PHILOSOPHY 20100/30000
ELEMENTARY LOGIC
Autumn 2006
Answers to practice problems, problem set 5

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† This problem set is due by 10:00am on Thursday, November 30 in the Philosophy Department office (Stuart 202).

† Either type your work or write it neatly and legibly. (Information for producing the logical symbols in Microsoft Word can be found on the course's Chalk web site in the Assignments folder.) *Any writing that takes effort to decipher will be marked incorrect, with no possibility of a do-over.*

† Problems marked with check marks ("✓") are practice problems. It is strongly advised that you do them, *but don't turn your work on them in.* Answers to these problems are available on the course's Chalk web site.

† Problems marked with bullets ("•") are extra-credit problems.

† Throughout, *DL* stands for Goldfarb's *Deductive Logic.*

1. **Instances.** For each of these pairs of schemata, is the second an instance of the first?

✓(a) $(\forall x)(\exists z)(Gyx \supset Hxz)$ $\quad$ $(\exists z)(Gyy \supset Hyz)$

Yup.

✓(b) $(\forall x)(\exists z)(Gyx \supset Hxz)$ $\quad$ $(\exists z)(Gyz \supset Hzz)$

Nah.

2. **Truth-functional implication.** For each of these pairs of schemata, does the first truth-functionally imply the second? If the answer is yes, provide a pair of truth-functional schemata, the first of which implies the second, of which the schemata in question are joint substitution instances.

✓(a) $(\forall y)Fy \supset (\forall y)(Gy \cdot Hy)$ $\quad$ $(\forall y)Fy \supset (\forall y)Hy$

No.

✓(b) $(\exists x)Fyx \equiv \neg(\exists x)Gx$ $\quad$ $(\exists x)Gx \vee (\exists x)Fyx$

Yes. "$p \equiv \neg q$" implies "$q \vee p$".

3. **Deduction.** For each of these pairs of schemata, deduce the second from the first:

✓(a) $\neg(\exists x)(Hx \cdot Gx)$ $\quad$ $(\forall y)(Hy \supset \neg Gy)$

[1]	(1)	$\neg(\exists x)(Hx \cdot Gx)$	P
[1]	(2)	$(\forall x)\neg(Hx \cdot Gx)$	(1) CQ
[1]	(3)	$\neg(Hy \cdot Gy)$	(2) UI
[1]	(4)	$Hy \supset \neg Gy$	(3) TF
[1]	(5)	$(\forall y)(Hy \supset \neg Gy)$	(4) UG

This deduction illustrates the basic strategy when one begins with a quantified schema: instantiating the schema and then working with the instance before generalizing back to a quantified conclusion. The one wrinkle here is that we need an application of **CQ** at the outset in order to get a quantified schema. This deduction illustrates as well the use of rule **UG**: the use of that rule at line (5) is legitimate because "y" does not occur free in (1), which is the only premise upon which (4) depends.

✓(b) $(\forall x)(\forall y)(Fxy \supset \neg Fyx)$ $\qquad$ $(\forall x)\neg Fxx$

[1]	(1)	$(\forall x)(\forall y)(Fxy \supset \neg Fyx)$	P
[1]	(2)	$(\forall y)(Fxy \supset \neg Fyx)$	(1) UI
[1]	(3)	$Fxx \supset \neg Fxx$	(2) UI
[1]	(4)	$\neg Fxx$	(3) TF
[1]	(5)	$(\forall x)\neg Fxx$	(4) UG

The trick for arriving at this deduction is to choose the right instance of the schema on line (2). The instance we opt for truth-functionally implies "Fxx"; all that remains is to apply rule **UG**.

✓(c) $(\forall x)(p \vee Fx)$ $\qquad$ $p \vee (\forall x)Fx$

[1]	(1)	$(\forall x)(p \vee Fx)$	P
[1]	(2)	$p \vee Fx$	(1) UI
[3]	(3)	$\neg p$	P
[1, 3]	(4)	Fx	(2)(3) TF
[1, 3]	(5)	$(\forall x)Fx$	(4) UG
[1]	(6)	$\neg p \supset (\forall x)Fx$	[3](5) D
[1]	(7)	$p \vee (\forall x)Fx$	(6) TF

Here we see the often useful strategy of introducing an auxiliary premise. In line (3) we introduce "p" as a premise; coupled with our initial premise, this enables us to isolate "Fx" on a line by itself and thus in turn to apply **UG** to it. Discharging the auxiliary premise at line (6) then gives us a schema that truth-functionally implies our desired result.

