Overinvestment and the Fear of Missing Out*

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January 20, 2016

Abstract

I show that investors skip due diligence to quickly fund uncertain projects at high valuations because they rationally fear missing out. Investors may either fund projects immediately or engage in cost-justified but time-consuming learning to evaluate project quality. Time spent learning puts the investor at risk of losing his investment opportunity, leading to an inefficiently low level of learning in equilibrium. Inefficient learning leads to overinvestment in unprofitable projects and often underinvestment in profitable projects. This offers a rational explanation of boom-like overinvestment and bust-like underinvestment for venture capital, mortgage lending, and mergers along price, quantity, and funding speed dimensions.

*Thanks to Professors Zhiguo He, Stavros Panageas, William Lin Con, Harald Uhlig, and Casey Mulligan for guidance, and to Yiyao Wang for help with the idea that led to this paper. Thanks also to Johnathan Loudis, Michael Barnett, Klakow Akepanidtaworn, students from Econ 33603, Winter 2015, and Sarah Kang.

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1 Introduction

“Valuations...” the New York Times reported in 2015, “are inflating, leading some people to worry that investment decisions are being guided by something venture capitalists call FOMO—the fear of missing out.” 2015 saw at least 73 private technology companies valued above $1 billion, compared to 41 the previous year. One venture capitalist noted in the Wall Street Journal that private technology firms were receiving “sums of money usually reserved for IPO offerings... with the kind of confidence usually associated with investors who’ve perused regulatory filings for detailed financial information.”

These stories of high-valuation, low-information venture fundings resemble those of the late 1990s tech bubble. Pets.com, an infamous case study of that era, received roughly $130 million in funding despite its investors conducting almost no due diligence regarding the firm’s business viability; the firm was liquidated less than a year after its IPO. In 2015, investors wondered if it was déjà vu all over again. Yet despite the lessons of the late 1990s and the apparent foolishness of investment without due diligence, investors may not always have a choice: Facing gobs of rival capital looking for investment opportunities, can an investor afford the time spent distinguishing a future Facebook or Amazon from a future Pets.com, or will a fear of missing out entirely make uncertain ventures attractive even at sky-high valuations?

This paper asks if investors will skip due diligence to quickly fund uncertain projects at high valuations because they rationally fear missing out. Here, due diligence is cost-justified but time-consuming, and careful evaluation puts the investor at risk of losing his investment opportunity altogether. I show that when there are many investors relative to investment opportunities, and

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3 http://en.wikipedia.org/wiki/Pets.com

Facts that rudimentary due diligence would have revealed include: Pets.com was selling merchandise for one-third of its cost, spent 1,900% of its revenues on advertising, and tried to absorb the costs of shipping heavy bags of cat litter, which elsewhere sold at razor-thin margins.
investors have sufficient bargaining power, the equilibrium amount of due diligence will be inefficiently low. This gives rise to bubble-like high-quantity, high-valuation investment behavior with many funded projects failing. Foregoing due diligence is not irrational exuberance but rather a calculated investment rush motivated by the fear of losing investment opportunities. Moreover, the same mechanism that drives overinvestment in unprofitable projects may lead investors to underfund profitable projects: Insufficient bargaining power may discourage them from funding enough projects of any type, or in a rush to invest without due diligence, they may deploy too much capital on unprofitable projects and have none remaining for the profitable projects.

The fear of missing out arises in settings with matching frictions between investors and borrowers such as venture capital financing, mortgage lending, and mergers and acquisitions. These settings share four important features. First, investors do not have ex-ante exclusive rights to fund particular projects. Second, both investors and potential projects are in limited supply and projects are not scalable. Third, investors choose investment timing. Fourth, projects choose when to accept funding. These features will generate investment inefficiencies driven by the fear of missing out and lead to a price, quantity, and speed theory of rational bubbles.

The economic mechanism behind the fear of missing out is as follows: Because investors face limited investment opportunities, they will try to preempt each other by sacrificing due diligence and investing earlier than they otherwise would. Projects can choose whether to accept offers, and therefore can demand high valuations for early investment. The resulting price mechanism does not lead to an efficient amount of due diligence, however, because the matching market forces price competition to occur only across time. Rushing investors can offer higher valuations to projects than what patient investors could commit to offering the following period, even though patient due diligence matches are more productive in expectation. Valuations, rather than inducing an efficient allocation, merely serve as a way for early investors and projects to appropriate value from patient investors.

The paper first gives conditions under which too little due diligence occurs
in the simple case of perfect matching—that is, when the technology matching investors and projects forms all possible matches. In this case, there may be an inefficient rush to fund projects that results in overinvestment in projects that fail ex-post. That the inefficiency occurs in the case of perfect matching demonstrates that the externality is distinct from well-known congestion externalities in the employment search literature. Instead, the inefficiency arises because the rushing investor and project jointly appropriate value from the hypothetical non-rushing investor.

The paper’s main result is to characterize conditions under which there is too little or too much due diligence in the case of imperfect matching. In this case, congestion externalities in search make some early investment—investment without due diligence—socially desirable, but the private equilibrium will not achieve the efficient outcome in general. A wedge arises between the private equilibrium and social optimum for two reasons: First, as in the case of perfect matching, early investment removes investment opportunities from late investors, and marginal early investors do not internalize this cost when making decisions. This wedge leads to too much early investment. Second, in order to compensate projects for their outside option of waiting for late funding, early investors must offer projects high valuations. While these transfers impact the investors’ decision making, they do not impact aggregate welfare and consequently do not impact the social planner’s allocation. This wedge leads to too little early investment. Which wedge dominates depends on the relative bargaining strength of investors and projects, with high investor bargaining power leading to excess early investment.

The paper offers a rational explanation of investment bubbles along price, quantity, and funding speed dimensions consistent with recent data and makes the following predictions. First, when investors have strong bargaining positions—for instance, following good performance—there will be more high-speed, high-valuation, and low-information investment leading to overinvestment into bad projects due to the fear of missing out. Second, large inflows of savings into a FOMO-prone investment sector exacerbate the high-speed, high-valuation, and low-information associated with the fear of missing out. Third, tech-
nological improvements in matching investors to savers exacerbates the fear of missing out problem. Fourth, improvements in the unconditional pool of projects—for example, advancements in nearly instant, high-speed screening—exacerbates the problem and can reduce social welfare. Consistent with these predictions, data show that as flows into the venture capital sector have increased, new companies seeking funding have received it more quickly. Figure 1 compares hazard rates for founded companies receiving funding by company age between 2004-2009 to 2010-2014. The figure shows that both the absolute probability of funding as well as the speed at which funding occurs has increased. According to Pitchbook Data, this increase has been accompanied with an increase in median valuation. As the funding speed and valuation has increased, however, the probability of success as measured by IPOs or acquisition has decreased. This paper argues that this increased speed and valuation and lower quality of funded project is driven partly by VC investors’ rational fear of missing out.

Figure 1: Dotted lines are for the period 2004-2009. Solid lines are for the period 2010-2014. The top panel shows the probability of a company receiving venture funding $t$ years after being founded conditional on not yet being funded (and founding + $t$ date being in the sample). The bottom panel shows the probability of a successful exit $t$ years after receiving funding conditional on not having exited yet (and funding + $t$ date being in the sample). The figure shows that (1) the probability of being funded is higher recently, (2) the speed at which funding takes place is faster recently, and (3) the quality of firms receiving funds, as measured by probability of a successful exit—IPO or acquisition—is lower recently. Founding/funding/exit data from crunchbase.com.

1.1 Literature

This paper proposes a theory of over and underinvestment centered on an inefficient timing-of-investment decision. A large literature offers alternate theories of waves of over and underinvestment. For example, papers such as Moore and Kiyotaki (1997), He and Krishnamurthy (2008), Brunnermeier and Sannikov (2014), and many others\footnote{See He and Kondor (2012) for a complete discussion.} emphasize firm-level financial frictions leading to aggregate inefficiencies. In these models, individual firms fail to internalize the fact that their borrowing in booms leads to fire-sale prices for other firms during busts. He and Kondor (2012) emphasize a firm-level liquidity manage-
ment channel. Rather than focusing on within-firm financial frictions and how they impact aggregates, this paper focuses on how one investor’s investment timing decision impacts the decisions of others.

The starting place for this paper’s theory of over and underinvestment relates most closely to the tragedy of the commons in natural resource extraction. This large literature, starting with Hardin (1968) studies the harvesting of common-pool, often renewable resources. For example, Huang and Smith (2014), find evidence of dynamic overharvesting in fisheries. Over-extraction of non-renewable natural resources prior to learning about extraction-site quality is a prediction of a special case of this paper. In particular, when investment projects in this model do not have the power to reject funding offers, they can be interpreted as passive resources in the commons waiting to be claimed. For example, by calling investors “prospectors,” projects “potential gold mining sites,” and requiring the sites to accept any prospector’s digging, this paper models a Gold Rush. If the quality of mining sites is discoverable only with time-consuming mineral studies, prospectors may dig before evaluating a site or risk being claim jumped by a rival. The Diamond and Dybvig (1983) style bank run has a similar mechanism: The pressure to claim jump, interpreted here as withdrawing deposits before illiquid projects are complete, can trigger society-wide claim jumping—a run. In the bank run, depositors do not have particular property rights to withdrawal deposits; here, investors do not have particular rights to fund specific projects. I extend the analysis past the classical run by introducing a search friction and bargaining in the spirit of Mortensen and Pissarides (1994). Interestingly, I find that a less severe search friction leads to more severe claim jumping.

A small number of papers apply these common-pool resource models specific

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The range of applications to natural resource extraction is voluminous and well-summarized in Huang and Smith (2014): Oil extraction (Libecap and Wiggins (1984)), ground water (Provencher and Burt (1993)), hunting (Smith (1975)), and forestry (Mendelsohn (1994) and Linde-Rahr (2003)), to name only a few. Clay and Wright (2005) document this phenomenon and the legal context during the California Gold Rush: Potential extraction sites were limited, and the law encouraged miners to mine in rivals’ inactive sites. In this setting, a miner wanting to evaluate a mining site before digging risked losing out to a less prudent miner.
ically to project funding. Jovanovic and Szentes (2013), Toxvaerd (2008), and Xie (2015) study investment entry or exit decisions in settings without property rights over limited extractable resources. These resources are interpreted as projects or ideas for Xie or merger targets for Toxvaerd and typically generate inefficiently early investment decisions and a “quantity” theory of investment bubbles. This result is analogous to my results when projects lack the right to reject offers. An important difference between natural resources and projects, however, is that projects and merger targets possess the legal right to reject an early investor’s offer in anticipation of a better offer from a patient investor. Models relating investment bubbles to natural resource extraction therefore miss an important aspect of the investor-investee relationship, which is that investees have the agency to reject inefficient offers in order to capture some gains to waiting for themselves. My model contributes to this literature by giving projects this agency and showing when and why inefficient investment still takes place. In doing so, the model generates “price” predictions in addition to the “quantity” and “speed” predictions of existing literature.

