefficient matching

$$\begin{aligned}\tilde{A} \equiv a + b\tilde{D} &= \frac{\bar{d}}{1+r} \\ \tilde{B} \equiv b\tilde{F} &= \frac{1}{1+r}\frac{\sigma^2}{\sigma^2 + \sigma_\epsilon^2 + \tilde{C}^2\sigma_u^2}\end{aligned}$$

Combining this result with Equations A.5 delivers

$$\begin{aligned}\tilde{F} &= \frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} \\ \tilde{D} &= \frac{\bar{d}}{1+r} - \frac{S\alpha\sigma_{d|s}^2}{(1+r)\,N_I} \\ \tilde{G} &= \frac{\alpha\sigma_\epsilon^2}{N_I} \\ a &= \frac{1}{\sigma^2+\sigma_\epsilon^2+\tilde{C}^2\sigma_u^2}\left[\frac{\bar{d}}{1+r}\tilde{C}^2\sigma_u^2 + \frac{S\alpha}{1+r}\frac{\sigma^2\sigma_\epsilon^2}{N_I}\right] \\ b &= \frac{\sigma^2+\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2+\tilde{C}^2\sigma_u^2}\end{aligned}$$

So the price functions in the PD-NREE with price transparency are

$$\begin{aligned}p^I &= \left[\frac{\bar{d}}{1+r} - \frac{S\alpha\sigma_{d|s}^2}{(1+r)\,N_I}\right] + \frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\left[\left(s-\bar{d}\right) - \frac{\alpha\sigma_\epsilon^2}{N_I}u\right] \\ p^U &= \frac{\bar{d}}{1+r} + \frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\frac{\sigma^2+\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2+\tilde{C}^2\sigma_u^2}\left[\left(s-\bar{d}\right) - \frac{\alpha\sigma_\epsilon^2}{N_I}u\right]\end{aligned}$$

The price functions are linear in the signal and supply noise, confirming the conjectured linearity. As in the other NREEs presented, non-linear pricing equilibria may also exist.

A.6.1 Linearly Independent Pricing Functions

If prices are linearly independent, then the signal of the informed agent can be perfectly extracted by taking a linear combination of p^U and p^I to solve for $s - \bar{d}$, namely

$$s - \bar{d} = \left(\frac{\tilde{G}}{\tilde{G}-\tilde{C}}\right)\left[\frac{p^U - \tilde{A}}{\tilde{B}} - \frac{\tilde{C}}{\tilde{G}}\left(\frac{p^I - \tilde{D}}{\tilde{F}}\right)\right]$$

The uninformed agents would then submit an identical demand schedule as the informed agents by identicality of the agents' information sets and the monopolist would no longer be able to distinguish agent types and price discriminate. This equilibrium is logically inconsistent as there will be no p^U's for the uninformed agents to observe if they submit indistinguishable demand schedules from the informed agents.

Allowing a measure zero set of uninformed agents to err by submitting uninformed demand schedules resolves the price observability paradox, but raises the concern that the monopolist might prefer to charge linearly independent prices rather than linearly dependent prices. The informed pooling equilibrium demands and prices would be given by

$$\begin{aligned} x\left(p^I_{\text{pool}}\right) &= \frac{\bar{d} + \left(s - \bar{d}\right)\beta_s - (1+r)\,p^I_{\text{pool}}}{\alpha\sigma^2_{d|s}} \\ p^I_{\text{pool}} &= \left[\frac{\bar{d}}{1+r} - \frac{S\alpha\sigma^2_{d|s}}{(1+r)\,N}\right] + \frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma^2_\epsilon}\left[\left(s-\bar{d}\right) - \frac{\alpha\sigma^2_\epsilon}{N}u\right] \end{aligned}$$

where p^U_{pool} is excluded as the price is given to only a measure zero set of agents and is otherwise unimportant other than in revealing $s - \bar{d}$. In fact, the monopolist would select the pooling equilibrium for very high signal or very low supply realizations.[18] Pooling equilibria are discussed in additional detail in Appendix C.

B Pricing Inequalities

B.1 Average Prices

Claim 1. For risk-averse traders ($\alpha > 0$), average prices for the informed agents D, the uninformed agents A, and for all agents in the non-price discriminating NREE A_{NREE} are related by

$$\tilde{D} < D < A_{NREE} < A < \tilde{A}$$

when $\sigma^2 + \sigma^2_\epsilon < C^2\sigma^2_u < 2\left(\sigma^2+\sigma^2_\epsilon\right)$, i.e., when the relative and bounded noise conditions hold. Furthermore

$$\tilde{D} < A_{NREE} < \tilde{A}$$

and

$$D < A$$

hold more generally and do not require additional noise boundedness assumptions.

[18] To see this, consider

$$\begin{aligned} \left[\frac{N_I}{N}p^I + \frac{N_U}{N}p^U\right] - p^I_{\text{pool}} &= \frac{1}{N}\left[N_I\left(p^I - p^I_{\text{pool}}\right) + N_U\left(p^U - p^I_{\text{pool}}\right)\right] \\ &= \frac{N_U}{(1+r)\,N}\frac{\sigma^2}{\sigma^2+\sigma^2_\epsilon}\left[2\frac{2S\alpha\sigma^2_\epsilon}{N} - \left(\frac{\tilde{C}^2\sigma^2_u}{\sigma^2+\sigma^2_\epsilon+\hat{C}^2\sigma^2_u}\right)\left[\left(s-\bar{d}\right) - \frac{\alpha\sigma^2_\epsilon}{N_I}u\right]\right] \end{aligned}$$

which is decreasing without bound in $s - \bar{d}$ and $-u$.

Proof. First, I will show that the average price for the informed agents D is less than that of the uninformed agents A for $\alpha > 0$. The unconditional average prices paid by the informed and uninformed agents are given by A and D. The difference between these is

$$\begin{aligned} E\left[p^I - p^U\right] &= (D - A) + 0 + 0 \\ &= \frac{-\frac{\sigma^2}{2} S \frac{\alpha}{1+r} \frac{\sigma^2 \sigma_\epsilon^2}{\sigma^2 + \sigma_\epsilon^2}}{\frac{1}{2}\sigma^2 N_I - \left(\sigma^2 - (1+r) B\left[\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right]\right)\left[N_I + N_U \frac{\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2)}{(\sigma^2+\sigma_\epsilon^2)(\sigma_\epsilon^2+C^2\sigma_u^2)}\right]} \end{aligned}$$

The numerator is negative for $\alpha > 0$. Hence, $D - A$ is negative if and only if

$$\begin{aligned} \frac{1}{2}\sigma^2 N_I - \left(\sigma^2 - (1+r) B\left[\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right]\right)\left[N_I + N_U \frac{\sigma_\epsilon^2\left(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right)}{\left(\sigma^2+\sigma_\epsilon^2\right)\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)}\right] &> 0 \\ \Leftrightarrow \frac{1}{2} N_I \frac{C^2\sigma_u^2}{\sigma^2 + \sigma_\epsilon^2} &> 0 \end{aligned}$$

which holds so long as $\alpha \neq 0$. Note that $D > A$ if $\alpha < 0$ because the numerator becomes positive while the denominator's sign is preserved.

