Philosophy 20100/30000
Elementary Logic
Autumn 2006
Problem set 3

Instructor: Jason Bridges

† This problem set is due on November 9 in class.

† Either type your work or write it neatly and legibly. (Information for producing the logical symbols in Microsoft Word can be found on the course's Chalk web site in the Assignments folder.) *Any writing that takes effort to decipher will be marked incorrect, with no possibility of a do-over.*

† Problems marked with check marks ("✓") are practice problems. It is strongly advised that you do them, *but don't turn your work on them in.* Answers to these problems are available on the course's Chalk web site.

† Problems marked with bullets ("•") are extra-credit problems.

† Throughout, *DL* stands for Goldfarb's *Deductive Logic*.

✓1. **Paraphrase.** Do the following problems from Part II, Section A of *DL* (pp. 265-267).

1.(a)	2.(a)	3.(a)	4.(a)	5.(b)
1.(c)	2.(c)	3.(c)	4.(c)	
	2.(e)		4.(i)	
			4.(m)	
			4.(o)	

2. **Paraphrase.** Do the following problems from Part II, Section A of *DL* (pp. 265-267).

1.(b)	2.(b)	3.(b)	4.(b)	5.(c)
1.(d)		3.(d)	4.(d)	
			4.(f)	
			4.(g)	
			4.(k)	
			4.(n)	

3. **Paraphrase.** Paraphrase using the following predicate letters: "P①" for "① is a painting", "S①" for "① is surrealist", "I①" for "① is impressionist", "W①" for "① is well-liked", "L①" for "① is large", "N①" for "Nick admires ①", and "F①" for "Francesca admires ①". Assume an unrestricted universe of discourse.

   (a) Francesca admires impressionist paintings if and only if they're either large or well-liked, and Nick admires any impressionist painting Francesca admires.

   (b) There are impressionist paintings that neither Nick nor Francesca admires only if there are surrealist paintings of which the same can be said.

   (c) All surrealist paintings except the large ones are well-liked.

(d) Francesca doesn't admire a painting unless Nick does—unless it's large, in which case she admires it whether or not Nick does.

4. **Interpreting monadic schemata.** Do problem 1. on pp. 267-268 of *DL*. (b) is a practice problem; (a) is to be turned in; skip (c).

5. **Validity.** For each of the following, determine whether the schema is valid. If it is, provide an informal argument showing that it is. If it isn't, give an example of a structure in which the schema is false.

   ✓(a) $(\forall x)Fx \vee (\forall x)\neg Fx$

   (b) $(\forall x)(Fx \vee \neg Fx)$

   ✓(c) $(\exists x)Fx \cdot (\exists x)\neg Fx \supset \neg((\forall x)Fx \vee (\forall x)\neg Fx)$

   (d) $(\forall x)(Fx \supset Gx) \supset (\exists x)(Fx \cdot Gx)$

✓6. **Paraphrase and implication.** In the following argument, the premises monadically imply the conclusion. Show this by schematizing the argument and then providing an informal argument that any interpretation that makes the premise-schemata true must also make the conclusion-schema true.

Premises:	If no country singers have rhythm, then no one who has rhythm can dance.
	Jazz musicians have rhythm.
Conclusion:	Therefore, if any jazz musician can dance, some country singers have rhythm.

7. **Syllogisms.** Do problem 2. on pp. 268-269 of *DL*. Skip (a) and (d).

•8. **More on syllogisms.** Do problem 3. on p. 272 of *DL*.

9. **Disproving implications.** Do problem 3. on pp. 269-270 of *DL*. (b) and (d) are practice problems; (a) and (f) are to be turned in; skip the remainder.

10. **Properties of natural-language quantifiers.** Contemporary linguists classify natural-language quantifiers—words like "all", "some", "no" and "only"—on the basis of whether they possess certain properties.[1] For example, a quantifier "$\mathscr{Q}$" is *symmetric* iff no matter what predicates we substitute for "F" and "G", "$\mathscr{Q}$ F's are G's" is true iff "$\mathscr{Q}$ G's are

[1] A *natural language* is a language, like English or French, spoken in everyday life ; it is opposed to an *artificial language*, such as the logical symbolism we are learning in this course. We'll ignore the fact that many linguists deny that "only" is a quantifier (partly on the ground that an expression "$\mathscr{Q}$" counts as a quantifier only if "$\mathscr{Q}$ F's are G's" is an intelligible answer to the question "How many F's are G's?").

F's" is true.[2] Granting the analyses we have proposed in this course, "some" and "no" are symmetric. (Both "Some F's are G's" and "Some G's are F's" are true iff there is at least one object in the universe of discourse that is in the extension of both "F①" and "G①". And both "No F's are G's" and "No G's are F's" are true iff there is no object in the universe of discourse that is in the extension of both "F①" and "G①".) On the other hand, neither "all" nor "only" is symmetric. ("All dogs are animals" is true but "All animals are dogs" is not; "Only men are bachelors" is true but "Only bachelors are men" is not.) Below are descriptions of various properties that a quantifier might possess. In each case, indicate which of these four quantifiers—"all", "some", "no" and "only"—have the property and which do not. If you deem that a given quantifier does *not* have a given property, then give a counterexample demonstrating this.