The fear of missing out drives claimants to forgo information-producing activities that are efficient but time-consuming because they risk losing out on the claim altogether. The option to invest only after learning is a valuable real option, but that option belongs in the commons. The strategic interaction coming from claim jumping elaborates on the option value of waiting literature that started with McDonald and Siegel (1986) and includes a large literature from Industrial Organization, such as Fudenberg et al. (1983), regarding patent races. Jovanovic and Rousseau (2001) consider the optimal investment time of a firm that acquires information about its production function. Jovanovic (2009) considers a model where new investment requires “seeds” and calculates the optimal policy for planting a potentially constrained measure of seeds. Garleanu et al. (2012) examines the role and exercise of real options in making technological investments on the business cycle and technology booms. In this literature, the real options are privately owned, and their value is mediated—sometimes driven to zero—by a price mechanism in output. Unlike much of this literature, here the option value of waiting is positive, but this value is in
the commons. Models of option exercise and patent races, moreover, do not capture an important aspect of project funding in that the project itself has control over when “exercise” takes place: Venture capital or merger targets, for instance, can be strategic and calculating in which offers they accept or reject; buried treasure or a particular—potentially monopolized—market cannot make this choice. Ownership of the intellectual property reduces or reverses the claim jumping problem but rarely eliminates it. Many policy prescriptions regarding patent or innovation races focus on intellectual property-based solutions. This paper shows that the financial structure of an innovating industry is also important even if intellectual property rights exist.

The model offers a rational explanation of speculative over-investment observed in venture capital booms and mortgage lending. Harris et al. (2010) and Harris et al. (2014) show that while both venture capital and private equity inflows spiked in the late 1990s, venture capital returns plummeted while private equity returns did not. Similarly, Gompers and Lerner (2000) show high valuations accompanied high inflows into venture capital between 1987 and 1995. In my model, early and late investing differ in the amount of information available when investment takes place. In venture capital, large quantities of information are produced about new firms near the beginning of their lives, which makes early and late investing meaningfully different. In private equity, take-private targets are already subject to heightened information disclosures and therefore the information differences between early and late investing is less relevant. Thus, venture capital investment is prone to claim jumping behavior while private equity investment is not. The fear of missing may explain the difference in returns of these sectors following large capital inflows. The fear of missing out may also be partly behind changing practices in mortgage lending. Rajan et al. (2010) and Keys et al. (2010) show that over 2002-2006, mortgage lenders engaged in more mechanical and faster screening techniques at the expense of softer and often slower techniques. Iyer et al. (2015) show that these soft techniques aid in screening, but my model predicts that lenders will rationally forgo these slow-but-effective screening techniques because they risked being claim jumped by faster screeners.
Finally, my paper has implications for the financial technology literature. Much of this literature, e.g., Philippon (2013) and Philippon and Reshef (2013), treats financial technology as essentially a factor-augmenting technology whose improvement is welfare-enhancing to society. My model demonstrates how the particular form that financial innovation takes is important for outcomes and welfare. In particular, financial innovations improving investor and project matching—online social media, for instance—increase the tendency for rushed investment and can be welfare-reducing. Additionally, new instant screening technologies like automated mortgage screening can improve the unconditional pool of investment opportunities and make investing without additional screening activities profitable. This may cause investors to forgo more careful screening activities, even if they are efficient, for fear of missing out. While the automated screening will avoid some of the worst projects, the adoption of faster screening can overall lead to more bad projects being funded and a decrease in social welfare. In these examples, financial innovation takes on a darker quality and raises questions, like Zingales (2015), about what sorts of financial innovation benefit society, especially when the technologies are chiefly about rent allocation among various intermediaries.

The paper proceeds as follows. In Section 2 I set up and discuss the model. In Section 3 I characterize the equilibrium and social planner’s solution. In Section 4 I discuss the model and its implications, and in Section 5 I conclude.

2 Model Setup

The model combines (1) projects with potentially profitable ideas requiring funding from investors, and (2) a search market structure where investors choose the timing of funding.

2.1 Projects

There is a continuum of measure p of unscalable projects. Each project has one speculative idea with unproven business potential and must receive one
unit of outside funding to execute the idea. Ideas are good with probability $\pi$; good ideas produce $R$ gross returns with certainty; bad ideas produce nothing. Idea quality cannot be detected without investor due diligence. In period one—the *early* period—good and bad ideas are indistinguishable. In period two—the *late* period—participants’ due diligence activities allow good and bad ideas to be distinguished. After the second period, projects’ ideas become widely known and copyable, and neither project nor investor can profit from investment.

Projects may receive offers of funding in both early and late periods, but because ideas are not scalable, if funded early, a project will not require (nor will it accept) late funding. A project may choose to reject an early offer if its outside option of waiting is better. In equilibrium, it will never reject a late offer because its outside option is obsolescence in the next period. Motivated by the often one-off nature of these deals requiring bespoke sourcing, negotiating, and legal work, I assume that projects receive at most one offer per period.

A match between an investor and project in the *early* period produces $\pi R$ in expected gross output; a match between an investor and a project in the *late* period produces $R$ with certainty. A riskless outside savings technology produces gross return $r$. I assume that projects are unconditionally positive net present value investments, i.e., $\pi R - r > 0$. Denote by $r_e$ and $r_l$ the expected production from an early or late match, net of the outside savings technology. These quantities are given by

$$r_e \equiv \pi R - r \tag{1}$$

$$r_l \equiv R - r \tag{2}$$

Note that $r_l > \pi r_l > r_e$ and that $r_l - r_e = (1 - \pi)r$, equal to the outside production from bad projects avoided when investing late.
2.2 Matching Market

Investors and projects must match for funding to occur. With $s$ atomistic investors, each with enough capital to fund one project, and $p$ available projects matching within a period, their efforts produce $m(s, p)$ total matches. Following the matching literature, I assume that $m(s, p)$ has a constant returns to scale Cobb-Douglas form, with the addition of a technical truncation so that the measure of formed matches exceeds neither $s$ nor $p$\textsuperscript{8}. Under this assumption,

$$m(s, p) = \min \{ s, p, \mu s^\alpha p^{1-\alpha} \} \quad (3)$$

where $\mu$ and $\alpha$ are technological parameters. As $\mu$ becomes large, matching becomes perfect in the sense that all possible matches are formed, and $m(s, p)$ becomes the perfect matching function, $\tilde{m}(s, p)$, where

$$\tilde{m}(s, p) = \min \{ s, p \} \quad (4)$$

Formed matches are allocated among investors uniformly and among projects uniformly. The probability of an investor or project receiving a match is $q_s(s, p)$ and $q_p(s, p)$, defined as follows:

$$q_s(s, p) \equiv \frac{m(s, p)}{s} = \min \left\{ 1, \frac{p}{s}, \mu \left[ \frac{p}{s} \right]^{1-\alpha} \right\} \quad (5)$$

$$q_p(s, p) \equiv \frac{m(s, p)}{p} = \min \left\{ \frac{s}{p}, 1, \mu \left[ \frac{p}{s} \right]^{-\alpha} \right\} \quad (6)$$

Note that when $s > p$, even perfect matching does not guarantee that a particular investor will match. In this case, only $p$ matches can occur, and they must be rationed to the over-abundant investors. To avoid rationing, investors will be tempted to claim jump.

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\textsuperscript{8}The truncation is necessary in the discrete-time setup. Though the results hold for all values of $m(s, p)$, including those in the “truncation region,” for clarity of exposition I present results under the assumption that the truncation does not bind. Proofs in the appendix treat the general case with truncation.
2.2.1 Claim Jumping in the Matching Market

Investors choose at the outset whether they will try to learn, invest late, and fund only projects with good ideas, or whether they will claim jump—forsgo learning, invest early, and risk funding projects with bad ideas. Both activities are time-consuming, so investors must choose only one. Let $s$ and $p$ be the economy-wide measures of investors and projects, respectively. Let $s_e$ denote the endogenous measure of early investors to be determined in equilibrium. Early investors and projects match first and produce $m(s_e, p)$ matches, which are allocated among the $s_e$ investors. In equilibrium investors will only attempt an early match if they can make offers that will be accepted, so the $m(s_e, p)$ matches are funded and removed from the potential project pool.

In the late investing round, investors can distinguish good ideas and will therefore only attempt to match with good-idea projects. After the early projects have been funded and idea quality has been revealed, there are $\pi(p - m(s_e, p))$ projects with good ideas remaining to be funded. The $s - s_e$ remaining investors attempt to match with the $\pi(p - m(s_e, p))$ projects, matches are allocated, and the good projects are funded. The measure of matches formed in each market given the measure of early matchers $s_e$ is

- Early Matches: $m(s_e, p)$
- Late Matches: $m(s - s_e, \pi(p - m(s_e, p)))$

Successful matches are allocated among attempted matches pro-rata, so that the probabilities of matching are as follows:

- Early: Investor Probability $\frac{m(s_e, p)}{m(s - s_e, \pi(p - m(s_e, p)))}$, Project Probability $\frac{m(s_e, p)}{\pi(p - m(s_e, p))}$
- Late: Investor Probability $\frac{s_e}{s - s_e}$, Project Probability $\frac{\pi(p - m(s_e, p))}{\pi(p - m(s_e, p))}$

2.2.2 Bargaining in the Marching Market

A matched investor and project engage in Nash Bargaining, where the investor’s weight in the negotiations is $\beta$. Having made his decision regarding
the timing of his investment, his outside option is the outside storage technology $r$. To calculate the project’s outside option, we work backwards from the late period.

In the late period, quality is observed so investors only match with good projects. A funded project produces $R$ with certainty. A project rejecting funding will not have another chance to get funding before the idea enters the public domain, so its outside option is zero. Total surplus from a late match is therefore

\[
\text{Late Surplus: } r_l = R - r
\]

Under the Nash Bargaining assumption, total payments to project and investor from a late match are

\[
\text{Late Investor Payment : } \xi_s = \beta r_l \\
\text{Early Investor Payment : } \xi_p = (1 - \beta) r_l
\]

In the early period, idea quality is unobserved. A project receiving an offer of funding has the option to reject it, determine its idea’s quality, and wait for funding in the late period. The rejecting project will match in the late period with probability $q_p(s - s_e, \pi(p - m(s_e, p)))$, and conditional on funding, will receive $(1 - \beta) r_l$. Expected surplus from an early match is therefore

\[
\text{Early Surplus: } r_e - \pi q_p(s - s_e, \pi(p - m(s_e, p))) \xi_p \\
= r_e - \pi q_p(s - s_e, \pi(p - m(s_e, p)))(1 - \beta) r_l
\]

Importantly, the early surplus is not simply the difference between what is produced early, $\pi R$, and what is produced late, $R$. Rather, the surplus relevant to an early match is the difference between what is produce early and what the project would receive in a late match.