The average prices in the price transparent PD-NREE are given by

$$\begin{aligned} \tilde{A} &= \frac{\bar{d}}{1+r} \\ \tilde{D} &= \frac{\bar{d}}{1+r} - \frac{S\alpha\sigma_{d|s}^2}{(1+r) N_I} \end{aligned}$$

and the average prices in the price opaque PD-NREE are

$$\begin{aligned} A &= \frac{\bar{d}}{1+r} + \frac{S \frac{\alpha}{1+r} \frac{\sigma^2\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}}{\frac{\frac{1}{2}\sigma^2 N_I}{\sigma^2 - (1+r)B[\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2]} - N_I - N_U \frac{\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2)}{(\sigma^2+\sigma_\epsilon^2)(\sigma_\epsilon^2+C^2\sigma_u^2)}} \\ D &= A - \frac{S\alpha\sigma_{d|s}^2}{(1+r)\left[N_I \frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2}\right]} \end{aligned}$$

$\tilde{A} > A$ if and only if the second term is negative, which is true when

$$\begin{aligned} \frac{\frac{1}{2}\sigma^2 N_I}{\sigma^2 - (1+r) B\left[\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right]} - N_I - N_U \frac{\sigma_\epsilon^2\left(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right)}{\left(\sigma^2+\sigma_\epsilon^2\right)\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)} &< 0 \\ \Leftrightarrow \frac{C^2\sigma_u^2}{\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2}\left[N_I + N_U \frac{\sigma_\epsilon^2\left(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right)}{\left(\sigma^2+\sigma_\epsilon^2\right)\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)}\right] &< 0 \end{aligned}$$

which holds when the relative noise condition does. Additionally, $\tilde{D} < D$ if and only if

$$\begin{aligned}
\frac{S\alpha\sigma_{d|s}^2}{(1+r)\left[N_I \frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2}\right]} - \frac{S\frac{\alpha}{1+r}\frac{\sigma^2\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}}{\frac{\frac{1}{2}\sigma^2 N_I}{\sigma^2-(1+r)B[\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2]} - N_I - N_U\frac{\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2)}{(\sigma^2+\sigma_\epsilon^2)(\sigma_\epsilon^2+C^2\sigma_u^2)}} - \frac{S\alpha\sigma_{d|s}^2}{(1+r)\,N_I} &< 0 \\
\Leftrightarrow \frac{1}{N_I\frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2}} - \frac{1}{\frac{\frac{1}{2}\sigma^2 N_I}{\sigma^2-(1+r)B[\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2]} - N_I - N_U\frac{\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2)}{(\sigma^2+\sigma_\epsilon^2)(\sigma_\epsilon^2+C^2\sigma_u^2)}} - \frac{1}{N_I} &< 0 \\
< \frac{1}{\frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2}} - \frac{1}{\frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2}} - 1 &= 0
\end{aligned}$$

that is, so long as $N_U > 0$, as is required for non-pooling to obtain, regardless, $\tilde{D} < D$.

Finally, the average price in the standard NREE is given by

$$A_{NREE} = \frac{\bar{d}}{1+r} - \frac{S\alpha\sigma_{d|s}^2}{(1+r)\left(N_I + N_U\frac{\sigma_{d|s}^2}{\sigma_{d|p}^2}\right)}$$

Note that $\tilde{A} > A_{NREE}$ is immediate, as the second term in the expression for A_{NREE} is negative. Furthermore, we have that $\tilde{D} < A_{NREE}$ as the number of uninformed agents is positive. Hence, the result

$$\tilde{D} < A_{NREE} < \tilde{A}$$

does not depend on the relative noise condition as do the comparisons between PD-NREE concepts.

For use throughout this proof, define

$$\begin{aligned}
K_0 &\equiv N_I + N_U\frac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}\left(1+\frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right) \\
K_1 &\equiv N_U\left[\frac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}\left[\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2} - \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right] - \frac{\sigma_\epsilon^2\left[\frac{N_I}{N_U}\left(\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}\right)+\left(\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)\right]}{\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2}\right] \\
K_2 &\equiv \frac{S\alpha\sigma_{d|s}^2}{1+r}\times N_I\frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2}\times K_0\times K_1
\end{aligned}$$

For the uninformed agents, the average price difference between the price opaque PD-NREE and NREE is

$$\begin{aligned}
& A - A_{NREE} \\
= & \frac{S\alpha\sigma_{d|s}^2}{(1+r)\left(N_I+N_U\frac{\sigma_\epsilon^2\left(\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2\right)}{(\sigma^2+\sigma_\epsilon^2)\left(\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2\right)}\right)} - \frac{S\alpha\sigma_{d|s}^2}{(1+r)\left[N_I+N_U\frac{\sigma_\epsilon^2(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2)}{(\sigma^2+\sigma_\epsilon^2)(\sigma_\epsilon^2+C^2\sigma_u^2)}\right] - \frac{\frac{1}{2}\sigma^2 N_I(1+r)}{\sigma^2-(1+r)B[\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2]}} \\
= & \frac{S\alpha\sigma_{d|s}^2}{1+r}\left[\frac{1}{K_0} - \frac{1}{K_0+K_1}\right]
\end{aligned}$$

As the non-K_1 component of each term is identical and positive, the overall expression will be positive if and only if either $K > 0$ or the second term is negative. Rearranging the second term obtains

$$K_0 + K_1 = -\frac{C^2\sigma_u^2}{\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2}\left[\frac{N_I}{N_U} + \frac{\sigma_\epsilon^2}{\sigma^2 + \sigma_\epsilon^2}\left(\frac{\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2}{\sigma_\epsilon^2 + C^2\sigma_u^2}\right)\right] \tag{B.1}$$

Which is negative if and only if $\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2 > 0$. Hence, $A > A_{NREE}$ if the relative noise condition fails. If the relative noise condition holds, the denominator will be positive and $A - A_{NREE}$ will be the same sign as K_1. As the only other cases to be considered require the relative noise condition to hold, I assume $\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2 < 0$ for the remainder of the uninformed agent portion of this proof. Rewriting K_1 obtains

$$K_1 = \frac{N_U}{\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2}\left[\frac{\sigma^2}{\sigma^2 + \sigma_\epsilon^2}\left[\frac{-\frac{3}{4}C^2\sigma_u^2\left(\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2\right)}{\sigma_\epsilon^2 + \frac{1}{4}C^2\sigma_u^2}\right] - \left[\frac{N_I}{N_U}\left(\frac{\sigma^2 + \sigma_\epsilon^2}{\sigma_\epsilon^2}\right)\left(\sigma_\epsilon^2 + C^2\sigma_u^2\right) + \sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right]\right]$$