(a) A quantifier "$\mathscr{Q}$" is *reflexive* iff no matter what predicate we substitute for "F", "$\mathscr{Q}$ F's are F's" is true.

✓(b) A quantifier "$\mathscr{Q}$" is *right upward monotonic* iff no matter what predicates we substitute for "F" "G" and "H", the following conditional holds: if "$\mathscr{Q}$ F's are G's" is true and "All G's are H's" is true, then "$\mathscr{Q}$ F's are H's" is true.

✓(c) A quantifier "$\mathscr{Q}$" is *right downward monotonic* iff no matter what predicates we substitute for "F" "G" and "H", the following conditional holds: if "$\mathscr{Q}$ F's are G's" is true and "All H's are G's" is true, then "$\mathscr{Q}$ F's are H's" is true.

(d) A quantifier "$\mathscr{Q}$" is *left upward monotonic* iff no matter what predicates we substitute for "F" "G" and "H", the following conditional holds: if "$\mathscr{Q}$ F's are G's" is true and "All F's are H's" is true, then "$\mathscr{Q}$ H's are G's" is true.

(e) A quantifier "$\mathscr{Q}$" is *left downward monotonic* iff no matter what predicates we substitute for "F" "G" and "H", the following conditional holds: if "$\mathscr{Q}$ F's are G's" is true and "All H's are F's" is true, then "$\mathscr{Q}$ H's are G's" is true.

•(f) A quantifier "$\mathscr{Q}$" is *conservative* iff no matter what predicates we substitute for "F" and "G", the following biconditional holds: "$\mathscr{Q}$ F's are G's" is true iff "$\mathscr{Q}$ F's are F's and G's" is true.

11. **Non-standard quantifiers.** Consider the following statement:

"Most dogs are friendly."

Clearly, there is no hope of paraphrasing this statement as either a universal or existential quantification. But it's natural to feel that "most" belongs to the same grammatical category as "all", "some", "any" and "no": it seems to play the same kind of role in the sentences in which it occurs.[3] Thus we might reasonably hypothesize that it will admit of an analogous logical treatment.

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[2] In fact, it is not the quantifier itself, but rather the mathematical relation said to be the meaning of the quantifier, to which linguists ascribe properties like symmetry. We can ignore that complication here.

[3] Other expressions that appear to fit into the same class are "few", "many", and "several".

Suppose, to this end, that we introduce a third quantifier into our logical symbolism, "⅃", which we contextually define as follows:

"(⅃x)Fx" is true in a universe of discourse U iff most values for "x" in U are in the extension of "F①".

We may then paraphrase:

"Most things are friendly."

as:

"(⅃x)(x is friendly)"

But just how useful is "⅃", so defined, as a tool for logical paraphrase? Let's return to our earlier statement: "Most dogs are friendly."

If we assume an unrestricted universe of discourse, can we use "⅃", as just defined, to paraphrase this statement?

(a) Taking a cue from our treatment of "some", we might try to construe the statement as a quantified conjunction:

(⅃x)(x is a dog $\cdot$ x is friendly)

Explain why this suggestion is incorrect.

(b) Taking a cue from our treatment of "all", we might try to construe the statement as a quantified conditional:

(⅃x)(x is a dog $\supset$ x is friendly)

Explain why this suggestion is incorrect.

•(c) At this point you might be beginning to suspect that no other truth-functional compound will do the job, and that sentences containing "most" simply cannot be logically paraphrased in a manner analogous to our treatment of sentences containing "some" and "all". And indeed, they cannot. Nor can sentences containing any of the other expressions listed in footnote 3. Why is that? Can you identify a particular feature of our method of paraphrasing quantified statements that renders it incapable of handling these 'non-standard' quantifiers?[4]

•(d) Logicians and linguists have suggested various methods for paraphrasing statements involving "most" (and other non-standard quantifiers). One proposal introduces a notation such that "Most dogs are friendly." would be rendered as follows:

(⅃x : x is a dog)(x is friendly)

Give a contextual definition for (⅃ :) such that the schema above correctly paraphrases "Most dogs are friendly." A contextual definition of (⅃ :) is a sentence that begins "(⅃x : Fx)(Gx) is true iff . . . ".

[4] Note that the problem has nothing to do with the alleged fact that the meanings of "most", "few" and so forth are in some sense vague or imprecise. Even if we supposed that "most" were exactly synonymous with "more than half", the problem we have uncovered would still arise.

Explain the major difference between this approach to quantification and the one we've been pursuing in the course.