Finally, given aggregate behavior $s_e$, expected surplus from the early match is divided among investor and project according to their bargaining strength,
so the surplus going to each party net of the outside option is

\[
\xi^e(s_e) = \beta \max \left\{ \left( r_e - \pi q_p(s - s_e, \pi(p - m(s_e, p))) \xi^l_p \right), 0 \right\} \\
\xi^p(s_e) = (1 - \beta) \max \left\{ \left( r_e - \pi q_p(s - s_e, \pi(p - m(s_e, p))) \xi^l_p \right), 0 \right\}
\]  

(7)  
(8)

2.3 Savings Timing Decision

Returns to early and late investment, net of the investor’s outside option on failing to match, are

\[
\begin{align*}
\rho_e(s_e) &= q_s(s_e, p)\xi^e(s_e) \\
\rho_l(s_e) &= q_s(s - s_e, \pi(p - m(s_e, p)))\xi^l
\end{align*}
\]

(9)  
(10)

Investors are risk-neutral and given aggregate behavior \( s_e \), pick the greater of (9) and (10).

Before characterizing the equilibrium, notice that while conditional on funding a project, late investing offers a higher return than early investing, an investor’s probability of receiving a late match may be so low that it is worth accepting a lower-quality early match with higher match probability.

3 Equilibrium

I look for the equilibrium quantity of early investors, \( s_e \), who choose to enter the early search market. The remaining \( s - s_e \) search in the late market among the remaining \( \pi(p - m(s_e, P)) \) projects with good ideas. I use \( s^p_e \) to denote a private equilibrium early investment level, and \( s^s_e \) to denote the social planner’s optimal early investment level. Before moving forward, I formally define the equilibrium condition, which is simply a Nash equilibrium. There are three varieties of equilibria:

**Definition 1.** \( s^p_e \) is an all-early equilibrium if for \( s^p_e = s \), \( \rho_e(s^p_e) \geq \rho_l(s^p_e) \).

**Definition 2.** \( s^p_e \) is an all-late equilibrium if for \( s^p_e = 0 \), \( \rho_e(s^p_e) \leq \rho_l(s^p_e) \).
Definition 3. \( s_e \) is a **mixed** equilibrium if for \( s_e \in (0, s) \), \( r_e(s_e) = r_l(s_e) \).

If, when all others choose early investment, an individual prefers to invest early, an **all-early** equilibrium occurs. If, instead all others choose late investment, an individual prefers to invest late, an **all-late** equilibrium occurs. Finally, if at a given interior early investment level, returns to early and late investment are equal, a **mixed equilibrium** occurs.

Of particular interest will be **stable** mixed equilibria, defined below.

**Definition 4.** A mixed equilibrium is **stable** if given the equilibrium quantity of early investment \( s_e \in (0, s) \), \( r'_e(s_e) < r'_l(s_e) \).

A **stable mixed equilibrium** occurs if adding a marginal amount of early investors makes early investment strictly worse than late investment: A small deviation encourages correction towards the equilibrium. Graphically, a mixed stable equilibrium occurs when as a function of \( S_e \), returns to late investment cross returns to late investment from below.

Finally, in the case of perfect matching technology, I consider a modification of the all early equilibrium, the **strong all-early** equilibrium:

**Definition 5.** \( s_e \) is a **strong all-early** equilibrium if for \( s_e = s \), \( r_e(s_e) > r_l(s_e) \) and projects receiving early offers accept them, even if the project could surely match with a late investor.

A **strong all-early** equilibrium has the interpretation that an \( \epsilon \) measure of trembling early investors will instead match late. In an ordinary all-early equilibrium, projects’ outside options in the early period are zero because they will have no investors with whom to match late. Lacking a non-zero outside option, they will accept any non-negative offer from investors. The strong all-early equilibrium considers the case where even if projects had a better outside option—a sure-thing late match—early investors would still be able to offer acceptable valuations to projects. The strong all-early equilibrium will illustrate the important role that the appropriation of late matching investors has in the inefficiency.
Finally, I define the social planner’s problem. The social planner faces the same matching friction but can allocate early and late savings as she pleases. Social welfare is equal to the total production from all matches plus outside production of unmatched investors ($r$ per investor) and outside production of unmatched projects (0 per project):

**Definition 6.** Given aggregate savings $s$ and early savings $s_e$, social welfare is given by:

$$w(s_e) = m(s_e, p)r_e + m(s - s_e, \pi(p - m(s_e, p)))r_l + sr$$  \hspace{1cm} (11)

The social planner’s solution $s^*_e$ satisfies

$$s^*_e = \arg \max_{s_e \in [0, s]} w(s_e)$$  \hspace{1cm} (12)

For purposes of interpretation, notice the following:

**Lemma 1.** The Social Planner’s Problem, given in (12), is equivalent to maximizing the total ex-ante expected welfare of investors and projects with equal Pareto weights.

**Proof.** See Appendix, Section 6.1.1.

I first characterize the social planner’s solution and private equilibrium with perfect matching, as it illustrates simply and starkly the frictions arising from the opportunity to claim jump. I will then characterize private equilibria and the social planner’s problem with imperfect matching technology, which provides meaningful notions of valuations, overinvestment, and underinvestment.

### 3.1 Perfect Matching Technology

The following proposition characterizes the social planner’s solution with perfect matching from (11):

**Proposition 1.** The social planner’s solution is $s^*_e = 0$. 

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Proof. See Appendix 6.1.2.

The following proposition characterizes private equilibria with perfect matching:

**Proposition 2.** With perfect matching technology, the existence and uniqueness of private equilibria are as follows:

1. **All-late:** An all-late equilibrium exists if
   \[ \frac{r_e}{r_l} - \pi(1 - \beta) \leq \frac{\pi p}{s} \]

2. **All-early:** An all-early equilibrium exists if \( p \leq s \) and
   \[ r_e \geq 0 \]

3. **Strong all-early:** A strong all-early equilibrium exists if \( p \leq s \) and
   \[ r_e - \pi(1 - \beta)r_l \geq 0 \]

4. **Mixed:** A mixed equilibrium exists if all-early and all-late equilibria exist.

Proof. See Appendix 6.1.3.
Figure 2: Perfect matching. The solid and dashed lines are the expected returns to early and late investors, respectively, as functions of aggregate early savings $s_e$. The dotted line is social welfare. The first panel shows a unique all-late equilibrium because late investment dominates for all $s_e$. The second panel shows a unique all-early equilibrium because early investment dominates for all $s_e$. The last panel shows all three types: At $s_e = 0$, late dominates—an all-late equilibrium; at $s_e = s$, early dominates—an all-early equilibrium. There is also a mixed equilibrium at the $s_e$ where early and late returns are equal. Note in all cases, the socially optimal level is $s_e = 0$.

Broadly, the existence of all-late depends primarily on project scarcity, and the existence of all-early depends on project returns and bargaining power. Figure 2 shows the possibilities graphically. First, absent a savings glut, investors can learn and thus invest late without fear of losing opportunities to rival late matchers or claim jumpers. Because they will surely match, they choose the larger return. On the other hand, a sufficiently severe savings glut can rule out the late equilibrium altogether. For $p$ much smaller than $s$, an late investor may be so unlikely to match with a project that he would prefer the lower conditional early payment in exchange for a sure match.

Once $s \geq p$, an all-early equilibrium can exist so long as early investing is NPV-justified. This is because if all invest early, all projects are consumed.
so the returns to late investment is zero. More interestingly, the condition for a strong all-early equilibrium illustrates both why investors choose to invest early and why it is socially inefficient. The strong early equilibrium condition is equivalent to $(1 - \pi) r \leq \pi \beta r_1$. The left-hand side is what society saves by avoiding bad ideas. The right-hand side is what the project would pay to a late investor if it rejected an early offer and matched with one of the $\epsilon$ trembling investors. This is a division of late surplus between project and investor. The early investor and project have the opportunity to appropriate this transfer from the absent third party and will do so if it is larger than the expected savings from avoiding bad ideas. Their gain is merely a transfer, but their loss is a net loss to society. All-early equilibria are self-defeating from the investors’ perspectives. The motivation for deviation is to avoid crowdedness and expropriation from others, but when all claim jump, the market is equally crowded but projects have lower expected returns.

Finally, early, late, and mixed equilibria are possible for a moderate savings glut. This has a similar intuition to a bank run in Diamond and Dybvig (1983), where claim jumping—running—can become a self-fulfilling prophesy. An individual who fears a claim jumping “run” will himself claim jump or lose out on the investment opportunities altogether. This triggers a rush to early investment.

For the social planner, all-late investment is always optimal. The social planner wants (1) if there are not enough projects, to make sure that each project gets matched late, and (2) if there are not enough investors, to make sure that each investor matches late. Claim jumping occurs in equilibrium because investors and projects can appropriate future payments from late investors and because investors fear missing out on investment opportunities altogether. These fears concern, from the social planner’s perspective, mere transfers. There is scope for divergence between the social planner’s solution and the private equilibrium in a moderate savings glut, and there is necessarily divergence in a severe savings glut.

Perfect matching starkly illustrates the early investment option’s dangers—it can lead to a rush to fund projects before their payoffs are known due to the
fear of missing out, which in turn leads to overinvestment. I next consider the more general case with imperfect matching. The results will illustrate clearly how the wedge between the private equilibrium and social planner’s solution arises and show when overinvestment and underinvestment occur.

3.2 Imperfect Matching Technology

A key difference with imperfect matching technology is that the social planner, in order to fund more good ideas, will want some speculative early investment along with safe late investment. As before, there will be scope for overinvestment in bad ideas in the private equilibrium, but there will also be scope for underinvestment in good ideas, because private investors may (1) fail to engage in enough speculation or (2) use too much of their capital funding bad ideas early, both of which will lead to unfunded good ideas after period two. Generally, an inefficient amount of early investing will occur in the private equilibrium, and whether there is too much or too little will depend on the investors’ bargaining power.

3.2.1 Social Planner’s Solution

I begin by characterizing the social optimum. The following proposition characterizes the social planner’s solution and gives a simple criterion for determining whether a given allocation with $s_e$ has more early investment than the social planner would choose.

**Proposition 3.** With imperfect matching technology, the social planner’s solution $(12)$ is characterized as follows:

1. **All-late:** If $w’(0) = r_e - [m_s(s, \pi p) + \pi m_p(s, \pi p)] r_l < 0$ then $s_e^a = 0$ is socially optimal.

2. **FOC:** If $\exists s_e \in [0, s]$ such that $w'(s_e) = 0$, then such an $s_e$ is unique and $s_e^a = s_e$ is socially optimal.

3. **Corner:** If (1) and (2) are not satisfied, then the unique $s_e^c$ satisfying
   $$\mu(s_e^c)^{\alpha} p^{1-\alpha} = s_e^c$$
   is socially optimal.
Moreover, if $w'(s_e)$ is the left-side derivative of $w$, if for any $s_e \in (0, s]$, $w'_-(s_e) < 0$, then $s_e^* < s_e$.

**Proof.** See Appendix, 6.1.4. □

**Figure 3:** The possible solutions of the social planner’s problem. Each panel shows the social welfare function for different values of $\mu$ in the matching function. The upper left shows that for high $\mu$, the social planner’s optimum is to set $s_e = 0$ and match all late. The upper right shows for a moderate $\mu$, some early investment is optimal, and the optimal point occurs when $\mu s_e^* p^{1-\alpha} = s_e$. The lower left shows that for a low $\mu$, the optimum occurs at a location where $w'(s_e) = 0$. Finally, the lower right panel shows the social welfare functions from each other panel over the entire range of $s_e$, to give a sense of the shape of the function.