Therefore, when the relative noise condition holds, $A - A_{NREE} > 0$ if and only if

$$\begin{aligned}
\frac{\sigma^2}{\sigma^2 + \sigma_\epsilon^2}\left[\frac{\frac{3}{4}C^2\sigma_u^2\left(\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2\right)}{\sigma_\epsilon^2 + \frac{1}{4}C^2\sigma_u^2}\right] + \left[\frac{N_I}{N_U}\left(\frac{\sigma^2 + \sigma_\epsilon^2}{\sigma_\epsilon^2}\right)\left(\sigma_\epsilon^2 + C^2\sigma_u^2\right) + \sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right] &> 0 \\
\Leftrightarrow 3\frac{\sigma^2}{\sigma^2 + \sigma_\epsilon^2}\left[\left(\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2\right) - \frac{\sigma_\epsilon^2\left(\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2\right)}{\sigma_\epsilon^2 + \frac{1}{4}C^2\sigma_u^2}\right] &+ \\
\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2 + \frac{N_I}{N_U}\frac{\left(\sigma^2 + \sigma_\epsilon^2\right)\left(\sigma_\epsilon^2 + C^2\sigma_u^2\right)}{\sigma_\epsilon^2} &> 0
\end{aligned} \tag{B.2}$$

Equation B.2 cannot be signed without making further assumptions. To see this, I can rewrite B.2 as

$$\begin{aligned}
\frac{-\frac{3}{4}C^2\sigma_u^2\left(\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2\right)}{\left(\sigma_\epsilon^2 + \frac{1}{4}C^2\sigma_u^2\right)} &< \frac{\sigma^2 + \sigma_\epsilon^2}{\sigma^2}\left(\sigma^2 + \sigma_\epsilon^2 + C^2\sigma_u^2\right) \\
\frac{C^2\sigma_u^2}{\sigma_\epsilon^2 + \frac{1}{4}C^2\sigma_u^2}\left[\frac{3}{4}C^2\sigma_u^2 - \frac{\sigma^2 + \sigma_\epsilon^2}{\sigma^2}\left(\sigma_\epsilon^2 + \frac{1}{4}C^2\sigma_u^2 + \frac{3}{4}\sigma^2\right)\right] &< \frac{\sigma^2 + \sigma_\epsilon^2}{\sigma^2}\left(\sigma^2 + \sigma_\epsilon^2\right)
\end{aligned}$$

As $C^2\sigma_u^2 \to \infty$, $\frac{C^2\sigma_u^2}{\sigma_\epsilon^2 + \frac{1}{4}C^2\sigma_u^2} \to 4$ and the bracketed expression becomes infinite. Hence, the inequality will fail for $C^2\sigma_u^2$ large enough relative to the other noise sources in the model. Imposing the bounded noise condition delivers

$$\begin{aligned}
0 &< 2\left(\sigma^2+\sigma_\epsilon^2\right)-C^2\sigma_u^2 \\
-3\left(\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2\right) &< \sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2 \\
-3\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\left(\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2\right) &< \sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2 \\
-3\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\left(\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2\right) &< \sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2 \\
&+ 3\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\left[-\frac{\sigma_\epsilon^2\left(\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2\right)}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]+\frac{N_I}{N_U}\frac{\left(\sigma^2+\sigma_\epsilon^2\right)\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)}{\sigma_\epsilon^2} \qquad \text{(B.3)}
\end{aligned}$$

That is, if supply noise is not too dominant and the relative noise condition holds, $A > A_{NREE}$. Equation B.2 holds under looser conditions, but the intuition driving these is less clear.

Likewise for the informed agents, the difference in average price between equilibrium concepts is given by

$$\begin{aligned}
& D - A_{NREE} \\
= \ & (A - A_{NREE}) + \frac{\sigma^2}{2}\left[\frac{\frac{\bar{d}}{1+r}-A}{\sigma^2-(1+r)B\left[\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right]}\right] \\
= \ & (A - A_{NREE}) - S\frac{\alpha\sigma_{d|s}^2}{1+r}\frac{1}{N_I}\left[\frac{\sigma^2+\sigma_\epsilon^2}{C^2\sigma_u^2}\right] \\
= \ & \frac{S\alpha\sigma_{d|s}^2}{1+r}\left[\frac{1}{K_0}-\frac{1}{K_0+K_1}-\frac{1}{N_I\frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2}}\right] \\
= \ & -K_2\times\left[\left[N_I\left(\frac{\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2}\right)+N_U\frac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}\left(1+\frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right)\right]K_1+K_0^2\right]
\end{aligned}$$

Rewriting $D - A_{NREE}$ in terms of N_U^2, $N_I N_U$, and N_I^2 obtains:

$$\begin{aligned}
& -N_U K_2\times\left[N_I\frac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}\left[\left[\frac{\sigma^2+\sigma_\epsilon^2-2C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2}\right]\left(1+\frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right)\right.\right. \\
& +\left(\frac{\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2}\right)\left[\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}-\frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]-\left.\left(\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)\right] \\
& \left.-N_U\frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2}\left[\frac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}\left(1+\frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right)\right]\left[\frac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]\right]
\end{aligned}$$

Any relation derived for $D - A_{NREE}$ should hold for all (N_I, N_U) pairings, so the sign on the N_I and N_U terms should be the same. The N_U term is positive if and only if $\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2<0$. I consider three cases for the sign on the N_I term. The N_I term is proportional to

$$-\left[\frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2}\left(1+\frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right)+\left(1+\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)\right]$$

which in turn implies that the N_I term has sign

$$\operatorname{sign}(N_I) = \begin{cases} -, & \sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2 > 0 \\ +, & \sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2 < 0 \end{cases}$$

The relative noise condition alone thus determines the sign of $D - A_{NREE}$ as the signs on N_I and N_U agree.

Equations B.1 and B.2 deliver that $-K_1$ and hence $-K_2$ has sign

$$\operatorname{sign}(-K_2) = \begin{cases} +, & \sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2 > 0 \\ -, & \sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2 < 0 < 2\left(\sigma^2 + \sigma_\epsilon^2\right) - C^2\sigma_u^2 \\ \text{ambiguous}, & 2\left(\sigma^2 + \sigma_\epsilon^2\right) - C^2\sigma_u^2 < 0 \end{cases}$$

Therefore, $D - A_{NREE}$ will be the product of negative and positive terms so long as the bounded noise condition holds. In summary, $D < A_{NREE} < A$ if the relative and bounded noise condition obtains and is otherwise equivocal. □

B.2 Signal Sensitivity

Claim 2. Price sensitivity to the signal s is higher for the informed agents than for the uninformed agents in the PD-NREE. The price sensitivity in the non-price discriminating NREE B_{NREE} is in the intermediate range. That is,

$$0 \quad < \quad B < \tilde{B} < B_{NREE} < F < \tilde{F}$$

Additionally, $B < B_{NREE} < F$ holds more generally and does not require boundedness assumptions on $C^2\sigma_u^2$.

