Philosophy 20100/30000
Elementary Logic
Autumn 2006
Answers to practice problems, problem set 2

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† This problem set is due on October 26 in class.

† Either type your work or write it neatly and legibly. (Information for producing the logical symbols in Microsoft Word can be found on the course's Chalk web site in the Assignments folder.) *Any writing that takes effort to decipher will be marked incorrect, with no possibility of a do-over.*

† Problems marked with check marks ("✓") are practice problems. It is strongly advised that you do them, *but don't turn your work on them in.* Answers to these problems are available on the course's Chalk web site.

† Problems marked with bullets ("•") are extra-credit problems.

† Throughout, *DL* stands for Goldfarb's *Deductive Logic*.

1. **Validity.** Use truth tables to test the following schemata for validity:

✓(a) $p \subset p$

p	$p \supset p$
T	T
F	T

The schema is valid.

(b) $(p \cdot q) \vee (p \cdot \neg q) \vee (\neg p \cdot \neg q)$

✓(c) $(p \supset q) \supset (p \vee \neg q)$

p	q	$p \supset q$	$p \vee \neg q$	$(p \supset q) \supset p \vee \neg q$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F			

The schema is not valid. (We didn't need to complete the last line, because as soon as we find one truth assignment under which the schema is false, we've shown that it's not valid.)

(d) $(p \supset \neg(q \vee r)) \vee (p \supset (q \cdot r))$

(e) $(p \equiv \neg q) \vee (p \equiv \neg r) \vee (q \equiv r)$

2. **Implication.** Use truth tables to check for implication, both ways.

✓(a) $p \cdot r \equiv q \cdot r$

$p \equiv q$

p	q	r	$p \cdot r$	$q \cdot r$	$p \cdot r \equiv q \cdot r$	$p \equiv q$
T	T	T	T	T	T	T
T	T	F	F	F	T	T
T	F	T	T	F	F	F
T	F	F	F	F	T	F
F	T	T	F	T	F	F
F	T	F	F	F	T	F
F	F	T	F	F	T	T
F	F	F	F	F	T	T

"$p \equiv q$" implies "$p \cdot r \equiv q \cdot r$". But not vice versa—see lines 4 and 6.

(b) $p \supset q$
$q \supset (r \supset q)$

(c) $p \cdot \neg q \supset (r \supset s)$
$p \supset q \vee s$

✓3. **Paraphrase and implication.** Schematize the following argument and determine whether the premises truth-functionally imply the conclusion:

Premises:	Mick will go the party only if Charlie does but Keith doesn't.
	Charlie go the party only if Mick and Keith do.
Conclusion:	Mick won't go to the party.

p: Mick will go to the party.

q: Charlie will go to the party.

r: Keith will go to the party.

Premises:	$p \supset q \cdot \neg r$
	$q \supset p \cdot r$
Conclusion:	$\neg p$

p	q	r	$p \supset q \cdot \neg r$	$q \supset p \cdot r$	$\neg p$
T	T	T	F		
T	T	F	T	F	
T	F	T	F		
T	F	F	F		
F	T	T	T	F	
F	T	F	T	F	
F	F	T	T	T	T
F	F	F	T	T	T

The premises truth-functionally imply the conclusion. Note that we were free to truncate several lines because to test for implication, we need to look at the truth value of the conclusion only in those cases where both premises are true.

4. **Paraphrase and implication.** Do problem 6. on pp. 256-257 of *DL*.

5. **Implication.** There is a land where every sentence asserted by an inhabitant on Monday is false, and every sentence asserted by an inhabitant on any other day is true.[1]

   ✓(a) One day an inhabitant says to you, "It's Monday and I'm married." Is she married? Explain in a few sentences how you know.

   If it's not Monday, then the first conjunct of the sentence is false, and thus the whole sentence is false. By stipulation, inhabitants don't utter false sentences on days that aren't Monday. It follows from these two facts that the sentence is uttered on a Monday. By stipulation, inhabitants don't utter true sentences on Mondays. Hence the sentence is false. As its first conjunct is true, it's second conjunct must be false. Hence the inhabitant is not married.

   (b) One day an inhabitant says to you, "It's Monday or I'm married." Is she married? Explain in a few sentences how you know. Does it matter whether the disjunction is interpreted inclusively or exclusively?

   (c) One day an inhabitant says to you, "It's Monday if and only if I'm married." Is she married? Explain in a few sentences how you know.

   •(d) One day an inhabitant says to you, "If today is Monday then today is Tuesday. If today is Wednesday than today is Thursday. It's not the case that if today is Friday than today is Saturday." Interpret the first two sentences as material conditionals and the third as a negation thereof. What day is it? How do you know?

6. **Implication.** Do problem 13. on p. 259 of *DL*. Show your work.