Though there are several cases, the intuition is simple: The social planner wants to fund as many of the good projects as possible while avoiding bad projects. All-late funding, which will avoid bad projects, may fail to fund all good projects due to the overconcentration of investors in the matching market. The social planner therefore trades off better conditional returns against less crowded matching and picks the optimal. If the matching friction mild relative to the differences in returns, she chooses all late, and otherwise chooses an interior quantity of early investment. An intuitive corollary of
Proposition 3 is that if \( m(s, p) = p \), then \( s^e = 0 \) is optimal. If the social planner can match all projects in a single round, he would prefer to match them all late. Figure 3 illustrates the cases.

The last line of the proposition is helpful in comparing the private equilibria to the social planner’s solution. In particular, to determine whether \( s^e \leq s^s \), it will be sufficient to compute the sign of the derivative of the social welfare function at \( s^p \): If social welfare is increasing at \( s^p \), then there is insufficient early investment; if social welfare is decreasing at \( s^p \), then there is too much early investment; if social welfare is flat at \( s^p \), then \( s^p \) and \( s^s \) are equal.

### 3.2.2 Private Equilibrium

I first give necessary and sufficient conditions for the existence of corner equilibria where \( s^p = 0 \) or \( s^p = s \):

**Proposition 4.** Given imperfect matching technology \( m(s, p) \),

1. An all early equilibrium exists if and only if \( m(s, p) = p \).

2. An all late equilibrium exists if and only if \( q_s(s, \pi p) > \frac{\pi R - r - \pi(1-\beta)q_p(s, \pi p)}{R-r} \).

**Proof.** See Appendix 6.1.5

The intuition for this proposition is similar to that of perfect matching. If it is possible for all projects to match in a single matching market—\( m(s, p) = p \)—then all-early is an equilibrium because no projects will remain for late matching. If any projects remain, some investors will invest late because they can match with certainty and get the larger late payment. The all-late equilibrium can exist so long as the crowding effects are not worse than the difference in payments. Notice that when an all-early equilibrium is possible corresponds exactly to when the social planner would choose all-late investment: When it is possible to match all projects, an early equilibrium is possible because it sends late returns to zero; at the same time, this is precisely when the social planner would only like to invest late. This illustrates how too much savings can make the inefficiency worse.
For the remainder of the paper, I will rule out pure private equilibria. These equilibria are analogous to the perfect matching technology case and so the analysis is similar. To that end, I assume the following:

**Assumption 1. Ruling out corner equilibria:**

1. No all-early equilibrium: \( m(s, p) < p \).
2. No all-late equilibrium: \( q(s, \pi p) > \frac{\pi R - r - \pi (1 - \beta) q(s, \pi p)}{R - r} \).

Having ruled out corner equilibria, the following proposition establishes the existence of stable mixed equilibria.

**Proposition 5.** Under Assumption 1, there is a stable mixed equilibrium quantity of early investment \( s^p_e \) satisfying

\[
    r_e(s^p_e) = r_l(s^p_e)
\]

**Proof.** See Appendix 6.1.6 \( \square \)
Figure 4: Imperfect matching. Along the horizontal axis is the (not necessarily equilibrium) amount of early investment. The solid and dashed lines are expected returns to early and late investors, respectively. The first and second panels show unique all-late and all-early equilibria. The third panel shows an interior, stable, mixed equilibrium.

Figure 4 shows the possibilities. Assumption 1 rules out the first two panels, and Proposition 5 guarantees the crossing point shown in the last panel. In focusing on stable mixed equilibria, we are considering points at which early investing is a strategic substitute. Marginal early investment has two effects: (1) early investment becomes more crowded relative to late investment, pushing towards strategic substitutability, and (2) future late investment opportunities are consumed, pushing towards strategic complementarity. In the perfect matching case, latter effect dominates and the only mixed equilibrium is unstable. In the imperfect case, the former dominates. Part of the force pushing towards inefficiently high early investment in the case of perfect matching is this strategic complementarity that creates the Diamond and Dybvig (1983) run-like equilibrium. Imperfect matching dulls this force: When bank runners may fail to reach the bank, other depositors are less worried about a run. When claim jumpers may fail to find the claim, investors who want to learn
are less worried about having their claims disturbed while they learn. This previews a result to come: improvements in the matching technology generate more claim jumping.

Having characterized the social planner’s solution and the private equilibria, we come to the main proposition of this section:

**Proposition 6.** Let $s^p_e$ denote the private equilibrium, and $s^s_e$ denote the social planner’s level of early investment. The efficiency of equilibrium early investment depends on the investors’ borrowing strength $\beta$ relative to their share in the matching function $\alpha$. In particular, under the maintained Assumptions[7]

1. **Efficient**: $s^p_e = s^s_e$ if $\beta = \alpha$.

2. **Too much early**: $s^p_e > s^s_e$ if $\beta > \alpha$.

3. **Too little early**: $s^p_e < s^s_e$ if $\beta < \alpha$.

*Proof.* See Appendix 6.1.7.

[7]
Figure 5: Comparison of the private equilibrium and the social planner’s solution. Solid and dashed curves show early and late expected returns, respectively. The dotted curve shows the social surplus. The solid vertical line marks the equilibrium $s_{pe}$. The dotted vertical line marks the socially optimal level $s_{se}$. When $\beta = \alpha$, $s_{pe} = s_{se}$. When $\beta > \alpha$, the investor’s high bargaining power leads to $s_{pe} > s_{se}$. When $\beta < \alpha$, the investor’s low bargaining power leads to $s_{pe} < s_{se}$.

Figure 5 shows a graphical comparison of the private equilibrium and the social planner’s solution. The efficiency of the private outcome depends on how the investors’ bargaining power $\beta$ compares to their importance in matching with projects $\alpha$. To see why, the following is instructive. Define

$$h(s_e) \equiv (1 - q_p(s_e, p))^{1-\alpha}$$

Interpret $h(s_e)$ as a transformation of the probability that a project is not funded early. Then, the $\beta \leq \alpha$ condition is equivalent to the following before
simplification:

\[ \beta > \alpha \iff \frac{-h'(s_e)m(s - s^p_e, \pi_p)r_l}{m(s^p_e, p)} > \frac{\partial m(s^p_e, p)}{\partial s^p_e} \pi q^l_p(s^p_e)(1 - \beta)r_l \]

\[ \text{marginal interference externality} \quad \text{marginal transfer to project} \]

The left-hand side measures the marginal interference of early investment on late investment. The right-hand side the transfer that an early investor must pay to a project in order to compensate it for its outside option of matching late. These generate opposing wedges in determining whether there will be too much or too little early investment.

The interference term arises because early investment prevents some late matches from forming at a marginal rate of \( h'(s_e) < 0 \). Marginal early investors impart this external cost on society. The social planner takes this into account but the private participants do not. The interference term is a wedge between the social planner and private investors that pushes the equilibrium towards too much early investment. The magnitude of the interference term is decreasing in \( \alpha \), because \( 1 - \alpha \) measures projects’ share in matching. When \( 1 - \alpha \) is large, the removal of the early-matched projects prevents a relatively large amount of late matches from forming.

The transfer term occurs because early investors must pay projects above their outside option of matching late. A project could refuse an early offer and obtain \( \pi q^l_p(s^p_e)(1 - \beta)r_l \) in expectation from matching late. The early investor must therefore compensate the project for this amount. Multiplying the per-investor transfer payment by the marginal number of early matches formed yields the total transfer payments that the additional early investors must pay to projects.\(^9\)

Because the social planner cares about aggregate welfare, this transfer payment does not impact her optimal allocation to early investment. Nonetheless, private investors must make the transfer, and they take this into account.

\(^9\)Notice that neither marginal early investor nor social planner care about the pecuniary externality coming from the fact that increased early investing changes the valuation other early investors must pay to projects.
account when deciding whether to match early. This makes early investing less desirable and reduces the amount of early investing. The transfer term is a wedge between the social planner and private investor and pushes the equilibrium towards too little early investment. The magnitude of the transfer term is decreasing in $\beta$: When $1 - \beta$ is small, the project will receive little in a late match, which makes the necessary transfer payment in an early match small.

The relative sizes of the marginal interference externality and the marginal transfer payment determine whether the amount of early matching is efficient. Too many projects match early if the private costs of matching early are less than the public externalized costs of matching early. Conversely, too few projects match early when the private costs of matching early are greater than the public costs of matching early. In the knife-edge case where they are equal, the amount of early investing is efficient.

The number of good and bad projects funded are:

- **Good Fundings**: $\pi m(s_e, p) + m(s - s_e, \pi(p - m(s_e, p)))$
- **Bad Fundings**: $(1 - \pi)m(s_e, p)$

While the social planner will fund some ex-post bad projects to insure matching of good projects, *overinvestment* occurs when private investors fund more bad projects than the social planner would. *Underinvestment* occurs when private investors fail to fund as many good projects as the social planner would. This can happen for two reasons: First if $\alpha > \beta$, there will be insufficient early investment, and crowding in the late period will leave good projects unfunded. In this case, there is *underinvestment* in both good and bad projects. Second, if $\alpha < \beta$, there will be too much early investment, and if the amount of early investment is so large, investors will use most of their capital on speculative early projects, leaving too little capital to fund known good projects in the late period. In this case, there is *overinvestment* in bad projects and

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10 This relationship between bargaining $\beta$ and share in the match production function $\alpha$ is similar to the result in Hosios (1990) regarding labor markets.
underinvestment in good projects.

4 Discussion and Comparative Statics

The fear of missing out can lead to a rush to claim investment opportunities before conducting due diligence. This rush leads to overinvestment in bad projects and often underinvestment in good projects. The problem is due to the separation of ownership between patent holder and financier: Hence, it is a problem of financial property rights rather than intellectual property rights. Models with projects-in-the-commons setups often prescribe intellectual property rights as the solution, but as shown here, such property rights do not entirely address the problem. Here, projects have the intellectual property rights over their ideas and therefore the power to accept or reject early financing. Nevertheless, an inefficient rush to investment will often occur. If the intellectual property belonged to the financier, or if the IP-owner did not require external finance, the problem would be solved. However, as Arora et al. (2015) show, investment in new technology is increasingly taking place outside large corporations rather than through internal capital markets. This suggests we are likely to see these problems continue or even grow going forward.

I now discuss three comparative statics shaping the size and direction of the inefficiency: (1) the investor’s bargaining power, (2) aggregate savings in the sector, and (3) technology. These comparative statics will help illuminate the model and offer predictions.

4.1 Investor’s Bargaining Power

Whether there is too much or too little early investment depends on the bargaining strength of investors, $\beta$. Interpret $\beta$ as representing the parties’ education levels, bargaining abilities, or un-modeled institutional backing. Recall from the discussion of Proposition 6 that $\beta$ impacts investment decisions through how much an early investor must compensate a project to accept his offer. In particular, $\beta > \alpha$ determines when there is too much or too little early
investment. This in turn determines whether there is over or underinvestment.