Proof. The price sensitivity to the signal in the NREE, B_{NREE}, is given by

$$B_{NREE} = \frac{N_I \frac{\sigma^2}{\sigma_\epsilon^2} + N_U \frac{\sigma^2}{\sigma_\epsilon^2 + \frac{1}{4}C^2\sigma_u^2}}{(1+r)\left[N_I \frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2} + N_U \frac{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]}$$

Maximizing (minimizing) B_{NREE} with respect to N_U/N_I obtains

$$\begin{aligned} \max_{N_U/N_I} B_{NREE} &= \frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} = \tilde{F} \\ \min_{N_U/N_I} B_{NREE} &= \frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2} = \tilde{B} \end{aligned}$$

delivering $\tilde{B} < B_{NREE} < \tilde{F}$ when N_U and N_I are positive. Note that the extrema are obtained by taking limits as N_U/N_I go to 0 and ∞ as no interior optima exist. Similarly, maximizing B with respect to N_U/N_I obtains

$$\max_{N_U/N_I} B_{NREE} = \frac{1}{1+r}\max\left\{\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}, \frac{1}{2}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\right\}$$

where the first argument is smaller if the relative noise condition holds. Clearly the first argument is less than $\tilde{B}$, as the denominator for $\tilde{B}$ is smaller. If the relative noise condition holds and the second argument is larger

$$\max_{N_U/N_I} B_{NREE} = \frac{1}{2}\frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} < \frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2} \Leftrightarrow \frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2} < 4$$

That is, $B < \tilde{B}$ is true if the bounded noise and relative noise conditions are true. Likewise, maximizing $F - \tilde{F}$ with respect to N_I/N_U

$$\begin{aligned}
\max_{N_U/N_I} F - \tilde{F} &= \max_{N_U/N_I} B + \frac{1}{2}\frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} - \frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} \\
&= \left[\max_{N_U/N_I} B\right] - \frac{1}{2}\frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} \\
&= \frac{1}{1+r}\max\left\{\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2} - \frac{1}{2}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}, \frac{1}{2}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} - \frac{1}{2}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\right\} \\
&= \frac{1}{1+r}\max\left\{\frac{\frac{1}{2}\sigma^2\left(\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2\right)}{\left(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right)\left(\sigma^2+\sigma_\epsilon^2\right)}, 0\right\}
\end{aligned}$$

which is zero when the relative noise condition holds and positive otherwise. Thus, under the relative noise condition, $\tilde{F} > F$ for any other choice of N_I/N_U, as B is thus decreasing in N_U/N_I–the latter argument corresponds to $\lim_{N_U/N_I} \to \infty$.

Comparing the NREE sensitivity with that for the uninformed agents in the price opaque PD-NREE, B, delivers

$$\begin{aligned}
& B_{NREE} - B \\
= \; & \frac{N_I\frac{\sigma^2}{\sigma_\epsilon^2} + N_U\frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}}{(1+r)\left[N_I\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2} + N_U\frac{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]} - \frac{N_I\frac{\sigma^2}{2\sigma_\epsilon^2} + N_U\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}}{(1+r)\left[N_I\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2} + N_U\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]} \\
= \; & \frac{\sigma^2\left[\frac{1}{2}\frac{N_I}{\sigma_\epsilon^2}\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)\left(\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2\right)\left(N+N_I\frac{\sigma^2}{\sigma_\epsilon^2}\right) + \frac{N_I}{\sigma_\epsilon^2}N_U\sigma^2\left(\frac{1}{2}\sigma_\epsilon^2-\frac{1}{4}C^2\sigma_u^2\right) + \frac{3}{4}N_UC^2\sigma_u^2\left(N+N_I\frac{\sigma^2}{\sigma_\epsilon^2}\right)\right]}{(1+r)\left(N+N_I\frac{\sigma^2}{\sigma_\epsilon^2}+N_U\frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right)\left(N+N_I\frac{\sigma^2}{\sigma_\epsilon^2}+N_U\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)\left(\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2\right)}
\end{aligned}$$

The denominator of $B_{NREE} - B$ is positive. Turning attention to the numerator, the expression is positive if and only if

$$\frac{1}{2}\frac{N_I}{\sigma_\epsilon^2}\left(\sigma_\epsilon^2 + C^2\sigma_u^2\right)\left(\sigma_\epsilon^2 + \frac{1}{4}C^2\sigma_u^2\right)\left(N + N_I\frac{\sigma^2}{\sigma_\epsilon^2}\right) + \frac{1}{2}N_I N_U\sigma^2 + \frac{3}{4}N_U C^2\sigma_u^2\left(N + N_I\frac{\sigma^2}{\sigma_\epsilon^2}\right) > \frac{1}{4}C^2\sigma_u^2\frac{N_I}{\sigma_\epsilon^2}N_U\sigma^2$$

$$\Leftrightarrow \frac{1}{2}\frac{N_I}{\sigma_\epsilon^2}\left(\sigma_\epsilon^2 + C^2\sigma_u^2\right)\left(\sigma_\epsilon^2 + \frac{1}{4}C^2\sigma_u^2\right)\left(N + N_I\frac{\sigma^2}{\sigma_\epsilon^2}\right) + \frac{1}{2}N_I N_U\sigma^2 + \frac{3}{4}N_U C^2\sigma_u^2 N + \frac{1}{2}N_U C^2\sigma_u^2 N_I\frac{\sigma^2}{\sigma_\epsilon^2} > 0$$

which holds as all terms are positive. Repeating the analogous calculation for B_{NREE} and the price sensitivity to the signal for the informed agents in the price opaque PD-NREE, F gives

$$\begin{aligned}
& B_{NREE} - F \\
= \;& B_{NREE} - B - \frac{1}{2}\frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} \\
= \;& \frac{\left[\frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right] - \left[\frac{1}{2}\frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]}{(1+r)\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]} \\
- \;& \frac{\frac{1}{2}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]}{(1+r)\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]} \\
= \;& \frac{\frac{1}{2}\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]\left[-\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}\left(1 + \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)\right] + \left(1 + \frac{N_I}{N_U}\right)\left[\frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2} - \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]}{(1+r)\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]} \\
= \;& \frac{-\frac{1}{2}\left(\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)\left[\frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right] + \frac{N}{N_U}\left[\frac{-\frac{1}{2}\sigma_\epsilon^2\left(\sigma^2+\sigma_\epsilon^2\right)+\frac{1}{8}C^2\sigma_u^2\left[\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2\right]}{\left(\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2\right)\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)}\right]}{(1+r)\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right]\left[\frac{N}{N_U} + \frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right]\frac{\sigma^2+\sigma_\epsilon^2}{\sigma^2}}
\end{aligned}$$

The denominator is positive. All terms in the numerator are negative except for the second multiplying N/N_U. Restricting attention to the numerator and hypothesizing that $B_{NREE} - F < 0$ obtains

$$\begin{aligned}
\frac{N}{N_U}\left[\frac{-\frac{1}{2}\sigma_\epsilon^2\left(\sigma^2+\sigma_\epsilon^2\right)+\frac{1}{8}C^2\sigma_u^2\left[\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2\right]}{\left(\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2\right)\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)}\right] &< \frac{1}{2}\left(\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)\left[\frac{N_I}{N_U}\frac{\sigma^2}{\sigma_\epsilon^2} + \frac{\sigma^2}{\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}\right] \\
\Leftrightarrow \frac{1}{4}C^2\sigma_u^2\left[\sigma_\epsilon^2 - 3\sigma^2 - C^2\sigma_u^2 - \frac{\sigma^2}{\sigma_\epsilon^2}\frac{N_I}{N}\left(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right)\right] &< \left(\sigma^2+\sigma_\epsilon^2\right)^2
\end{aligned}$$