   Hint. Given the way the problem is set up, we know that Jack is a knight if and only if what he says is true. And what he says is that either he is a knight or Zach is a knave. Thus we know that Jack is a knight if and only if either Jack is a knight or Zach is a knave. We can construct analogous biconditionals for Mack's and Zach's assertions. And then we can schematize these biconditionals and think things through from there.

7. **Implication.** A man in an advertisement says, "If I'm not playing golf, I'm watching it. And if I'm not watching it, I'm reading about it." Assume that you can't perform more than one of these activities at the same time. Also assume, absurdly, that the two sentences the man utters are both literally true. What does he spend every moment of his life doing? Show your work.[2]

   Hint. The assumption that the activities can't be simultaneously performed can be understood as the assumption that only one of the sentences, "The man is playing golf," "The man is watching golf," and "The man is reading about golf," can be true at any given time.

[1] The general idea for these problems is due to Raymond Smullyan.

[2] Problem due to Herbert Enderton.

8. **Expressive adequacy.** "$\cdot$", "$\vee$", "$\supset$" and "$\equiv$" are all *binary* or *2-place* connectives: they join with two schemata to form a new schema. "$\neg$" is a *unary* or *1-place* connective: it joins with one schema to form a new schema. The following problems concern connectives that are neither unary nor binary.

   (a) There is nothing to stop us from introducing connectives with more than two places into our symbolism. We might, for example, supplement our set of truth-functional connectives with one or more *ternary* or *3-place* truth-functional connectives. But there is little point in doing so, because for any schema containing one or more of these ternary connectives, we can construct a truth-functionally equivalent schema containing only members of our original pool of unary and binary connectives. In this sense, anything that can be 'said' using a ternary truth-functional connective can equally be said using connectives with less than three places.

   Below I introduce a ternary connective "T" and define it in various ways. For each definition, find a schema equivalent to "$T(p, q, r)$" that uses only connectives drawn from the following list: $\{\neg, \cdot, \vee, \supset, \equiv\}$.

   ✓i. "$T(p, q, r)$" is true under an assignment iff at least one of "p", "q", and "r" is true under that assignment.

   $p \vee q \vee r$

   ii. "$T(p, q, r)$" is true under an assignment iff at least two of "p", "q", and "r" are true under that assignment.

   iii. "$T(p, q, r)$" is true under an assignment iff either exactly one or exactly three of "p", "q", and "r" are true under that assignment.

   •iv. Do ii. without using "$\vee$".

   •v. Do iii. without using "$\vee$", "$\cdot$" or "$\neg$".

   •(b) There are two zero-place truth-functional connectives, usually written "$\top$" and "$\bot$". As their name suggests, these connectives do not join with schemata to form a new schema. Rather, each counts as a schema all by itself. (Granted, this makes it a bit misleading to call them 'connectives'.) The connectives are defined thusly: "$\top$" is true under any assignment and "$\bot$" is false under any assignment.

   Like all other truth-functional schemata, "$\top$" and "$\bot$" can be joined with unary and binary connectives to form new schemata. Thus, e.g., "$\top \vee p$" is a well-formed schema. As it happens, it is valid: since "$\top$" is true under any assignment, so is any disjunction of which it is a disjunct.

   Now, as discussed in §16 of Part I of *DL*, $\{\neg, \cdot, \vee, \supset, \equiv\}$ is an expressively adequate set of truth-functional connectives, but $\{\cdot, \vee, \supset, \equiv\}$ is not. You may appeal to these two facts in doing the following problems.

   •i. Explain why $\{\bot, \cdot, \vee, \supset, \equiv\}$ is an expressively adequate set.

   •ii. Explain why $\{\top, \cdot, \vee, \supset, \equiv\}$ is not an expressively adequate set.

✓9. **Informal proof of general laws about schemata.** Explain in a few sentences why the following is true: any schema implied by a satisfiable schema is satisfiable.

Let A be a satisfiable schema and B a schema implied by A. As A implies B, B is true under any truth assignment under which A is true. Hence if there is at least one assignment under which A is true, there is at least one assignment under which B is true. Since A is satisfiable, there is at least one interpretation under which A is true. Thus there is at least one interpretation under which B is true. Thus B is satisfiable. Generalizing, any schema implied by a satisfiable schema is satisfiable.

10. **Informal proof of general laws about schemata.** A set of schemata is said to be *inconsistent* if and only if there is no truth-assignment under which all of the schemata in the set are true.

(a) Explain in a few sentences why the following is true: if a set of schemata is inconsistent, then that set implies every schema.

•(b) Explain in a few sentences why the following is true: for any two schemata A and B, if the set $\{A, B\}$ is inconsistent, then A implies the negation of B.