Figure 6 shows equilibrium early investment, and number of projects of each type funded as a function of bargaining power. Increases in $\beta$ lead to more early investment. More early investment leads one-to-one to more bad projects being funded, with overinvestment in bad projects when $\beta > \alpha$. Interestingly, the number of good projects funded is not monotonic in $\beta$: For low $\beta$ and below-efficient early investment, too many investors match late and due to matching frictions, many good projects go unfunded. For moderately-high $\beta$, more good projects are funded than the social planner would fund, but this is because there is higher-than-efficient early investment and more projects of all types are funded. For very-high $\beta$, there is again underinvestment in good projects because so much early investment leaves little savings to fund good projects in period two.

![Figure 6](image.png)

**Figure 6:** The equilibrium effects of increases investor bargaining power $\beta$. The top panel shows the total quantity of early investment. The middle panel shows the number of good projects funded. The bottom panel shows the number of bad projects funded.

These results show the importance of the bargaining process between in-
vestors and projects, especially if this bargaining power is time-varying. For instance, if following a VC bust investors have an institutionally weak bargaining position due to demands from worried clients and hence a low $\beta$, there will be underinvestment and the market will recover even more slowly. If, on the other hand, VC firms have been producing good returns and can bargain confidently, the high $\beta$ will exacerbate subsequent overinvestment in bad projects. This is the model’s first prediction: There will be slow funding and underinvestment in good projects when investors have a weak bargaining position and fast funding and overinvestment in bad projects when they have a strong bargaining position.

4.2 Aggregate Savings

With intellectual-property-owner and financier separate, investors compete for project funding opportunities. Projects enable and benefit from this competition, which leads to inefficient quantities of early investment. When total savings in the sector increases, there is more between-investor competition, leading to greater inefficiencies. Data suggests that increased inflows have corresponded with faster and lower quality fundings, and the comparative statics presented here generate the same result. Figure 7 shows how equilibrium early investment, returns, and project valuations—the amount paid to a project in early matching—vary with increases in society-wide savings, $s$, under the assumption that $\beta > \alpha$. 

32
Figure 7: The equilibrium effects of increases in total savings $s$, with $\beta > \alpha$. For all $s$, $s^p_e > s^e_e$, and differences grow for increasing $s$. The top panel shows social planner and private allocation to early investment. The middle panel shows return to investors. The bottom panel shows the valuation of early projects—how much early projects receive when funded.

The key result is that the wedge between equilibrium early investment and the social planner’s early investment is increasing in total savings. Valuations increase with allocation to early investment: More savings mean better outside options for waiting projects, and hence investors must pay more to invest early. Increasing prices, however, are not sufficient to push early investment back to the efficient level. This is the model’s second prediction: Greater savings in a rush-prone investment sector leads to increased speed and valuations, and decreased quality. The model stops short of explaining entry into the sector into the first place, but increased savings in the sector is consistent, for example, with an economy-wide increase in the supply of savings or poorer investment opportunities in other sectors.
4.3 Technological Improvements

Perversely, technological innovations can make worsen inefficiencies and reduce welfare. I consider two types of technology improvements: First, technological improvements in matching technology, and second, technological improvements in rudimentary screening technology.

4.3.1 Improvements in Matching Technology

Recent innovations in social media and other online communications tools facilitate matching, which I interpret as increases in $\mu$, the scale parameter in the matching technology (3). Interestingly, these matching tools exacerbate the inefficiencies in the model and can even decrease equilibrium welfare. Figure 8 shows how private equilibria and the social planner’s solution change as matching technology improves and how unconditional returns change in both cases.
Figure 8: The equilibrium effects of improvements in matching technology. The top panel shows equilibrium early investment and social planner’s early investment as $\mu$ increases. The bottom panel shows equilibrium returns to savings and the returns to an unconditional investor’s returns in the social planner’s solution. Note that equilibrium returns can decrease when $\mu$ increases.

With bad matching technology, social planner and private individual allocate similarly to early investment because the preemption effect is weak. As technology improves, however, the allocations rapidly diverge. Eventually, the private equilibrium has all early investment, and the social planner has all late investment. The bottom panel shows returns to these allocations. Note that the social planner is always able to weakly increase unconditional returns as technology improves. Private returns initially increase with the matching technology, but when marginally better technology prompts large switches to early investing, technological improvements reduce returns to private individuals.

We saw from Proposition 6 that the social planner and private equilibrium diverge because (1) early matching interferes with late matching, leading to too much early matching, and (2) early investors must make transfer payments to projects, leading to too little early matching. Improved matching technology
worsens the interference effect directly because more projects match early and are hence interfered with. Improved matching technology similarly worsens the bargaining power effect because if projects have a higher chance of matching late, they demand greater compensation early. Which dominates depends on $\alpha$ relative to $\beta$. Interestingly, improvements in matching technology can decrease welfare by triggering a rush to early investment. This gives the model’s third prediction: Improved matching rates between investors and projects increases the speed of financing and decreases the quality of funded projects.

4.3.2 Improvements in Project Screening

Next, I consider small improvements in technology that pre-screens projects. That is, I assume that the unconditional pool of projects have good ideas with probability $\tilde{\pi} > \pi$. Interpret increases $\pi$ as being improvements in baseline mechanical screening—a computer or robo-signer that does fast, cheap, and rudimentary screening to screen out obviously bad ideas or scammers from the project pool. Figure 9 shows how private equilibria and the social planner’s solution change as $\pi$ increases, and how unconditional returns change in both cases.

In the top panel, with a relatively bad unconditional project pool, neither private investors nor the social planner choose to invest early. However, as the project pool gets better, private investors start to rush to invest early. This reduces welfare because investors are now funding both good ideas and bad ideas, even though there more good ideas overall. Here, technological advances in screening make early investing feasible in equilibrium, but the effect of the technology is only to enable—and necessitate—a rush to invest. The technology does not create a new investment opportunity; it merely provides a chance to take on an existing project in a less efficient, more opportunistic way. This is the model’s fourth prediction: Higher pre-screened project quality leads to faster funding and can decrease unconditional project returns. This is largely consistent with the rise of computerized screening and robo-sign programs in the run-up to the 2008 financial crisis.
Figure 9: The equilibrium effects of improvements in preliminary screening. The top panel shows equilibrium early investment and social planner’s early investment as $\pi$ increases. The bottom panel shows equilibrium returns to savings and the returns to an unconditional investor in the social planner’s solution. Note that equilibrium returns can decrease when the unconditional project pool improves.

These situations are particularly likely to arise in many financial contexts where matching frictions result in investors earning rents. In a setting rife with rents and opportunities to wastefully cut in line, as Zingales (2015) warns us, we should be cautious before concluding that the widespread adoption of a new financial technology is efficient and beneficial simply because it is an equilibrium outcome. Often times, the technologies may simply allow investors to obtain rents faster—and less prudently—than competitors, which makes everyone worse off.

5 Conclusion

Investors will skip due diligence to quickly fund uncertain projects because they rationally fear missing out. Early investors and projects can appropriate
value from learning investors, and in order to protect themselves, investors ra-
tionally choose to forgo learning. The model applies naturally to the context
of venture capital funding: Projects must fund unproven ideas, matching be-
tween investors and projects is difficult, and there are no property rights over
the financing opportunities. Unlike settings concerning resource extraction or
firm entry, projects have the opportunity to reject early offers in anticipation
of better ones. Nevertheless, early investors can offer high valuations to induce
acceptance, and a glut of savings relative to projects makes paying the high
valuation worthwhile for investors in order to avoid congestion later on. This
phenomenon generates rational boom-like overinvestment with bad projects
funded in large quantities and at high valuations, and often underinvestment
in good projects. The model’s predictions are consistent with past episodes in
tech financing, mortgage lending, and recent venture funding data.
References


6 Appendix for Online Publication

6.1 Proofs of Propositions

6.1.1 Proof of Lemma 1

Begin with the social welfare function

\[ w(s_e) = m^e(s_e)\pi R + m^l(s_e)R + (s - m^e(s_e) - m^l(s_e))r \]

Adding and subtracting

\[ m^e(s_e)\pi R = m^e(s_e)\left[ \pi R - r + r - \pi q^l_p(s_e)(1 - \beta)(R - r) + \pi q^l_p(s_e)(1 - \beta)(R - r) \right] \]

\[ = m^e(s_e)\left[ \beta (\pi R - r - \pi q^l_p(s_e)(1 - \beta)(R - r)) + r \right] \]

\[ + m^e(s_e)[(1 - \beta)(\pi R - r - \pi q^l_p(s_e)(1 - \beta)(R - r)) + \pi q^l_p(s_e)(1 - \beta)(R - r)) \]

\[ = s_e q^e_s(s_e)\xi^e_s(s_e) + p q^e_p(s_e)\xi^e_p(s_e) \]

Then,

\[ m^e(s_e)(\pi R - r) = s_e q^e_s(s_e)\xi^e_s(s_e) - r + p q^e_p(s_e)\xi^e_p(s_e) \]

Next, using similar algebra,

\[ m^l(s_e)(R - r) = (s - s_e)q^l_s(s_e)\xi^l_s - r + \pi(p - m(s_e, p))q^l_p(s_e)\xi^l_p \]

Replace these in the social welfare function,

\[ m^e(s_e)(\pi R - r) + m^l(s_e)(R - r) + sr = s_e q^e_s(s_e)\xi^e_s(s_e) + (s - s_e)q^l_s(s_e)\xi^l_s \]

\[ + p q^e_p(s_e)\xi^e_p(s_e) + \pi(p - m(s_e, p))q^l_p(s_e)\xi^l_p \]

Rearrange the top and bottom lines to

\[ s_e q^e_s(s_e)\xi^e_s(s_e) + (s - s_e)q^l_s(s_e)\xi^l_s = s \left[ \frac{s_e q^e_s(s_e)\xi^e_s(s_e) + s - s_e q^l_s(s_e)\xi^l_s}{s} \right] \]

\[ p q^e_p(s_e)\xi^e_p(s_e) + \pi(p - m(s_e, p))q^l_p(s_e)\xi^l_p = p \left[ q^e_p(s_e)\xi^e_p(s_e) + (1 - p q^e_p(s_e))\pi q^l_p(s_e)\xi^l_p \right] \]
The top line is the total ex-ante expected welfare of investors. The bottom line is the total ex-ante expected welfare of projects: the first term is the probability of matching early times the payments of matching early; the second term is the probability of matching late times the payments of matching late—$1 - q_p^e(s_e)$ is the probability the project does not match early; $\pi$ is the probability that the project is good, and $q_p^l(s_e)$ is the probability of matching conditional on not matching early and being good.

### 6.1.2 Proof of Proposition 1

The social planner’s welfare function is

$$
\min(s_e, p)(\pi R - r) + \min(s - s_e, \max(p - s_e, 0))(R - r) + sr
$$

Because $\pi R - r < R - r$, this is maximized at $s_e = 0$, and social welfare is

$$
\min(s, p)(R - r) + sr
$$

### 6.1.3 Proof of Proposition 2

First, suppose that $p > s$. For all $s_e$, an investor choosing early matching will surely form a match and earn at most $\beta(\pi R - r)$. For all $s_e$, an investor choosing late matching will match with a project with probability $\frac{\pi(p - s_e)}{s - s_e} \geq \pi$ and a match will pay $\beta(R - r)$. Expected returns to early matching are at most $\beta(\pi R - r)$. Expected returns to late matching are at least $\beta\pi(R - r)$. Hence, late matching strictly dominates for $p > s$.