Increasing σ^2 or N_I only decreases the left-hand-side and makes this condition easier to satisfy. I set $\sigma^2 = N_I = 0$ to show the tighter bound of

$$\begin{aligned}
\frac{1}{4}C^2\sigma_u^2\left[\sigma_\epsilon^2 - C^2\sigma_u^2\right] &< \left(\sigma_\epsilon^2\right)^2 \leq \left(\sigma^2+\sigma_\epsilon^2\right)^2 + \frac{1}{4}C^2\sigma_u^2\left(3\sigma^2 + \frac{\sigma^2}{\sigma_\epsilon^2}\frac{N_I}{N}\left(\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2\right)\right) \\
\Leftrightarrow 0 &< \left(2\sigma_\epsilon^2 - C^2\sigma_u^2\right)^2 + 3\sigma_\epsilon^2 C^2\sigma_u^2
\end{aligned}$$

which holds because all variances are positive, confirming the hypothesis. Hence $B_{NREE} < F$. □

B.3 Price Informativeness

Claim 3. *Price informativeness conditional on the uninformed agents' price is lower in the price opaque PD-NREE than in the NREE. The precision of the next-period payoff conditional on the informed agent's price is higher in the price opaque PD-NREE than in the NREE if* $\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2<0$. *Price informativeness in the price transparent PD-NREE is equal to that of the standard NREE. Taken together*

$$\begin{cases} \tau_{p^U} < \tilde{\tau}_{p^U} = \tau_{NREE} = \tilde{\tau}_{p^I} < \tau_{p^I}\,, & \sigma^2+\sigma_\epsilon^2 - C^2\sigma_u^2 < 0 \\ \tau_{p^U} < \tilde{\tau}_{p^U} = \tau_{NREE} = \tilde{\tau}_{p^I} \lesseqgtr \tau_{p^I}\,, & \sigma^2+\sigma_\epsilon^2 - C^2\sigma_u^2 > 0 \end{cases}$$

Proof. Price informativeness in the canonical NREE is given by the variance of the next-period dividend conditional on the price

$$\tau_{NREE} = \frac{1}{\sigma^2_{d|p}} = \frac{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}{\sigma^2\left(\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2\right)}$$

In the price transparent PD-NREE, price informativeness is given by

$$\tilde{\tau}_{p^U} = \frac{1}{\sigma^2_{d|p^U}} = \frac{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}{\sigma^2\left(\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2\right)} = \frac{1}{\sigma^2_{d|p^I}} = \tilde{\tau}_{p^I} = \tau_{NREE}$$

which is equal to that of the NREE. Subtracting τ_{NREE} from the informativeness measure based on the price observed by the uninformed agent in the price opaque PD-NREE obtains

$$\tau_{p^U} = \frac{1}{\sigma^2_{d|p^U}} = \frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma^2\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)}$$

Comparing this measure to that from the NREE obtains

$$\begin{aligned} \tau_{p^U} - \tau_{NREE} &= \frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma^2\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)} - \frac{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}{\sigma^2\left(\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2\right)} \\ &= \frac{\left(\frac{1}{4}C^2 - C^2\right)\sigma_u^2\sigma^2}{\sigma^2\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)\left(\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2\right)} < 0 \end{aligned} \tag{B.4}$$

So the price informativeness from the perspective of the uninformed trader is less in the price opaque PD-NREE. Computing τ_{p^I} obtains, by analogy

$$\tau_{p^I} = \frac{1}{\sigma^2_{d|p^I}} = \frac{\sigma^2+\sigma_\epsilon^2+G^2\sigma_u^2}{\sigma^2\left(\sigma_\epsilon^2+G^2\sigma_u^2\right)}$$

Comparing with τ_{NREE}, it remains only to be shown whether $G > \frac{1}{4}C$. $\tau_{p^I} - \tau_{NREE} > 0$ if and only if $G < \frac{1}{4}C$. Rewriting G and working through the inequality obtains

$$
\begin{aligned}
C\frac{B}{B + \frac{1}{2}\frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}} &\lesseqgtr \frac{1}{4}C \\
6B &\lesseqgtr \frac{1}{1+r}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} \\
\frac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}N_U\left(\frac{\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right) &\lesseqgtr -\frac{2}{3}\left[N_I + \frac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}N_U\left(\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)\right] \\
5\sigma^2+5\sigma_\epsilon^2-C^2\sigma_u^2 &\lesseqgtr -2\frac{N_I}{N_U}\frac{\sigma_\epsilon^2\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)}{\sigma^2+\sigma_\epsilon^2}
\end{aligned}
$$

From the penultimate line, $\tau_{p^I} > \tau_{NREE}$ if the relative noise condition holds, i.e., if $\sigma^2+\sigma_\epsilon^2 - C^2\sigma_u^2$. The relative noise condition is sufficient, but not necessary. For $\tau_{p^I} > \tau_{NREE}$ for all ${}^{N_I}/{}_{N_U}$, only a weak version of the bounded noise condition need hold, that is

$$5\sigma^2+5\sigma_\epsilon^2-C^2\sigma_u^2 > 0$$

The average price informativeness cannot be signed

$$\tau_{p^{AVE}} - \tau_{NREE} = \left[\frac{N_I}{N}\frac{\sigma^2+\sigma_\epsilon^2+G^2\sigma_u^2}{\sigma^2\left(\sigma_\epsilon^2+G^2\sigma_u^2\right)} + \frac{N_U}{N}\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma^2\left(\sigma_\epsilon^2+C^2\sigma_u^2\right)}\right] - \frac{\sigma^2+\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2}{\sigma^2\left(\sigma_\epsilon^2+\frac{1}{4}C^2\sigma_u^2\right)}$$

as the relation depends on the relative size of N_I and N_U. □

B.4 Distributional Consequences

Claim 4. Prices (quantities) for the uninformed agents are higher (lower) on average in the PD-NREE than in the non-price discriminating NREE if $2\left(\sigma^2+\sigma_\epsilon^2\right) - C^2\sigma_u^2 > 0$, *but are lower (higher) for sufficiently high realizations of* s *or* u. *Prices (quantities) for the informed agents are lower (higher) on average in the PD-NREE if* $2\left(\sigma^2+\sigma_\epsilon^2\right) - C^2\sigma_u^2 > 0$, *but are higher (lower) for sufficiently high realizations of* s *or low realizations of* u.