✓(d) $(\forall x)[(\exists y)Fy \supset Gx]$ $\qquad$ $(\forall x)(\forall y)(Fy \supset Gx)$

[1]	(1)	$(\forall x)[(\exists y)Fy \supset Gx]$	P
[2]	(2)	Fy	P
[2]	(3)	$(\exists y)Fy$	(2) EG
[1]	(4)	$(\exists y)Fy \supset Gx$	(1) UI
[1, 2]	(5)	Gx	(3)(4) TF
[1]	(6)	$Fy \supset Gx$	[2](5) D
[1]	(7)	$(\forall y)(Fy \supset Gx)$	(6) UG
[1]	(8)	$(\forall x)(\forall y)(Fy \supset Gx)$	(7) UG

What suggests to us to introduce "Fy" as an auxiliary premise? Answer: the schema we're trying to deduce is a quantified conditional that has "Fy" as its antecedent. We reason as follows: if we introduce "Fy" as an auxiliary premise and then use it along with the main premise to arrive at "Gx", discharging the auxiliary premise will yield "$Fy \supset Gx$". And this puts us just a couple of applications of **UG** away from our goal. (Note that having "y" free in premise (2) does not prevent our application of **UG** at line (7), for "y" does not appear free in any premise upon which (6) depends.)

✓(e) $(\exists x)(Fx \cdot p)$ $(\exists x)Fx \cdot p$

[1]	(1)	$(\exists x)(Fx \cdot p)$	P
[1, 2]	(2)	$Fx \cdot p$	(1)x EII
[1, 2]	(3)	Fx	(2) TF
[1, 2]	(4)	$(\exists x)Fx$	(3) EG
[1, 2]	(5)	$(\exists x)Fx \cdot p$	(2)(4) TF
[1]	(6)	$(\exists x)Fx \cdot p$	[2](5) EIE

This deduction illustrates the use of rules **EII** and **EIE**.

Note that the use of **EII** at line (2) meets all the requirements for the use of that rule: the schema on line (2) is an instance of the schema on line (1), and "x" has not appeared free in any line up to and including (1). (Of course, there aren't any lines earlier than (1). But had there been, we would have needed to make sure that "x" was not free in *any* of those lines, even lines that were not themselves premises of (1).) Furthermore, (2) is added as an additional premise of (2). **EII** is the only rule besides **P** that involves the introduction of a premise. Finally, the instantial variable "x" is flagged in the citation of (2).

Note also that the use of **EIE** at line (6) meets the requirement for the use of that rule: the instantial variable "x" does not occur free in the schema on line (5), nor does it occur free in any premise of (5) other than line (2), the line that was obtained by **EII**.

One might have been tempted to apply **EIE** immediately after line (4). That would yield the following alternative denouement:

[1, 2]	(4)	$(\exists x)Fx$	(3) EG
[1]	(5^*)	$(\exists x)Fx$	[2](4) EIE
[1, 2]	(6^*)	$(\exists x)Fx \cdot p$	(2)(5) TF

The use of **EIE** at line (5^*) is perfectly correct. The problem is that when we infer (6^*) from (2) and (5^*) we are obliged to take on (2) again as a premise: schemata reached with **TF** inherit all the premises of the schemata from which they were inferred, and (2) is a premise of (2). Thus we can't stop at line (6^*). Our aim is to deduce "$(\exists x)Fx \cdot p$" from "$(\exists x)(Fx \cdot p)$"; to do that, we must produce a deduction in which "$(\exists x)Fx \cdot p$" appears on a line whose *sole* premise is "$(\exists x)(Fx \cdot p)$". Thus we would need one more line:

$$[1] \quad (7^*) \quad (\exists x)Fx \cdot p \quad [2](6) \text{ EIE}$$

But now it is clear that the application of **EIE** are line (5^*), while not incorrect, was useless.

4. **Deduction.** For each…

5. **Deduction.** Do problem…

✓6. **Deduction.** Do problem 6. on p. 278 of *DL*.

[1]	(1)	$(\forall x)(\forall y)(Gxy \supset Gxx \cdot Gyy)$	P
[2]	(2)	$(\exists z)Gxz$	P
[2, 3]	(3)	Gxz	(2)z EII
[1]	(4)	$(\forall y)(Gxy \supset Gxx \cdot Gyy)$	(1) UI
[1]	(5)	$Gxz \supset Gxx \cdot Gzz$	(4) UI
[1, 2, ~~3~~]	(6)	Gxx	(3)(5) TF; [3] EIE
[1]	(7)	$(\exists z)Gxz \supset Gxx$	[2](6) D
[8]	(8)	$(\exists z)Gzx$	P
[8, 9]	(9)	Gwx	(8)w EII
[1]	(10)	$(\forall y)(Gwy \supset Gww \cdot Gyy)$	(1) UI
[1]	(11)	$Gwx \supset Gww \cdot Gxx$	(10) UI
[1, 8, ~~9~~]	(12)	Gxx	(9)(11) TF; [9] EIE
[1]	(13)	$(\exists z)Gzx \supset Gxx$	[8](12) D
[1]	(14)	$(\exists z)Gxz \vee (\exists z)Gzx \supset Gxx$	(7)(13) TF
[1]	(15)	$(\forall x)[(\exists z)Gxz \vee (\exists z)Gzx \supset Gxx]$	(14) UG