Before characterizing the equilibria for $p < s$, some preliminary helpful facts. Returns to early and late investing are

$$
r_e(s_e) = \min \left\{ 1, \frac{p}{s_e} \right\} \beta \left( r_e - \pi \min \left\{ 1, \frac{s - s_e}{\pi \max\{0, p - s_e\}} \right\} (1 - \beta)r_l \right)
$$

$$
r_l(s_e) = \min \left\{ 1, \frac{\pi \max\{0, p - s_e\}}{s - s_e} \right\} \beta r_l
$$
Note that when \( p \leq s \), there are two cases for matching probabilities given \( s_e \).

**Case 1:** \( s_e \leq p \):

\[
\min \left\{ 1, \frac{p}{s_e} \right\} = 1 \quad \text{Early investor matching probability}
\]

\[
\min \left\{ 1, \frac{\pi \max\{0, p - s_e\}}{s - s_e} \right\} = \frac{\pi(p - s_e)}{s - s_e} \quad \text{Late investor matching probability}
\]

\[
\min \left\{ 1, \frac{s - s_e}{\pi \max\{0, p - s_e\}} \right\} = 1 \quad \text{Late project matching probability}
\]

**Case 2:** \( s > s_e > p \):

\[
\min \left\{ 1, \frac{p}{s_e} \right\} = \frac{p}{s_e} \quad \text{Early investor matching probability}
\]

\[
\min \left\{ 1, \frac{\pi \max\{0, p - s_e\}}{s - s_e} \right\} = 0 \quad \text{Late investor matching probability}
\]

\[
\min \left\{ 1, \frac{s - s_e}{\pi \max\{0, p - s_e\}} \right\} = 1 \quad \text{Late project matching probability}
\]

**Case 2:** \( s = s_e > p \):

\[
\min \left\{ 1, \frac{p}{s_e} \right\} = \frac{p}{s_e} \quad \text{Early investor matching probability}
\]

\[
\min \left\{ 1, \frac{\pi \max\{0, p - s_e\}}{s - s_e} \right\} = 0 \quad \text{Late investor matching probability}
\]

\[
\min \left\{ 1, \frac{s - s_e}{\pi \max\{0, p - s_e\}} \right\} = 0 \quad \text{Late project matching probability (uncoordinated)}
\]

\[
q_p^{l*} = 1 \quad \text{Late project matching probability (coordinated)}
\]

Importantly, when \( s_e = s \), mechanically the probability of a project matching late is zero because there is no potential funding to match with, but in
evaluating the *strong* all early equilibrium, we consider the possibility of a project/investor coordinating to wait, in which case the matching probability is one.

To characterize when equilibria exist, I go through each case in turn.

**All late**: For an all late equilibrium to exist, it must be that for aggregate behavior $s_e = 0$, investing late is weakly preferred. Using the above facts, when $s_e = 0$, returns to early and late are:

$$r_e(0) = \beta (r_e - \pi (1 - \beta) r_l)$$
$$r_l(0) = \frac{\pi p}{s} \beta r_l$$

All late exists when

$$\frac{\pi p}{s} \beta r_l \geq \beta (r_e - \pi (1 - \beta) r_l)$$

$$\iff \frac{r_e}{\pi r_l} - (1 - \beta) \frac{p}{s}$$

**All early**: For an all early equilibrium to exist, it must be that for aggregate behavior $s_e = s$, investing early is weakly preferred. When $s_e = s$, the requirement is

$$\frac{\pi p}{s} \beta r_e \geq 0$$

$$\iff r_e \geq 0$$

The above is true because when $p < s$, so long as early investing is NPV justified, there is no incentive for an investor to deviate because there will be no projects for the investor to invest in.

**Strong all early**: For a strong all early equilibrium to exist, it must be that for aggregate behavior $s_e = s$, even if a project and investor could coordinate to meet in the late period, one of the parties to the agreement would
deviate—either the project would accept and early offer, or the investor would make an early offer. The investor would not deviate because he could get a guaranteed late match which is by assumption the highest return. The project, on the other hand, could receive an early offer that it would accept so long as

\[ r_e \geq \pi(1 - \beta) r_l \]

surplus available to early investor

\[ \pi(1 - \beta) r_l \]

expected value of late match

\[ \iff r_e \geq \pi(1 - \beta) r_l \]

Note that the above being true implies \( r_e > 0 \), that is, existence of a strong all early equilibrium implies existence of an all early equilibrium.

**Mixed**: \( r_e(s_e) \) and \( r_l(s_e) \) are continuous in \( s_e \), and therefore so is \( r_e(s_e) - r_l(s_e) \). If an all-early equilibrium exists and an all-late equilibrium exists, then \( r_e(s) - r_l(s) \geq 0 \) and \( r_e(0) - r_l(0) \leq 0 \). In this case, by the intermediate value theorem, there is a value \( s_e^* \in (0, s) \) satisfying \( r_e(s_e^*) = r_l(s_e^*) \), whence \( s_e^* \) is a mixed equilibrium.

**6.1.4 Proof of Proposition 3**

I first rule out \( s_e = s \) as a possible equilibrium so that candidate optima are (1) \( s_e = 0 \), (2) points where \( w'(s_e) = 0 \), and (3) points where \( w'(s_e) \) does not exist.

Note that

\[ \pi m(s, p) \leq m(s, \pi p) \]

To see, case one: \( m(s, p) = s \), then \( s \leq p \) and \( s \leq \mu s^\alpha p^{1-\alpha} \). Note then that \( \pi s \leq \pi p \) and \( \pi s < \mu s^\alpha (\pi p)^{1-\alpha} \), so \( \pi m(s, p) \leq m(s, \pi p) \).

Case two: \( m(s, p) = p \). Then \( p \leq s \) and \( p \leq \mu s^\alpha p^{1-\alpha} \). Note then that \( \pi p \leq s \) and \( \pi p < \pi \mu s^\alpha (\pi p)^{1-\alpha} \), so \( \pi m(s, p) \leq m(s, \pi p) \).

Case three: \( m(s, p) = \mu s^\alpha p^{1-\alpha} \). Then \( \mu s^\alpha p^{1-\alpha} \leq s \) and \( \mu s^\alpha p^{1-\alpha} \leq p \). Note
then that \( \pi \mu s^\alpha p^{1-\alpha} \leq \pi s \), \( \pi \mu s^\alpha p^{1-\alpha} \leq \pi p \), and \( \pi \mu s^\alpha p^{1-\alpha} \leq \mu s^\alpha (\pi p)^{1-\alpha} \), so that \( \pi m(s, p) \leq m(s, \pi p) \).

Next, if \( m(s, p) = p \), then \( s^*_e = 0 \) is the socially optimal level of early investment. Moreover, \( s^*_e \neq s \) in general, because a solution with \( s^*_e = s \) can be trivially improved upon by setting \( S^*_e = 0 \), and observing that

\[
m^*(s)r_e \equiv m(s, p)r_e \\
< m(s, p)\pi r_l \\
\leq m(s, \pi p)r_l \\
= m^l(s)r_l
\]

Hence, candidate optima are (1) \( s = 0 \), (2) points where \( w'(s_e) = 0 \) and (3) points where \( w'(s_e) \) does not exist.

The outline is as follows. We need to consider separately two connected intervals for \( s_e \). Define,

\[
s^c_e \equiv s_e \text{ such that } s_e = \mu s^e p^{1-\alpha} \\
\bar{s}_e \equiv s_e \text{ such that } \mu(s - \bar{s}_e)^\alpha (\pi(P - m(\bar{s}_e, p)))^{1-\alpha} = \pi(p - m(\bar{s}_e, p))
\]

\( s^c_e \) is the amount of early investment such that early investment becomes unstuck from being at the upper bound of \( m(s_e, p) = s_e \). It is the first potential kink in the objective function. \( \bar{s}_e \) is the point at which late matching begins to consume all projects available for late matching, i.e., when the late matching function becomes stuck at the number of projects limitation.
We consider two intervals:

\[ I_1 \equiv \{ s_e | s_e \in (0, s_e^c) \} \]
\[ I_2 \equiv \{ s_e | s_e \in (s_e^c, \bar{s}_e) \} \]

I show (1) that in the region \( s_e \in (\bar{s}_e, s] \), \( f'(s_e) < 0 \) and therefore that the optimum cannot occur in \([\bar{s}_e, s]\). I show (2) that inside each of \( I_1 \) or \( I_2 \), \( f(s_e) \) has at most one point satisfying \( w'(s_e) = 0 \) and that it is a local maximum. I show (3) that if an \( s_e \) satisfying \( f'(s_e) = 0 \) occurs in one of the intervals, it cannot occur in the other. I show (4) that if such an \( s_e \) occurs, it is the unique maximum. I show (5) that if \( w'(0) \leq 0 \) then \( s_e = 0 \) is the maximizer, and finally I show (6) that if \( f'(0) > 0 \) and no \( s_e \) exists with \( f'(s_e) = 0 \), then \( s_e = s_e^c \) is the maximizer. In making these arguments, I will show along the way that whenever \( f'_-(s_e) < 0 \) then \( s_e^s < s_e \).

For (1). I want to show that \( w''(s_e) < 0 \) for \( s_e \in (\bar{s}_e, s] \) where it exists, and if not that the derivative on both sides is negative. In this region, the objective function and right derivative is always

\[
\begin{align*}
    w(s_e) &= m(s_e,p)r_e + \pi(p - m(s_e,p))r_l \\
    w'_-(s_e) &= \frac{\partial m(s_e,p)}{\partial s_e} [r_e - \pi r_l] < 0
\end{align*}
\]

Hence, the social planner can always reduce early investment to increase social welfare. Note that this shows that the objective function is decreasing over this region.

For (2), we want to show there is at most one \( s_e \in I_1 \) satisfying \( w'(s_e) = 0 \) and one \( s_e \in I_2 \) satisfying \( w(s_e) = 0 \) and that in either case, these points are local maxima. Take \( s_e \in I_1 \). The goal is to show that if \( w'(s_e) = 0 \) then \( w''(s_e) < 0 \),

\[11\text{If one interval is empty the result goes through and we simply rule out an interior point in that interval.}\]
whence $s_e$ is a local maximizer, and moreover, only one $s_e$ satisfies $w'(S_e) = 0$ in this region. In this region,

$$w(s_e) = s_e r_e + m(s - s_e, \pi(p - s_e)) r_l$$

If $m(s - s_e, \pi(p - s_e)) = s - s_e$ then decreasing $s_e$ then reducing $s_e$ increases social welfare by $r_l - r_e > 0$, whence it is not a maximum. If $m(s - s_e, \pi(p - s_e)) = \pi(p - s_e) \neq \mu(s - s_e) \alpha (\pi(p - s_e))^{1-\alpha}$, then decreasing $s_e$ increases social welfare by $\pi r_l - r_e > 0$. Hence, the only possibility for a local maximizer in this region is that $m(s - s_e, \pi(p - s_e)) = \mu(s - s_e) \alpha (\pi(p - s_e))^{1-\alpha}$.