Proof. Comparing prices in the PD-NREE to those of the standard NREE obtains

$$
\begin{aligned}
p^U - p_{NREE} &= \left(A - A_{NREE}\right) + \left(B - B_{NREE}\right)\left(s - \bar{d}\right) - \left(BC - B_{NREE}C_{NREE}\right)u \\
p^I - p_{NREE} &= \left(D - A_{NREE}\right) + \left(F - B_{NREE}\right)\left(s - \bar{d}\right) - \left(FG - B_{NREE}C_{NREE}\right)u
\end{aligned}
$$

For the uninformed agents, the first term is negative if the bounded noise condition holds. The second coefficient is unambiguously negative and the third coefficient is negative (positive) in the price opaque and price transparent PD-NREEs, respectively. Hence, the price paid by the uninformed agents may be higher or lower in the price discriminating REE, on average, but will tend toward being lower when the signal or supply realizations are high (or low for supply realizations in the price transparent case). The informed agents have a lower average price in the PD-NREEs if the bounded noise condition is true, but face a larger magnitude slope with respect to s and u than in the NREE. Hence, under the bounded noise condition, $p^I - p_{NREE}$ will be positive for sufficiently high signal or low supply realizations.

Quantities for the informed agents vary inversely with prices. The informed agents' demand is given by

$$x^I\left(p^I\right) = \frac{E\left[d_{t+1}|s_t\right] - \left(1 + r_{t+1}\right)p_t^I}{\alpha\sigma_{d|s}^2}$$

under both equilibrium concepts. As all non-price parameters are invariant across NREE types, I can compare prices directly to establish demand relations. Hence, for $x^I > x_{NREE}^I$ for $p^I < p_{NREE}$. As the bond issue supply is fixed, the uninformed agents capture the residual supply by the market clearing condition. Therefore $x^U > x_{NREE}^U$ for $p^I > p_{NREE}$. On average, then, informed agents will accrue more of the bond issue in the PD-NREE than in the NREE. $\square$

C Equilibrium Selection

In this section, I introduce a more general form of the PD-NREE in which uninformed agents and the monopolistic dealer observes a partially informative signal

$$\hat{s} = \rho s + \sqrt{1-\rho^2}z,\ z \overset{i.i.d.}{\sim} N\left(0, \sigma^2 + \sigma_\epsilon^2\right)$$

Note that this model is asymptotically equivalent to the PD-NREE when $\rho \to 0$. I show that for ρ in an ϵ-neighborhood of 0, the equilibrium price coefficients will be close to their PD-NREE counterparts. Taking $\epsilon \to 0$ while preserving $\rho \neq 0$ obtains approximately identical pricing functions that preclude the equilibrium in which informed agents choose to pool in their demand functions. The intuition for eliminating the pooling equilibrium is that, by introducing an iota of noise in the inference of the uninformed agent, the informed agents no longer know the uninformed agents' conditional expectation of the next-period value of the risky asset and cannot perfectly mimic their demand schedules. Crucially, the monopolist sees the signal of the uninformed agent and can therefore identify the agent type by the demand schedule she submits.[19][20]

The proof proceeds as follows. I first outline the form of the PD-NREE with the auxiliary signal. I then show that z cannot be inferred by the informed agents under either equilibrium concept. I next conjecture, but do not completely show, that pricing functions are continuous in ρ and that $\rho = 0$ is associated with a different set of pricing functions from $\rho > 0$. I conclude the proof by demonstrating that $\rho = 0$ reduces to the PD-NREE.

C.1 Preliminaries

I provide conditional expectations and variances of the next-period payoff, as I use these frequently in solving the price discriminating noisy rational expectations equilibrium with the additional signal. The informed agents condition on their signal and see, as before

$$\begin{aligned} E\left[d|s,p^I\right] &= \bar{d} + \left(s-\bar{d}\right)\beta_s \\ \sigma^2_{d|s} &= var\left(d\right) - \beta_s^2 var\left(s\right) = \frac{\sigma^2\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2} \\ \beta_s &= \frac{cov\left(d,s\right)}{var\left(s\right)} = \frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} \end{aligned}$$

whereas the uninformed agents condition on their price p^U and noisy signal $\hat{s}$ and perceive

$$\begin{aligned} E\left[d|\hat{s},p^U\right] &= \bar{d} + \left(p^U - E\left[p^U\right]\right)\beta_{p^U} + \left(\hat{s} - E\left[\hat{s}\right]\right)\beta_{\hat{s}} \\ &= \bar{d} + \left(p^U - E\left[p^U\right]\right)\beta_{p^U} + \left(\rho\left(s-\bar{d}\right) + \sqrt{1-\rho^2}z\right)\beta_{\hat{s}} \\ \sigma^2_{d|p^U} &= var\left(d\right) - \beta_{p^U}^2 var\left(p^U\right) - \beta_{\hat{s}}^2 var\left(\hat{s}\right) - 2\beta_{p^U}\beta_{\hat{s}} cov\left(p^U,\hat{s}\right) \end{aligned}$$

I conjecture that the price for the uninformed agents p^U is linear in signal and supply innovations as in the standard NREE

$$p^U = A + B\left[\left(s-\bar{d}\right) - Cu + Hz\right]$$

[19] The alternative case being that the informed agent mimics the demand schedule of a hypothetical uninformed agent but the monopolist cannot distinguish between the agent classes. Price discrimination will obtain in modified form with the outcome being two separating equilibria rather than one equilibrium of each variety.

[20] Supplementing the information set of the uninformed agents with an ancillary signal has no effect on the monopolistic dealer's pooling of pricing functions; the dimensionality of independent stochastic terms (s,u,z) is equal to the dimensionality of signals (p^U, p^I, $\hat{s}$), so linearly independent pricing functions are still fully revealing.

Computing the constants used in the expressions above assuming this linear pricing function obtains

$$
\begin{aligned}
var\left(\hat{s}\right) &= var\left(\rho s+\sqrt{1-\rho^2}z\right)=var\left(s\right)=\left(\sigma^2+\sigma_\epsilon^2\right)\\
cov\left(p^U,\hat{s}\right) &= B\left(\sigma^2+\sigma_\epsilon^2\right)\left[\rho+H\sqrt{1-\rho^2}\right]\\
cov\left(\hat{s},d\right) &= \rho\sigma^2\\
var\left(p^U\right) &= B^2\left[\left(1+H^2\right)\left(\sigma^2+\sigma_\epsilon^2\right)+C^2\sigma_u^2\right]\\
cov\left(p^U,d\right) &= B\sigma^2
\end{aligned}
$$