[1]	(1)	$(\forall x)[(\exists z)Gxz \vee (\exists z)Gzx \supset Gxx]$	P
[2]	(2)	Gxy	P
[2]	(3)	$(\exists z)Gxz$	(2) EG
[1]	(4)	$(\exists z)Gxz \vee (\exists z)Gzx \supset Gxx$	(1) UI
[2]	(5)	$(\exists z)Gzy$	(2) EG
[1]	(6)	$(\exists z)Gyz \vee (\exists z)Gzy \supset Gyy$	(1) UI
[1, 2]	(7)	$Gxx \cdot Gyy$	(3)(4)(5)(6) TF
[1]	(8)	$Gxy \supset Gxx \cdot Gyy$	[2](7) D
[1]	(9)	$(\forall y)(Gxy \supset Gxx \cdot Gyy)$	(8) UG
[1]	(10)	$(\forall x)(\forall y)(Gxy \supset Gxx \cdot Gyy)$	(9) UG

The inference to line (7) is sanctioned by the fact that "p", "$p \vee q \supset r$", "t", and "$s \vee t \supset u$" jointly imply "$r \cdot u$".

✓7. **Paraphrase and deduction.** Do problem 9 on p. 279 of *DL*.

UD: set of people

"A①②": "① admires ②"

"L①②": "① loves ②"

Premises: $(\forall x)(\neg Axx \supset (\forall y)\neg Ayx)$

$(\forall y)(\forall x)(\neg Ayx \supset \neg Lyx)$

Conclusion: $(\forall x)[(\forall y)\neg Axy \supset (\forall y)\neg Lyx]$

[1]	(1)	$(\forall x)(\neg Axx \supset (\forall y)\neg Ayx)$	P
[2]	(2)	$(\forall y)(\forall x)(\neg Ayx \supset \neg Lyx)$	P
[3]	(3)	$(\forall y)\neg Axy$	P
[3]	(4)	$\neg Axx$	(3) UI
[1]	(5)	$\neg Axx \supset (\forall y)\neg Ayx$	(1) UI
[1, 3]	(6)	$(\forall y)\neg Ayx$	(4)(5) TF
[1, 3]	(7)	$\neg Ayx$	(6) UI
[2]	(8)	$(\forall x)(\neg Ayx \supset \neg Lyx)$	(2) UI
[2]	(9)	$\neg Ayx \supset \neg Lyx$	(8)UI
[1, 2, 3]	(10)	$\neg Lyx$	(7)(9) TF
[1, 2, 3]	(11)	$(\forall y)\neg Lyx$	(10) UG
[1, 2]	(12)	$(\forall y)\neg Axy \supset (\forall y)\neg Lyx$	[3](11) D
[1, 2]	(13)	$(\forall x)[(\forall y)\neg Axy \supset (\forall y)\neg Lyx]$	(12) UG

✓8. **Paraphrase and deduction.** Do problem 10 on p. 279 of *DL*.

UD: set of people

"L①②": "① loves ②"

"R①②": "① respects ②"

Premise: $(\forall x)(\forall y)(Lxy \supset (\forall z)(Ryz \supset Rxz))$

Conclusion: $(\forall x)(Rxx \supset (\forall y)(Lyx \supset Ryx))$

[1]	(1)	$(\forall x)(\forall y)(Lxy \supset (\forall z)(Ryz \supset Rxz))$	P
[2]	(2)	Rxx	P
[1]	(3)	$(\forall y)(Lwy \supset (\forall z)(Ryz \supset Rwz))$	(1) UI
[1]	(4)	$Lwx \supset (\forall z)(Rxz \supset Rwz)$	(3) UI
[5]	(5)	Lwx	P
[1, 5]	(6)	$(\forall z)(Rxz \supset Rwz)$	(4)(5) TF
[1, 5]	(7)	$Rxx \supset Rwx$	(6) UI
[1, 2, 5]	(8)	Rwx	(2)(7) TF
[1, 2]	(9)	$Lwx \supset Rwx$	[5](8) D
[1, 2]	(10)	$(\forall y)(Lyx \supset Ryx)$	(9) UG
[1]	(11)	$Rxx \supset (\forall y)(Lyx \supset Ryx)$	[2](10) D
[1]	(12)	$(\forall x)(Rxx \supset (\forall y)(Lyx \supset Ryx))$	(11) UG