It is sufficient to show then that for $m(s_e, p) = s_e$ and $m(s_e - s, \pi(p - m(s_e, p))) = m(s - s_e, \pi(p - s_e))^{1-\alpha}$, that $w'(s_e) = 0 \implies w''(s_e) < 0$. Note that here, $m(s - s_e, \pi(p - s_e)) = \pi^{1-\alpha} m(s - s_e, p - s_e)$ so we first factor the $\pi^{1-\alpha}$. This leaves

$$w(s_e) = s_e r_e + m(s - s_e, p - s_e) \pi^{1-\alpha} r_l$$

$$w'(s_e) = r_e - m_s(s - s_e, p - s_e) \pi^{1-\alpha} r_l - m_p(s - s_e, p - s_e) \pi^{1-\alpha} r_l$$

Take derivatives to obtain:

$$w''(s_e) = \pi^{1-\alpha} r_l \left[ m_{ss}(s - s_e, p - s_e) + 2 m_{sp}(s - s_e, p - s_e) + m_{pp}(s - s_e, p - s_e) \right] < 0$$

$$\iff -\alpha (1-\alpha) \mu(s - s_e)^{\alpha-2}(p - s_e)^{1-\alpha} + 2 \alpha (1-\alpha) \mu(s - p_e)^{\alpha-1}(p - s_e)^{-\alpha} - \alpha (1-\alpha) \mu(s - s_e)^{\alpha}(p - s_e)^{-1-\alpha} < 0$$

$$= -\alpha (1-\alpha) (s - s_e)^{\alpha-2}(p - s_e)^{-1-\alpha} [(p - s_e)^2 - 2(s - s_e)(p - s_e) + (s - s_e)^2]$$

$$= -\alpha (1-\alpha) (s - s_e)^{\alpha-2}(p - s_e)^{-1-\alpha} [p - s]^2 < 0$$

And the last line is negative because by assumption, $p - s_e < 0$. This shows that $w(s_e)$ is weakly concave inside the region and hence has at most one location where the derivative is equal to zero, and whenever it is equal to zero,

\[\text{If this occurs at the corner of the late matching function, the derivative must be changing sign here in which case this is still the relevant critical point.}\]
Similarly, if \( s_e \in [s^e_e, \bar{s}_e] \), then 
\[
m(s - s_e, p - m(s_e, p)) = \mu(s - s_e)^\alpha(\pi(p - m(s_e, p)))^{1-\alpha} = \mu(s - s_e)^\alpha(p - m(s_e, p))^{1-\alpha}\pi^{1-\alpha}
\]
the objective function is
\[
w(s_e) = m(s_e, p)r_e + m(s - s_e, p - m(s_e, p))\pi^{1-\alpha}r_l
\]
\[
w'(s_e) = m_s(s_e, p)[r_e - m_p(s - s_e, p - m(s_e, p))\pi^{1-\alpha}r_l] - m_s(s - s_e, p - m(s_e, p))\pi^{1-\alpha}\pi^{1-\alpha}r_l
\]
If \( w'(s_e) = 0 \), because \( m_s > 0 \), this means that
\[
r_e - m_p(s - s_e, p - m(s_e, p))\pi^{1-\alpha}r_l > 0
\]
The second derivative is
\[
w''(s_e) = m_{ss}(s_e, p)[r_e - m_p(s - s_e, p - m(s_e, p))\pi^{1-\alpha}r_l]
\[
+ \left[ m_{ss}(s_e - s, p - m(s_e, p)) + 2m_sp(s_e, p) 
\right.
\[
+ m_{pp}(s_e - s, p - m(s_e, p))m_s(s_e, p)^2 \right] \pi^{1-\alpha}r_l
\]
The first term is negative when \( f'(w_e) = 0 \). Next, performing the same factoring trick as above the second term is non-positive iff:
\[
0 \geq +m_{ss}(s_e - s, p - m(s_e, p)) + 2m_sp(s_e, p) + m_{pp}(s_e - s, p - m(s_e, p))m_s(s_e, p)^2
\]
\[
\iff 0 \geq -\left[ m_s(s_e, p)^2(p - m(s_e, p)^2) - 2m_s(s_e, p)(p - m(s_e, p))(s - s_e) + (s - s_e)^2 \right]
\]
\[
\iff 0 \geq -\left[ m_s(s_e, p)(p - m(s_e, p)) - (s - s_e)^2 \right]
\]
And the last line is true because it is the negative of a number squared. This shows that when \( w'(s_e) = 0 \) for \( s_e \in [s^e_e, \bar{s}_e] \), it is a maximum, and because the derivative is continuous in this region, it happens at most once. Moreover, a maximum can occur in this region only if \( w'(s^e_e) > 0 \).

(3) Now I show that if there is an \( s_e \in I_l \) satisfying \( w'(s_e) = 0 \) then there
is no \( s_e \) in \( I_{-i} \) satisfying \( w'(s_e) = 0 \). Consider the derivative immediately to the left and right of \( s_e = s_e^c \). We have,

\[
\begin{align*}
w'_-(s_e) &= 1 \left[ r_e - m_p(s - s_e, (p - s_e))\pi^{1-\alpha} r_l \right] - m_s(s - s_e, (p - s_e))\pi^{1-\alpha} r_l \\
w'_+(s_e) &= m_s(s_e, p) \left[ (r_e - m_p(s - s_e, (p - s_e))\pi^{1-\alpha} r_l \right] - m_s(s - s_e, (p - s_e))\pi^{1-\alpha} r_l
\end{align*}
\]

Suppose there is an \( s_e \in I_2 \) where \( w'(s_e) = 0 \). Then it must be that at the beginning of the interval, \( w'_-(s_e) = 0 \). If \( w'_+(s_e) > 0 \) then it must be that \( r_e - m_p(s - s_e, \pi(p - s_e))\pi^{1-\alpha} r_l > 0 \). Since \( m_s(s_e, p) \leq 1 \), we therefore must have that \( w'_-(s_e) \leq w'_+(s_e) > 0 \). Inside \( I_1 \), \( w' \) is weakly decreasing as shown in step (2), which means that there could not have been an \( s_e \in I_1 \) satisfying \( w'(s_e) = 0 \).

Hence, if there is a zero of the derivative in \( I_2 \) then there is no zero of the derivative in \( I_1 \), so a zero can occur in at most one of these regions.

Taking stock, we have so far showed that \( s_e \) satisfying \( w'(s_e) \) can only occur in \( I_1 \) or \( I_2 \) and if it does occur in one it cannot occur in the other. This tells us there is at most one unique \( w'(s_e) = 0 \) for all \( s_e \in [0,s] \).

(4) Now I show that if \( \exists s_e \) such that \( w'(s_e) = 0 \), it is the unique maximizer. Other candidate points are \( s_e = s_e^c \) and \( s_e = 0 \). The former cannot occur: Suppose \( \exists w'(s_e) = 0 \) but the maximizer is \( s_e^c = 0 \). \( s_e^c \) can only be a maximum if \( w_-(s_e^c) > 0 \) and \( w_+(s_e^c) < 0 \). The derivative cannot go from negative to positive on the left side of \( s_e^c \) so no \( w'(s_e) = 0 \) exists on the left side. The derivative cannot go from negative to positive on the right-hand side in \( s_e \in [s_e^c, s_e^r] \), so no \( w'(s_e) = 0 \) exists on the right side. Hence, there is no \( s_e \) such that \( w'(s_e) = 0 \) if \( s_e^c \) is a maximizer, which is a contradiction.

If \( s_e = 0 \) is a maximizer, then \( w'(0) \leq 0 \). Again, because the derivative cannot cross from negative to positive inside either interval, it must be strictly negative on the interior of \( I_1 \) and therefore strictly negative on \( I_2 \). Moreover
it must decrease going across \( \bar{s}_e \), there can be no \( s_e \) satisfying \( f'(s_e) = 0 \) if \( s_e = 0 \) is a maximizer.

(5) Next I show that if \( w'(0) \leq 0 \) then \( s_e = 0 \) is the maximizer. I showed this in the above argument. If \( w'(0) \leq 0 \) the derivative can only be negative thereafter.

Finally, (6), if \( w'(0) > 0 \) and there is no \( s_e \) satisfying \( w'(s_0) = 0 \), then \( s_e = \bar{s}_e \) is the maximizer. This is true simply because at this point we have ruled out all other candidates for maxima.

Notice that in making these arguments, whenever we found a maximizer, we saw that the objective function must be always increasing to the left of the point and decreasing to the right thereafter: If \( s_e = 0 \) is the maximizer, \( w'(0) \leq 0 \) and arguments above showed that the derivative can never become positive thereafter. Hence, if \( s_e = 0 \) is the maximizer, any \( s_e \) with a negative derivative (aside from \( s_e = 0 \)) satisfies \( s_e > s_e^* \). If \( s_e = \bar{s}_e \) is the maximizer, we argued that on the left side \( f(s_e) \) must be always increasing and on the right side \( w(s_e) \) must be always decreasing. Hence if we find any \( s_e \) with \( w'(s_e) < 0 \), we know \( s_e > s_e^* \). If the maximizer is when \( w'(s_e^*) = 0 \), again, to the right the function must be always decreasing and to the left it must be always increasing. Whence, \( w'(s_e) > 0 \implies s_e > s_e^* \).

This completes the proof.

6.1.5 Proof of Proposition 4

All early: If \( m(s,p) = p \), then when \( s_e = s \), the probability of matching late is zero, and hence, a private investor prefers to match early. Hence, \( m(s,p) = p \) implies the existence of an early equilibrium. Next, suppose that \( s_e = s \) is a private equilibrium. This implies that \( q_l^s(s)(\pi R - r) \geq q_l^s(0)(R - r) \). But \( q_l^l(s) = 1 \) unless \( m(s,p) = p \), and if \( q_l^s(s) = 1 \), then \( q_l^e(s)(\pi R - r) < q_l^l(s)(R - r) \) because \( R - r > \pi R - r \). Hence, it must be that that \( m(s,p) = p \).
All late: This follows directly from the definition of the all late equilibrium.

6.1.6 Proof of Proposition 5

Consider the function \( f(s_e) \equiv q_e^c(s_e)(\pi R - r) - q_e^l(s_e)(R - r) \). Under the maintained assumption that all late investment is not an equilibrium, \( f(0) > 0 \). Under the maintained assumption that all early investment is not an equilibrium, \( f(s_e) < 0 \). \( f(\cdot) \) is continuous so by the intermediate value theorem there exists at least one \( s_e^p \) where \( f(s_e^p) = 0 \). This shows a mixed equilibrium exists. There must be a stable mixed equilibrium because \( f(0) \) must cross zero from above, i.e., returns to late investing cross returns to early investing from below, at at least one point.

6.1.7 Proof of Proposition 6

The maintained assumptions rule out \( s_e^p = 0 \) and \( s_e^p = s \), and hence I only consider interior equilibria. From Proposition 3, it is sufficient to check the derivative of the social welfare function at the candidate private equilibria \( s_e^p \). If it is negative, there is too much early investing; if it is positive, there is too little. Due to the technical truncation of \( m(s, p) \), I go through the function in a piecewise manner. The “main” case is Case 5, but the others are shown here as well for completeness.