C.2 General Demand Functions

As the unconditional expectation of the signal is $\bar{d}$ and of the supply innovation is 0, $E\left[p^U\right]=A$. Plugging in the expression for p^U and $\hat{s}$ gives the next-period payoff sensitivity to price β_{p^U} and to the noisy signal $\hat{s}$ as

$$
\begin{aligned}
\beta_{p^U} &= \frac{var\left(\hat{s}\right)cov\left(p^U,d\right)-cov\left(p^U,\hat{s}\right)cov\left(\hat{s},d\right)}{var\left(p^U\right)var\left(\hat{s}\right)-cov\left(p^U,\hat{s}\right)cov\left(p^U,\hat{s}\right)}\\
&= \frac{M}{\sqrt{\sigma^2+\sigma_\epsilon^2}}\left(\frac{1}{B}\left(1-\rho K_2\right)\right)\\
\beta_{\hat{s}} &= \frac{var\left(p^U\right)cov\left(\hat{s},d\right)-cov\left(p^U,\hat{s}\right)cov\left(p^U,d\right)}{var\left(p^U\right)var\left(\hat{s}\right)-cov\left(p^U,\hat{s}\right)cov\left(p^U,\hat{s}\right)}\\
&= \frac{M}{\sqrt{\sigma^2+\sigma_\epsilon^2}}K_3
\end{aligned}
$$

where

$$
\begin{aligned}
M &\equiv \frac{\sigma^2\sqrt{\sigma^2+\sigma_\epsilon^2}}{\left(\sigma^2+\sigma_\epsilon^2\right)\left[\rho H+\sqrt{1-\rho^2}\right]^2+C^2\sigma_u^2}\\
K_1 &= H^2+\frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2}\\
K_2 &= \rho+H\sqrt{1-\rho^2}\\
K_3 &= K_1\rho-H\sqrt{1-\rho^2}
\end{aligned}
$$

Finally, the conditional variance $\sigma_{d|p^U}^2$ of the next-period payoff given the uninformed agents' price is

$$
\begin{aligned}
\sigma_{d|p^U}^2 &= var\left(d\right)-\beta_{p^U}^2 var\left(p^U\right)-\beta_{\hat{s}}^2 var\left(\hat{s}\right)-2\beta_{p^U}\beta_{\hat{s}}cov\left(p^U,\hat{s}\right)\\
&= \sigma^2-M^2\left[\left(1-\rho K_2\right)^2\left(1+K_1\right)+K_3^2+2\left(1-\rho K_2\right)K_2K_3\right]
\end{aligned}
$$

Substituting p^U into the expression for $E\left[d|p^U\right]$ gives

$$\begin{aligned}
E\left[d|\hat{s},p^U\right] &= E\left[d\right]+\left(p^U-E\left[p^U\right]\right)'\beta_p+\left(\rho\left(s-\bar{d}\right)+\sqrt{1-\rho^2}z\right)'\beta_{\hat{s}}\\
&= \bar{d}+\left(p^U-E\left[p^U\right]\right)\beta_{p^U}+\left(\rho\left(s-\bar{d}\right)+\sqrt{1-\rho^2}z\right)\beta_{\hat{s}}\\
&= \bar{d}+\left[\left(s-\bar{d}\right)\left(1-\rho K_2+\rho K_3\right)-\left(1-\rho K_2\right)Cu+\left(H\left(1-\rho K_2\right)+K_3\sqrt{1-\rho^2}\right)z\right]\frac{M}{\sqrt{\sigma^2+\sigma_\epsilon^2}}\\
&= \bar{d}+\left[\left(s-\bar{d}\right)\left(\left(1-\rho\right)-\rho K_1\right)-\left(1-\rho K_2\right)Cu+\rho\sqrt{1-\rho^2}\frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2}z\right]\frac{M}{\sqrt{\sigma^2+\sigma_\epsilon^2}}
\end{aligned}$$

CARA utility delivers immediately the optimal demand of the uninformed $x^U\left(p^U\right)$ in terms of expected payoffs, payoff variance, and risk aversion

$$\begin{aligned}
x^U\left(p^U\right) &= \frac{E\left[d|p\right]-\left(1+r\right)p^U}{\alpha\sigma_{d|p^U}^2}\\
&= \frac{\bar{d}+\left[\left(s-\bar{d}\right)\left(\left(1-\rho\right)-\rho K_1\right)-\left(1-\rho K_2\right)Cu+\rho\sqrt{1-\rho^2}\frac{C^2\sigma_u^2}{\sigma^2+\sigma_\epsilon^2}z\right]\frac{M}{\sqrt{\sigma^2+\sigma_\epsilon^2}}-\left(1+r\right)p^U}{\alpha\left(\sigma^2-M^2\left[\left(1-\rho K_2\right)^2\left(1+K_1\right)+K_3^2+2\left(1-\rho K_2\right)K_2K_3\right]\right)}
\end{aligned}$$

Note that so long as A, B, C, and H are constants, x^U will be linear in p^U.

Likewise, I conjecture that for the informed agents, the price p^I is linear in signal and supply innovations

$$p^I = D+F\left[\left(s-\bar{d}\right)-Gu+Kz\right]$$

For the informed agents, conditioning on the signal gives a demand function $x^I\left(p^I\right)$ only of model primitives except for where the price explicitly enters

$$x^I\left(p^I\right)\equiv x^I\left(p^I;s\right)=\frac{E\left[d|s\right]-\left(1+r\right)p^I}{\alpha\sigma_{d|s}^2}=\frac{\bar{d}+\left(s-\bar{d}\right)\beta_s-\left(1+r\right)p^I}{\alpha\sigma_{d|s}^2}$$

As in the standard PD-NREE, the PD-NREE with $\hat{s}$ applies market clearing and profit-maximization to obtain nonlinear equations for the pricing function unknowns

$$\begin{aligned}
N_I x^I\left(p^I\right)+N_U x^U\left(p^U\right) &= S+u\\
\left(p^I,p^U\right)=\arg\max_{\left\{p^I,p^U\right\}} N_I x^I\left(p^I\right)p^I+N_U x^U\left(p^U\right)p^U \quad \text{s.t.} \quad & N_I x^I\left(p^I\right)+N_U x^U\left(p^U\right)=S+u
\end{aligned}$$

By coefficient matching on 1, s, u, and z, the two equations deliver a system of eight nonlinear equations in eight unknowns.

C.3 Inference of z

In the price opaque PD-NREE, the informed agents observe p^I and s. As p^I is a function of s, u, and z with non-degenerate coefficients, the two-dimensional space of signals observable to the informed agents does not span the three-dimensional space of stochastic outcomes, as s, u, and z are independent random variables. Hence u and z are subject to a joint inference problem by the informed agent by which neither can be precisely recovered. Analogously, in the price transparent PD-NREE, the informed agent observes p^I, p^U, and s. As before, p^I and p^U are linearly dependent by assumption, reducing the signal space to p^I and s and the price transparent PD-NREE's inference problem to the price opaque one.

C.4 Continuity in ρ

Consider the system of eight equations in eight unknowns augmented by an auxiliary equation

$$\rho = \rho^*$$

where ρ^* parameterizes the pricing functions in terms of information content of the uninformed agents' signals. Then, by the implicit function theorem, the nine-by-nine system will have a solution and be continuous in ρ in the neighborhood of $\rho = 0$ if

$$|JF(A, B, C, H, D, F, G, K; \rho)| \neq 0$$

and $F = [F_1, F_2, \ldots, F_9] \in \left(C^1\right)^9$, that is, if the system linearized around a solution with respect to the pricing function constants and ρ has non-zero determinant, where $\rho = \rho^*$ is given exogenously. Although not shown here, the market clearing condition, first-order conditions for the profit maximization condition, and $\rho = \rho^*$ indeed give $F \in \left(C^1\right)^9$. Unfortunately, the system is too complex for this conjecture to be readily verified.

