Philosophy 20100/30000
Elementary Logic
Autumn 2006
Problem set 4

Instructor: Jason Bridges

† This problem set is due on November 21 in class.

† Either type your work or write it neatly and legibly. (Information for producing the logical symbols in Microsoft Word can be found on the course's Chalk web site in the Assignments folder.) *Any writing that takes effort to decipher will be marked incorrect, with no possibility of a do-over.*

† Problems marked with check marks ("✓") are practice problems. It is strongly advised that you do them, *but don't turn your work on them in.* Answers to these problems are available on the course's Chalk web site.

† Problems marked with bullets ("•") are extra-credit problems.

† Throughout, *DL* stands for Goldfarb's *Deductive Logic*.

✓1. **Paraphrase.** Do the following problems from Part III, Section A of *DL* (pp. 273-275).
1.(a) 2.(a) 3.(a) 4.(e) 6.(c)
1.(c) 2.(e) 3.(b)
1.(e)
1.(h)
1.(k)

2. **Paraphrase.** Do the following problems from Part III, Section A of *DL* (pp. 273-275).
1.(b) 2.(b) 3.(c) 4.(b) 5.(a) 6.(a)
1.(d) 2.(f) 5.(b) 6.(b)
1.(f)
1.(i)
1.(l)

3. **Paraphrase.** A *set*, as we discussed in class, is a collection of objects. The objects in a set are said to be the *members* of that set. In set theory, we concern ourselves with no objects other than sets; thus the members of sets are always themselves sets. (An original aim of set theory was to show how much of mathematics could be reconstructed from this limited basis.) So for the following, take the universe of discourse to consist entirely of sets. Using "① ∈ ②" for "① is a member of ②", the identity predicate "① = ②", and no other predicate letters, paraphrase the following standard axioms of set theory:

✓(a) For all sets x and y, x is identical to y iff x and y have the same members.

(b) There exists a set with no members.

(c) For any two distinct objects x and y, there exists a set whose only members are x and y.

(d) For every set x, there exists a set y whose members are the subsets of x.

Note: A set x is a subset of y iff every member of x is a member of y.

4. **Paraphrase.** Paraphrase, with the universe of discourse restricted to persons.

✓(a) Someone someone loves loves someone.

(b) Everyone everyone loves loves everyone.

•(c) Everyone everyone everyone loves loves loves everyone.

(d) No one loves people who only love people who love them.

5. **Paraphrase.** Do problem 8. on p. 276 of *DL*. (b) and (c) are practice problems; (a), (d) and (e) are to be turned in. For those hazy on genealogical jargon: half-sisters share exactly one parent. One's first cousins are the children of one's aunts and uncles. (Also, assume that sisters and brothers need share only one parent. Thus, for example, half-sisters count as sisters.)

6. **Paraphrase.** For the following problems, take the universe of discourse to be the set of non-negative integers (i.e., $\{0, 1, 2, 3, \ldots\}$), and use "A①②③" for "①+② = ③", "M①②③" for "① × ② = ③" and no other predicate letters (not even "① = ②").

✓(a) Find an open schema that's true iff $x = 0$.

(b) Find an open schema that's true iff $x = 1$.

(c) Find an open schema that's true iff $x = 2$.

(d) Find an open schema that's true iff $x < y$.

•(e) Find an open schema that's true iff x is prime.

•(f) Find an open schema that's true iff $\sqrt[5]{x} = y$.

•7. **Paraphrase.** The following sentence appeared in a well-known logic text a few decades ago, where it was claimed that the sentence cannot be paraphrased using first-order logic (the logic we've learned in this course). That claim was mistaken. Paraphrase the sentence.

Some critics admire one another and no one else.

Hint. This is hard. The main difficulty is that the paraphrase must respect the fact that it is not specified how many critics are involved in this mutual admiration society. Here's a big hint: start by schematizing the open sentence, "Everyone x admires admires everyone x admires."

8. **Interpreting polyadic schemata.** Do problem 1. on p. 276 of *DL*. (c) is a practice problem; (a) and (b) are to be turned in; skip (d).

9. **Disproving implication.** Do problem 2. on p. 277 of *DL*. (a) and (c) are practice problems; (b), (e) and (f) are to be turned in; skip (d).