Case 1: \( m(s_e^p, p) = s_e^p; m(s - s_e, \pi(p - s_e^p)) = s - s_e^p \). Because the matching probability for early and late is one in this case, \( r_l > r_e \) implies this is only an equilibrium if \( s_e = 0 \), which is ruled out by the maintained assumption.

Case 2: \( m(s_e^p, p) = s_e^p; m(s - s_e, \pi(p - s_e^p)) = \mu(s - p_e^c)^\alpha(\pi(p - s_e))^{1-\alpha} \). Consider the social planner’s first-order condition evaluated at this point. I
will show that the FOC is negative:

\[ FOC = r_e - \left[ \mu (s - s_e)^{\alpha-1}(p - s_e)^{1-\alpha} - \mu(1 - \alpha)(s - s_e)^\alpha(p - s_e)^{-\alpha} \right] \pi^{1-\alpha} r_l \]

\[ = r_e - \mu(s - s_e)^\alpha(p - s_e)^{-\alpha} \left[ \frac{p - s_e}{s - s_e} + (1 - \alpha) \right] \pi^{1-\alpha} r_l \]

Using the equilibrium condition that

\[ r_e = \mu \left[ \frac{p - s_e}{s - s_e} \right]^{1-\alpha} \pi^{1-\alpha} r_l + \pi^{1-\alpha}(1 - \beta) \mu \left[ \frac{p - s_e}{s - s_e} \right]^{-\alpha} r_l \]

\[ = \pi^{1-\alpha} r_l \mu \left[ \frac{p - s_e}{s - s_e} \right]^{-\alpha} \left[ \frac{p - s_e}{s - s_e} - (1 - \beta) \right] \]

inject this into the FOC:

\[ w'(s_e) = \pi^{1-\alpha} r_l \mu \left[ \frac{p - s_e}{s - s_e} \right]^{-\alpha} \left[ \frac{p - s_e}{s - s_e} - (1 - \beta) \right] \]

\[ - \mu(s - s_e)^\alpha(p - s_e)^{-\alpha} \left[ \frac{p - s_e}{s - s_e} + (1 - \alpha) \right] \pi^{1-\alpha} r_l \]

\[ = \mu \pi^{1-\alpha} r_l \left[ \frac{p - s_e}{s - s_e} \right]^{-\alpha} \left[ (1 - \alpha) \frac{p - s_e}{s - s_e} - (1 - \beta) - (1 - \alpha) \right] \]

\[ = \mu(1 - \alpha) \pi^{1-\alpha} r_l \left[ \frac{p - s_e}{s - s_e} \right]^{-\alpha} \left[ \frac{p - s_e}{s - s_e} - \frac{1 - \beta}{1 - \alpha} - 1 \right] \]

Efficiency depends on whether the following is positive:

\[ \frac{p - s_e}{s - s_e} \frac{1 - \beta}{1 - \alpha} - 1 \leq 0 \]

\[ \iff \frac{p - s_e}{s - s_e} \leq \frac{(1 - \alpha) + (1 - \beta)}{1 - \alpha} \]

In the case of \( \beta = 1 \), the LHS is greater than the RHS and the social welfare function is decreasing at \( s_e \). As \( \beta \) becomes larger, this may go the other way.

**Case 3:** \( m(s_e^p, p) = s_e^p; m(s - s_e, \pi(p - s_e^p)) = \pi(p - s_e) \). The social wel-
fare function is
\[ s_e r_e + \pi (p - s_e) r_l \]
which because \( r_e < \pi r_l \) has a corner solution of \( s_e = 0 \). Hence, the private equilibrium is \( s^p_e > 0 = s^s_e \).

**Case 4:** \( m(s_e, p) = \mu s^\alpha_e p^{1-\alpha} \) and \( m(s - s_e, \pi (p - m(s_e, p))) = s - s_e \). No equilibrium with \( s_e > 0 \) exists because it would require
\[ q^e_e(s_e)(r_e - \pi q^l_p(s_e)(1 - \beta)r_l) \geq r_l \]
and since \( q^e_e(s_e) \leq 1 \), this cannot happen. No corner equilibrium exists by the maintained assumptions.

**Case 5:** \( m(s_e, p) = \mu s^\alpha_e p^{1-\alpha} \) and \( m(s - s_e, \pi (p - m(s_e, p))) = \mu (s - s_e)\alpha (\pi (p - \mu s^\alpha_e p^l(1 - \alpha)))^{1-\alpha} \). Consider the social planner’s first-order condition evaluated at the equilibrium condition:

In this case we can write
\[
m(s - s^p_e, \pi (p - m(s^p_e, p))) = (1 - q^e_e(s^e_e))^{1-\alpha} m(s - s^p_e, \pi p) \\
= h(s^p_e) m(s - s^p_e, \pi p)
\]
Interpret \( h(s^p_e) \): \( 1 - q^e_e(s^e_e) \) is the probability that a project is *not* matched with an early investor. This is a measure of interference of early matching with late matching—a higher \( h(s^p_e) \) means less interference. Note that \( h'(s^p_e) < 0 \), so that more early matching causes more interference.

Next, the social planner’s problem as:
\[
m^e(s^p_e) r_e + h(s^p_e) m(s - s^p_e, \pi p) r_l
\]
Which we can rewrite as

\[ s_p^e q_s(s_p^e, p)r_e + (s - s_p^e)h(s_p^e)q_s(s - s_p^e, \pi p)r_l \]

The derivative of this is

\[ q_s(s_p^e, p)r_e - h(s_p^e)q_s(s - s_p^e, \pi p)r_l + s_p^e \frac{\partial q_s(s_p^e, \pi p)}{\partial s} r_e + (s - s_p^e) \left[ h'(s_p^e)q_s(s - s_p^e, \pi p) - h(s_p^e) \frac{\partial q_s(s - s_p^e, \pi p)}{\partial s} \right] r_l \]

Next, we use two facts. First, given the value of \( m \) in this region, we have

\[ s_e \frac{\partial q_s(s_p^e, p)}{\partial s} = -(1 - \alpha)q_s(s_p^e, p) \quad \text{and} \quad (s - s_p^e) \frac{\partial q_e(s - s_p^e, \pi p)}{\partial s} = (1 - \alpha)q(s - s_p^e, \pi p). \]

Plug this in to simplify the above derivative:

\[ \alpha [q_s(s_p^e, p)r_e - h(s_e)q(s - s_p^e, \pi p)r_l] + (s - s_p^e)h'(s_e)q_s(s - s_p^e, \pi p)r_l \quad \text{(13)} \]

Next, \( s_p^e \) being a private equilibrium implies that

\[ q_p^e(s_p^e)(r_e - \pi q_p^e(s_p^e)(1 - \beta)r_l) = q_p^e(s_p^e)r_l \]

Now, add and subtract the project’s outside option term into (13):

\[ \alpha [q_s(s_p^e, p)r_e - h(s_e)q(s - s_p^e, \pi p)r_l] + (s - s_p^e)h'(s_e)q(s - s_p^e, \pi p)r_l \]

\[ = \alpha [q_s(s_p^e, p)(r_e - \pi q_p^e(s_p^e)(1 - \beta)r_l) - h(s_e)q(s - s_p^e, \pi p)r_l] \]

\[ + (s - s_p^e)h'(s_e)q(s - s_p^e, \pi p)r_l + \alpha \pi q_s(s_p^e)q_p^e(s_p^e)(1 - \beta)r_l \]

Now, the first line gets canceled out with the equilibrium condition, and the important question is whether the second line is negative or positive. Cancel
the $r_l$ on each term and expand all the expressions:

$$(s - s_e^p)h'(s_e)q(s - s_e^p, \pi p) = \frac{\partial}{\partial s_e} \left[ \frac{p - m(s_e, p)}{p} \right]^{1-\alpha} m(s - s_e, \pi p)$$

$$= -(1 - \alpha) \frac{m(s_e, p)}{p} \left[ \frac{p - m(s_e, p)}{p} \right]^{-\alpha} m(s - s_e, \pi p)$$

$$= -\alpha(1 - \alpha) \mu s_e^{\alpha-1} p^{-\alpha} m(s - s_e, \pi p) \left[ \frac{p - m(s_e, p)}{p} \right]^{-\alpha}$$

And

$$\pi \alpha q(s_e^p)q'_p(s_e^p)(1 - \beta) = \pi \alpha (1 - \beta) \frac{m(s_e, p)}{s_e} \frac{m(s - s_e, \pi(p - m(s_e, p)))}{\pi(p - m(s_e, p))}$$

$$= \pi \alpha (1 - \beta) \mu s_e^{\alpha-1} p^{1-\alpha} \mu(s - s_e)^{1-\alpha} (\pi(p - m(s_e, p)))^{-\alpha}$$

$$= \pi \alpha (1 - \beta) \mu s_e^{\alpha-1} p^{1-\alpha} (\pi p)^{1-\alpha} (\pi p)^{-1} \left[ \frac{(p - m(s_e, p))}{p} \right]^{-\alpha}$$

$$= \pi \alpha (1 - \beta) \mu s_e^{\alpha-1} p^{1-\alpha} (\pi p)^{1-\alpha} m(s - s_e, \pi p) \left[ \frac{(p - m(s_e, p))}{p} \right]^{-\alpha}$$

$$= \alpha (1 - \beta) \mu s_e^{\alpha-1} p^{-\alpha} m(s - s_e, \pi p) \left[ \frac{(p - m(s_e, p))}{p} \right]^{-\alpha}$$

After canceling stuff out, we’re left with,

$$(1 - \beta) - (1 - \alpha) \leq 0$$

That is, the derivative is negative if

$$\beta > \alpha$$

and positive if

$$\beta < \alpha$$
Case 6: \( m(s_e, p) = \mu s_e^\alpha p^{1-\alpha} \) and \( m(s - s_e, \pi(p - \mu s_e^\alpha p^{1-\alpha})) = \pi(p - \mu s_e^\alpha p^{1-\alpha}) \).

If the equilibrium is interior, notice that the social welfare function is

\[
\mu s_e^\alpha p^{1-\alpha} r_e + \pi(p - \mu s_e^\alpha p^{1-\alpha}) r_l
\]

\[
FOC: \quad \alpha \mu s_e^{\alpha - 1} p^{1-\alpha} (r_e - \pi r_l)
\]

This is strictly negative, whence, the social planner strictly prefers less early investment.

Cases 7, 8, 9: If \( m(s_e, p) = p \), then the private equilibrium must be \( s_e = s \) because there are no projects remaining for late investment. Next, if it is the case that \( m(s, p) = p \), then the social planner strictly prefers all late investment because he can surely match all good projects in the late matching market.

Finally, if at any time the social welfare function evaluated at the private equilibrium is not differentiable, i.e., it occurs on a corner of the piecewise matching function, because the derivative is negative in all of these cases, the left derivative will be negative which is sufficient to show that the social optimal occurs at a lower value \( s_e^s < s_e^p \).