C.5 Reduction to PD-NREE

As pricing function coefficients are continuous in ρ, prices converge to their standard PD-NREE forms as $\rho \to 0$

$$\begin{aligned}
p^I &= D_\rho + F_\rho \left[\left(s-\bar{d}\right) - G_\rho u + K_\rho z\right] \\
&\overset{\rho\downarrow 0}{\to} D_0 + F_0 \left[\left(s-\bar{d}\right) - G_0 u + K_0 z\right] \\
&= D + F \left[\left(s-\bar{d}\right) - G u\right] \\
p^U &= A_\rho + B_\rho \left[\left(s-\bar{d}\right) - C_\rho u + H_\rho z\right] \\
&\overset{\rho\downarrow 0}{\to} A_0 + B_0 \left[\left(s-\bar{d}\right) - C_0 u + H_0 z\right] \\
&= A + B \left[\left(s-\bar{d}\right) - C u\right]
\end{aligned}$$

Correspondingly, demand functions for the informed and uninformed agents converge to their unperturbed counterparts

$$\begin{aligned}
x^I\left(p^I_\rho\right) &= \frac{E\left[d|s\right] - (1+r)\, p^I_\rho}{\alpha\sigma^2_{d|s}} \\
&\overset{\rho\downarrow 0}{\to} \frac{\bar{d} + \left(s-\bar{d}\right)\beta_s - (1+r)\, p^I_0}{\alpha\sigma^2_{d|s}} \\
&= x^I\left(p^I\right) \\
x^U\left(p^U_\rho, z\right) &= \frac{E\left[d|p\right] - (1+r)\, p^U_\rho}{\alpha\sigma^2_{d|p^U}} \\
&\overset{\rho\downarrow 0}{\to} \frac{\bar{d} + B\left[\left(s-\bar{d}\right) - Cu\right]\frac{\sigma^2}{B\left[\sigma^2+\sigma^2_\epsilon+C^2\sigma^2_u\right]} - (1+r)\, p^U}{\alpha\frac{\sigma^2\left(\sigma^2_\epsilon+C^2\sigma^2_u\right)}{\sigma^2+\sigma^2_\epsilon+C^2\sigma^2_u}} \\
&= \frac{\bar{d} + \beta_{p^U}\left(p^U_0 - A\right) - (1+r)\, p^U_0}{\alpha\sigma^2_{d|p^U}} \\
&= x^U\left(p^U\right)
\end{aligned}$$

Likewise, the price transparent PD-NREE with noise converges to the price transparent PD-NREE without noise when taking $\rho \to 0$.

I have shown that informed agents cannot emulate uninformed agents' demand schedules and that price functions with infinitesimally informative signals for the uninformed agents will be asymptotically equivalent to the PD-NREE without noise. Thus only the separating equilibrium presented in the paper obtains as $\rho \downarrow 0$.[21] $\diamond$

[21] For notational convenience, I assume throughout that the PD-NREE discussed is the modified variant with $\rho \downarrow 0$ but suppress additional clarifying subscripts.

D Relative Noise Condition

Claim 5. *Price sensitivity to the informed agents' signal increases (decreases) with the number of informed (uninformed) agents if and only if $\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2 < 0$ in the PD-NREE. Price sensitivity to the informed agents' signal unambiguously increases (decreases) with the number of informed (uninformed) agents in the non-price discriminating NREE.*

Proof. Taking first-order conditions for the price sensitivities to information B, F, and B_{NREE} obtains directly

$$\begin{aligned}
\frac{\partial B}{\partial N_I} = \frac{\partial F}{\partial N_I} &= -\left(\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2\right) N_U K \\
\frac{\partial B}{\partial N_U} = \frac{\partial F}{\partial N_U} &= \left(\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2\right) N_I K \\
\frac{\partial B_{NREE}}{\partial N_I} &= \left(2C^2\sigma_u^2\right) N_U K \\
\frac{\partial B_{NREE}}{\partial N_U} &= -\left(2C^2\sigma_u^2\right) N_I K
\end{aligned}$$

where

$$K\left(N_I, N_U\right) = \frac{\frac{1}{2}\frac{1}{1+r}\frac{\sigma^2}{\sigma_\epsilon^2}\left(\frac{1}{\sigma_\epsilon^2 + C^2\sigma_u^2}\right)}{\left(N_I\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2} + N_U\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)^2} > 0$$

□

Claim 6. *In the price opaque PD-NREE, the desirability of the asset from the perspective of the informed agents is increasing in the signal s for all (N_U, N_I)-pairs if and only if $\sigma^2 + \sigma_\epsilon^2 - C^2\sigma_u^2 < 0$.*

Proof. Desirability of the asset is strictly increasing in s if and only if demand is increasing in s, namely when

$$\begin{aligned}
\frac{\partial\left(\frac{\bar{d}+\left(s-\bar{d}\right)\beta_s-(1+r)p^I}{\alpha\sigma_{d|s}^2}\right)}{\partial s} &= \frac{1+r}{\alpha\sigma_{d|s}^2}\left[\frac{\beta_s}{1+r} - F\right] \\
&= \frac{1}{\alpha\sigma_{d|s}^2}\left[\frac{1}{2}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} - \frac{\left(\frac{\sigma^2}{2\sigma_\epsilon^2}\right) + \frac{N_U}{N_I}\left(\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)}{\left(\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}\right) + \frac{N_U}{N_I}\left(\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)}\right] \\
&> \frac{1}{\alpha\sigma_{d|s}^2}\left[\frac{1}{2}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} - \max_{N_U/N_I}\frac{\left(\frac{\sigma^2}{2\sigma_\epsilon^2}\right) + \frac{N_U}{N_I}\left(\frac{\sigma^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)}{\left(\frac{\sigma^2+\sigma_\epsilon^2}{\sigma_\epsilon^2}\right) + \frac{N_U}{N_I}\left(\frac{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}{\sigma_\epsilon^2+C^2\sigma_u^2}\right)}\right] \\
&= \frac{1}{\alpha\sigma_{d|s}^2}\left[\frac{1}{2}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2} - \max_{N_U/N_I}\left\{\frac{1}{2}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}, \frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2+C^2\sigma_u^2}\right\}\right] \\
&= 0
\end{aligned}$$

where the maximum is weakly less than $\frac{1}{2}\frac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}$ if and only if $\sigma^2+\sigma_\epsilon^2-C^2\sigma_u^2<0$. Note that in the price opaque equilibrium

$$\frac{\partial\left(\frac{\bar{d}+\left(s-\bar{d}\right)\beta_s-(1+r)p^I}{\alpha\sigma_{d|s}^2}\right)}{\partial s}=\frac{1+r}{\alpha\sigma_{d|s}^2}\left[\frac{\beta_s}{1+r}-\tilde{F}\right]=0$$

regardless of (N_U,N_I). $\square$